$\int f'(x)dx = f(x)+C$ indefinite integral

$\int_a^b f'(x)\,dx = f(b)-f(a)$ definite integral (Fundamental theorem of Calculus)

Example: $\int x^2 dx = \frac{1}{3}x^3 + C$ indefinite integral

$$\int_1^2 x^2 dx = \left(\frac{1}{3}x^3\right)\Big|_1^2 = \left(\frac{1}{3}2^3 - \frac{1}{3}1^3\right) = \frac{1}{3}\left(2^3-1^3\right) = \frac{1}{3}(8-1) = \frac{7}{3}$$

$$\int_1^3 x^2 e^{x^3} dx = \frac{1}{3}\int_1^3 e^{x^3}(3x^2dx) = \frac{1}{3}\int_1^3 e^{x^3} dx^3 \overset{u=x^3}{=\!=\!=} \frac{1}{3}\int_{1^3}^{3^3} e^u du$$

$$= \frac{1}{3}\int_1^{27} e^u du = \frac{1}{3}\left(e^{27} - e^1\right)$$

$$\int_0^\pi \sin x\,dx = (-\cos x)\Big|_0^\pi = \left(-\cos(\pi) - (-\cos(0))\right) = \left(-(-1)-(-1)\right) = 1+1 = 2$$

$$\int_2^6 \frac{1}{\sqrt{4x+1}}dx = \frac{1}{4}\int_2^6 \frac{1}{\sqrt{4x+1}} d4x = \frac{1}{4}\int_2^6 \frac{1}{\sqrt{4x+1}} d(4x+1) \overset{u=4x+1}{=\!=\!=} \frac{1}{4}\int_{4\times2+1}^{4\times6+1} \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{4}\int_9^{25} u^{-\frac{1}{2}} du = \frac{1}{2}\int_9^{25} \frac{1}{2}u^{-\frac{1}{2}} du = \frac{1}{2} u^{\frac{1}{2}}\Big|_9^{25} = \frac{1}{2}\left(\sqrt{25}-\sqrt{9}\right) = \frac{1}{2}(5-3) = 1$$

$$\int_0^\pi e^{\sin x}\cos x\,dx = \int_0^\pi e^{\sin x} d\sin x \overset{u=\sin x}{=\!=\!=} \int_{\sin(0)}^{\sin(\pi)} e^u du = \int_0^0 e^u du = e^u\Big|_0^0$$

$$= e^0 - e^0 = 0$$

$$\int_0^1 \frac{x}{x^2+3}dx = \frac{1}{2}\int_0^1 \frac{1}{x^2+3}(2xdx) = \frac{1}{2}\int_0^1 \frac{1}{x^2+3}dx^2 = \frac{1}{2}\int_0^1 \frac{1}{x^2+3} d(x^2+3)$$

$$\overset{u=x^2+3}{=\!=\!=} \frac{1}{2}\int_{0^2+3}^{1^2+3} \frac{1}{u}du = \frac{1}{2}\int_3^4 \frac{1}{u}du = \frac{1}{2}\ln u\Big|_3^4 = \frac{1}{2}(\ln 4 - \ln 3) = \frac{1}{2}\ln\frac{4}{3}$$