# Estimating the Number of Active Devices Within a Fixed area Using Wi-Fi Monitoring

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# Outline

- Problem Description
- System Model
- Estimation Scheme
- Simulation and Result
- Experimental Implementation and Result
- Conclusion

# Introduction

Occupancy estimation has several applications

- Smart home management
- Emergency situation

Camera based approach is costly and limited by resolution

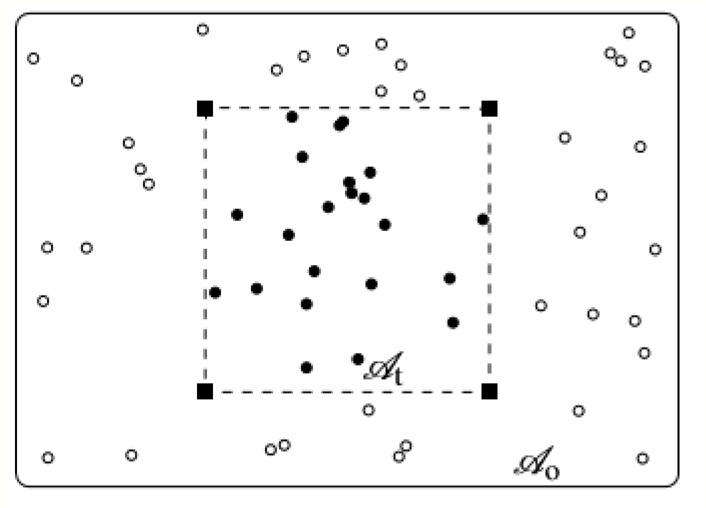
RF signal based approach consumes less power

The increasing mobile penetration rate and widely distributed access point suggest Wi-Fi a good source for occupancy estimation

# Problem Description

- Estimating number of active wireless units using Wi-Fi monitoring with statistical inference
- Increase the accuracy by deploying proper directional antennas

# System Model




$A_t$ is the area of target region.
$A_O$ is the area of complement region.

Location vector:

$$\underline{\mathbf{U}} = (\mathbf{U}_1, \ldots, \mathbf{U}_{n_a})$$

Received signal vector:

$$\underline{\mathbf{P}} = (\mathbf{P}_1, \ldots, \mathbf{P}_{n_a})$$

# Channel Model

Log-normal channel model:

$$P_d[\mathrm{dBm}] = A + B\log_{10}(d) + L_s + G_a$$

Log-normal distribution:

$$f_{L_s}(\ell) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{s}}}\exp\left(-\frac{\ell^2}{2\sigma_{\mathrm{s}}^2}\right)$$

# Find parameter values

Using least squares method:

$$y = \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix} \quad X = \begin{bmatrix} 1 & \log_{10}(d_1) \\ \vdots & \vdots \\ 1 & \log_{10}(d_N) \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = (X^T X)^{-1} X^T y \qquad \sigma_s^2 = \frac{1}{N-1} \boldsymbol{y}^T (I - X(X^T X)^{-1} X^T) \boldsymbol{y}$$

# System Model

We assume the number and locations of wireless units form a Poisson point process

$$\Pr(R_{\mathrm{t}} = r_{\mathrm{t}}) = \frac{(\lambda_{\mathrm{t}} A_{\mathrm{t}})^{r_{\mathrm{t}}}}{r_{\mathrm{t}}!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}}} \quad r_{\mathrm{t}} = 0, 1, \ldots$$

$$\Pr(R_{\mathrm{o}} = r_{\mathrm{o}}) = \frac{(\lambda_{\mathrm{o}} A_{\mathrm{o}})^{r_{\mathrm{o}}}}{r_{\mathrm{o}}!} e^{-A_{\mathrm{o}}\lambda_{\mathrm{o}}} \quad r_{\mathrm{o}} = 0, 1, \ldots$$

# Estimation Scheme

Bayes Estimation:

- Applied when we know the Poisson parameters $\lambda_t$ and $\lambda_o$
- Prior probabilities are given by Poisson distributions

Maximum Estimation:

- Applied when we Poisson parameters $\lambda_t$ and $\lambda_o$ are unknown
- Likelihood function: $\mathcal{L}\left(\lambda_t, \lambda_o; \underline{\mathbf{p}}, \underline{\mathbf{u}}\right) = f_{\underline{\mathbf{p}},\underline{\mathbf{u}}}\left(\underline{\mathbf{p}}, \underline{\mathbf{u}}; \lambda_t, \lambda_o\right)$

# Bayes Estimation

Distribution of $\underline{\mathbf{U}}$

$$f_{\underline{\mathrm{U}}}(\underline{\mathbf{u}}) = \frac{1}{A_{\mathrm{t}}^{R_{\mathrm{t}}(\underline{\mathbf{u}})}} \frac{(\lambda_{\mathrm{t}} A_{\mathrm{t}})^{R_{\mathrm{t}}(\underline{\mathbf{u}})}}{(R_{\mathrm{t}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}}} \times \frac{1}{A_{\mathrm{o}}^{R_{\mathrm{o}}(\underline{\mathbf{u}})}} \frac{(\lambda_{\mathrm{o}} A_{\mathrm{o}})^{R_{\mathrm{o}}(\underline{\mathbf{u}})}}{(R_{\mathrm{o}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{o}}\lambda_{\mathrm{o}}} = \frac{\lambda_{\mathrm{t}}^{R_{\mathrm{t}}(\underline{\mathbf{u}})}}{(R_{\mathrm{t}}(\underline{\mathbf{u}}))!} \frac{\lambda_{\mathrm{o}}^{R_{\mathrm{o}}(\underline{\mathbf{u}})}}{(R_{\mathrm{o}}(\underline{\mathbf{u}}))!} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}} - A_{\mathrm{o}}\lambda_{\mathrm{o}}}.$$

Conditional distribution of $P_j$ $given$ $U_j$

$$f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j) = \prod_{i=1}^{n_{\mathrm{s}}} f_{L_{ij}}(p_{ij} - A - B\log_{10}(d_{ij}) - G_i(\phi_{ij})) = \frac{1}{(2\pi\sigma_{\mathrm{s}}^2)^{\frac{n_{\mathrm{s}}}{2}}} \prod_{i=1}^{n_{\mathrm{s}}} e^{-\frac{(p_{ij}-A-B\log_{10}(d_{ij})-G_i(\phi_{ij}))^2}{2\sigma_{\mathrm{s}}^2}} = (2\pi\sigma_{\mathrm{s}}^2)^{-\frac{n_{\mathrm{s}}}{2}} e^{-\frac{\sum_{i=1}^{n_{\mathrm{s}}}(p_{ij}-A-B\log_{10}(d_{ij})-G_i(\phi_{ij}))^2}{2\sigma_{\mathrm{s}}^2}}$$

# Bayes Estimation

Conditional distribution of $\underline{\mathbf{P}}$ given $\underline{\mathbf{U}}$

$$f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) = \prod_{j=1}^{n_a} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)$$

Marginal distribution of $\underline{\mathbf{P}}$

$$f_{\underline{\mathbf{P}}}\left(\underline{\mathbf{p}}\right) = \int_{\{\underline{\mathbf{u}}:R_t(\underline{\mathbf{u}})+R_o(\underline{\mathbf{u}})=n_a\}} f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}(\underline{\mathbf{u}})d\underline{\mathbf{u}}$$

$$= \sum_{(r_t,r_o):r_t+r_o=n_a} \sum_{\{\mathbb{I}\subset[n_a]:|\mathbb{I}|=r_t\}} \frac{\lambda_t^{r_t}\lambda_o^{r_o}}{r_t!r_o!} e^{-A_t\lambda_t-A_o\lambda_o}$$

$$\times \prod_{j\in\mathbb{I}} \mathscr{I}_{\mathscr{A}_t}(j) \prod_{j\in\mathbb{I}^c} \mathscr{I}_{\mathscr{A}_o}(j).$$

