1. (a) $\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx = \tan x - x + C$

(b) $\int (7x-4)e^{2x}dx \overset{u=2x}{=\!=\!=} \int \left(\frac{7}{2}u-4\right)e^u \frac{1}{2}du = \int \frac{7}{4}ue^u du - \int 2e^u du$

$= \frac{7}{4}(ue^u - e^u) - 2e^u + C = \frac{7}{4}ue^u - \frac{15}{4}e^u + C = \frac{7}{2}xe^{2x} - \frac{15}{4}e^{2x} + C$

(c) $\int \frac{8x}{e^{x^2}}dx = \int 8xe^{-x^2}dx \overset{u=x^2}{=\!=\!=} 4\int e^{-u}du = -4e^{-u} + C = -4e^{-x^2} + C$

(d) $\int_0^{\pi} x\sin(8x)\,dx \overset{u=8x}{=\!=\!=} \frac{1}{8}\int_0^{8\pi} u\sin u\,du = \frac{1}{8}\left[-u\cos u + \sin u\right]\Big|_0^{8\pi} = -\pi$

2. (a) $f(0)=0$, $f'(0) = e^0 + f(0) = 1$

(b) $\frac{dy}{dt} = \frac{t^2y^2}{t^3+8} \Rightarrow \frac{dy}{y^2} = \frac{t^2dt}{t^3+8} \Rightarrow -\frac{1}{y} = \frac{1}{3}\ln(t^3+8) + C \Rightarrow y = -\frac{1}{\frac{1}{3}\ln(t^3+8)+C}$

(c) $y'+y=e^{2t} \Rightarrow (e^t y)' = e^{3t} \Rightarrow e^t y = \frac{1}{3}e^{3t} + C$, plug in $y(0)=-1$, $C = -\frac{4}{3}$

thus $y(t) = \frac{1}{3}e^{2t} - \frac{4}{3}e^{-t}$

3. (a) $\sum_{k=0}^{\infty} \frac{3^k}{4^{k+1}} = \frac{1}{4}\sum_{k=0}^{\infty}\left(\frac{3}{4}\right)^k = \frac{1}{4}\cdot\frac{1}{1-\frac{3}{4}} = 1$

(b) $2.7\overline{18} = 2.7 + 0.0\overline{18} = 2.7 + \frac{0.018}{1-0.01} = 2.7 + \frac{2}{110} = \frac{299}{110}$

(c) Let $f(x) = \frac{1}{x\sqrt{\ln x}}$, $x \geq 2$, $f(x)$ is continuous and decreasing , thus we can use the integral test : $\int_2^{\infty} f(x)dx = \lim_{b\to\infty}\int_2^b \frac{1}{x\sqrt{\ln x}}dx \overset{u=\ln x}{=\!=\!=} \lim_{b\to\infty}\int_2^{\ln b} u^{-\frac{1}{2}}du =$

$\lim_{b\to\infty}\left[2u^{\frac{1}{2}}\right]\Big|_2^{\ln b} = \lim_{b\to\infty}\left[2(\ln b)^{\frac{1}{2}} - 2\sqrt{2}\right] = \infty$

Thus $\sum_{k=2}^{\infty}\frac{1}{k\sqrt{\ln k}}$ does not converge

(d) $\frac{2}{x^{-2}e^{x^3}} = 2x^2e^{-x^3} = 2x^2\left(1 + (-x^3) + \frac{1}{2!}(-x^3)^2 + \frac{1}{3!}(-x^3)^3 + \cdots\right)$

$= 2x^2\left(1 - x^3 + \frac{1}{2}x^6 - \frac{1}{6}x^9 + \cdots\right) = 2x^2 - 2x^5 + x^8 - \frac{1}{3}x^{11} + \cdots$

4. (a) Let X be the money you gain

Choose bet A : $E(X) = 5000\cdot 0.1 + (-2000)\cdot 0.2 + 0\cdot 0.7 = 100$

Choose bet B : $E(X) = 1000\cdot 0.28 + (-100)\cdot 0.72 = 208$

Don't bet : $E(X) = 200\cdot 1 = 200$

But then $208 > 200 > 100$

Therefore you should bet for B

5. The probability density function is $f(x) = F'(x) = \frac{3}{125}x^2$

(a) $E(X) = \int_0^5 x f(x)\,dx = \int_0^5 \frac{3}{125}x^3\,dx = \frac{3}{500}x^4\Big|_0^5 = \frac{15}{4}$

(b) $Var(X) = \int_0^5 x^2 f(x)\,dx - E(X)^2 = \int_0^5 \frac{3}{125}x^4\,dx - \left(\frac{75}{4}\right)^2 = \frac{3}{625}x^5\Big|_0^5 - \left(\frac{15}{4}\right)^2 = \frac{15}{16}$

6. X satisfies geometric distribution with $p = \frac{5}{6}$, where p is the probability of a Red taxi showing up

(a) $Pr(X=n) = \left(\frac{5}{6}\right)^n\left(\frac{1}{6}\right)$

(b) $Pr(X \geq 4) = 1 - Pr(X \leq 3) = 1 - P_0 - P_1 - P_2 - P_3 = 1 - \frac{1}{6} - \frac{5}{6}\frac{1}{6} - \left(\frac{5}{6}\right)^2\frac{1}{6} - \left(\frac{5}{6}\right)^3\frac{1}{6} = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

(c) $E(X) = \frac{p}{1-p} = \frac{\frac{5}{6}}{1-\frac{5}{6}} = 5$

7. Since X satisfies a normal distribution with expected value 3 and variance 4, $\frac{X-3}{2} =: Z$ satisfies standard normal distribution

(a) $Pr(X \geq 7) = Pr\left(\frac{X-3}{2} \geq \frac{7-3}{2}\right) = Pr(Z \geq 2) = Pr(Z \geq 0) - Pr(0 \leq Z \leq 2)$

$= \frac{1}{2} - A(2) = 0.5 - 0.4772 = 0.228$

(b) $Pr(X \leq -1) = Pr\left(\frac{X-3}{2} \leq \frac{-1-3}{2}\right) = Pr(Z \leq -2) = Pr(Z \geq 2) = 0.228$