# Channel Estimation in Time and Frequency Domain in OFDM Systems

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Abstract—In Orthogonal Frequency Division Multiplexing (OFDM) systems over fading channels, channel estimation is carried out by transmitting pilot symbols in different patterns. The process consists of two steps. First, the estimation of channel at pilot frequencies is done based on LS criteria. Then, this estimation is interpolated using linear interpolation, cubic interpolation, and spline interpolation. In this paper, we propose a two dimensional (2-D) pilot-aided channel estimator for OFDM systems, whose purpose is to reduce bit error rate by using different pilot arrangement in fast fading channels. The major concept behind the proposed scheme is to estimate the channel in time and frequency domain. So we first obtain the channel frequency response of a number of subcarriers via before-mentioned interpolators in time domain. Next, using this estimations and the channel responses, which obtained in pilot frequencies we estimate the channel in frequency domain and achieve the channel in all frequencies. Consequently, using the fewer pilots, the more accurate channel estimation will be obtained. We have compared the performance of our proposed 2-D channel estimator with conventional channel estimators.

Keywords-OFDM; channel estimation; frequency domain interpolation; pilot arrangement; time domain interpolation.

### I. INTRODUCTION

Orthogonal Frequency division multiplexing (OFDM) has attracted a lot of interest in wireless communication studies because of its spectrum efficiency and robustness to multi-path fading channel characteristics. The nature of radio channel is frequency selective and time invariant, therefore, dynamic channel estimation is necessary on OFDM signals [1].

The channel estimation can be performed by different pilot patterns. In block type, pilot tones are inserted in all subcarriers of an OFDM symbols, periodically. This type is suitable for slow-fading channels that channel characteristics are assumed stationary during one OFDM data block. In practice, the characrestics of a radio channel is changing within an OFDM block. Therefore, channel transfer function should be estimated in each OFDM symbol of a data block. The comb type pilot arrangement is generally based on inserting pilot tones in each individual OFDM data block and channel is estimated in all OFDM symbols. The estimation can be done based on Least Square (LS) or Minimum Mean Square Error (MMSE) in both block type and comb type. It has been shown that MMSE estimate provide a significant improvement in signal-to-noise ratio (SNR) for the same

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BER in LS estimate [2]. Also, the complexity of MMSE estimate can be reduced by using a low-rank approximation by singular value decomposition (SVD) [4].

After channel estimation in comb type pilot arrangement at pilot frequencies, interpolation process between pilot tones is carried out to give the channel transfer function at data frequencies. The interpolation for comb type pilot arrangement can be done by linear interpolation, cubic interpolation and spline interpolation. It has been shown that spline interpolation give better performance compared to linear interpolation and cubic interpolation [5].

In this paper, we proposed an additional interpolation in time domain. Then we compare 1-D estimation in frequency domain with 2-D estimation in both time and frequency domain. And it will be shown that 2-D estimation give lower bit error rate in compare to 1-D estimation. In section II, OFDM system description is given. In section III, channel estimation based on Least Square (LS) is discussed. In section IV, the system model and OFDM parameters in simulation is described. Section V, includes the conclusion of the paper.

# II. SYSTEM DESCRIPTION

Fig.1 depicts a typical block diagram of an OFDM system with pilot signal assisted. After pilot insertion, modulated data  $\{X(k)\}$  are grouped to IDFT block. Therefore, the time domain signal x(n) is given as follows:

$$x(n) = IDFT \{X(k)\}$$

$$= \sum_{k=0}^{N-1} X(k) e^{j(2\pi kn/N)}$$

$$n = 0, 1, 2, ..., N-1$$
(1)

Where N is the number of subchannels. After that, the guard band is added to prevent inter-symbol interference. The guard band is the cyclically extended of the OFDM symbol and the output of this block is  $x_{GI}(n)$ . Then, the transmitted signal is sent to the multi-path fading channel and the signal  $y_{GI}(n)$  in receiver can be represented by:

$$y_{GI}(n) = x_{GI}(n) \otimes h(n) + w(n)$$
(2)



