Definition 0.0.1. $\{A_i\} \subseteq \mathcal{P}(X)$, $X \xrightarrow{f} Y$ is a map. f **separates** A_i if $\bigcap_i f(A_i) = \varnothing$. f **completely separates** A_i if $f(A_i) = f(a_i)$ for some distinct $a_i \in A_i$. f **perfectly separates** A, B if $A_i = f^{-1}(a_i)$ for some $a_i \in A_i$

Zorn's lemma

Lemma 0.0.2 (Zorn's lemma)**.** P is a nonempty poset and every chain has an upper bound, then P contains a maximal element

Theorem 0.0.3 (Schröder–Bernstein theorem)**.** $A \xrightarrow{f} B$ and $B \xrightarrow{g} A$ are injective, then there exists $A \xrightarrow{h} B$ bijective

Inclusion-exclusion principle

Theorem 0.0.4 (Inclusion-exclusion principle)**.** $A_1, \cdots, A_n \subseteq S$ are of finite cardinality, then

$$\left|\bigcup_{i=1}^{n} A_n\right| = \sum_{k=1}^{n}(-1)^k \sum_{i_1<\cdots<i_k} |A_{i_1} \cap \cdots \cap A_{i_k}|$$

Definition 0.0.5. A **lattice** is a partially ordered set in which the supremum and infinimum of any two elements exists uniquely

Lemma 0.0.6. Trees are bipartite

Proof. Take some $v \in T$ as the root, and label the nodes that are even distance away from 2 and odd distance away from 1 $\square$

Definition 0.0.7. A **Hasse diagram** is a mathematical diagram used to represent a partially ordered set

$$\begin{tikzcd}
 & \{x,y,z\} & & 2 \\
\{x,y\} \arrow[ur] & \{x,z\} \arrow[u] & \{y,z\} \arrow[ul] & 1 \\
\{x\} \arrow[u] \arrow[ur] & \{y\} \arrow[ul] \arrow[ur] & \{z\} \arrow[u] \arrow[ul] & 0 \\
 & \{\} \arrow[ul] \arrow[u] \arrow[ur] & & -1
\end{tikzcd}$$

Definition 0.0.8. The *Bermoulli numbers* B_n are defined by

$$\frac{t}{e^t - 1} = \sum_{k=0}^{\infty} \frac{B_k t^k}{k!}$$

For instance, $B_0 = 1$, $B_1 = -\frac{1}{2}$, $B_2 = \frac{1}{6}$, $B_3 = 0$, $B_4 = -\frac{1}{30}$, etc.