Section 12.4a

November 22, 2019

Motivating example:
Definition: $f(x) = ke^{-kx}(k > 0)$ for $x \geq 0$ is called an **exponential density function**, if X is a random variable with an exponential density function, it is called an **exponential random variable**, cdf of X will be $F(x) = \int_0^x f(t)dt = \int_0^x ke^{-kt}dt = (-e^{-kt})\big|_0^x = (-e^{-kx}) - (-1) = 1 - e^{-kx}$, using integration by parts $E(X) = \frac{1}{k}$, and $Var(X) = \frac{1}{k^2}$
We say $X \sim Exp(k)$ meaning X is a random variable satisfying exponential distribution with parameter k
Problems:
1: Find the expected value and variance of the exponential random variable with the pdf given by $1.7e^{-1.7x}$

$X \sim Exp(1.7)$, $E(X) = \frac{1}{1.7}$, $Var(X) = \frac{1}{1.7^2}$

2: Suppose that in a large factory there is an average of two accidents per day and the time between accidents has an exponential density function with expected value of $\frac{1}{2}$ day. Find the probability that the time between two accidents will be more than $\frac{1}{2}$ day and less than 1 day

Let X denote the time between accidents, then $X \sim Exp(k)$ for some k, since $\frac{1}{2} = E(X) = \frac{1}{k}$, $k=2$, $Pr(\frac{1}{2} \leq X \leq 1) = F(1) - F(\frac{1}{2}) = (1-e^{-2\cdot 1}) - (1-e^{-2\cdot\frac{1}{2}})$
$= e^{-1} - e^{-2}$

3: The amount of time required to serve a customer at a fast food restaurant has an exponential density function with mean 5 minutes. Find the probability that a customer is served in less than 3 minutes

Let X denote the serving time, then $X \sim Exp(k)$ for some k
since $5 = E(X) = \frac{1}{k}$, $k = \frac{1}{5}$, $Pr(X \leq 3) = F(3) = 1 - e^{-\frac{1}{5}\cdot 3} = 1 - e^{-\frac{3}{5}}$

4: The amount of time required to serve a customer at a bank has an exponential density function with mean 4 minutes. Find the probability that a customer is served in more than 13 minutes

Let X denote the serving time, then $X \sim Exp(k)$ for some k,

since $4 = E(X) = \frac{1}{k}$, $k = \frac{1}{4}$, $Pr(X \geq 13) = 1 - Pr(X \leq 13) = 1 - (1 - e^{-\frac{1}{4} \cdot 13}) = e^{-\frac{13}{4}}$

5: During a certain part of the day, the time between arrivals of automobiles at the tollgate on a turnpike is an exponential random variable with expected value 10 seconds. Find the probability that the time between successive arrivals is more than 20 seconds

Let X denote the time between arrivals, then $X \sim Exp(k)$ for some k,

since $10 = E(X) = \frac{1}{k}$, $k = \frac{1}{10}$, $Pr(X \geq 20) = 1 - Pr(X \leq 20) = 1 - F(20) = 1 - (1 - e^{-\frac{1}{10} \cdot 20}) = e^{-2}$

6: Suppose that the average life span of an electronic component is 84 months and that the life spans are exponentially distributed

(a) Find the probability that a component lasts for more than 72 months

(b) The reliability function $r(t)$ gives the probability that a component will last for more than t months. Compute $r(t)$ in this case

Let X denote the life span, then $X \sim Exp(k)$ for some k,

since $84 = E(X) = \frac{1}{k} \Rightarrow k = \frac{1}{84}$,

(a) $Pr(X \geq 72) = 1 - Pr(X \leq 72) = 1 - F(72) = 1 - (1 - e^{-\frac{1}{84} \cdot 72}) = e^{-\frac{72}{84}} = e^{-\frac{6}{7}}$

(b) $r(t) = Pr(X \geq t) = 1 - Pr(X \leq t) = 1 - F(t) = 1 - (1 - e^{-\frac{1}{84} \cdot t}) = e^{-\frac{t}{84}}$