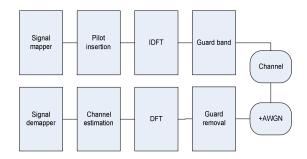


Fig.1. OFDM system block diagram

Where w(n) is Additive White Gaussian Noise (AWGN) and h(n) is the impulse response of channel. In the first step at receiver, the guard band is removed and the resultant signal is y(n). Then, y(n) pass through the DFT block:

$$Y(k) = DFT \{y(n)\}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j(2\pi k n/N)} \qquad k = 0,1,2,...,N-1$$
 (3)

Finally, the received pilot signals are extracted and channel estimation process can be performed.

### III. CHANNEL ESTIMATION BASED ON LS TECHNIQUE

The idea behind least square is to fit a model to measurements in a way that weighted errors between the model and measurements are minimized.

In block type pilot arrangement which pilots are inserted in all subcarriers of an OFDM symbol, LS estimate of channel transfer function can be written as:

$$\mathbf{h}_{ls} = \mathbf{X}^{-1}\mathbf{Y} = \left[\frac{y_0}{x_0} \ \frac{y_1}{x_1} \ \dots \ \frac{y_{N-1}}{x_{N-1}}\right]^T$$
 (4)

For comb type pilot subcarrier arrangement, the  $N_p$  pilot signals  $X_p = (m), m = 0, 1, ..., N_p - 1$  are uniformly inserted into X(k). The OFDM signal modulated on the k th subcarrier can be expressed as:

$$X(k) = X(mGI + l)$$

$$= \begin{cases} x_{p}(m), & l = 0 \\ data, & l = 1, ..., N - 1 \end{cases}$$
(5)

Where  $GI = N / N_p$  .If the received pilot signal vector expressed as [3]:

$$\mathbf{Y}_{p} = \mathbf{X}_{p}.\mathbf{H}_{p} + \mathbf{I}_{p} + \mathbf{W}_{p} \tag{6}$$

Where

$$\mathbf{Y}_{p} = [Y_{p}(0) \ Y_{p}(1) \dots Y_{p}(N_{p}-1)]^{T}$$
(7)

$$\mathbf{X}_{p} = diag[X_{p}(0) X_{p}(1) \dots X_{p}(N_{p}-1)]^{T}$$
(8)

And  $\mathbf{I}_p$  is a vector of ICI and  $\mathbf{W}_p$  is the vector of Gaussian noise in pilot subcarriers. If  $\mathbf{I}_p$  and  $\mathbf{W}_p$  assumed to be eliminated by guard band, the estimate of pilot signals based on least square (LS) criterion in comb type pilot arrangement, is given by:

$$\mathbf{H}_{p,ls} = \mathbf{X}_{p}^{-1} \mathbf{Y}_{p}$$

$$= \left[ \frac{Y_{p}(0)}{X_{p}(0)} \frac{Y_{p}(1)}{X_{p}(1)} \dots \frac{Y_{p}(N_{p}-1)}{X_{p}(N_{p}-1)} \right]^{T}$$
(9)

Because the channel responses of data subcarriers are obtained by interpolating the channel estimation of pilot subcarriers, the performance of OFDM system based on comb type pilot arrangement is highly dependent on the accuracy of the pilot signal estimation. Thus it is better to apply a better estimation on pilot signals.

#### IV. SYSTEM MODEL IN SIMULATION

#### A. system parameters and channel model

OFDM system parameters that used in the simulation are given in Table I [1]. Also, we use multipath fading channel with delays and gains as follows [1]:

$$h(n) = \alpha(n) + 0.3162\alpha(n-2) + 0.1995\alpha(n-17) + 0.1296\alpha(n-36) + 0.1\alpha(n-75) + 0.1\alpha(n-137)$$
(10)

In both block type and comb type pilot arrangement, transfer function of channel vary in each block according to the following autoregressive (AR) model:

$$h(n+1) = ah(n) + w(n)$$
 (11)

Where w(n) is AWGN noise vector and a is the fading factor which changes from 0.9 to 1.

