Estimating the Number of Active Devices Within a Fixed area Using Wi-Fi Monitoring

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Outline

- Background
- Problem Description
- System Model
- Estimation Schemes
- Simulations and Results
- Experimental Implementation and Results
- Conclusions

Background

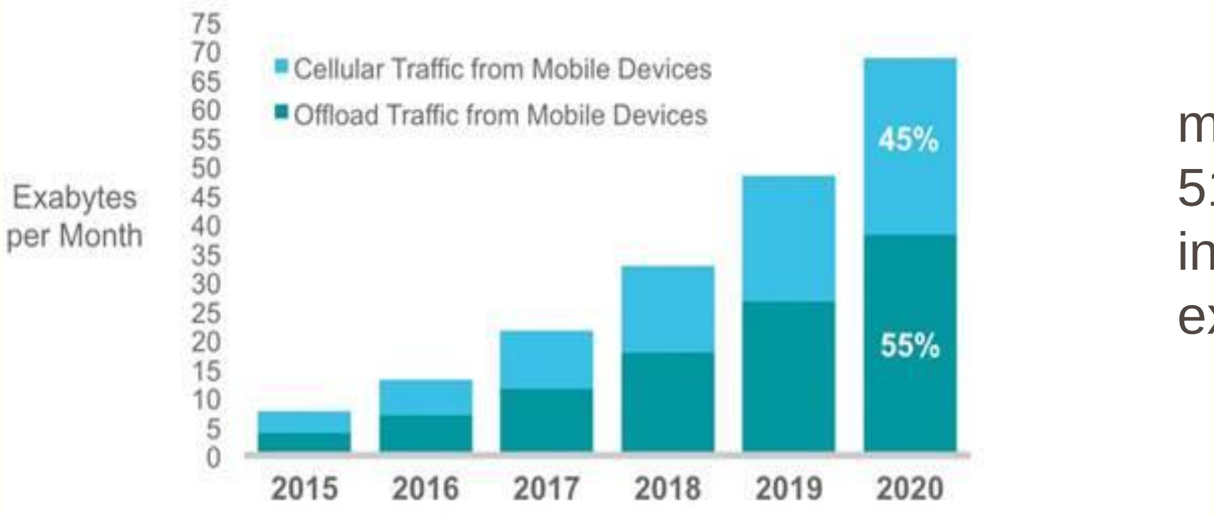
Occupancy estimation has several applications

- Smart home management
- Emergency situation

Camera based approach is costly and limited by resolution

RF signal based approach consumes less power

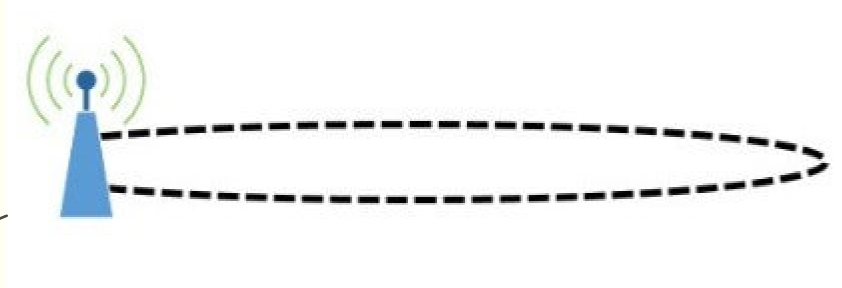
Wi-Fi Traffic Growth



mobile offload increases from 51 percent (3.9 exabytes/month) in 2015 to 55 percent (38.1 exabytes/month) by 2020

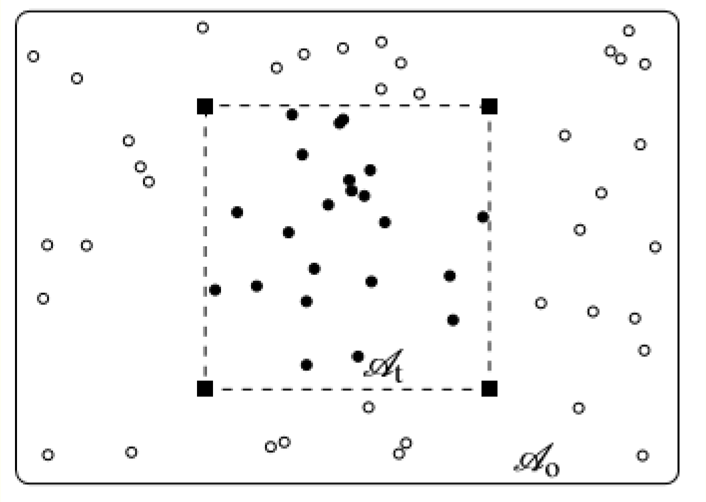
Problem Description

- Estimating number of active wireless units using Wi-Fi monitoring with statistical inference
- If we could increase the estimation accuracy by deploying proper directional antennas



Which one may perform better?

System Model



A_t is the area of target region.
A_O is the area of complement region.

Location vector:

$$\underline{\mathbf{U}} = (\mathbf{U}_1, \ldots, \mathbf{U}_{n_\mathrm{a}})$$

Received signal vector:

$$\underline{\mathbf{P}} = (\mathbf{P}_1, \ldots, \mathbf{P}_{n_\mathrm{a}})$$

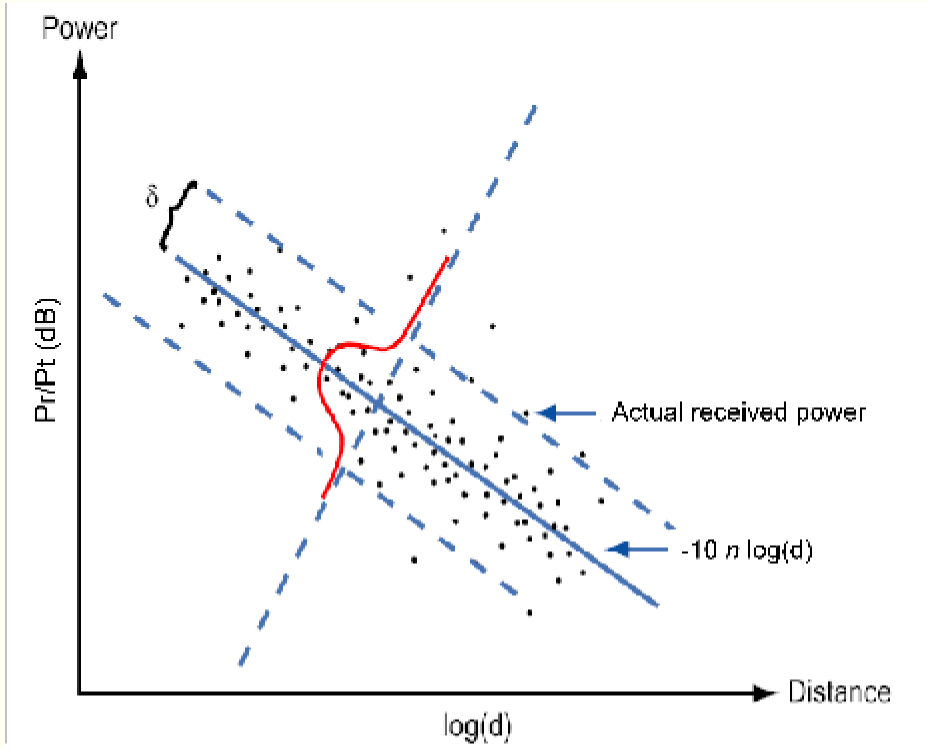
Channel Model

Log-normal channel model:

$$P_d[\mathrm{dBm}] = A + B\log_{10}(d) + L_s + G_a$$

Log-normal distribution:

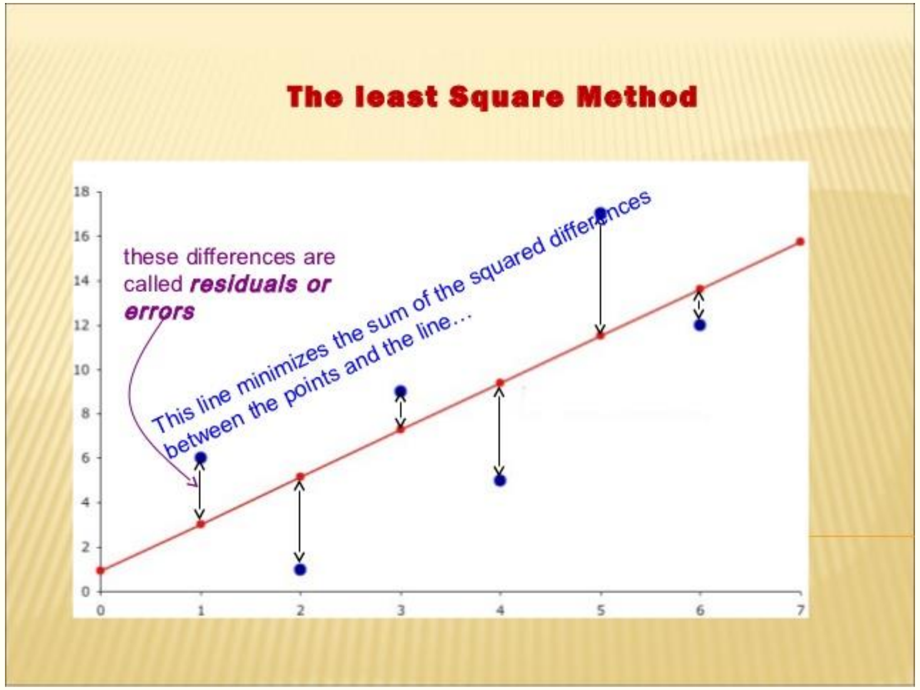
$$f_{L_s}(\ell) = \frac{1}{\sqrt{2\pi}\sigma_s}\exp\left(-\frac{\ell^2}{2\sigma_s^2}\right)$$



