Problem: $f(x) = 2x^2 + 3x^3 + 5x^5 + 7x^7 + 11x^{11} + 13x^{13} + 17x^{17} + 19x^{19}$

what is $f^{(19)}(0)$

Solution: $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(19)}(0)}{19!}x^{19} + \cdots$

$= 2x^2 + 3x^3 + 5x^5 + 7x^7 + 11x^{11} + 13x^{13} + 17x^{17} + 19x^{19}$

compare the coefficients, we get $19 = \frac{f^{(19)}(0)}{19!} \Rightarrow f^{(19)}(0) = 19 \cdot 19!$

how about $f^{(18)}(0)$?

by comparing the coefficients, we have $0 = \frac{f^{(18)}(0)}{18!} \Rightarrow f^{(18)}(0) = 0$

Section 12.1

November 11, 2019

Motivating example: Throwing dice

The outcome X is a **random variable**, X takes value from 1 to 6, with probability $\dfrac{1}{6}$ for each outcome, and each time you throw the dice is called an **experiment**, and the result of each experiment is called an **outcome**, the following table of all possible outcomes with corresponding probility is called a **probability table**

outcome	1	2	3	4	5	6
probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

if you throw the dice, what will be your expected value, which motivate the definition of **E(X):=expected value(or average or mean)**: $\dfrac{1}{6}\cdot 1+\dfrac{1}{6}\cdot 2+\dfrac{1}{6}\cdot 3+\dfrac{1}{6}\cdot 4+\dfrac{1}{6}\cdot 5+\dfrac{1}{6}\cdot 6=3.5$ meaning for each $\dfrac{1}{6}$ chance, the outcome maybe $1,\cdots,6$, so this is like a weighted sum, notice the expected value may not be an possible outcome

how to measure the variance, how severely the result is deviated away from the average**V(X):=Variance** is defined as follows, subtract the mean from each possible outcome, and then square, and then take weighted sum, namely, $V(X)=\dfrac{1}{6}(1-3.5)^2+\dfrac{1}{6}(2-3.5)^2+\dfrac{1}{6}(3-3.5)^2+\dfrac{1}{6}(4-3.5)^2+\dfrac{1}{6}(5-3.5)^2+\dfrac{1}{6}(6-3.5)^2=\dfrac{8.75}{3}$, why squared?(to avoid cancellation), for example $V(X)=\dfrac{1}{6}(1-3.5)+\dfrac{1}{6}(2-3.5)+\dfrac{1}{6}(3-3.5)+\dfrac{1}{6}(4-3.5)+\dfrac{1}{6}(5-3.5)+\dfrac{1}{6}(6-3.5)=0$, since we have squared, let's take a square root, the **standard deviation** $\sigma(X):=\sqrt{V(X)}=\sqrt{\dfrac{8.75}{3}}$

In general, let X be a random variable, suppose its possible outcomes are $a_1,\cdots,a_n$, with probability $p_1,\cdots,p_n$, the probability table look like

X	a_1	a_2	a_3	$\cdots$	a_n
P	p_1	p_2	p_3	$\cdots$	p_n

Note that we should certainly have $0\le p_i\le 1$ and $p_1+\cdots+p_n=1$

the expected value is $E(X):=a_1p_1+\cdots+a_np_n$, sometimes people denotes $\bar{a}=E(X)$, then variance $V(X):=(a_1-\bar{a})^2p_1+\cdots+(a_n-\bar{a})^2p_n$, and the standard deviation $\sigma(X):=\sqrt{V(X)}$

Problems:

1: The number of accidents per week at a busy intersection was recorded for a year. There were 5 weeks with no accidents, 30 weeks with one accident, 15 weeks with two accidents, and 2 weeks with three accidents. A week is to be selected at random and the number of accidents noted. Let X be the outcome. Then X is a random variable taking on the values 0, 1, 2, and 3

(a) Write out a probability table for X

(b) Compute $E(X)$

(c) Interpret $E(X)$

(d) How about $V(X),\sigma(X)$

All possible outcomes of X are $0, 1, 2, 3$ with probabilities $\frac{5}{52}, \frac{30}{52}, \frac{15}{52}, \frac{2}{52}$

(a) probability table for X is

X	0	1	2	3
P	$\frac{5}{52}$	$\frac{30}{52}$	$\frac{15}{52}$	$\frac{2}{52}$

(b) $E(X) = 0 \times \frac{5}{52} + 1 \times \frac{30}{52} + 2 \times \frac{15}{52} + 3 \times \frac{2}{52} = \frac{66}{52} = \frac{33}{26}$

(d) $V(X) = (0 - \frac{33}{26})^2 \times \frac{5}{52} + (1 - \frac{33}{26})^2 \times \frac{30}{52} + (2 - \frac{33}{26})^2 \times \frac{15}{52} + (3 - \frac{33}{26})^2 \times \frac{2}{52} = \frac{315}{676}$

$\sigma(X) = \sqrt{V(X)} = \sqrt{\frac{315}{676}}$

2: Consider a circle with radius 1

(a) What percentage of the points lie within $\frac{3}{4}$ unit of the center? **(b)** Let c be a constant with $0 < c < 1$. What percentage of the points lies within c units of the center?

(a) $\frac{\pi \cdot (\frac{3}{4})^2}{\pi \cdot 1^2} = \frac{9}{16} = 56.25\%$

(b) $\frac{\pi \cdot c^2}{\pi \cdot 1^2} = c^2 = 100c^2\%$

3: A citrus grower anticipates a profit of \$100,000 this year if the nightly temperatures remain mild. Unfortunately, the weather forecast indicates a 35% chance that the temperatures will drop below freezing during the next week. Such freezing weather will destroy 10% of the crop and reduce the profit to \$90,000. However, the grower can protect the citrus fruit against the possible freezing (using smudge pots, electric fans, and so on) at a cost of\$5,000. Should the grower spend the \$5,000 and thereby reduce the profit to \$95,000?

If you do nothing, let X be the loss, X can only be $100000 - 90000 - 10000$ with probability $35\% = 0.35$, and 0 with probability 0.65

Thus the expected value (expected loss) $E(X) = 10000 \times 0.35 + 0 \times 0.65 = 3500$

If you prevent, the loss is 5000 for sure

But $3500 < 5000$, therefore you should do nothing