# B. Block type and comb type channel estimation

We have sent a fix number of OFDM symbols in each block and pilot tones in all subcarriers of the first symbol of the block. At the first symbol of each block channel changes according to AR model and we apply this channel to the entire symbols of the block. Channel responses of all frequencies in the first OFDM symbol are estimated by using pilot signals that are known for receiver. Then, this estimation is used for entire symbols of a block.

In comb type channel estimation, we have estimated channel transfer function at pilot frequencies by using LS technique in each symbol of a block. Then, different interpolation techniques are used to estimate channel transfer function at data frequencies.

TABLE I. SIMULATION PARAMETERS

Parameters	Specifications
FFT size	1024
Number of active carriers(N)	128
Pilot ratio	1/4,1/8,1/16
Guard interval	256
Guard type	Cyclic extension
Sample rate	44.1kHz
Bandwidth	17.5kHz
Signal constellation	BPSK
Channel model	Rayleigh fading,AR model

# C. Comb type channel estimation with variable pilot position

The pilot pattern is different from previous and pilot position in each symbol is shifted from previous symbol. For the four first symbol of the block, interpolation technique is performed normally. But, for the entire symbol of the block we can use the pilot estimation from previous symbols and lessen the interpolation length. Because of the slow-fading channel model, we can transfer channel responses from three prevoius symbols to the current symbol. Then, we can apply interpolation techniques to the new pilot pattern. In fact, we increase the accuracy of the computation by using the effect of channel response at pilot frequencies from the previous symbols.

# D. Comb type channel estimation with variable pilot position and 2-D estimation

In this section OFDM blocks pass through fast fading channel. In such a fast fading channel, channel transfer function in each symbol of the block changes according to AR model. Thus, channel transfer function changes in both frequency and time domain. To have a better performance in fast fading channel, it is necessary to estimate channel in both frequency and time domain. In this paper we propose an algorithm to perform channel estimation in frequency and time domain.

Because of the higher delay of the pilot ratio of 1/8, we have used pilot ratio of 1/6 instead. In the first two symbol and the last two symbol of each block, we just interpolate in frequency domain. But in the third symbol, we wait to receive two following symbol. Then, channel responses in pilot frequencies in these symbols are estimated. Thus, we have estimated channel responses in first five OFDM symbol of the block. Afterthat, we interpolate between pilot responses in first and fourth symbols, and also between second and fifth symbols, in time domain. Fig. 2. illustrates the estimation of pilot responses in time domain. So, we have obtained the effect of previous and next pilot responses in the current (third symbol) symbol.

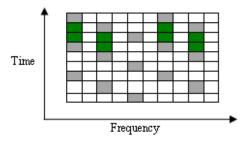


Fig.2. Pilot pattern in comb type channel estimation with variable pilot position and 2-D estimation



Fig.3. The resultant pilot pattern in comb type channel estimation with variable pilot position and 2-D estimation

Fig. 3. depictes the resultant pilot pattern. Now, we can interpolate in the third symbol in frequency domain. By using time interpolation, we apply the effect of previous and next pilot symbols to the current symbol instead of transfering the same previous pilot symbols to the current symbol. Then, we perform the algorithm for the entire OFDM symbols of the block.

This algorithm is helpful in fast fading channels that channel transfer function changes through OFDM symbols. Thus, time interpolation can follow channel transfer function changes between OFDM symbols.

# F. Simulation results

We performed channel estimation for all of the pilot patterns by using the Least Square (LS) technique. The parameters of OFDM system are as Table I. and Doppler frequency is 70 Hz. The simulation is done for BPSK modulation schemes. The legends "linear, cubic, spline" denotes the interpolation techniques of linear interpolation, cubic interpolation and spline interpolation.