Find Parameter Values

Using least squares method:

$$\operatorname{argmin}_{a,b} \left\| \underbrace{\begin{bmatrix} \vdots \\ p_{ij} - G_i(\phi_{ij}) \\ \vdots \end{bmatrix}}_{\mathbf{y}} - \underbrace{\begin{bmatrix} \vdots & \vdots \\ 1 & \log_{10}(d_{ij}) \\ \vdots & \vdots \end{bmatrix}}_{M} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 .$$



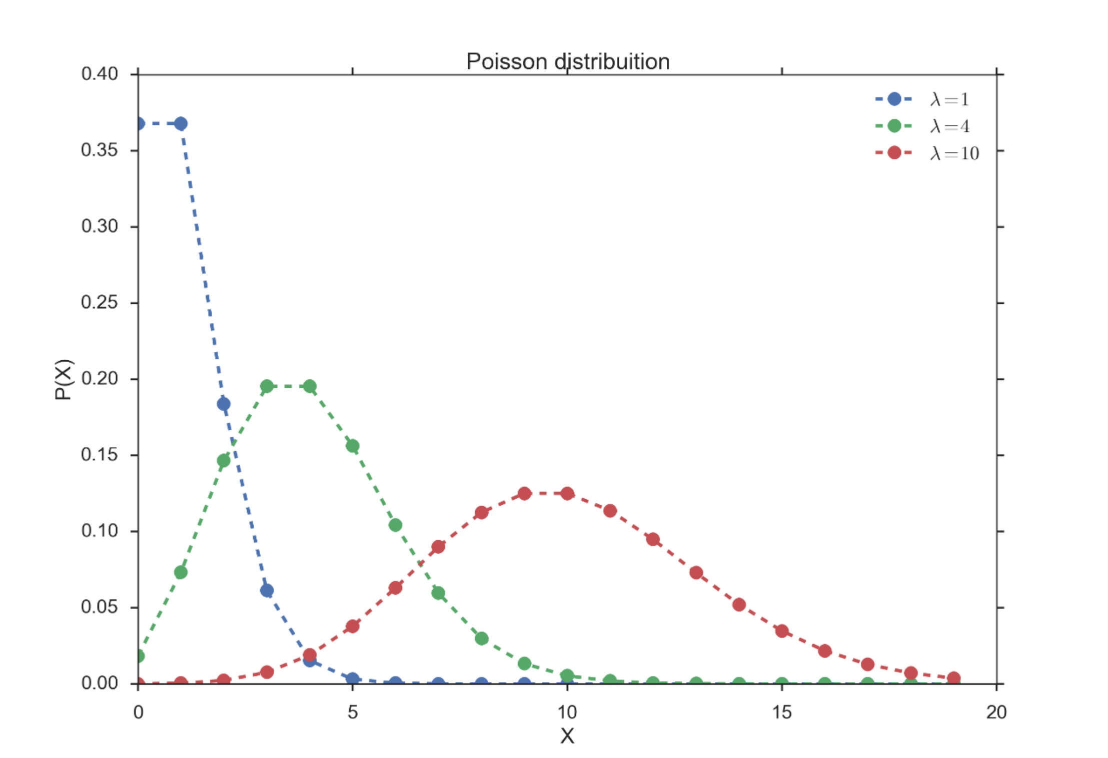
$$[A\ B]^T = (M^T M)^{-1} M^T \mathbf{y}. \qquad \sigma_{\mathrm{s}}^2 = \tfrac{1}{N-1} \mathbf{y}^T (I - M(M^T M)^{-1} M^T) \mathbf{y}.$$

System Model

We assume the number and locations of wireless units form a Poisson point process

$$\Pr(R_t = r_t) = \frac{(\lambda_t A_t)^{r_t}}{r_t!} e^{-A_t \lambda_t} \quad r_t = 0, 1, \ldots$$

$$\Pr(R_o = r_o) = \frac{(\lambda_o A_o)^{r_o}}{r_o!} e^{-A_o \lambda_o} \quad r_o = 0, 1, \ldots$$



Estimation Scheme

Bayes Estimation:

- Applied when we know the Poisson parameters λ_t and λ_o
- Prior probabilities are given by Poisson distributions

Maximum Estimation:

- Applied when Poisson parameters λ_t and λ_o are unknown
- Likelihood function: $\mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}, \underline{\mathbf{u}}\right) = f_{\underline{\mathbf{P}}, \underline{\mathbf{U}}}\left(\underline{\mathbf{p}}, \underline{\mathbf{u}}; \lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}\right)$

Bayes Estimation

Distribution of $\underline{\mathbf{U}}$

$$f_{\underline{\mathbf{U}}}(\underline{\mathbf{u}}) = \frac{1}{A_{\mathrm{t}}^{R_{\mathrm{t}}(\underline{\mathbf{u}})}} \frac{(\lambda_{\mathrm{t}} A_{\mathrm{t}})^{R_{\mathrm{t}}(\underline{\mathbf{u}})}}{(R_{\mathrm{t}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}}} \frac{1}{A_{\mathrm{o}}^{R_{\mathrm{o}}(\underline{\mathbf{u}})}} \frac{(\lambda_{\mathrm{o}} A_{\mathrm{o}})^{R_{\mathrm{o}}(\underline{\mathbf{u}})}}{(R_{\mathrm{o}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{o}}\lambda_{\mathrm{o}}} = \frac{\lambda_{\mathrm{t}}^{R_{\mathrm{t}}(\underline{\mathbf{u}})}}{(R_{\mathrm{t}}(\underline{\mathbf{u}}))!} \frac{\lambda_{\mathrm{o}}^{R_{\mathrm{o}}(\underline{\mathbf{u}})}}{(R_{\mathrm{o}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}} - A_{\mathrm{o}}\lambda_{\mathrm{o}}}.$$

Conditional distribution of P_j $given$ U_j

$$\begin{aligned}
& f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j) \\
&= \prod_{i=1}^{n_{\mathrm{s}}} f_{L_{ij}}(p_{ij} - A - B\log_{10}(d_{ij}) - G_i(\phi_{ij})) \\
&= \frac{1}{(2\pi\sigma_{\mathrm{s}}^2)^{\frac{n_{\mathrm{s}}}{2}}} \prod_{i=1}^{n_{\mathrm{s}}} e^{-\frac{(p_{ij} - A - B\log_{10}(d_{ij}) - G_i(\phi_{ij}))^2}{2\sigma_{\mathrm{s}}^2}} \\
&= \left(2\pi\sigma_{\mathrm{s}}^2\right)^{-\frac{n_{\mathrm{s}}}{2}} e^{-\frac{\sum_{i=1}^{n_{\mathrm{s}}}(p_{ij} - A - B\log_{10}(d_{ij}) - G_i(\phi_{ij}))^2}{2\sigma_{\mathrm{s}}^2}}.
\end{aligned}$$

Bayes Estimation

Conditional distribution of $\underline{\mathbf{P}}$ given $\underline{\mathbf{U}}$

$$f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) = \prod_{j=1}^{n_{\mathrm{a}}} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)$$

Marginal distribution of $\underline{\mathbf{P}}$

$$f_{\underline{\mathbf{P}}}\left(\underline{\mathbf{p}}\right) = \int_{\{\underline{\mathbf{u}}:R_{\mathrm{t}}(\underline{\mathbf{u}})+R_{\mathrm{o}}(\underline{\mathbf{u}})=n_{\mathrm{a}}\}} f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}(\underline{\mathbf{u}})d\underline{\mathbf{u}}$$

$$= \sum_{(r_{\mathrm{t}},r_{\mathrm{o}}):r_{\mathrm{t}}+r_{\mathrm{o}}=n_{\mathrm{a}}} \sum_{\{\mathbb{I}\subset[n_{\mathrm{a}}]:|\mathbb{I}|=r_{\mathrm{t}}\}} \frac{\lambda_{\mathrm{t}}^{r_{\mathrm{t}}}\lambda_{\mathrm{o}}^{r_{\mathrm{o}}}}{r_{\mathrm{t}}!r_{\mathrm{o}}!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}}-A_{\mathrm{o}}\lambda_{\mathrm{o}}}$$

$$\times \prod_{j\in\mathbb{I}} \mathscr{I}_{\mathscr{A}_{\mathrm{t}}}(j) \prod_{j\in\mathbb{I}^{\mathrm{c}}} \mathscr{I}_{\mathscr{A}_{\mathrm{o}}}(j).$$

The integral components

$$\mathcal{I}_{\mathcal{A}_{\mathrm{t}}}(j) = \int_{\mathcal{A}_{\mathrm{t}}} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)d\mathbf{u}_j$$

$$\mathcal{I}_{\mathcal{A}_{\mathrm{o}}}(j) = \int_{\mathcal{A}_{\mathrm{o}}} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)d\mathbf{u}_j.$$