The integral components

$$\mathscr{I}_{\mathscr{A}_t}(j) = \int_{\mathscr{A}_t} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)d\mathbf{u}_j$$

$$\mathscr{I}_{\mathscr{A}_o}(j) = \int_{\mathscr{A}_o} f_{\mathbf{P}_j|\mathbf{U}_j}(\mathbf{p}_j|\mathbf{u}_j)d\mathbf{u}_j.$$

# Bayes Estimation

The posterior distribution of $R_t$ $given$ $\underline{\mathbf{P}}$

$$\Pr\left(R_{\mathrm{t}} = r_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right) = \sum_{\{\mathbb{I} \subset [n_{\mathrm{a}}] : |\mathbb{I}| = r_{\mathrm{t}}\}} \frac{\lambda_{\mathrm{t}}^{r_{\mathrm{t}}} \lambda_{\mathrm{o}}^{r_{\mathrm{o}}} e^{-A_{\mathrm{t}}\lambda_{\mathrm{t}} - A_{\mathrm{o}}\lambda_{\mathrm{o}}}}{r_{\mathrm{t}}! r_{\mathrm{o}}! f_{\underline{\mathbf{P}}}(\underline{\mathbf{p}})} \times \prod_{j \in \mathbb{I}} \mathscr{I}_{\mathscr{A}_{\mathrm{t}}}(j) \prod_{j \in \mathbb{I}^{\mathrm{c}}} \mathscr{I}_{\mathscr{A}_{\mathrm{o}}}(j)$$

Bayes estimator

$$\hat{R}_{\mathrm{t}}\left(\underline{\mathbf{p}}\right) = \mathrm{E}\left[R_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right] = \sum_{r_{\mathrm{t}}=0}^{n_{\mathrm{a}}} r_{\mathrm{t}} \Pr\left(R_{\mathrm{t}} = r_{\mathrm{t}} | \underline{\mathbf{P}} = \underline{\mathbf{p}}\right)$$

Performance criterion

$$\mathrm{BMSE}\left[\hat{R}_{\mathrm{t}}\right] = \mathrm{E}\left[\left(\hat{R}_{\mathrm{t}}\left(\underline{\mathbf{P}}\right) - R_{\mathrm{t}}\right)^2\right] \qquad \mathrm{BMSE}\left[\hat{R}_{\mathrm{t}}\right] \approx \frac{1}{M} \sum_{m=1}^{M} \left(\hat{R}_{\mathrm{t}}^{(m)}\left(\underline{\mathbf{P}}^{(m)}\right) - R_{\mathrm{t}}^{(m)}\right)^2$$

# Maximum Likelihood Estimation

Likelihood function

$$\mathscr{L}\left(\lambda_t, \lambda_o; \underline{\mathbf{p}}, \underline{\mathbf{u}}\right) = f_{\underline{\mathbf{P}},\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}, \underline{\mathbf{u}}; \lambda_t, \lambda_o\right)$$

$$= f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}\left(\underline{\mathbf{u}}; \lambda_t, \lambda_o\right)$$

Marginal likelihood function of observed data $\underline{\mathbf{P}}$

$$\mathscr{L}\left(\lambda_t, \lambda_o; \underline{\mathbf{p}}\right) = \int_{\{\underline{\mathbf{u}}: R_t(\underline{\mathbf{u}}) + R_o(\underline{\mathbf{u}}) = n_a\}} f_{\underline{\mathbf{P}}|\underline{\mathbf{U}}}\left(\underline{\mathbf{p}}|\underline{\mathbf{u}}\right) f_{\underline{\mathbf{U}}}\left(\underline{\mathbf{u}}; \lambda_t, \lambda_o\right) d\underline{\mathbf{u}}$$

$$= e^{-\Lambda_t \lambda_t - \Lambda_o \lambda_o} \sum_{(r_t, r_o): r_t + r_o = n_a} \frac{\lambda_t^{r_t} \lambda_o^{r_o}}{r_t! r_o!} \sum_{\{\mathbb{I} \subset [n_a]: |\mathbb{I}| = r_t\}} \prod_{j \in \mathbb{I}} \mathscr{I}_{\mathscr{A}_t}(j) \prod_{j \in \mathbb{I}^c} \mathscr{I}_{\mathscr{A}_o}(j)$$

# Maximum Likelihood Estimation

Property of likelihood function

$$\mathcal{L}\left(c\lambda_{\mathrm{t}}, c\lambda_{\mathrm{o}}; \underline{\mathbf{p}}\right) = c^{n_{\mathrm{a}}} e^{-(A_{\mathrm{t}}\lambda_{\mathrm{t}} + A_{\mathrm{o}}\lambda_{\mathrm{o}})(c-1)} \mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}\right)$$

Set c

$$c = \frac{n_{\mathrm{a}}}{A_{\mathrm{t}}\lambda_{\mathrm{t}} + A_{\mathrm{o}}\lambda_{\mathrm{o}}}$$

$$\max_{\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}} \mathcal{L}\left(\lambda_{\mathrm{t}}, \lambda_{\mathrm{o}}; \underline{\mathbf{p}}\right) = \max_{\alpha} \mathcal{L}\left(\frac{n_{\mathrm{a}}}{A_{\mathrm{t}}}\alpha, \frac{n_{\mathrm{a}}}{A_{\mathrm{o}}}(1-\alpha); \underline{\mathbf{p}}\right)$$

# Maximum Likelihood Estimation

Maximum likelihood estimator

$$\hat{R}_\mathrm{t}\left(\underline{\mathbf{p}}\right) = \mathrm{E}_{\hat{\lambda}_\mathrm{t},\hat{\lambda}_\mathrm{o}}\left[R_\mathrm{t}|\underline{\mathbf{P}} = \underline{\mathbf{p}}\right]$$
$$= \sum_{r_\mathrm{t}=0}^{n_\mathrm{a}} r_\mathrm{t} \Pr\left(R_\mathrm{t} = r_\mathrm{t}|\underline{\mathbf{P}} = \underline{\mathbf{p}}; \hat{\lambda}_\mathrm{t}, \hat{\lambda}_\mathrm{o}\right)$$

