MCG5138 Fall 2020: Assignment 3

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Due Date: *Tuesday Dec. 1st*

Total Marks: 100

1. (20 marks) Consider the following matrix and column vector used in Assignment 1

$$A = \begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -2 & 2 \\ -1 & 2 & 3 & -1 \end{pmatrix}, \; X = \begin{pmatrix} 4 \\ 1 \\ -3 \\ 4 \end{pmatrix}. \tag{1}$$

   Assume we have a system with two CPUs, P1 and P2.

   (a) Perform explicitly the steps (e.g., each loop, each communication step showing which elements are sent and received) taken by each processor to generate the LU decomposition of A. Assume the matrix is stored in wrapped interleaved row format (show your work).

   (b) Perform explicitly the multiplication AX assuming A is stored in a block column-based format and X is stored accordingly (show your work).

2. (80 marks) (Computational Submission)

   Consider the $n \times n$ linear system

$$\begin{pmatrix} n & 1 & 1 & \cdots & 1 \\ 1 & n & 1 & \cdots & 1 \\ 1 & 1 & n & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{2}$$

   (a) Implement a parallel distributed algorithm to perform the LU decomposition and Forward/Backward solve using the storage format of your choice. Assume that the matrix size is a multiple of the number of CPUs used for execution.

   (b) Compute the solution to this system using your implementation for $n = 10, 100, 200, 500, 1000$. Determine the amount of wall-clock time required for each value n when using the following numbers of CPUs: 1, 2, and 4 (preferably you would try the problem on even more CPUs).

   (c) Select two problem sizes (e.g., 100 and 1000) and show how your code scales (i.e., strong scaling study) in each case. Plot the speed-up of the code vs. the ideal one, and the parallel efficiency.

   (d) Select a suitable problem size for a weak scalability study and show how your code performs.

Assignment Notes (in no particular order)

- The assignment must be done individually. You can modify existing algorithms and/or use algorithms that you find from reputable sources (which include books, but not your peers). When someone else's code is used like this, you must acknowledge the source explicitly in your written report.
- Submit the code to the dropbox on the course website.
- Ask questions if you encounter issues with solving the problems. Use also the Discussion forum on the course website.
- Students interested in more algorithm development should explore the use of OpenMP and vectorization in the implementation.
- If you have access to a computing cluster, I recommend experimenting with a larger number of CPUs than 4 (e.g., 20, 40, 80 etc.). You should also test your code when the computing cores coexist or not on the same node (i.e., distribute the simulation across multiple nodes by requesting a certain number of cores per node).