Bayes Estimation

The posterior distribution of R_t $given$ $\underline{\mathbf{P}}$

$$
\begin{aligned}
&\Pr\left(R_{\mathrm{t}} = r_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right) \\
&= \sum_{\{\mathbb{I} \subset [n_{\mathrm{a}}] : |\mathbb{I}| = r_{\mathrm{t}}\}} \frac{\lambda_{\mathrm{t}}^{r_{\mathrm{t}}} \lambda_{\mathrm{o}}^{r_{\mathrm{o}}} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}} - A_{\mathrm{o}}\lambda_{\mathrm{o}}}}{r_{\mathrm{t}}! r_{\mathrm{o}}! f_{\underline{\mathbf{P}}}(\underline{\mathbf{p}})} \\
&\quad \times \prod_{j \in \mathbb{I}} \mathscr{I}_{\mathscr{A}_{\mathrm{t}}}(j) \prod_{j \in \mathbb{I}^{\mathrm{c}}} \mathscr{I}_{\mathscr{A}_{\mathrm{o}}}(j)
\end{aligned}
$$

Bayes estimator

$$
\hat{R}_{\mathrm{t}}\left(\underline{\mathbf{p}}\right) = \mathrm{E}\left[R_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right] = \sum\nolimits_{r_{\mathrm{t}}=0}^{n_{\mathrm{a}}} r_{\mathrm{t}} \Pr\left(R_{\mathrm{t}} = r_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right).
$$

Performance criterion

$$
\mathrm{BMSE}\left[\hat{R}_{\mathrm{t}}\right] = \mathrm{E}\left[\left(\hat{R}_{\mathrm{t}}\left(\underline{\mathbf{P}}\right) - R_{\mathrm{t}}\right)^2\right]
$$

$$
\mathrm{BMSE}\left[\hat{R}_{\mathrm{t}}\right] \approx \frac{1}{M} \sum\nolimits_{m=1}^{M} \left(\hat{R}_{\mathrm{t}}^{(m)}\left(\underline{\mathbf{P}}^{(m)}\right) - R_{\mathrm{t}}^{(m)}\right)^2
$$

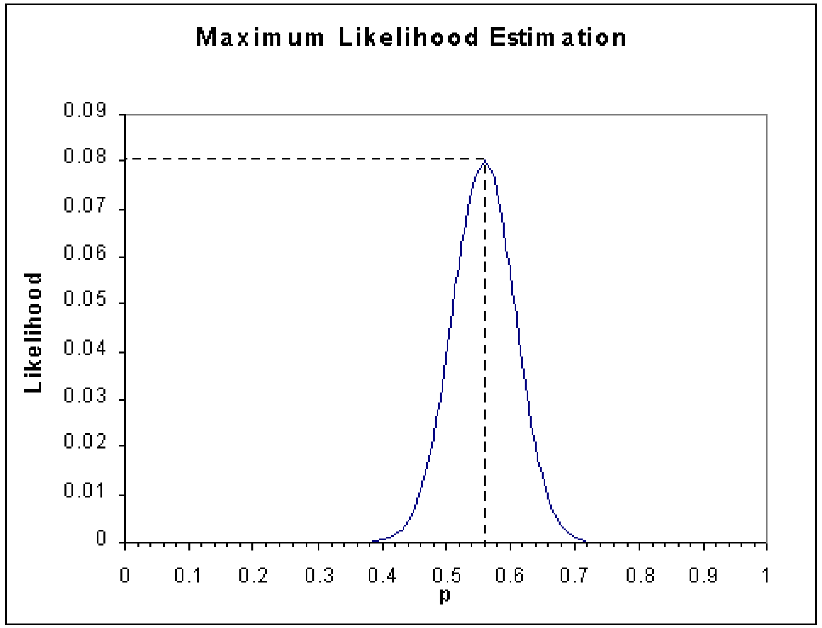
Maximum Likelihood Estimation

Likelihood function

$$
\begin{aligned}
&\mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}, \underline{\mathbf{u}}\right) \\
&= f_{\underline{\mathbf{P}}, \underline{\mathbf{U}}}\left(\underline{\mathbf{p}}, \underline{\mathbf{u}}; \lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}\right) \\
&= f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}\left(\underline{\mathbf{u}}; \lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}\right).
\end{aligned}
$$

Marginal likelihood function of observed data $\underline{\mathbf{P}}$

$$
\begin{aligned}
\mathscr{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}\right) &= \int_{\{\underline{\mathbf{u}}: R_{\mathrm{t}}(\underline{\mathbf{u}}) + R_{\mathrm{o}}(\underline{\mathbf{u}}) = n_{\mathrm{a}}\}} f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}\left(\underline{\mathbf{u}}; \lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}\right) d\underline{\mathbf{u}} \\
&= e^{-\Lambda_{\mathrm{t}}\lambda_{\mathrm{t}} - \Lambda_{\mathrm{o}}\lambda_{\mathrm{o}}} \sum_{(r_{\mathrm{t}}, r_{\mathrm{o}}): r_{\mathrm{t}} + r_{\mathrm{o}} = n_{\mathrm{a}}} \frac{\lambda_{\mathrm{t}}^{r_{\mathrm{t}}} \lambda_{\mathrm{o}}^{r_{\mathrm{o}}}}{r_{\mathrm{t}}! r_{\mathrm{o}}!} \sum_{\{\mathbb{I} \subset [n_{\mathrm{a}}]: |\mathbb{I}| = r_{\mathrm{t}}\}} \prod_{j \in \mathbb{I}} \mathscr{I}_{\mathscr{A}_{\mathrm{t}}}(j) \prod_{j \in \mathbb{I}^{\mathrm{c}}} \mathscr{I}_{\mathscr{A}_{\mathrm{o}}}(j)
\end{aligned}
$$



Maximum Likelihood Estimation

Property of likelihood function

$$\mathcal{L}\left(c\lambda_{\mathrm{t}}, c\lambda_{\mathrm{o}}; \underline{\mathbf{P}}\right) = c^{n_{\mathrm{a}}} e^{-(A_{\mathrm{t}}\lambda_{\mathrm{t}} + A_{\mathrm{o}}\lambda_{\mathrm{o}})(c-1)} \mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{P}}\right).$$

Set c

$$c = \frac{n_{\mathrm{a}}}{A_{\mathrm{t}}\lambda_{\mathrm{t}} + A_{\mathrm{o}}\lambda_{\mathrm{o}}}.$$

$$\max_{\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}} \mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}\right) = \max_{\alpha} \mathcal{L}\left(\frac{n_{\mathrm{a}}}{A_{\mathrm{t}}}\alpha, \frac{n_{\mathrm{a}}}{A_{\mathrm{o}}}(1-\alpha); \underline{\mathbf{p}}\right)$$

$$\lambda_{\mathrm{t}} = \alpha \frac{\lambda}{A_{\mathrm{t}}}$$

$$\lambda_{\mathrm{o}} = (1-\alpha)\frac{\lambda}{A_{\mathrm{o}}}$$

Maximum Likelihood Estimation

Maximum likelihood estimator

$$\begin{aligned}&\hat{R}_{\mathrm{t}}\left(\underline{\mathbf{p}}\right)\\ &= \mathrm{E}_{\hat{\lambda}_{\mathrm{t}},\hat{\lambda}_{\mathrm{o}}}\left[R_{\mathrm{t}}|\underline{\mathbf{P}} = \underline{\mathbf{p}}\right]\\ &= \sum_{r_{\mathrm{t}}=0}^{n_{\mathrm{a}}} r_{\mathrm{t}}\,\mathrm{Pr}\left(R_{\mathrm{t}} = r_{\mathrm{t}}|\underline{\mathbf{P}} = \underline{\mathbf{p}};\hat{\lambda}_{\mathrm{t}},\hat{\lambda}_{\mathrm{o}}\right).\end{aligned}$$

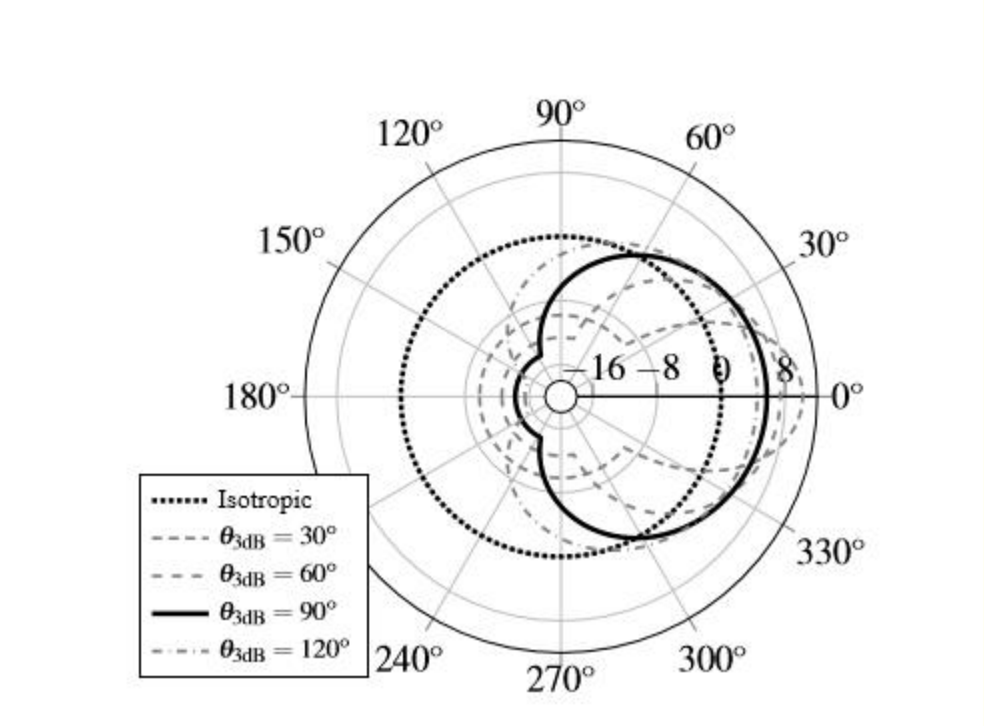
Performance criterion

$$\mathrm{MSE}\left[\hat{R}_{\mathrm{t}}\right] \approx \tfrac{1}{M}\sum_{m=1}^{M}\left(\hat{R}_{\mathrm{t}}^{(m)}\left(\underline{\mathbf{P}}^{(m)}\right) - R_{\mathrm{t}}^{(m)}\right)^{2}$$

