Reflections on Prismatic Constructions

Marc Coiffier

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The Calculus of Prismatic Constructions, upon which this platform is based, is an extension of the standard CoC with a mechanism for discriminating inductive constructors.

The Problem

Inductive types can be described as enumerations of constructors. In Coq (and similarly in other proof assistants), an inductive type must be declared along with its constructors, using a syntax like :

```
Inductive T : forall A..., Type :=
| t0 : forall x0..., T (f0... x0...)
...
| tn : forall xn..., T (fn... xn...)
.
```

Here, we declare the inductive type $T : \forall A..., Type$, and its constructors called t_i ($i \in \{0..n\}$).

As a more concrete example, here is how the type of Booleans can be defined inductively :

```
Inductive Boolean : Type := true : Boolean | false : Boolean.
```

The above definition is essentially a formal statement of the following description of Booleans : a Boolean can have one of two shapes, *true* or $false$, and cannot be any other thing.

This means that, if we want to prove a property $P\,x$ for some unknown Boolean x, all we need is to prove $P\,true$ and $P\,false$.

This information is summed up in what we call the *induction principle* for Booleans. In Coq, it will be given the name `Boolean_rect`, and have the type $\forall (P : Boolean \rightarrow Type),\ P\,true \ \rightarrow\ P\,false \ \rightarrow\ \forall (b : Boolean),\ P\,b$