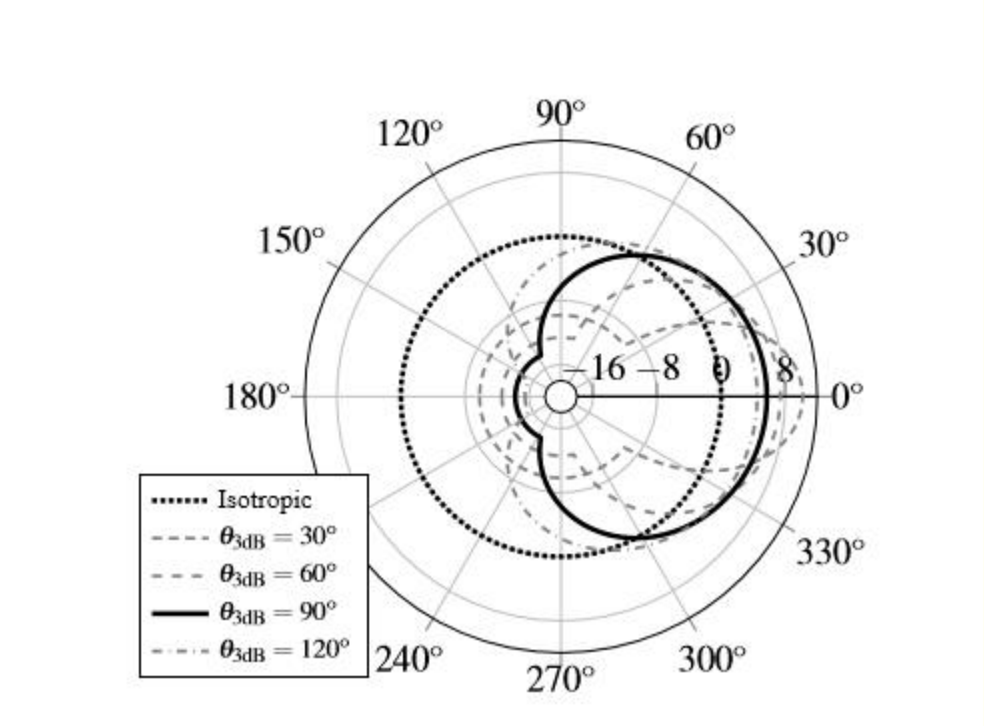
Performance criterion

$$\mathrm{MSE}\left[\hat{R}_\mathrm{t}\right] \approx \frac{1}{M}\sum_{m=1}^{M}\left(\hat{R}_\mathrm{t}^{(m)}\left(\underline{\mathbf{P}}^{(m)}\right) - R_\mathrm{t}^{(m)}\right)^2$$

# Numerical Simulation

3GPP antenna model

$$G_i(\phi_{ij}) = -\min\left\{12\left(\frac{\phi_{ij}-\theta_i}{\theta_{3\mathrm{dB}}}\right)^2, G_{\mathrm{floor}}\right\} - G_{\mathrm{avg}}$$



# Channel Parameters

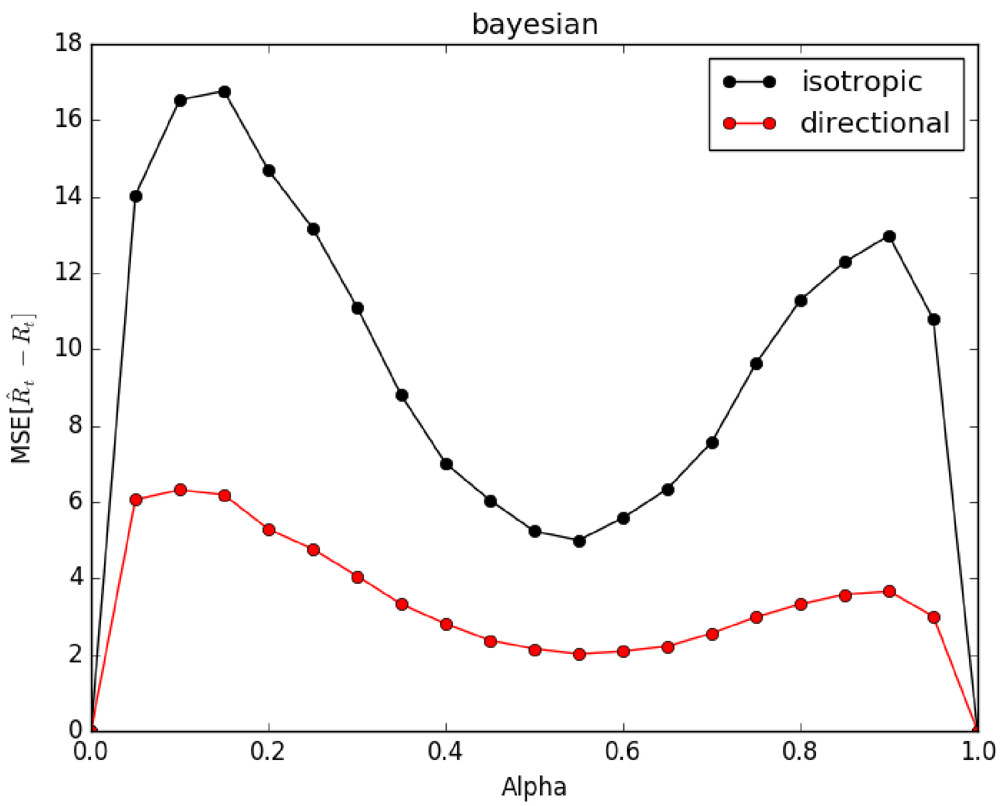
$$P[\mathrm{dBm}] = A + B\log_{10}(d) + L + G(\phi)$$

$$A = P_{\mathrm{t}} + 20\log_{10}\left(\frac{3\times10^8}{f_{\mathrm{carrier}}}\right) - 20\log_{10}(4\pi)$$

$$B = -20dBm$$

$$\sigma_s = 2.0dBm$$

# Simulation Results




Bayes estimator
Intensity $\lambda = 32$
21 points on each curve
50000 trials for each point