Numerical Simulation

3GPP antenna model

$$G_i(\phi_{ij}) = -\min\left\{12\left(\frac{\phi_{ij}-\theta_i}{\theta_{3\text{dB}}}\right)^2, G_{\text{floor}}\right\} - G_{\text{avg}}$$



Channel Parameters

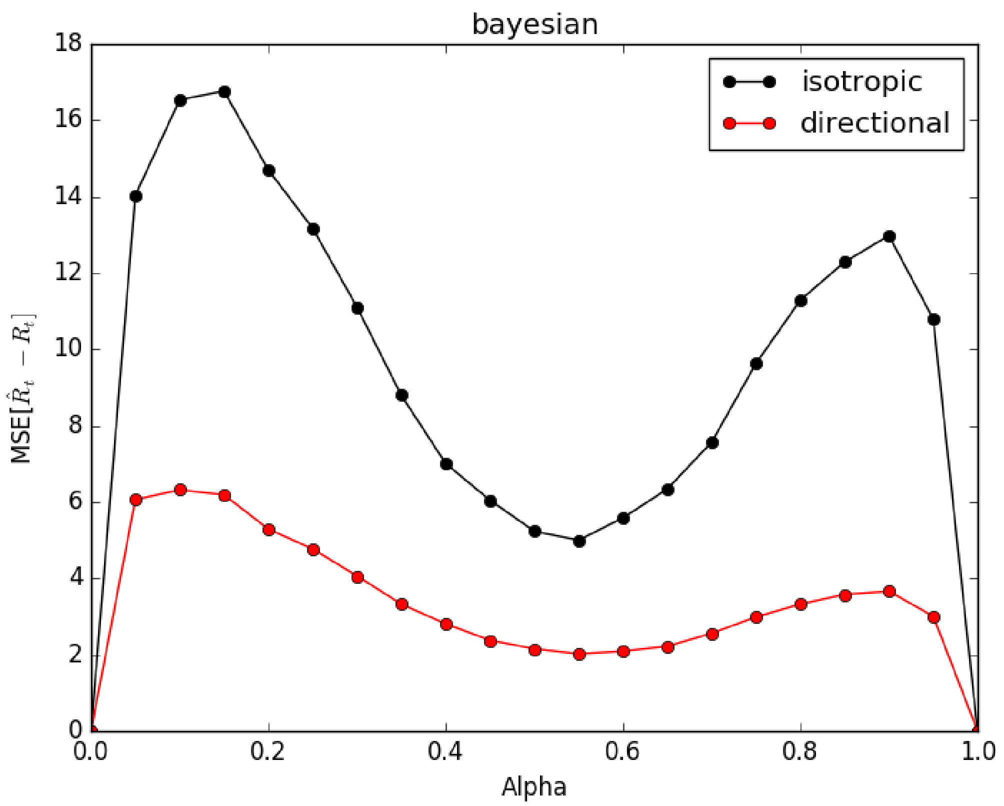
$$P[\mathrm{dBm}] = A + B\log_{10}(d) + L + G(\phi)$$

$$A = P_{\mathrm{t}} + 20\log_{10}\left(\frac{3\times10^8}{f_{\mathrm{carrier}}}\right) - 20\log_{10}(4\pi)$$

$$B = -20dBm$$

$$\sigma_s = 2.0dBm$$

Simulation Results



Bayes estimator
Intensity $\lambda = 32$
21 points on each curve
50000 trials for each point

Simulation Results

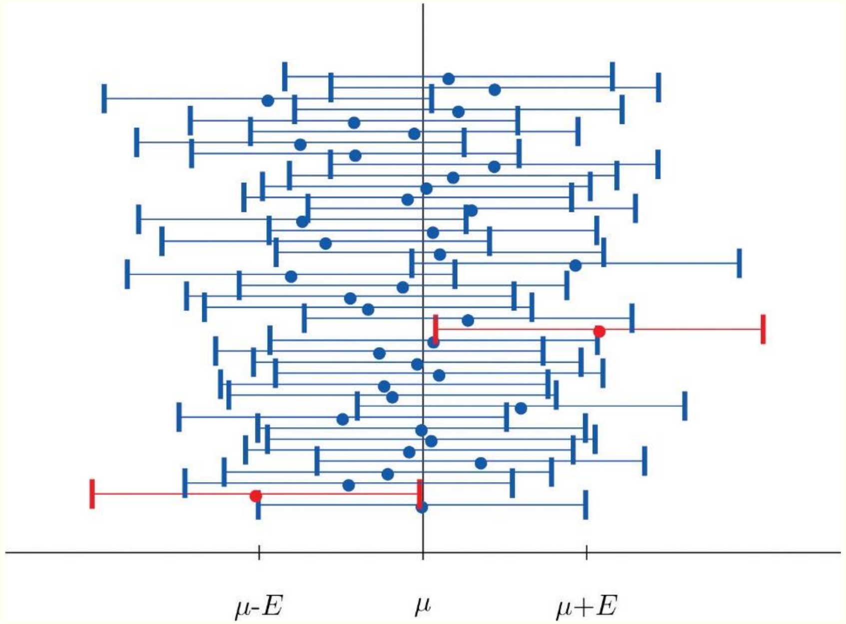
$$\alpha = 1 - (confidence\ level/100)$$

$$p^* = 1 - \alpha/2$$

$$df = N - 1$$

$$SE = \frac{\sigma}{\sqrt{N}}$$

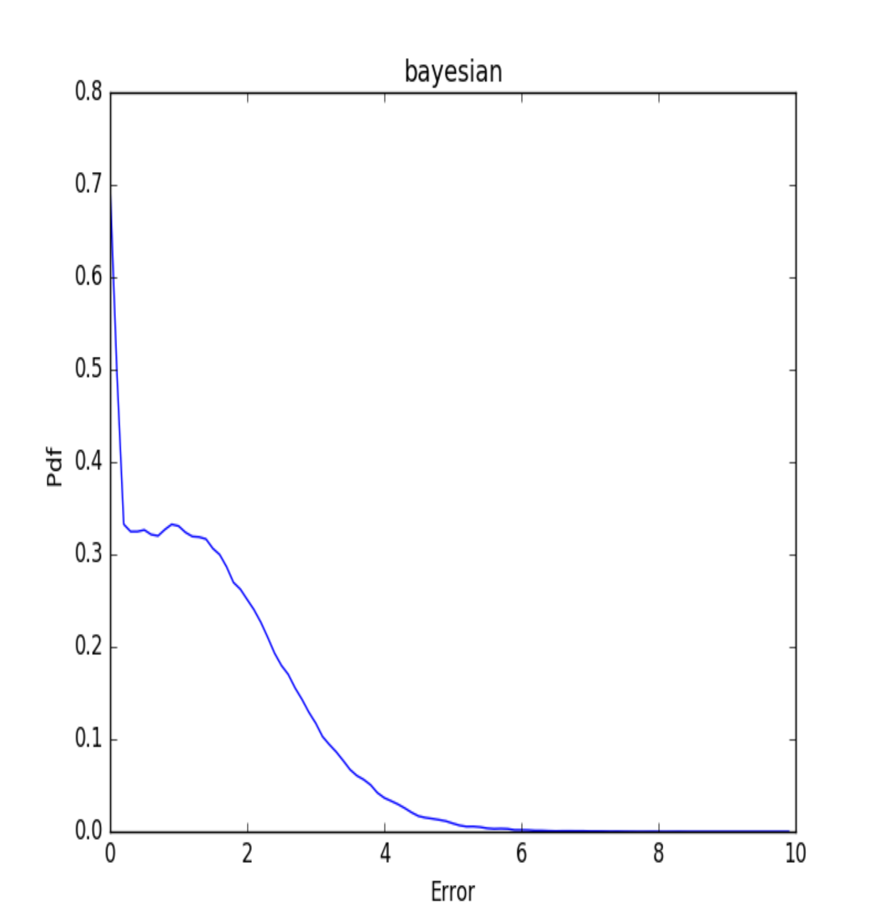
$$Confidence\ interval = \mu \pm Margin\ of\ error$$



