$$\int \cos 3t \, dt = \frac{1}{3}\int \cos 3t \, d(3t) = \frac{1}{3}\sin 3t + C$$

$$\int_0^{\pi} \sin t \, dt = \left[-\cos t\right]\Big|_0^{\pi} = \left[-(-1) - (-1)\right] = 2$$

$$\lim_{h\to 0} \frac{\sin h}{h} = \lim_{h\to 0} \frac{\sin(0+h) - \sin(0)}{h} = \cos 0 = 1$$

$$\int \tan t \, dt = \int \frac{\sin t}{\cos t} dt = -\int \frac{1}{\cos t} d(\cos t) = -\ln|\cos t| + C$$

$$\int \cos \frac{x-2}{2} dx = \int \cos \frac{x-2}{2} d(x-2) = 2\int \cos \frac{x-2}{2} \, d\,\frac{x-2}{2} =$$

$$2\sin \frac{x-2}{2} + C$$

Example 7 from Section 8.3 (Textbook)

$N(t) = 5000 + 2000\cos\left(\frac{2\pi t}{36}\right)$, evaluate

$$\int_0^{144} N(t)\, dt = \int_0^{144} 5000 + 2000\cos\left(\frac{\pi t}{18}\right) dt$$

$$= 1000\left[\int_0^{144} 5 + 2\cos\left(\frac{\pi t}{18}\right) dt\right]$$

$$= 1000 \cdot \frac{18}{\pi}\left[\int_0^{144} 5 + 2\cos\left(\frac{\pi t}{18}\right) d\left(\frac{\pi t}{18}\right)\right]$$

$$= \frac{18000}{\pi}\left[\int_0^{144\cdot\frac{\pi}{18}} 5 + 2\cos(u)\, du\right]$$

$$= \frac{18000}{\pi}\int_0^{8\pi} (5 + 2\cos u)\, du$$

$$= \frac{18000}{\pi}\left(\int_0^{8\pi} 5\, du + 2\int_0^{8\pi} \cos u \, du\right)$$

$$= \frac{18000}{\pi}\left(5u\Big|_0^{8\pi} + 2\sin u\Big|_0^{8\pi}\right)$$

$$= \frac{18000}{\pi}\left[(40\pi - 0) + 2(0 - 0)\right]$$

$$= \frac{18000}{\pi} \cdot 40\pi = 72000$$

$\sin(2t+1) = -\frac{\sqrt{3}}{2}$, $t = ?$

$2t+1 = -\frac{\pi}{3} + k\cdot 2\pi$ or $-\frac{2\pi}{3} + k\cdot 2\pi$

$\Rightarrow 2t = -\frac{\pi}{3} - 1 + k\cdot 2\pi$ or $-\frac{2\pi}{3} - 1 + k\cdot 2\pi$ $\quad k \in \mathbb{Z}$

$\Rightarrow t = -\frac{\pi}{6} - \frac{1}{2} + k\cdot\pi$ or $-\frac{\pi}{3} - \frac{1}{2} + k\cdot\pi$

Review of basic laws of differentiation and integration

$(x^n)' = nx^{n-1}$ (*including constants*) $\quad (\ln x)' = \frac{1}{x} = x^{-1}$

$$\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & (n \neq -1) \\ \ln|x| + C & (n=-1) \end{cases}$$

C is an arbitrary constant

$(e^x)' = e^x$, $\int e^x dx = e^x + C$

$(\sin x)' = \cos x$, $\int \cos x\,dx = \sin x + C$

$(\cos x)' = -\sin x$, $\int \sin x\,dx = -\cos x + C$

$$\frac{df}{dx} = f'(x) \Updownarrow df = f'(x)\,dx$$

$$\int f'(x)\,dx = \int df = f + C$$

differentiation and integration respect linear combination

examples: $(2\cos x - e^x)' = [2\cos x + (-1)e^x]' = 2(\cos x)' + (-1)(e^x)'$

$= 2(-\sin x) + (-1)e^x = -2\sin x - e^x$

$\int \sin x - \frac{1}{x}\,dx = \int \sin x\,dx - \int \frac{1}{x}\,dx$

$= -\cos x - \ln|x| + C$

$$\frac{d(c_1 f_1 + c_2 f_2)}{dx} = c_1 \frac{df_1}{dx} + c_2 \frac{df_2}{dx}$$

$$\Downarrow$$

$$d(c_1 f_1 + c_2 f_2) = c_1 df_1 + c_2 df_2$$

product rule for differentiation

$(\sin x\cos x)' = (\sin x)'\cos x + \sin x(\cos x)'$

$= \cos x\cos x + \sin x(-\sin x)$

$= \cos^2 x - \sin^2 x = \cos 2x$

$$\frac{d(f_1 f_2)}{dx} = \frac{df_1}{dx} f_2 + f_1 \frac{df_2}{dx}$$

$$\Downarrow$$

$$d(f_1 f_2) = f_2\,df_1 + f_1\,df_2$$

$(e^x\sin x)' = (e^x)'\sin x + e^x(\sin x)' = e^x\sin x + e^x\cos x$

composition law for differentiation

$[f(g(x))]' = f'(g(x))\,g'(x)$

$= f'(g)\,g'(x)$

$$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$$

$$df = \frac{df}{dg}dg = \frac{df}{dg}\frac{dg}{dx}dx = f'(g)g'(x)\,dx$$

$$(e^{2x})' \xlongequal{y=2x} (e^y)' = e^y \cdot y' = e^{2x}(2x)' = 2e^{2x}$$

$$(\cos x)' = \left(\sin\left(\frac{\pi}{2} - x\right)\right)' \xlongequal{y=\frac{\pi}{2}-x} (\sin y)' = \cos y \cdot y'$$

$$= \cos\left(\frac{\pi}{2} - x\right)\left(\frac{\pi}{2} - x\right)'$$

$$= (0 \cdot \cos x + 1 \cdot \sin x)(-1) = -\sin x$$

$$(e^{\sin x})' \xlongequal{y=\sin x} (e^y)' = e^y \cdot y' = e^{\sin x}(\sin x)' = e^{\sin x}\cos x$$

$$\left[\sin(e^{x^2})\right]' \xlongequal{y=e^{x^2}} (\sin y)' = \cos y \cdot y' = \cos(e^{x^2}) \cdot (e^{x^2})' \xlongequal{u=x^2}$$

$$\cos(e^{x^2}) \cdot (e^u)' = \cos(e^{x^2}) e^u \cdot u' = \cos(e^{x^2}) \cdot e^{x^2} \cdot (x^2)'$$

$$= \cos(e^{x^2}) \cdot e^{x^2} \cdot 2x = 2x e^{x^2} \cos(e^{x^2})$$

$$\frac{df}{dx} = \frac{df}{dy}\frac{dy}{du}\frac{du}{dx}$$

quotient rule for differentiation

$$\left[\frac{f(x)}{g(x)}\right]' = \left[f(x) \cdot \frac{1}{g(x)}\right]' = f'(x) \cdot \frac{1}{g(x)} + f'(x)\left[\frac{1}{g(x)}\right]$$

$$= \frac{f'(x)}{g(x)} + f'(x)\left[-\frac{g'(x)}{g(x)^2}\right.$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

example $(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$d\left(\frac{f}{g}\right) = \frac{g\,df - f\,dg}{g^2}$$