A DE is called autonomous if it is "independent" of $t$, i.e. of the form $y'=g(y)$

Example: $y'=3-\frac{1}{2}y$ is autonomous with $g(y)=3-\frac{1}{2}y$

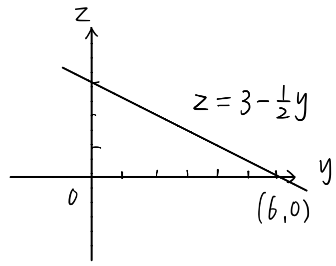
$y'=y^2-8$ is autonomous with $g(y)=y^2-8$

$y'=2t-y$ is not autonomous since it has $t$ in it

How to draw qualitative pictures of solution curves (the graph of $y(t)$ in $yt$-plane which is the solution of certain DE with IC)

First example: $y'=3-\frac{1}{2}y$, with IC: $y(0)=5$, $y(0)=7$

① Find all constant ("stable") solutions $\leftrightarrow$ solve $0=g(y)$ $\leftrightarrow$ intersections of the graph of $z=g(y)$ and $y$-axis

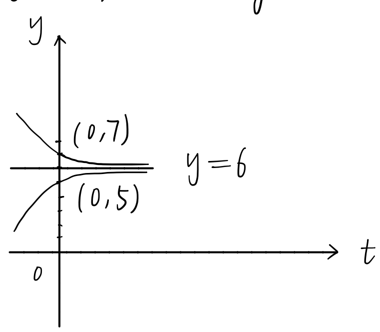



$0=3-\frac{1}{2}y \Rightarrow y=6$ or as you can see the intersection is only $(6,0)$, hence $y=6$ is the only constant solution

notice: If $y<6$, $z>0$, and $y\nearrow 6$, $z\searrow 0$ $\qquad y'=g(y)=z$

If $y>6$, $z<0$, and $y\searrow 6$, $z\nearrow 0$

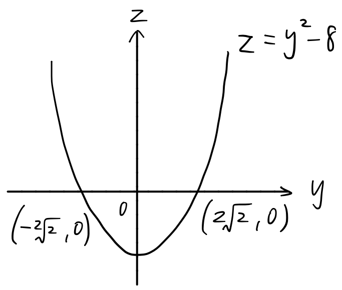
② Draw all the constant solutions on the $yt$-plane, and the use the graph of $g(y)$ as a reference to sketch the solution curves




Consider the autonomous equation, $y'=y^2-8$, Sketch the graph of solutions with IC: $y(0)=-4$, $y(0)=1$, $y(0)=5$

① find the constant solutions, i.e. solve $y^2-8=0 \Rightarrow y^2=8 \Rightarrow y=-\sqrt{8}$ or $\sqrt{8}$

thus $y=-\sqrt{8}$, $y=\sqrt{8}$ are the constant solutions



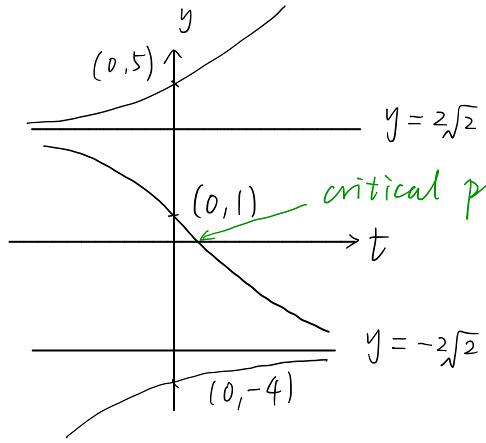

notice: If $y<-\sqrt{8}$, $z>0$, $y\nearrow -\sqrt{8}$, $z\downarrow 0$

If $y>\sqrt{8}$, $z>0$, $y\downarrow\sqrt{8}$, $z\downarrow 0$

If $-\sqrt{8}<y<\sqrt{8}$, $z<0$, $y=0$, $z$ obtain minimum, i.e. $y=0$ is a critical point, when $y$ decreases the fastest, $y\downarrow -\sqrt{8}$, $z\nearrow 0$ $\qquad y\nearrow 0$, $z\downarrow -8$

$y\nearrow\sqrt{8}$, $z\nearrow 0$ $\qquad y\downarrow 0$, $z\downarrow -8$

②



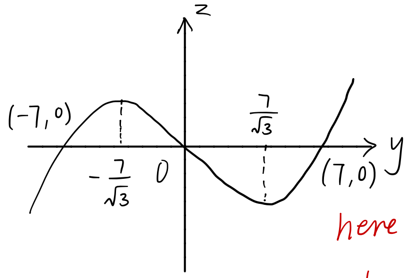

critical point $y=0$ when $y$ decreases the fastest

$y' = y^3 - 49y$ with IC $y(0)=-9$, $y(0)=-6$, $y(0)=1$, $y(0)=9$

① constant solutions: $0 = y^3 - 49y = y(y^2-49) = y(y+7)(y-7)$

$\Rightarrow y=0$ or $y+7=0$ or $y-7=0$

$\Rightarrow y=0$ or $y=-7$ or $y=7$




hence $y=0$, $y=-7$, $y=7$ are the constant solutions

here the value of critical points is found by solving $0=g'(y)$ which is not required, but can help you analyse better

notice: If $y<-7$, $z<0$, $y \nearrow -7$, $z \nearrow 0$

If $-7<y<0$, $z>0$, $y \searrow -7$, $z \searrow 0$ $\quad$ $y \nearrow 0$, $z \searrow 0$

If $0<y<7$, $z<0$, $y \searrow 0$, $z \nearrow 0$ $\quad$ $y \nearrow 7$, $z \nearrow 0$

If $y>7$, $z>0$, $y \searrow 7$, $z \searrow 0$