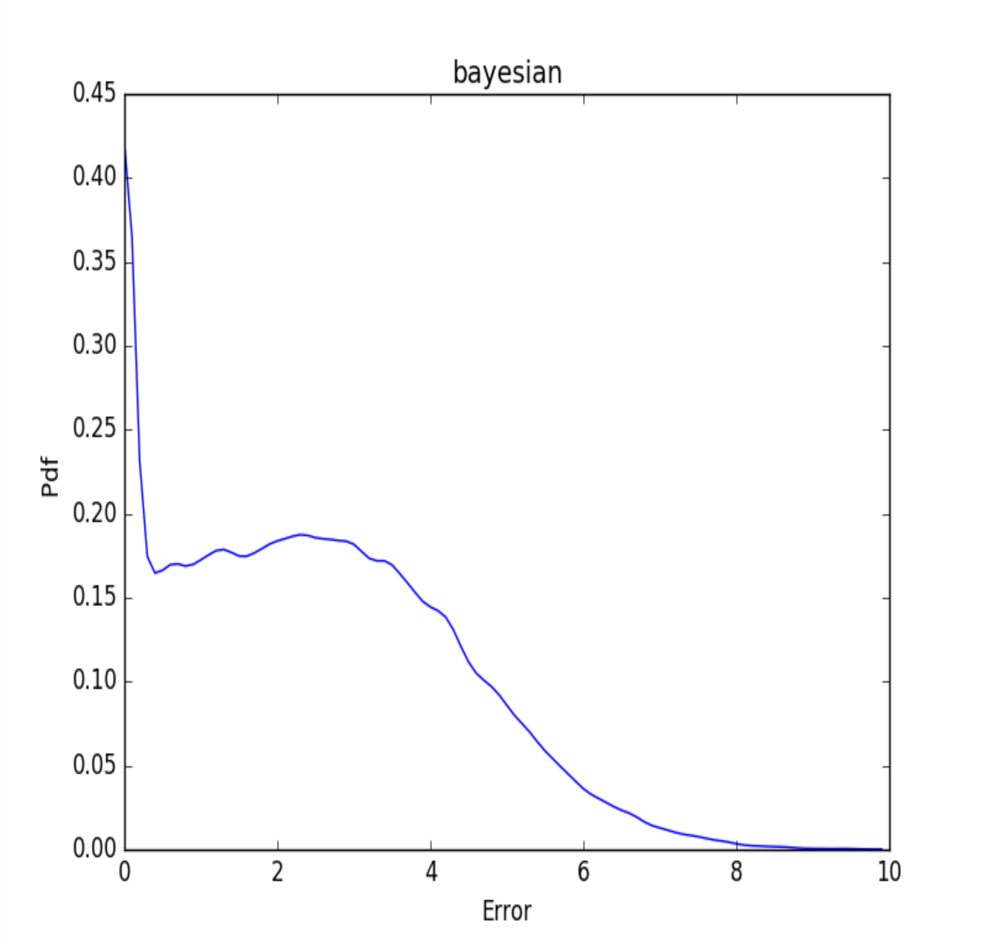
95% Confidence interval of $|r_t - \hat{r_t}|$

Antenna type	Confidence interval
Directional	1.412411 ± 0.002166
Isotropic	2.454623 ± 0.003450

Simulation Results

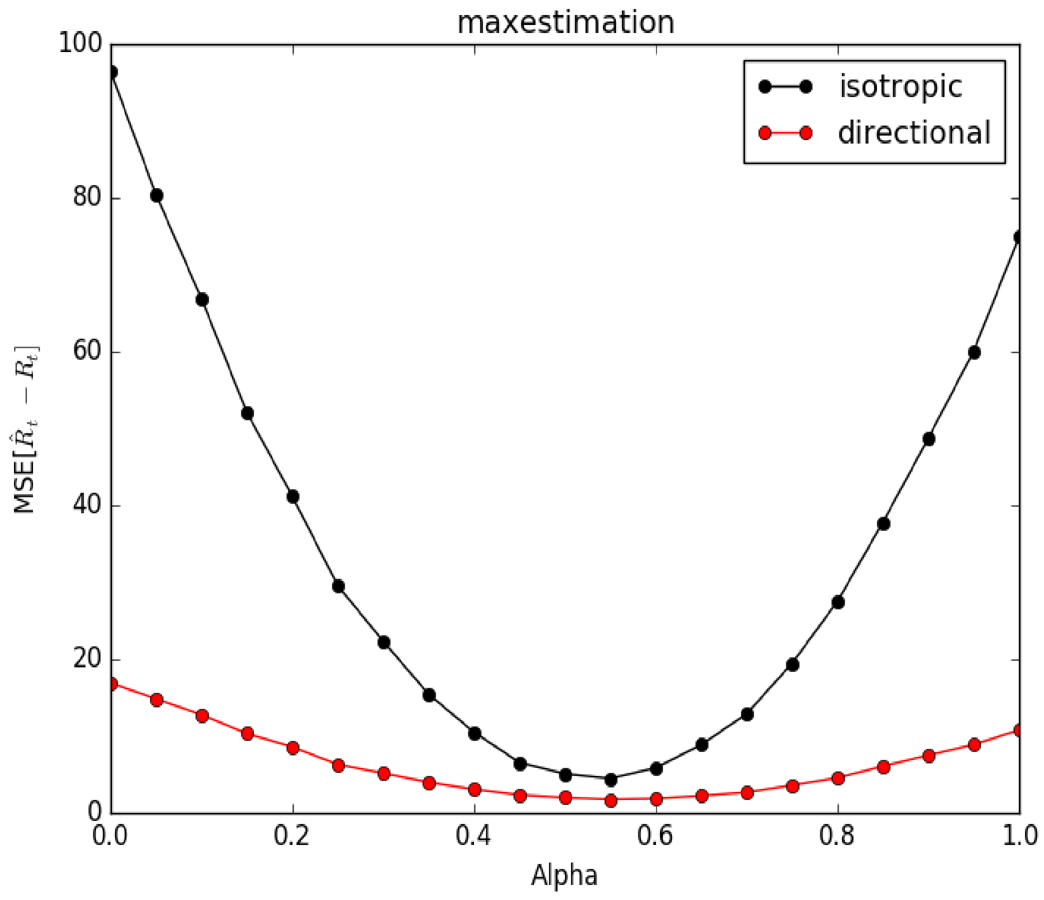


Pdf of $|r_t - \hat{r}_t|$
System with directional antennas



Pdf of $|r_t - \hat{r}_t|$
System with isotropic antennas

Simulation Results

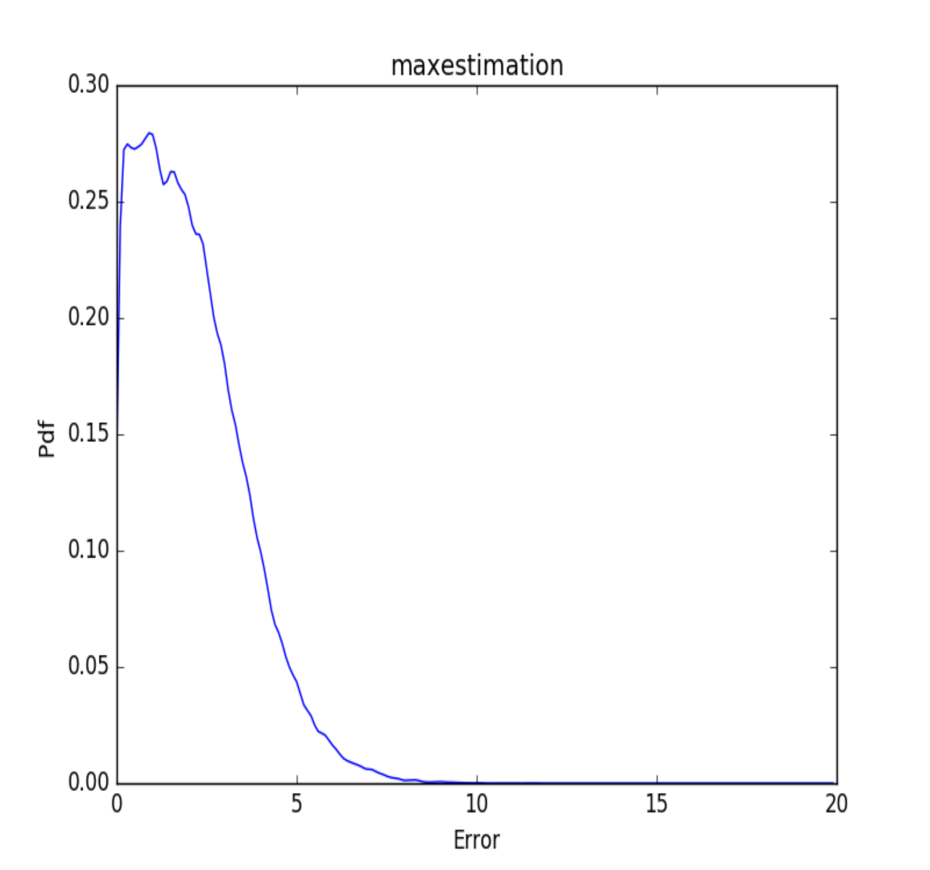


Maximum likelihood estimator
Intensity $\lambda = 32$
21 points on each curve
50000 trials for each point

