Minimum Cost Diet Problem

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Introduction

The idea of this project is to solve the problem of finding the minimum cost diet satisfying a set of "Recommended Daily Allowances" (RDAs) for a particular set of nutrients and possible kind of food.

History

- Problem first "solved" by Stigler (1945) for a set of foods and prices and RDA requirements (see Dantzig (1990) for an entertaining discussion of what "solved" meant in that context).
- Times have changed: the variety of different kinds of food, food prices, and RDA requirements are all quite different from what they were for our earlier generations.

Dietary Guidelines

A compilation of dietary guidelines are provided at in an appendix of Dietary Guidelines for Americans. These provide recommended levels of 31 different nutrients by age and sex, and are updated every five years. I've put some of these data into a google spreadsheet.

Diet Cost

Stigler's insight was that the minimum cost diet (MCD) problem was most naturally posed as a linear program.

- Suppose n different kinds of food;
- Represent quantities consumed of these as a *vector* x with n elements. Many elements may be zero!
- Each kind of food has price; call this vector of prices p.

Total Cost

A consumer's diet costs $p'x$, where the prime indicates the inner or dot product of the two vectors: p (prices) and x (quantities of food).

Nutritional Recommendations

Each unit of a given kind of food is assumed to provide a set of nutrients.

- Suppose m nutrients, then let A be a matrix with m rows and n columns describing the nutritional content of a single unit of each of kind of food.
- Different sources of "recommendations" regarding nutrition.
  - Equalities: A female in her twenties 'should' consume:
    - 2000 kilo-calories
    - 46 grams of protein
    - 28 grams of fiber per day
- Inequalities:
  - Less than 23 grams of sodium

Matrix notation

We can write these constraints as something like

$$Ax \geq \underline{b}$$

where $\underline{b}$ is a vector of recommendations about *minimum* amounts of different nutrients. Similarly, if there are some things we want to make sure we eat *less* of (e.g., mercury, sodium, calories), that can be written as a set of linear inequalities

$$Ax \leq \bar{b},$$

where $\bar{b}$ is a vector of recommendations about *maximum* amounts of different nutrients. Note that this constraint can *also* be expressed as a greater than constraint by multiplying both sides by -1.

Linear Program

Putting this all together, the linear program to compute the minimum cost diet looks like

$$\min_x p'x$$

such that

$$\begin{bmatrix} A \\ -A \end{bmatrix} x \geq \begin{bmatrix} \underline{b} \\ -\bar{b} \end{bmatrix}.$$

We can express this more succinctly by denoting the stacked A matrices and b vectors by $\tilde{A}$ and $\tilde{b}$, respectively.

Solution

We'll introduce code which can quickly solve linear programs given inputs $(\tilde{A}, \tilde{b}, c)$. Your job in this project has more to do with finding interesting cases in which to use methods such as these.