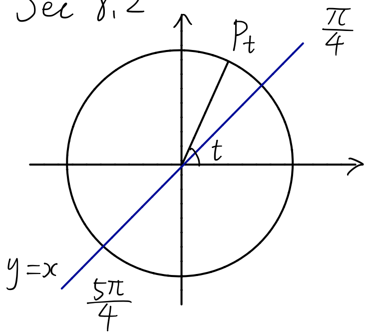
Review:

Sec 8.1 Conversion between radians and degrees

Examples: $\frac{7\pi}{12}$ rad $= \frac{7\pi}{12} \times \frac{180}{\pi} = 105^\circ$

$2^\circ = 2 \times \frac{\pi}{180} = \frac{\pi}{90}$ rad

Sec 8.2



$P_t = (\cos t, \sin t)$, $\tan t = \frac{\sin t}{\cos t}$ = slope of P_t

If $\sin t = \cos t$, what is t?

P_t will be on the intersection of the unit circle and the line $y=x$

thus $t = \frac{\pi}{4} + k2\pi$ or $\frac{5\pi}{4} + k2\pi$ (or rather $\frac{\pi}{4} + k\pi$), $k \in \mathbb{Z}$

Sec 8.3

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\qquad$ $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(-x) = -\sin x$ $\qquad$ $\cos(-x) = -\cos x$ $\qquad$ $\tan(-x) = -\tan x$

Example: $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right) = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$$\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

and $\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$$

Sec 8.4 $\sec t = \frac{1}{\cos t}$, $(\tan t)' = \sec^2 t$, $\cos^2 t + \sin^2 t = 1$, $1 + \tan^2 t = \sec^2 t$

Chapter 9

Foremost: $(x^n)' = nx^{n-1}$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$, $(\sin x)' = \cos x$

$(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$

Note $\left(\frac{1}{x^n}\right)' = (x^{-n})' = -n x^{-n-1}$ and

$\left(\sqrt[n]{x^m}\right)' = \left(x^{\frac{m}{n}}\right)' = \frac{m}{n} x^{\frac{m}{n}-1}$

$(te^t)' = t'e^t + t(e^t)' = e^t + te^t$

$(\sin(e^{t^2}))' \xlongequal{u=e^{t^2}} (\sin u)' = \cos u \cdot u' = \cos(e^{t^2})(e^{t^2})' \xlongequal{v=t^2} \cos(e^{t^2})(e^v)'$

$= \cos(e^{t^2}) e^v v' = \cos(e^{t^2}) e^{t^2}(t^2)' = \cos(e^{t^2}) e^{t^2} 2t = 2te^{t^2}\cos(e^{t^2})$

$\int \frac{\ln(2x)}{x}dx = \int \frac{\ln(2x)}{2x} d(2x) \xlongequal{u=2x} \int \frac{\ln u}{u} du = \int \ln u \frac{du}{u} = \int \ln u \, d\ln u$

$\xlongequal{v=\ln u} \int v dv = \frac12 v^2 + C = \frac12 \ln^2 u + C = \frac12\ln^2(2x)+C$

$\int_0^1 2x^3 e^{x^2}dx = \int_0^1 x^2 e^{x^2}\cdot 2x dx = \int_0^1 x^2 e^{x^2} dx^2 \xlongequal{u=x^2} \int_{0^2}^{1^2} ue^u du$

$= \int_0^1 ue^u du = \int_0^1 u de^u = ue^u\Big|_0^1 - \int_0^1 e^u du = [1\cdot e^1 - 0] - e^u\Big|_0^1$

$= e - (e^1 - e^0) = e - (e-1) = 1$

$\int te^{t^2}\cos(e^{t^2})dt = \frac12\int e^{t^2}\cos(e^{t^2})\cdot 2t dt = \frac12 \int e^{t^2}\cos(e^{t^2}) dt^2 \xlongequal{v=t^2}$

$\frac12\int e^v\cos(e^v)dv = \frac12\int\cos(e^v)\cdot e^v dv = \frac12\int \cos(e^v) de^v \xlongequal{u=e^v} \frac12 \int \cos u \, du$

$=\frac12 \sin u + C = \frac12\sin(e^v)+C = \frac12 \sin(e^{t^2})+C$

Some other examples:

Given $\tan t = \frac12$, what is $\frac{1}{\sin^2 t}$?

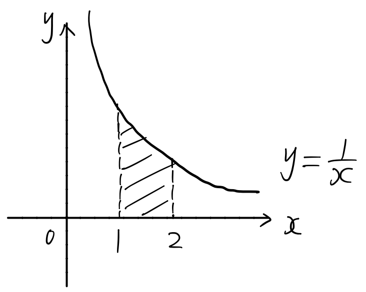
$\frac{\sin t}{\cos t} = \tan t \Rightarrow \sin t = \tan t\cos t = \frac12\cos t \Rightarrow \sin^2 t = (\frac12\cos t)^2 = \frac14\cos^2 t$

also notice $\frac{1}{\cos^2 t} = \sec^2 t = 1+\tan^2 t = 1+(\frac12)^2 = \frac54 \Rightarrow \cos^2 t = \frac45$

thus $\sin^2 t = \frac15$, $\frac{1}{\sin^2 t} = \frac15$ Or $\frac{1}{\sin^2 t} = \frac{\sin^2 t + \cos^2 t}{\sin^2 t} = 1 + \frac{\cos^2 t}{\sin^2 t}$

$= 1 + \frac{1}{\frac{\sin^2 t}{\cos^2 t}} = 1 + \frac{1}{\tan^2 t} = 1 + \frac{1}{(\frac12)^2} = 1 + \frac{1}{\frac14} = 1 + 4 = 5$

Computes the shaded area



By the definition of definite integral, we have

Area $= \int_1^2 \frac1x dx = \ln x\Big|_1^2 = \ln 2 - \ln 1 = \ln 2$