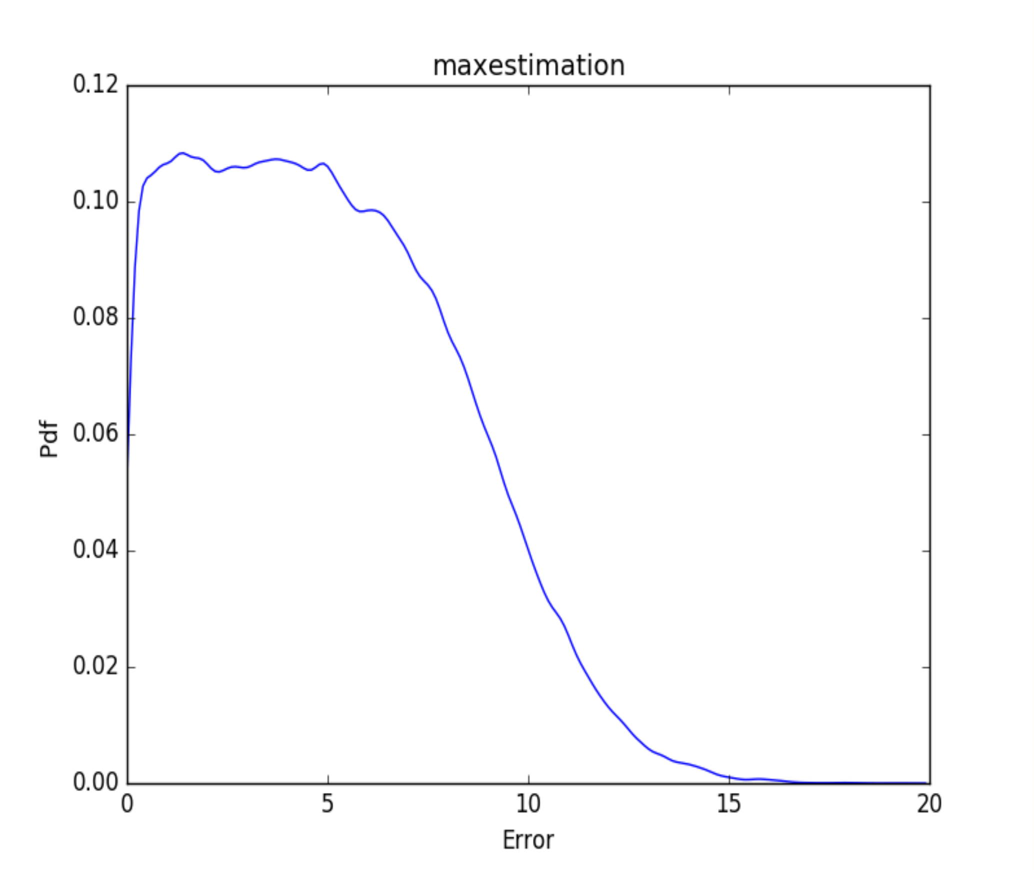
95% Confidence interval of $|r_t - \hat{r_t}|$

Antenna type	Confidence interval
Directional	2.080484 ± 0.002809
Isotropic	4.972427 ± 0.006020

Simulation Results



Pdf of $|r_t - \hat{r}_t|$
System with directional antennas



Pdf of $|r_t - \hat{r}_t|$
System with isotropic antennas

Simulation results

In both results, BMSE and MSE of directional antennas systems are smaller.

The error is of system with directional antennas distributed more closely to 0.

System with directional antennas perform better.

Experimental Implement

Experimental setup consists of three components

- Alpha network card AWUS036NHA
- Local processing units: Intel Next Unit of Computing
- Cloud server

NIC operates in monitor mode to capture Wi-Fi packets

Local processing units parses information: MAC addresses, RSSI values, time stamps

RSSI values are employed as basis for inference

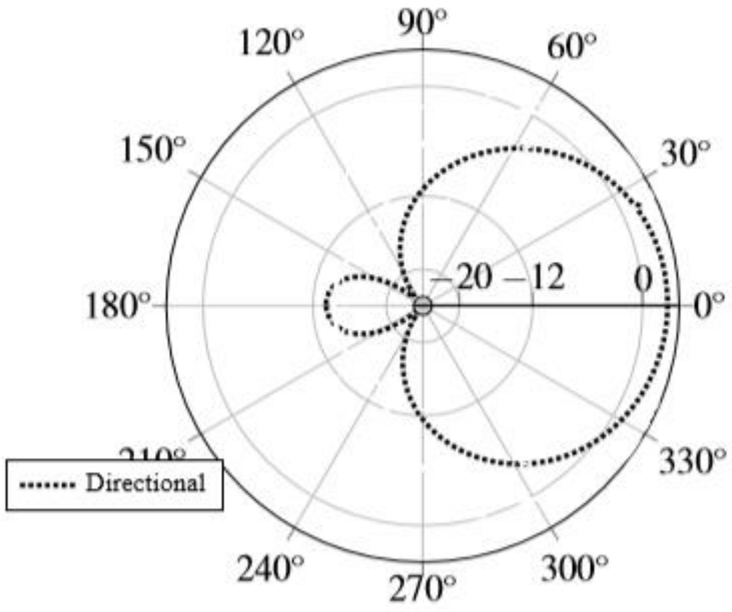


Fig. This graph depicts directional antenna radiation patterns

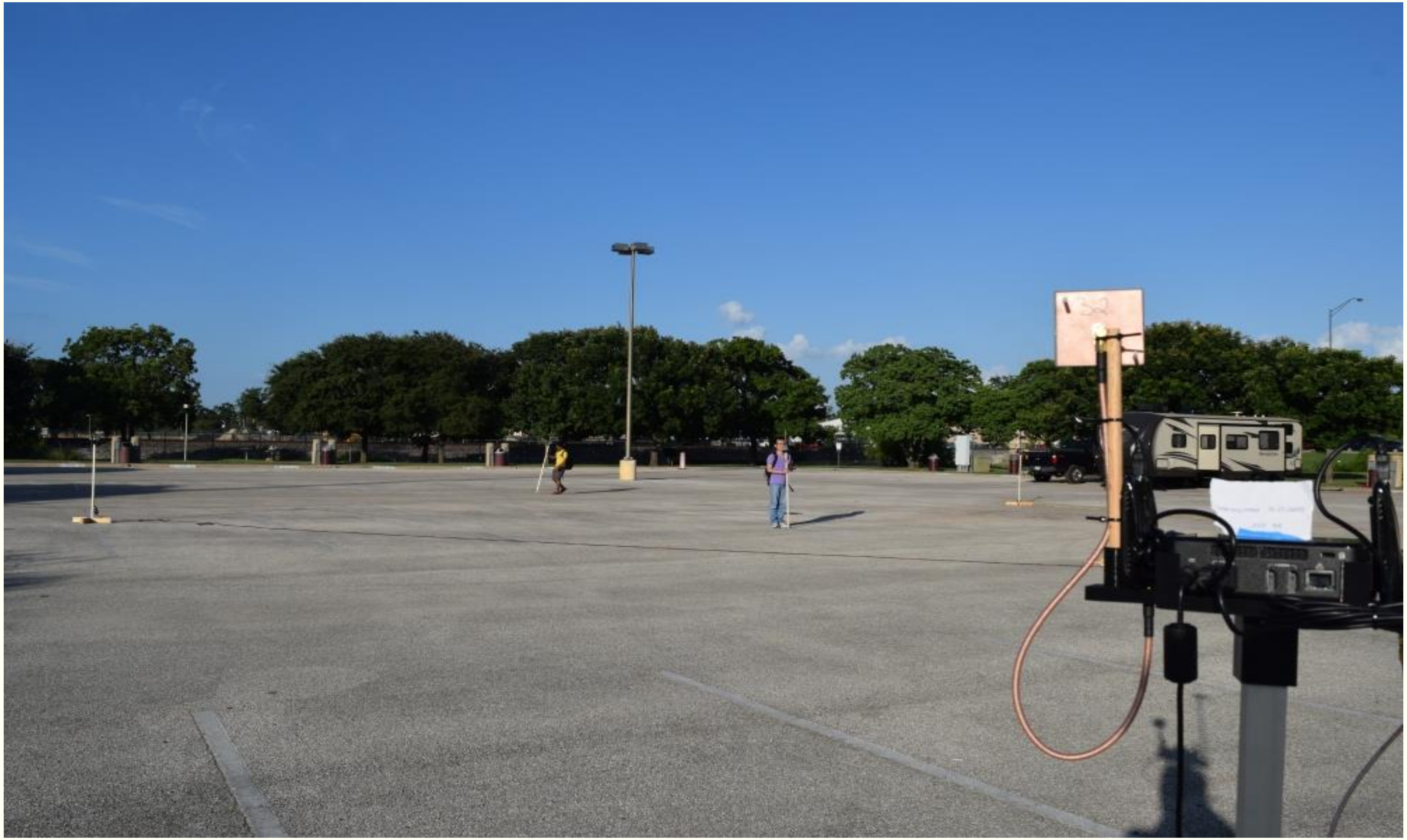
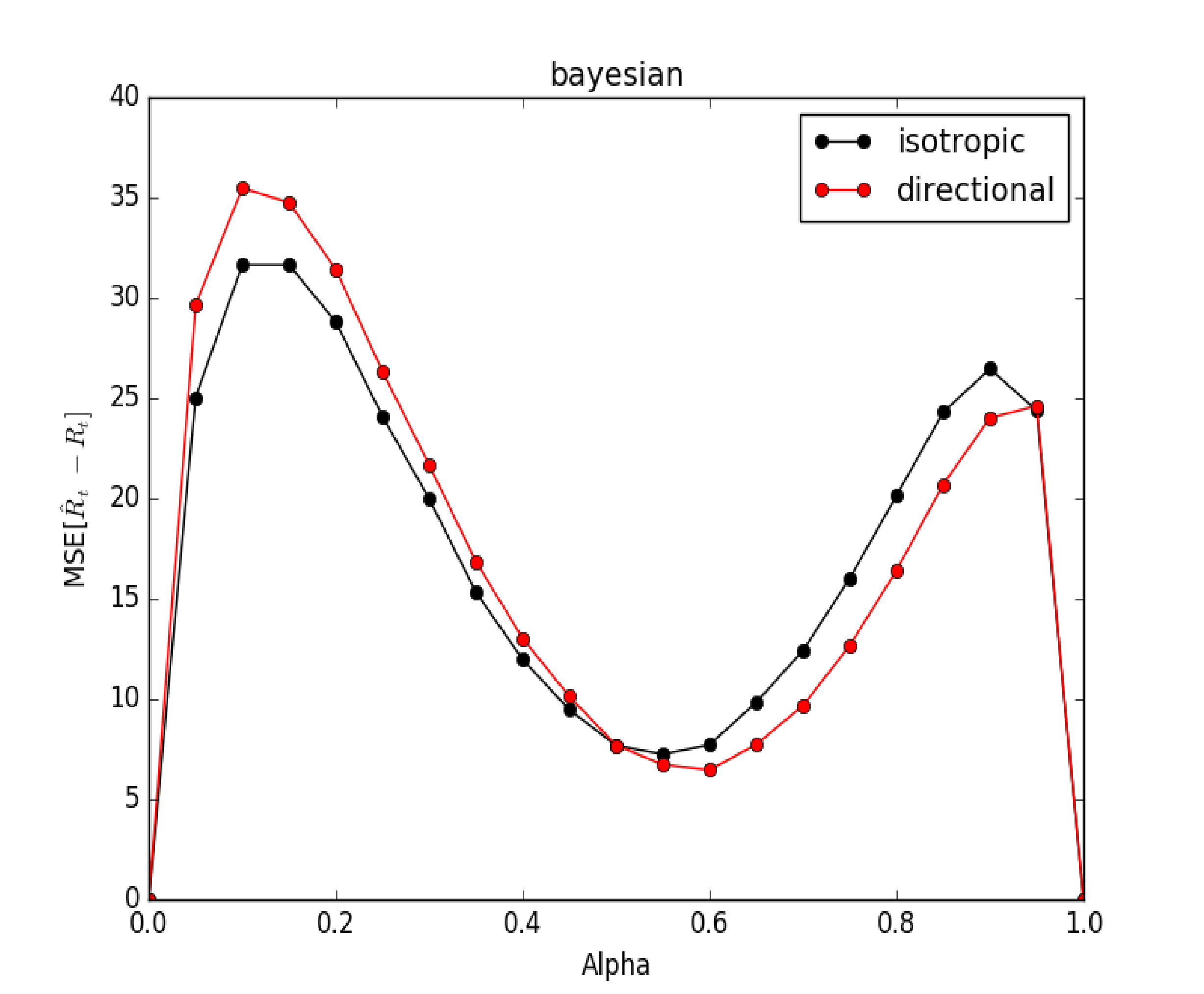