# Simulation Results

$$\alpha = 1 - (confidence\ level/100)$$

$$p^* = 1 - \alpha/2$$

$$df = N - 1$$

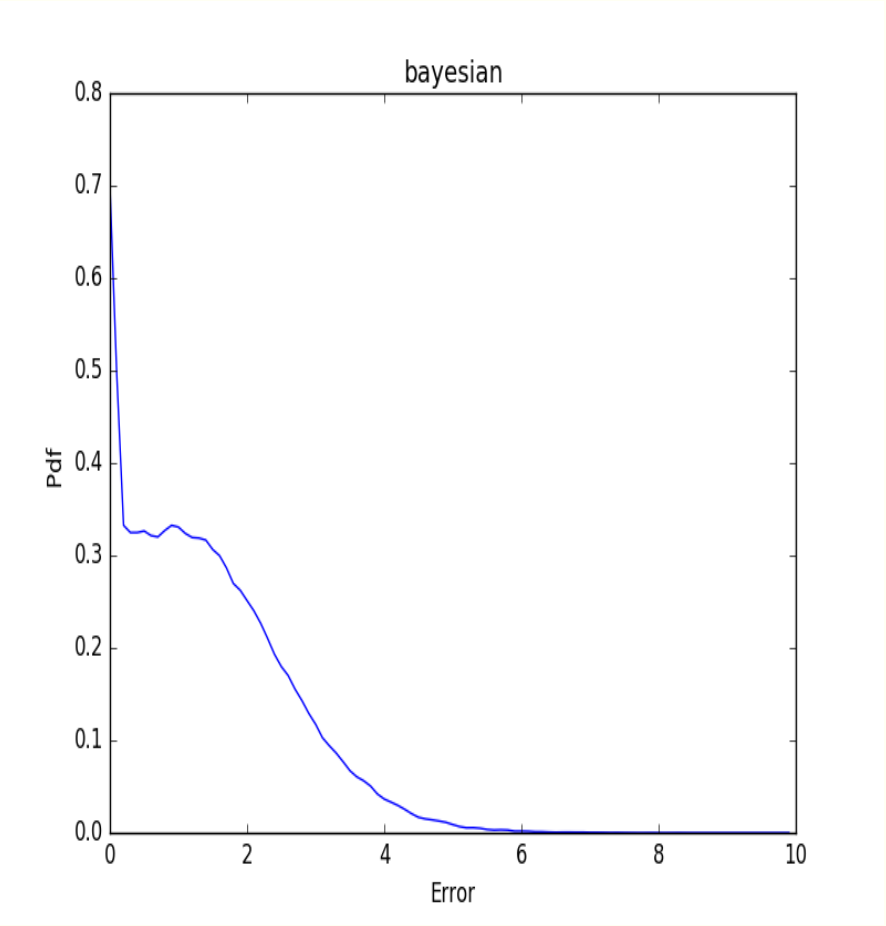
$$SE = \frac{\sigma}{\sqrt{N}}$$

$$Confidence\ interval = \mu \pm Margin\ of\ error$$

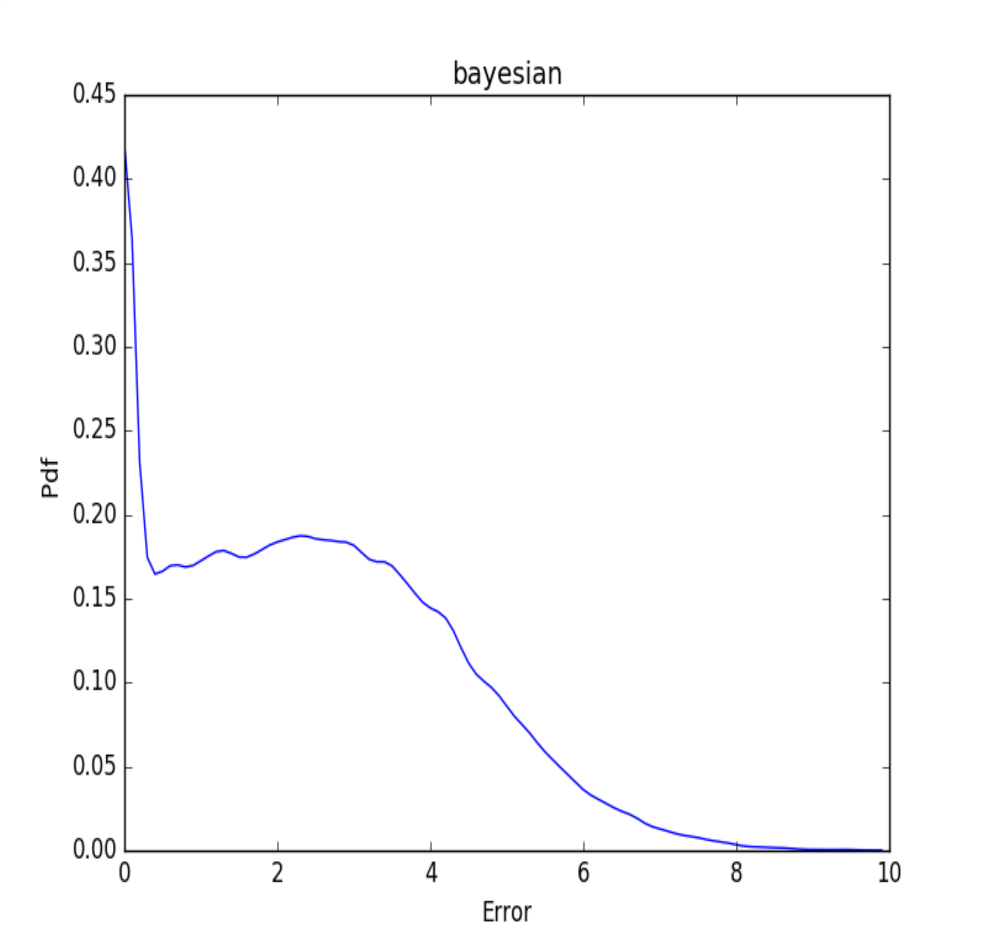
Confidence interval of $|r_t - \hat{r}_t|$

| Antenna type | Confidence interval |
|---|---|
| Directional | $1.412411 \pm 0.002166$ |
| Isotropic | $2.454623 \pm 0.003450$ |

# Simulation Results

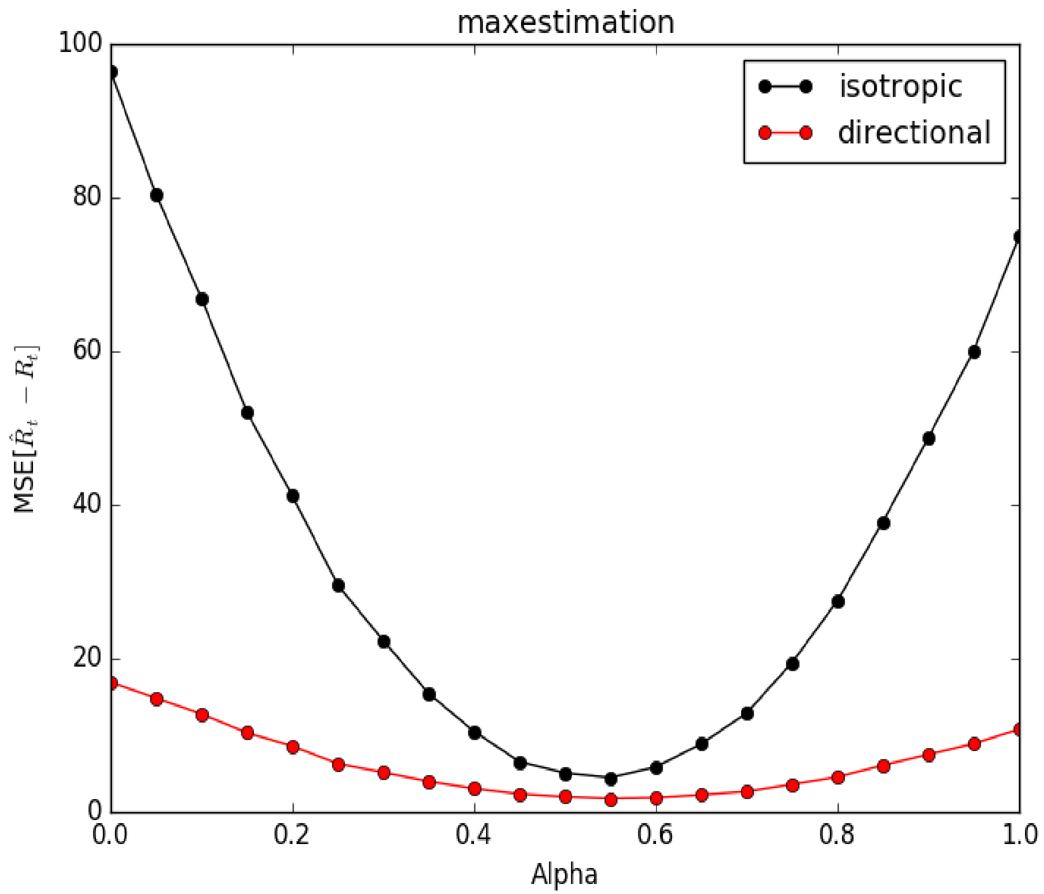



Pdf of $|r_t - \hat{r}_t|$
System with directional antennas




Pdf of $|r_t - \hat{r}_t|$
System with isotropic antennas

# Simulation Results




Maximum likelihood estimator
Intensity $\lambda = 32$
21 points on each curve
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# Simulation Results