Fig. 4 shows the BER performance of slow fading channel. These figure compares performance of block type pilot arrangement with comb type pilot arrangement by using different interpolation techniques and pilot ratio of 1/8. The result shows that the comb-type estimation BER is 10-15 dB lower than block type estimation. This is because block type estimation cannot follow channel transfer function changes between OFDM symbols of a block. Also, different interpolation schemes perform from the best to worst as follows: spline interpolation, cubic interpolation and linear interpolation.

Fig. 5 gives the performance of comb type pilot arrangement of pilot ratio of 1/8 for constant and variable pilot positions. Frequency domain interpolation is just used in this simulation. The figure shows better performance for variable pilot arrangement.

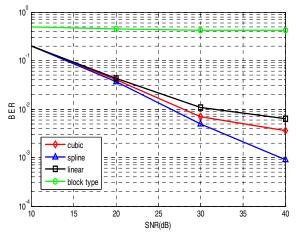


Fig. 4. BPSK modulation with slow fading channel

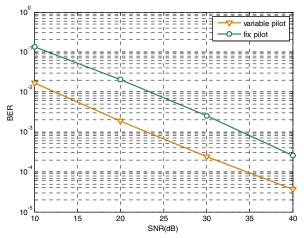


Fig.5. Variable and constant pilot position for pilot ratio 1/8

Fig . 6 give the performance of 2-D channel estimation for BPSK modulation scheme. In these figure 1-D channel estimation is compared to 2-D channel estimation for the pilot ratio of 1/6. Fast fading channel is applied to the transmitted data. The general behaviour of the plot is decreasing in BER by using 2-D channel estimation. The reason is that time domain interpolation is added to system which can track the changes in channel impulse response in time domain.

## V. CONCLUSION

In this paper, a full review of different pilot arrangement base on block type and comb type is given. Channel estimation based on Least Square technique is described. Channel estimation based on comb type pilot pattern is presented for constant pilot position and variable pilot position in each symbol of a block. The simulation results show that comb type pilot arrangement with variable pilot

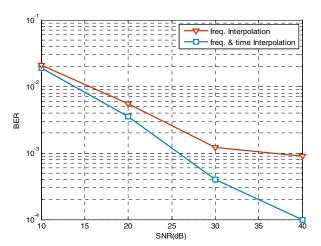


Fig.6. BPSK modulation with 2-D estimation in fast fading channel

positions performs better than comb type pilot arrangement with constant pilot positions. This is expected because we use the previous pilot estimation in current symbol. In addition, we proposed 2-D estimation which adds time domain interpolation to the system. We achieve better performance by using 2-D interpolation since we use the effect of previous and next pilot symbols in current symbol that allows tracking of fast fading channel. Also, comb type pilot arrangement with spline interpolation gives better performance among all channel interpolation techniques.

## ACKNOWLEDGMENT

This work was supported by Iran Telecommunication Research Center (ITRC).

### REFERENCES

- S. Coleri, M. Ergen, A. Puri, A. Bahai, "Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems", IEEE Trans. Broadcasting, VOL. 48, NO.3, September 2002, pp. 223-229.
- [2] J. J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P.O. Borjesson," On Channel Estimation in OFDM Systems," in Proc. IEEE 45th Vehicular Technology Conf., Chicago, IL, Jul. 1995,pp. 815-819.
- [3] K. Arshad, "Channel Estimation in OFDM Systems." Thesis in King Fahd Uni. Of Petroluem and Mineral. June 2003.
- [4] O. Edfors, M. Sandell, J. J. van de Beek, S. K. Wilson, and P. O. Brjesson, "OFDM Channel Estimation by Singular Value Decomposition," IEEE Trans. Commun., vol. 46,no. 7, pp. 931-939, Jul. 1998.M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.
- [5] M. Hsieh and C.Wei, "Channel Estimation for OFDM Systems Based on Comb-type Pilot Arrengement in Frequency Selective Fading Channels," IEEE Trans. Consumer Electron., vol. 44, no. 1, Feb. 1998.
- [6] L.Hanzo and M.Munster , "OFDM and MC-CDMA for Broadband Multi-User Communications", John Wiley & sons, 2003.