Figure: The place we conduct our experiment

Fig. This figure highlights the site used for the experiments and marks locations of experiment data.

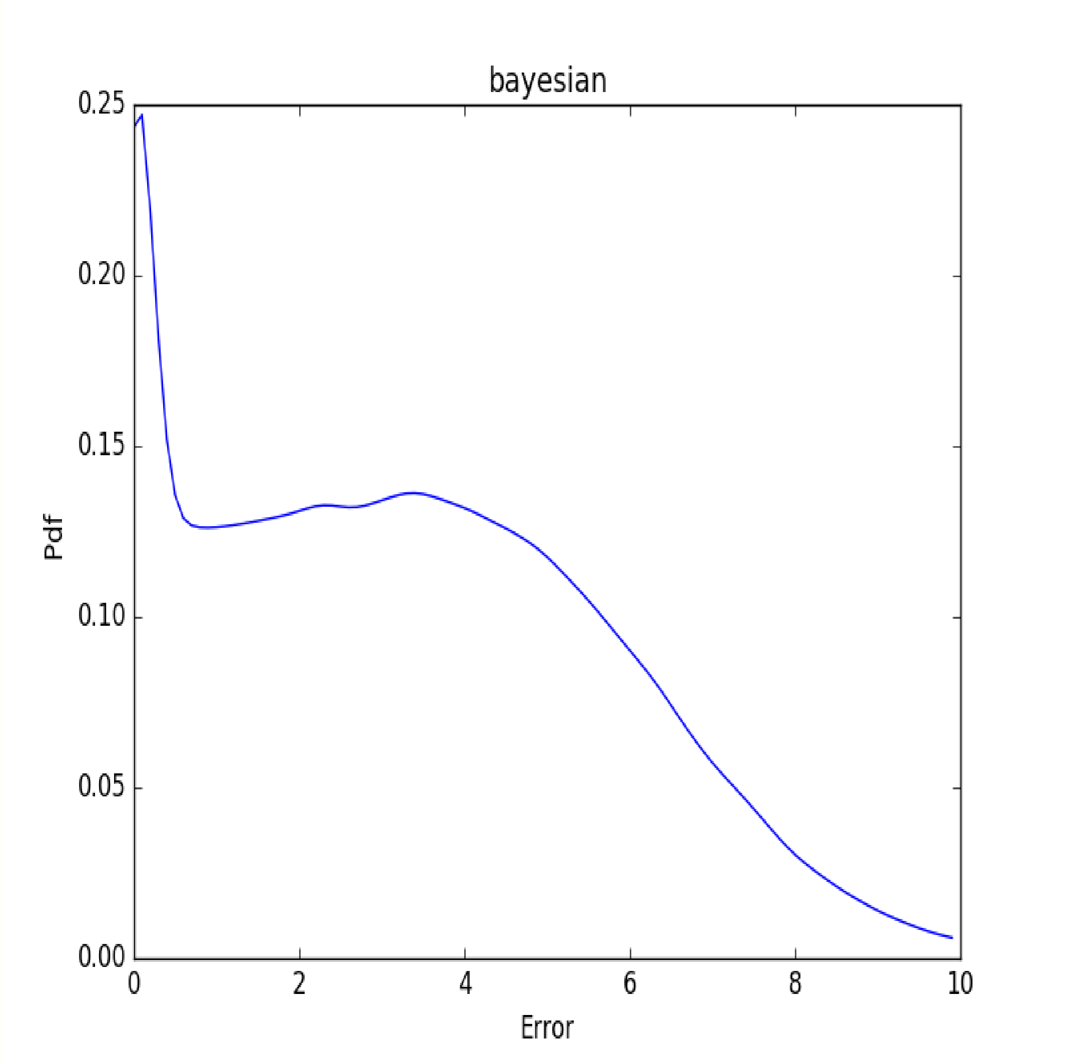
Experiment Results

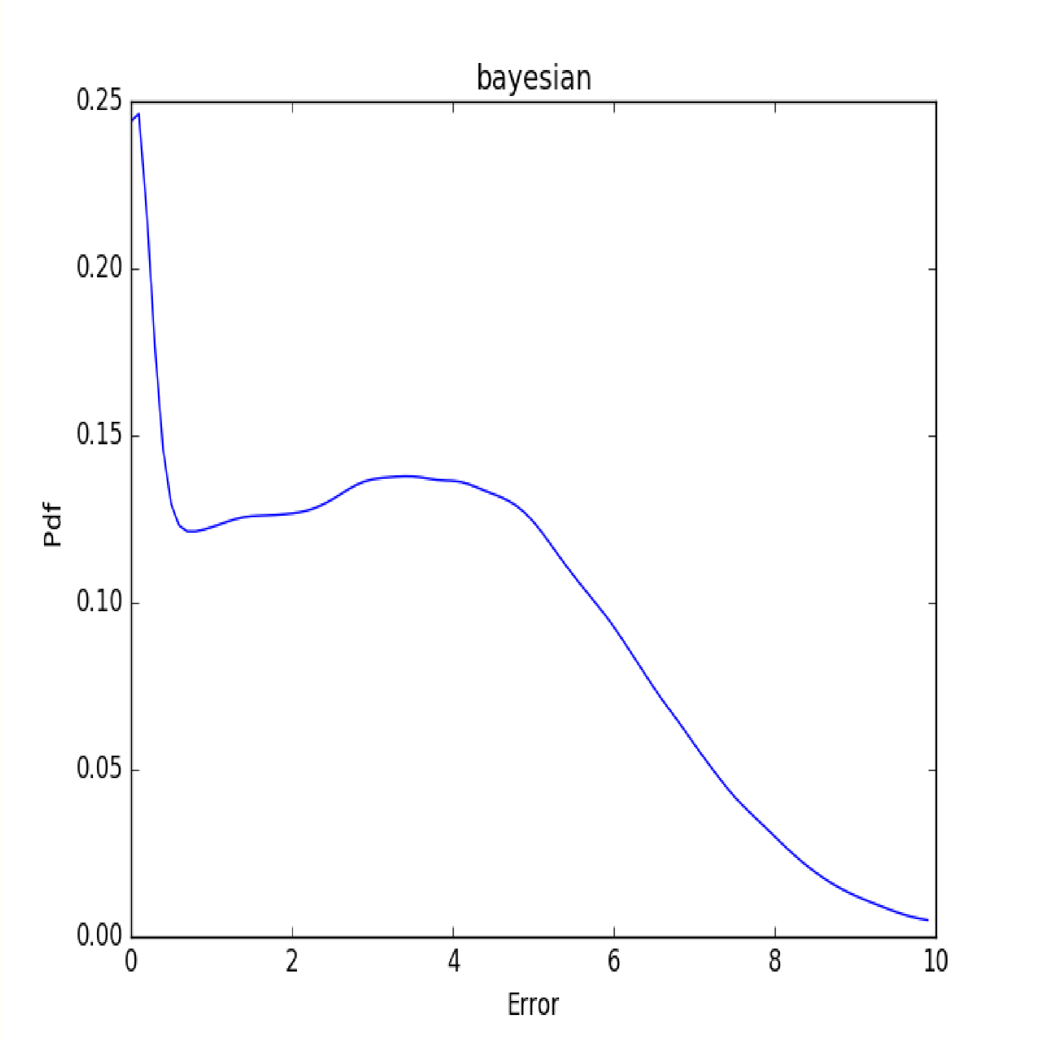


Bayes estimator
Intensity $\lambda = 32$
21 points on each curve
10000 trials for each point