$$\alpha = 1 - (confidence\ level/100)$$

$$p^* = 1 - \alpha/2$$

$$df = N - 1$$

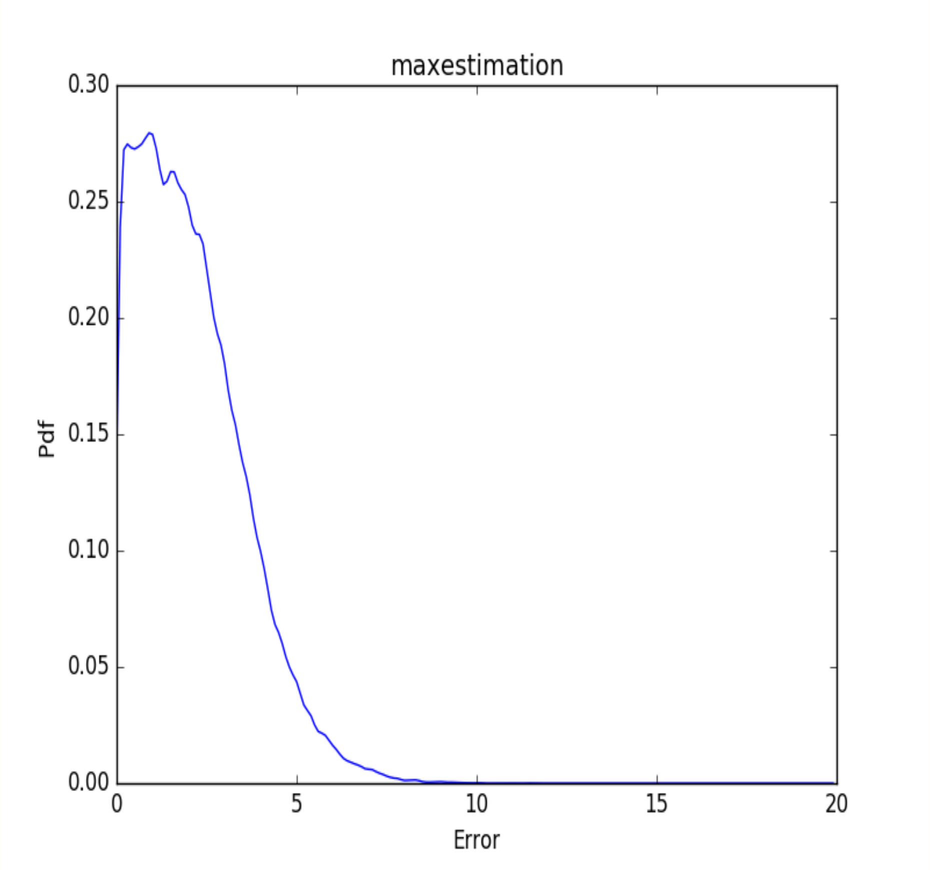
$$SE = \frac{\sigma}{\sqrt{N}}$$

$$Confidence\ interval = \mu \pm Margin\ of\ error$$

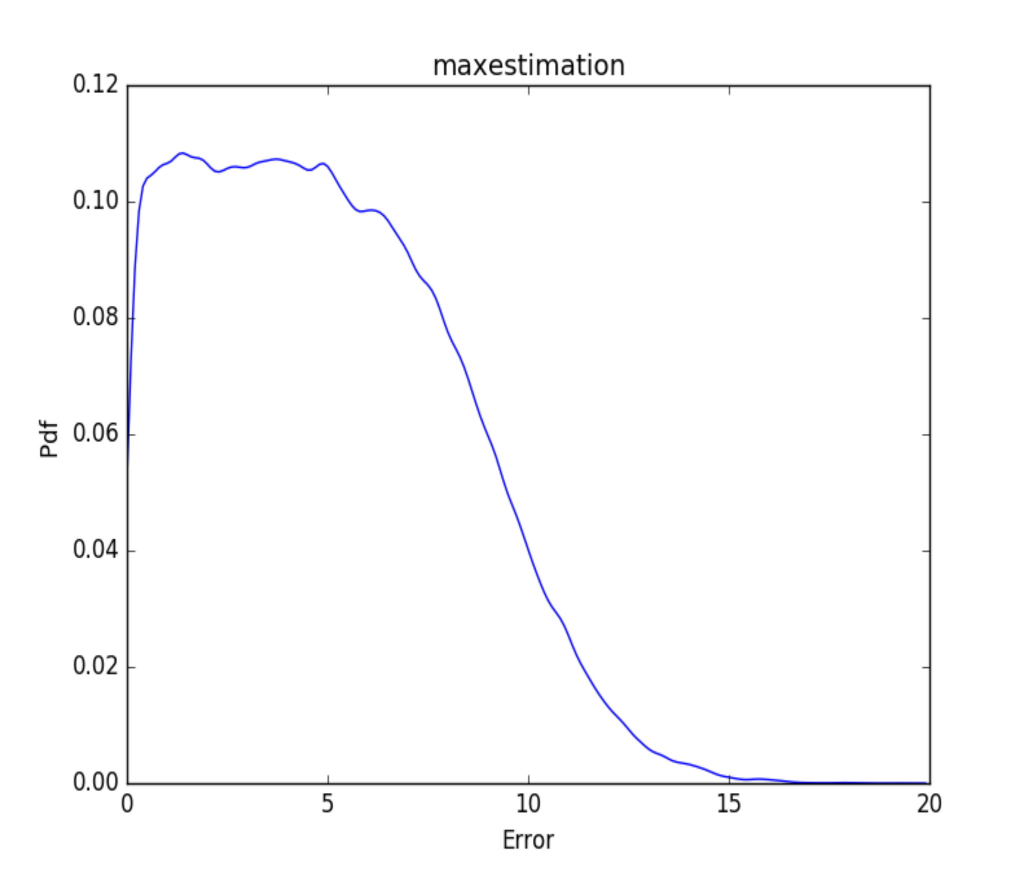
Confidence interval of $|r_t - \hat{r}_t|$

| Antenna type | Confidence interval |
|---|---|
| Directional | $2.080484 \pm 0.002809$ |
| Isotropic | $4.972427 \pm 0.006020$ |

# Simulation Results




Pdf of $|r_t - \hat{r}_t|$
System with directional antennas




Pdf of $|r_t - \hat{r}_t|$
System with isotropic antennas

# Simulation results

In both results, BMSE and MSE of directional antennas systems are smaller.

The error is of system with directional antennas distributed more closely to 0.

System with directional antennas perform better.

# Experimental Implement

Experimental setup consists of three components

- Alpha network card AWUS036NHA
- Local processing units: Intel Next Unit of Computing
- Cloud server

NIC operates in monitor mode to capture Wi-Fi packets

Local processing units parses information: MAC addresses, RSSI values, time stamps

