

1. The value of bit 20 is $2^{20} = 1,048,576$ Kr. To receive more than 1 million next month, one of solutions is he can let last bit be 1.

plain text

| | | | | | | | | | | |
|---|---|---|---|-----|-----|---|---|---|---|---|
| 0 | 0 | 1 | 1 | ... | ... | 0 | 1 | 0 | 0 | 1 |
|---|---|---|---|-----|-----|---|---|---|---|---|

key

| | | | | | | | | | | |
|---|---|---|---|-----|-----|---|---|---|---|---|
| 1 | 1 | 1 | 0 | ... | ... | 1 | 1 | 0 | 1 | 1 |
|---|---|---|---|-----|-----|---|---|---|---|---|

cipher text

| | | | | | | | | | | |
|---|---|---|---|-----|-----|---|---|---|---|---|
| 1 | 1 | 0 | 1 | ... | ... | 1 | 0 | 0 | 1 | 0 |
|---|---|---|---|-----|-----|---|---|---|---|---|

To let last five bits in decrypted message be 1, he can calculate in this way.

He knows that his own salary is less than 1 million, so the last bit of plaintext must be 0. No matter what the last bit is in ciphertext, he can just flip this bit. The procedure is shown below.

1. $\text{plaintext}[20] \text{ xor } k[20] = \text{ciphertext}[20]$ (encryption)
2. $\text{changed_ciphertext}[20] \text{ xor } k[20] = \text{changed_plaintext}[20]$ (decryption)
3. we know $\text{plaintext}[20] = 0$, which means $0 \text{ xor } k[20] = \text{ciphertext}[20]$, and to let $\text{changed_plaintext}[20]$ be 1, is to let $\text{changed_ciphertext}[20] \text{ xor } k[20] = 1$
4. so we want to let $k[20] = - \text{changed_ciphertext}$ and given that $k[20] = \text{ciphertext}[20]$
5. we need let $\text{ciphertext}[20] = - \text{changed_ciphertext}[20]$, so just flip last bit in ciphertext.

2. Both. He knows his salary range, which is some information towards plaintext($\text{plaintext}[20] = 0$), and he can intercept the traffic. So, he can actually derive the $\text{key}[20]$, which is not mean to be known by others. so confidentiality has been broken. He can modify bits on traffic and the receiver cannot detect this, which means authenticity has been broken.

3. Because he knows his own salary(or at least the range of his own salary), which means he knows (part of) plaintext.

4. Proof:

1. $\text{plaintext}[i] \text{ xor } k[i] = \text{ciphertext}[i]$ (encryption)
2. $\text{changed_ciphertext}[i] \text{ xor } k[i] = \text{changed_plaintext}[i]$ (decryption)
3. we want to let changed plaintext be 0, which lead to $\text{changed_ciphertext} \text{ xor } k[i] = 0$
4. so $k[i] = \text{changed_ciphertext}[i]$,
5. so $\text{plaintext}[i] \text{ xor } \text{changed_ciphertext}[i] = \text{ciphertext}[i]$
6. $\text{plaintext}[i] = \text{changed_ciphertext}[i] \text{ xor } \text{ciphertext}[i]$

if $p > 1-p$, we prefer to believe that $\text{plaintext}[i] = 1$, in this way, attacker can flip the $\text{ciphertext}[i]$. and success probability is p .

if $p < 1-p$, we believe that $\text{plaintext}[i] = 0$, attack can keep $\text{ciphertext}[i]$ the same. And success probability is $1-p$.

So, the adversary can make the receiver obtain a 0-bit in position i with probability $\max(p, 1 - p)$.