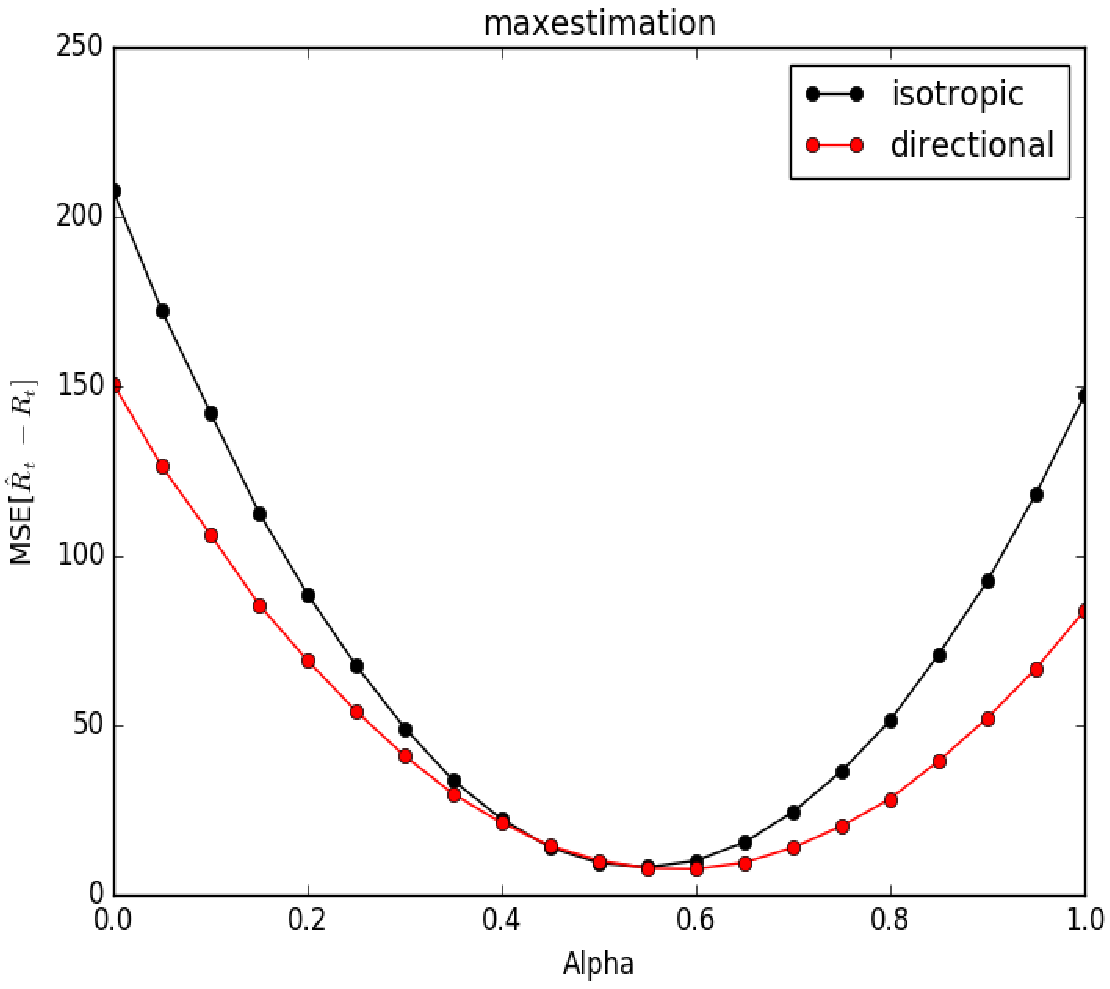
95% Confidence interval of $|r_t - \hat{r_t}|$

Antenna type	Confidence interval
Directional	3.317535 ± 0.010430
Isotropic	3.331094 ± 0.010274


bayesian
Pdf
Error
0.00
0.05
0.10
0.15
0.20
0.25
0
2
4
6
8
10


bayesian
Pdf
Error
0.00
0.05
0.10
0.15
0.20
0.25
0
2
4
6
8
10

Experiment result

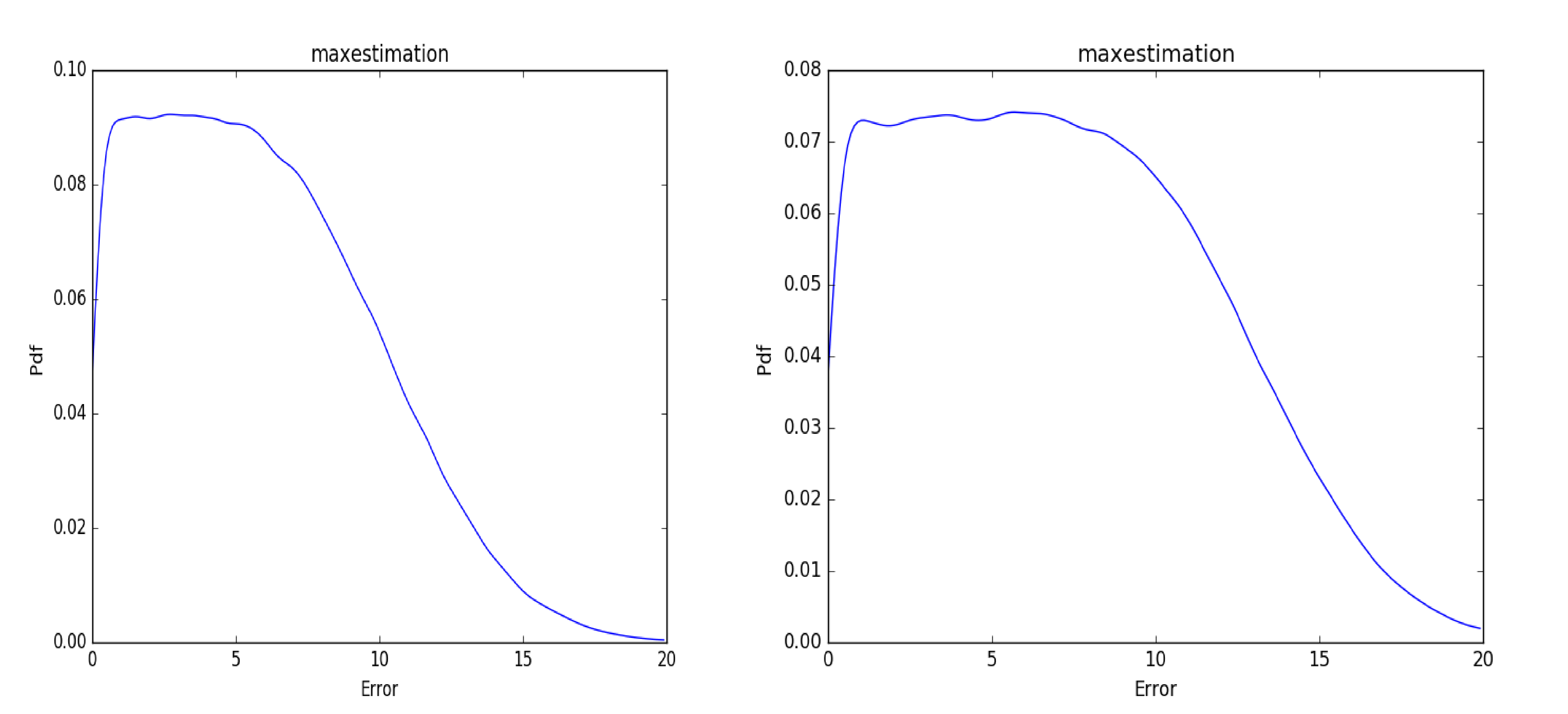


Maximum likelihood estimator
Intensity $\lambda = 32$
21 points on each curve
10000 trials for each point

95% Confidence interval of $|r_t - \hat{r}_t|$

Antenna type	**Confidence interval**
Directional	5.881027 ± 0.016484
Isotropic	7.144900 ± 0.019182

Experiment Results



Conclusion

The directional antenna do improve the performance of estimator accuracy.

This work may extend to be used in track the motion of specific device.

Thank You