RSSI values are employed as basis for inference

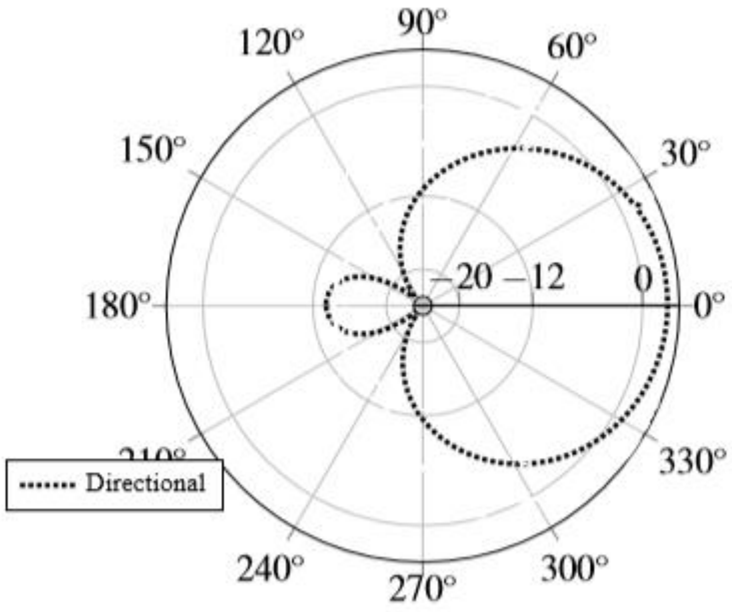



Fig. This graph depicts directional antenna radiation patterns

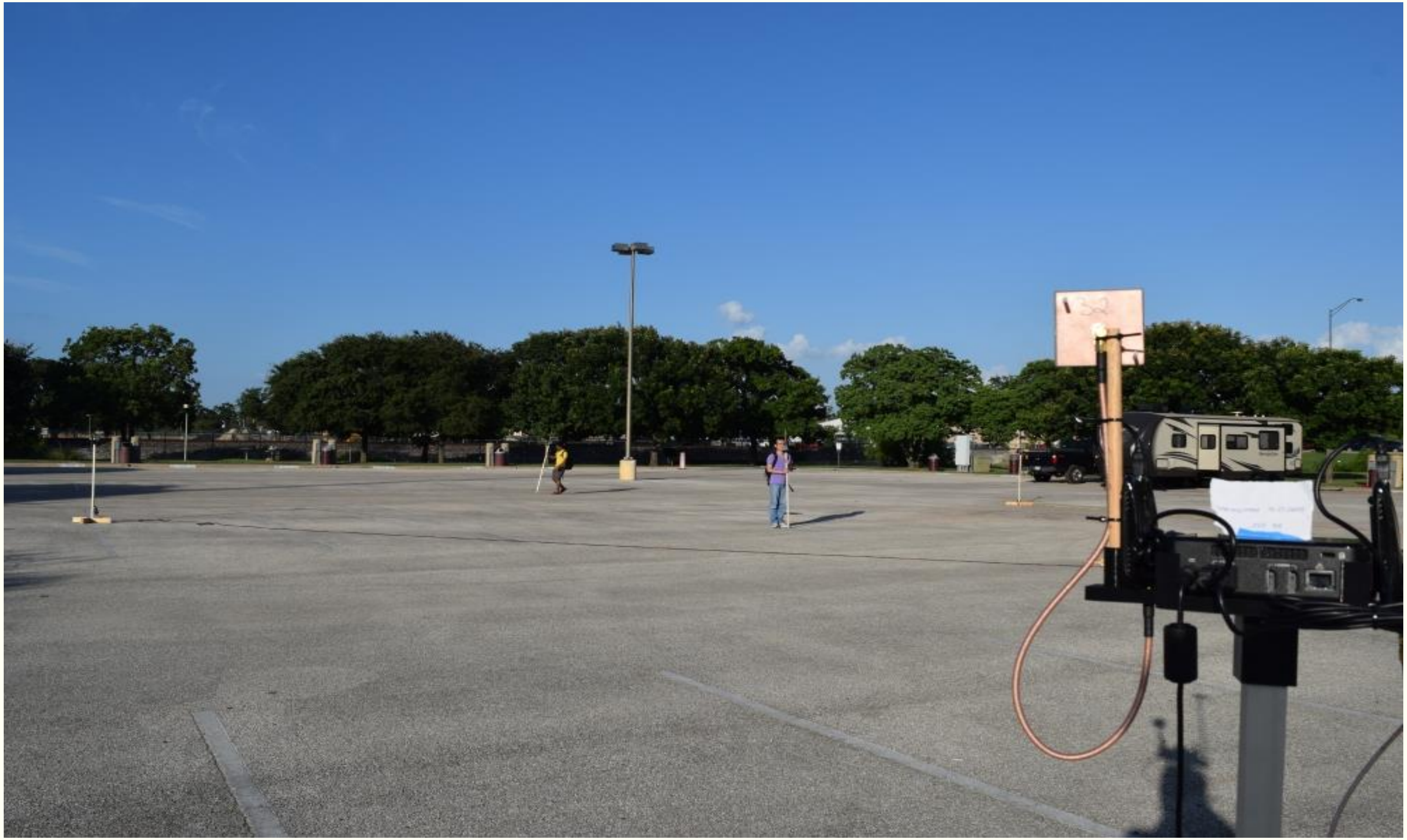
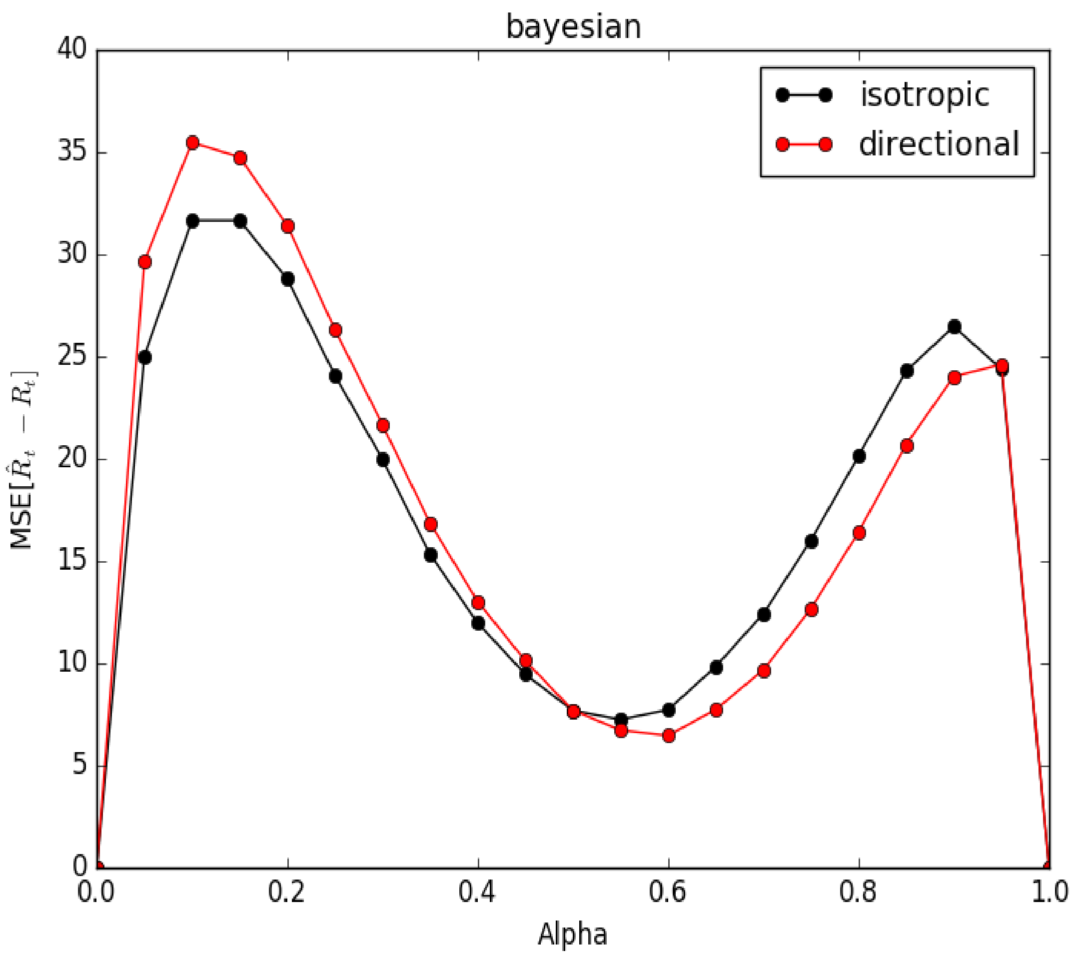


Figure: The place we conduct our experiment

Fig. This figure highlights the site used for the experiments and marks locations of experiment data.

