

# Lists : a general sequence of elements

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```
'utils require import
```

- Required module: [utils](#)

Sometimes in life, we want to do a series of tasks in a certain order, or sort objects in a sequence. In mathematical disciplines, lists play a similar semantic role, allowing us to express sequences of related elements.

## The elements of a sequence

The simplest kind of list is either an empty list, or a list containing one element, followed by another list. Given a type  $A$  of elements, we can define lists of type  $A$  in the following context :

```
'List_context {  
  Type '.List ->  
  A 'a -> .List 'l -> .List ? ? '.cons ->  
  .List '.nil -> } def
```

Armed with this context, defining the usual constructors for the List type and its members becomes easy :

```
'List List_context .List ? ? ? "List A" defconstr  
A 'a -> List 'l -> 'cons List_context  
  .cons ( a l ( .List .cons .nil ) ) ! ! ! "cons a l" defconstr ! !  
'nil List_context .nil ! ! ! "nil" defconstr  
[ 'List 'nil 'cons ] { export } each
```

The list recursor,  $\lambda(l : \text{List } A). \mu(l)$ , has type  $\forall(l : \text{List } A) (\text{List}^P : \text{List } A \rightarrow \text{Set}_1), (\forall(a : A) (l_0 : \text{List } A), \text{List}^P l_0 \rightarrow \text{List}^P (\text{cons } a \ l_0)) \rightarrow \text{List}^P \text{nil} \rightarrow \text{List}^P l$ . We can now start to define non-trivial combinators that work on lists, such as “map” and “append” :

## List combinators

```
!
'list_map Type 'A -> Type 'B -> A 'x -> B ? 'f -> List ( A ) 'l ->
  l (
    List ( B )
    A 'x -> List ( B ) 'l -> cons ( B f ( x ) l ) ! !
    nil ( B )
  ) 4 lambdas def
```

`list_map` has type  $\forall(A : \text{Set}_0) (B : \text{Set}_0), (A \rightarrow B) \rightarrow \text{List } A \rightarrow \text{List } B$ .

```
'list_append Type 'A -> List ( A ) dup 'x -> 'y -> x (
  List ( A )
  cons ( A )
  y ) ! ! ! def
```

`list_append` has the type  $\forall(A : \text{Set}_0), \text{List } A \rightarrow \text{List } A \rightarrow \text{List } A$ .