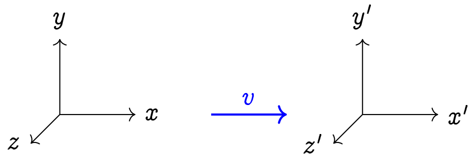
Definition 0.0.1 (Galilean group)**.** The **Galilean group** is the group of **Galilean transformations** generated by rotations in $\mathbb{R}^n$, translations in $\mathbb{R}^{n+1}$ and **Galilean boosts** $(x,t) \mapsto (x+tv,t)$

$$\begin{pmatrix} R & v & w \\ 0 & 1 & s \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \\ 1 \end{pmatrix} = \begin{pmatrix} Rx+tv+w \\ t+s \\ 1 \end{pmatrix}$$

Definition 0.0.2 (Lorentz group)**.** The **Lorentz group** is the group of **Lorentz transformations** generated by rotations in $\mathbb{R}^n$ and **Lorentz boosts** $(x,t) \mapsto (\sinh sx - \cosh st, \sinh st - \cosh sx)$

Definition 0.0.3 (Poincaré group)**.** The **Galilean group** is the isometry group of the Minkowski space $\mathbb{R}^{n+1}$

Definition 0.0.4. $(x,ct) \mapsto \left(\gamma(x-vt), \gamma\left(t-\frac{vx}{c^2}\right)\right)$ $\beta = \frac{v}{c}$, $\alpha = \sqrt{1-\beta^2}$. $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ is the **Lorentz factor** $\begin{cases} t' = \gamma\left(t-\frac{vx}{c^2}\right) \\ x' = \gamma(x-vt) \end{cases}$, where (x,t) and (x',t') are the coordinates of two frames, and frame (x',t') is moving towards the positive direction of the x axis with velocity v, and c is the speed of light, we can find the inverse transformation $\begin{cases} t = \gamma\left(t'+\frac{vx'}{c^2}\right) \\ x = \gamma(x'+vt') \end{cases}$ which makes perfect sense since relatively speaking, frame (x,t) is moving towards the negative direction of the x' axis with velocity v or rather moving towards the positive direction of the x' axis with velocity $-v$



More generally, if we consider $(\vec{x},t),(\vec{x}',t')$ are the coordinates of two frames, with frame $(\vec{x}',t')$ moving with velocity $\vec{v}$, then the Lorentz transformation will be

Deduction 0.0.5 (Time dilation)**.** A frame moving (x',t') is at a constant speed v, then $\Delta t = \gamma \Delta t'$. Suppose you are on the train with constant speed v and height h, and let light bouncing up and down perpendicularly, then we have

$$\begin{aligned} &2\sqrt{h^2+\left(\frac{\Delta t}{2}v\right)^2} = c\Delta t, v\Delta t' = h \\ \Rightarrow &\Delta t = \gamma \Delta t' \end{aligned}$$

Things happen simultaneously in one frame may not be simultaneous in another frame

Deduction 0.0.6 (Length contraction)**.** Suppose a train is moving with speed v, shed a beam light from one end to get to the other end
A in frame (x,t) send a signal when the left end of train passes, B in frame (x',t') on the right end of the train receives and return the signal, suppose the length of the train is l', and the length appears to be l in frame (x,t), then it takes time $\frac{l'}{c}$ for B to receive the signal in (x',t'), which takes time $\frac{l'\gamma}{c}$ in (x,t), when B should be in distance $l+\frac{vl'\gamma}{c}$ from A in (x,t) but distance $l'+\frac{vl'}{c}$ in (x',t') which take time $\frac{l+\frac{vl'\gamma}{c}}{c}$ and $\frac{l'+\frac{vl'}{c}}{c}$ to get back to A in (x,t) and (x',t'), hence we should have $\frac{l+\frac{vl'\gamma}{c}}{c} = \frac{l'+\frac{vl'}{c}}{c}\gamma \Rightarrow l = \gamma l'$