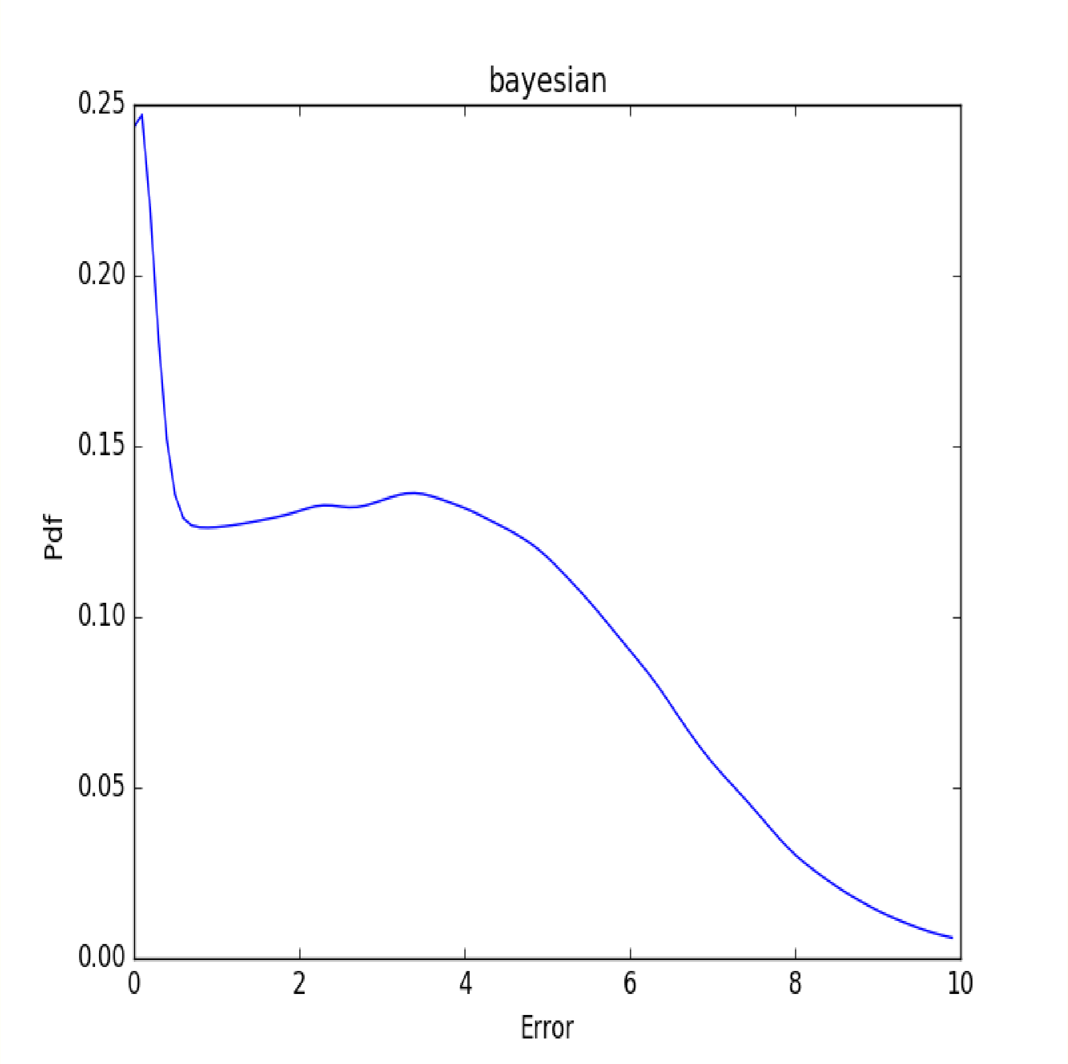
# Experiment Results

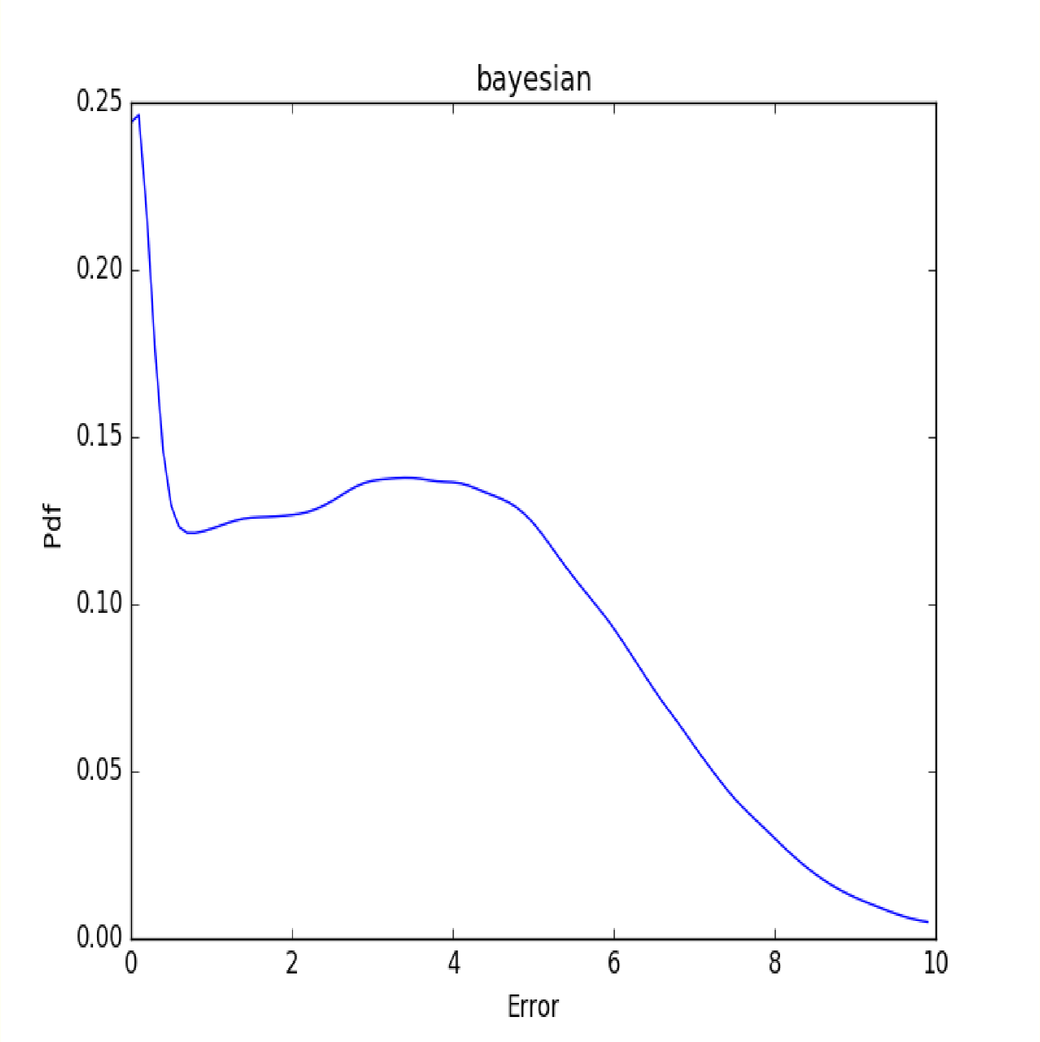



Bayes estimator
Intensity $\lambda = 32$
21 points on each curve
10000 trials for each point

