STAT400

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2020 Fall

Contents

1 Practice problems

1.1 Practice1 - 9/3/2020

Exercise 1.1 ([1] Section 2.1, Exercise 1.)**.**
Four universities (1, 2, 3, and 4) are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4)

a. List all outcomes in $\mathcal{S}$

b. Let A denote the event that 1 wins the tournament. List outcomes in A

c. Let B denote the event that 2 gets into the championship game. List outcomes in B

d. What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A'?

Solution. The first two digits represent the two winners of the first round, the last two digits represent the two losers of the first round, the order of the first two digits determine the first and second place from the second round, the order of the last two digits determine the third and fourth place from the second round. Note that what the first two digits are automatically determine what the last two digits are and vice versa

a. The first two digits must consist one from $\{1, 2\}$ and the other from $\{3, 4\}$

$$\mathcal{S} = \begin{Bmatrix} 1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, \\ 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231 \end{Bmatrix}$$

b. The first digit must be 1

$$A = \{1324, 1342, 1423, 1432\}$$

c. The first two digits must contain 2

$$B = \{2314, 2341, 3214, 3241, 2413, 2431, 4213, 4231\}$$

d. Note that if 1 wons the first place is equivalent of saying 2 doesn't make into the championship game, hence A, B are disjoint, therefore $A \cap B = \varnothing$

$$A \cup B = \{1324, 1342, 1423, 1432, 2314, 2341, 3214, 3241, 2413, 2431, 4213, 4231\}$$

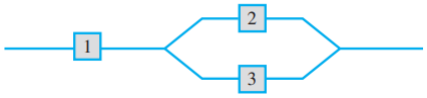
and

$$A' = \{2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231\}$$

□

Exercise 1.2 ([1] Section 2.1, Exercise 3.)**.**
Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2-3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2-3 subsystem. The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component]

a. Which outcomes are contained in the event A that exactly two out of the three components function?



b. Which outcomes are contained in the event B that at least two of the components function?

c. Which outcomes are contained in the event C that the system functions?

d. List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$

Solution. Write $T_1T_2T_3$ for the outcome, where $T_i \in \{S, F\}$ stands for whether component i is functioning successfully or not, thus the sample space is

$$\mathcal{S} = \{SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF\}$$

a.

$$A = \{SSF, SFS, FSS\}$$

b.

$$B = \{SSF, SFS, FSS, SSS\}$$

c. Since component 1 and subsystem 2-3 are connected in series, the whole system functions if both component 1 and subsystem function, and since component 2,3 are connected in parallel, subsystem 2-3 functions if at least one of components 2,3 works

$$C = \{SSF, SFS, SSS\}$$

d.

$$C' = \{FSS, SFF, FFS, FSF, FFF\}$$

$$A \cup C = \{SSSF, SFS, FSS, SSS\}$$

$$A \cap C = \{SSF, SFS\}$$

$$B \cup C = \{SSF, SFS, FSS, SSS\}$$

$$B \cap C = \{SSF, SFS, SSS\}$$

□

References

[1] *Probability and Statistics for Engineering and the Sciences* - Jay Devore