

**Definition 0.0.1.** A **Riemann surface** is a one dimensional complex manifold

**Theorem 0.0.2** (Riemann's removable singularity theorem).  $f$  is holomorphic on  $X \setminus \{a\}$  and bounded near  $a$ , then  $f$  is holomorphic on  $X$

**Theorem 0.0.3** (Principle of analytic continuation).  $X$  is connected,  $X \xrightarrow{f} Y$  is holomorphic and  $f \equiv c$  on some nondiscrete subset of  $X$ , then  $f \equiv c$  on  $X$

**Remark 0.0.4.** This does not apply to higher dimensions, for example,  $f(z, w) = z$ , but in higher dimensions, we have Theorem ??

**Theorem 0.0.5** (Local behaviour of holomorphic maps).  $X \xrightarrow{f} Y$  is a nonconstant holomorphic map,  $a \in X$ ,  $f(a) = b \in Y$ . There are local charts  $U \xrightarrow{\phi} \mathbb{C}$ ,  $V \xrightarrow{\psi} \mathbb{C}$  of  $a, b$  such that  $\psi f \phi^{-1} = z^k$  for some  $k \geq 1$

**Remark 0.0.6.** If the **multiplicity**  $k > 1$ ,  $a$  is a **branch point**

**Theorem 0.0.7.**  $X \xrightarrow{f} Y$  is a proper nonconstant holomorphic map between Riemann surfaces, there exists some  $n$  such that  $f$  take every value  $c \in Y$ , counting multiplicities,  $n$  times

**Theorem 0.0.8** (Radó's theorem). A connected Riemann surface is second countable  
Uniformization theorem

**Theorem 0.0.9** (Uniformization theorem). A simply connected Riemann surface is either  $\mathbb{C}$ ,  $\mathbb{P}$  or  $\mathbb{H}^2$