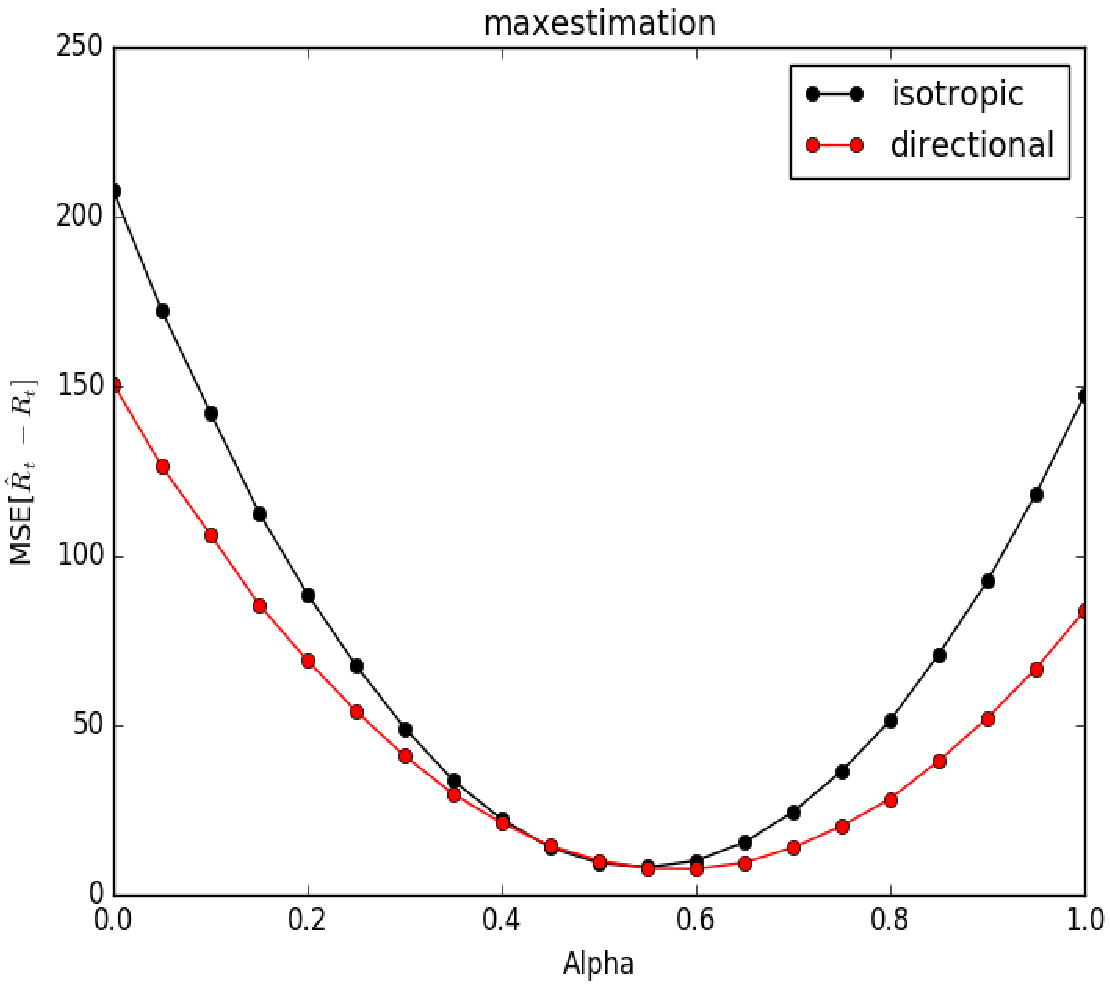
Confidence interval of $|r_t - \hat{r}_t|$

| Antenna type | Confidence interval |
|---|---|
| Directional | $3.317535 \pm 0.010430$ |
| Isotropic | $3.331094 \pm 0.010274$ |


bayesian
Pdf
Error
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10


bayesian
Pdf
Error
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10

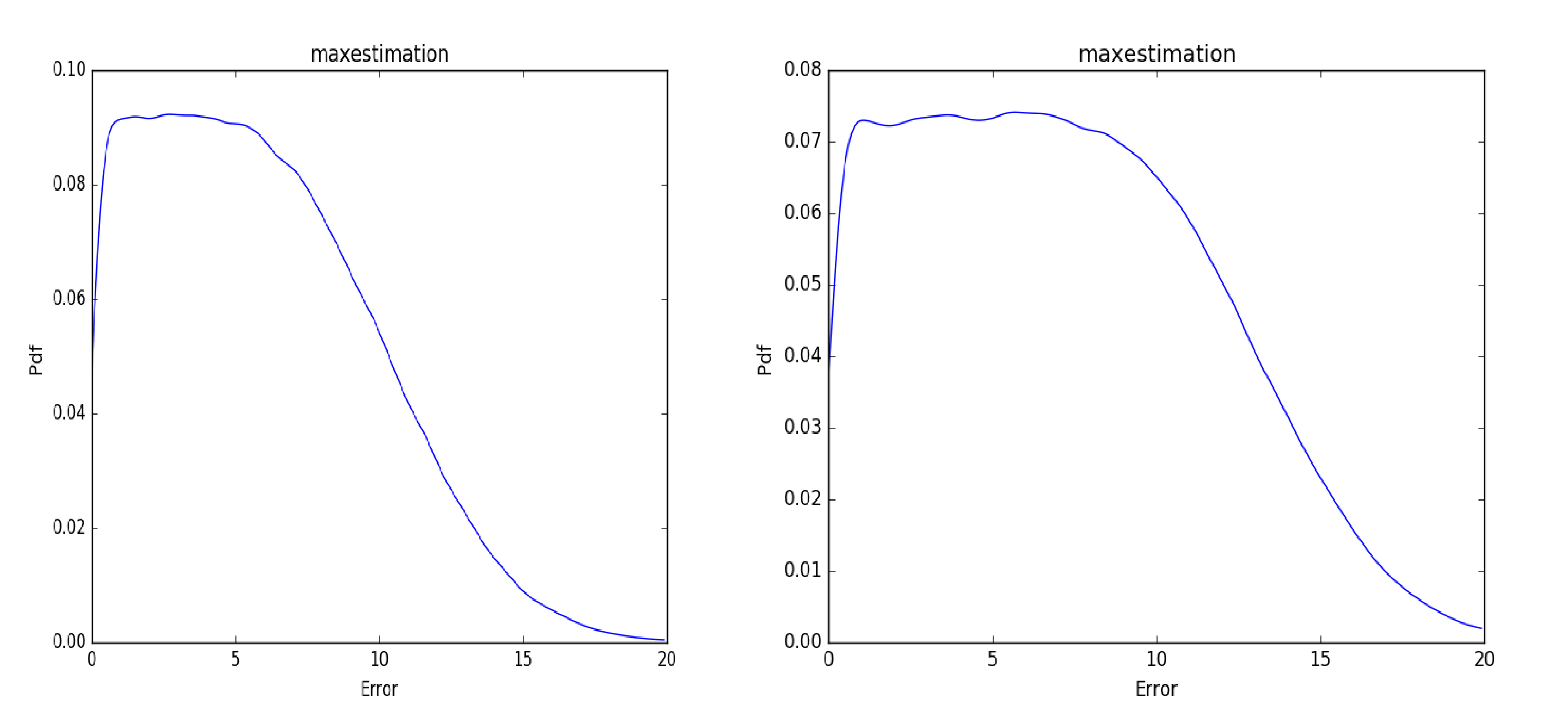
# Experiment result




Maximum likelihood estimator
Intensity $\lambda = 32$
21 points on each curve
10000 trials for each point

Confidence interval of $|r_t - \hat{r}_t|$

| **Antenna type** | **Confidence interval** |
|---|---|
| Directional | $5.881027 \pm 0.016484$ |
| Isotropic | $7.144900 \pm 0.019182$ |


maxestimation
Pdf
Error
0.10
0.08
0.06
0.04
0.02
0.00
0
5
10
15
20
maxestimation
Pdf
Error
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
5
10
15
20

# Conclusion

The directional antenna do improve the performance of estimator accuracy.

This work may extend to be used in track specific device.