

Lists : a general sequence of elements

Marc Coiffier

Contents

| | |
|--------------------------------------|---|
| The elements of a sequence | 1 |
| List combinators | 2 |

'utils require import

- Required module: utils

Sometimes in life, we want to do a series of tasks in a certain order, or sort objects in a sequence. In mathematical disciplines, lists play a similar semantic role, allowing us to express sequences of related elements.

The elements of a sequence

The simplest kind of list is either an empty list, or a list containing one element, followed by another list. Given a type A of elements, we can define lists of type A in the following context :

```
'List_context {  
  Type '.List ->  
  A 'a -> .List 'l -> .List ? ? '.cons ->  
  .List '.nil -> } def
```

Armed with this context, defining the usual constructors for the List type and its members becomes easy :

```
'List List_context .List ? ? ? "List A" defconstr  
A 'a -> List 'l -> 'cons List_context  
  .cons ( a l ( .List .cons .nil ) ) ! ! ! "cons a l" defconstr ! !  
'nil List_context .nil ! ! ! "nil" defconstr  
[ 'List 'nil 'cons ] { export } each
```

The list recursor, $\lambda(l : List\ A).\mu(l)$, has type $\forall(l : List\ A) (List^P : List\ A \rightarrow Set_1), (\forall(a : A) (l_0 : List\ A), List^P\ l_0 \rightarrow List^P\ (cons\ a\ l_0)) \rightarrow List^P\ nil \rightarrow List^P\ l$. We can now start to define non-trivial combinators that work on lists, such as “map” and “append” :

List combinators

```
!  
'list_map Type 'A -> Type 'B -> A 'x -> B ? 'f -> List ( A ) 'l ->  
  l (  
    List ( B )  
    A 'x -> List ( B ) 'l -> cons ( B f ( x ) l ) ! !  
    nil ( B )  
  ) 4 lambdas def
```

list_map has type $\forall(A : Set_0) (B : Set_0), (A \rightarrow B) \rightarrow List\ A \rightarrow List\ B$.

```
'list_append Type 'A -> List ( A ) dup 'x -> 'y -> x (  
  List ( A )  
  cons ( A )  
  y ) ! ! ! def
```

list_append has the type $\forall(A : Set_0), List\ A \rightarrow List\ A \rightarrow List\ A$.