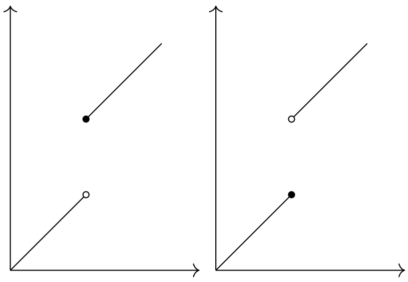
Definition 0.0.1 (Hyperbolic functions)**.** $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}$, $\cos z = \dfrac{e^{iz} + e^{-iz}}{2}$ $\sinh z = -i\sin(iz) = \dfrac{e^z - e^{-z}}{2}$, $\cosh z = \cos(iz) = \dfrac{e^z + e^{-z}}{2}$

Definition 0.0.2. X is a convex set, $X \xrightarrow{f} \mathbb{R}$ is **convex** if $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$ for $0 \leq t \leq 1$ and $x, y \in X$, f is **strictly convex** if $f(tx + (1-t)y) < tf(x) + (1-t)f(y)$ for $0 < t < 1$ and $x \neq y \in X$. f is **concave** if $-f$ is convex

Definition 0.0.3. $X \xrightarrow{f} [-\infty, \infty]$ is **upper semicontinuous** at x if for any $y > f(x)$, there exists a neighborhood U of x such that $f(U) < y$, i.e. f can only jump down at x. Thus $X \xrightarrow{f} [-\infty, \infty]$ is upper semicontinuous if $\{f < a\}$ are open. $X \xrightarrow{f} [-\infty, \infty]$ is **lower semicontinuous** if $-f$ is upper semicontinuous, i.e. f can only jump up



Lemma 0.0.4. $\{f_\alpha\}_{\alpha \in A}$ is a family of upper semicontinuous functions, $f = \inf\limits_{\alpha \in A} f_\alpha$ is also upper semicontinuous

Proof.
$$\{f < a\} = \bigcup_{\alpha \in A} \{f_\alpha < a\}$$
□

Lemma 0.0.5. f is upper semicontinuous, K is compact, then f attains maximum over K

Definition 0.0.6. $\Omega \xrightarrow{u} [-\infty, \infty)$ is **harmonic** at $x \in \Omega$ if u is continuous at x and for any ball $B(x,r)$, $u(x) = \dfrac{1}{|B(x,r)|}\displaystyle\int_{B(x,r)} u(y)dy$. $\Omega \xrightarrow{u} [-\infty, \infty)$ is **subharmonic** at $x \in \Omega$ if u is upper semicontinuous at x and for any ball $B(x,r)$, any continuous v harmonic on $B(x,r)$, $u \leq v$ on $\partial B(x,r) \Rightarrow u \leq v$ on $\overline{B(x,r)}$. $\Omega \xrightarrow{u} [-\infty, \infty)$ is **superharmonic** if $-u$ is subharmonic
Harmonic $\Leftrightarrow$ subharmonic and superharmonic

Lemma 0.0.7. $\Omega \xrightarrow{u} \mathbb{R}$ is subharmonic, $\mathbb{R} \xrightarrow{f} \mathbb{R}$ is convex, then $f \circ u$ is also subharmonic

f holomorphic => log|f| subharmonic

Example 0.0.8. If f is holomorphic, then $\log|f|$ is subharmonic