Section 8.1

Conversion between degrees and radians

On the unit circle, the radian of an angle is the length of the corresponding arc with the sign

Examples: $90° = \frac{\pi}{2}$ rad $\quad 270° = \frac{3\pi}{2}, \quad 450° = \frac{5\pi}{2}, \quad -360° = -2\pi$

generally $\quad d° = d \times \frac{2\pi}{360} = d \times \frac{\pi}{180} = r$

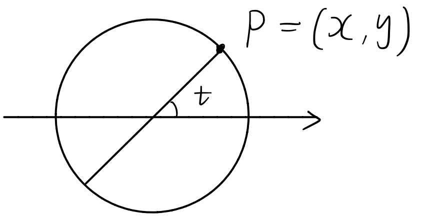
so conversely we get $\quad d = \frac{180}{\pi} r$

Exercise: $\frac{3\pi}{4}, \ -\frac{2\pi}{3}, \ 75°$

Homework: 1-17 odd for 8.1
1-33 odd for 8.2

Section 8.2

Given an angle of t radians, let the angle be on the unit circle, then $(\cos t, \sin t) = (x, y)$, $\tan t = \frac{\sin t}{\cos t}$



hence we have

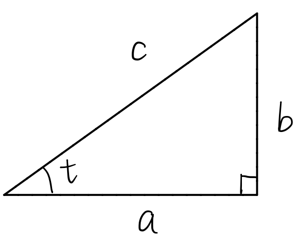
$$\cos^2 t + \sin^2 t = x^2 + y^2 = 1$$

$\cos(t \pm 2\pi) = \cos t$, $\sin(t \pm 2\pi) = \sin t$, $\cos(-t) = \cos t$

$\sin(-t) = -\sin t$, $\sin(s+t) = \sin s \cos t + \cos s \sin t$

For example
$$\begin{aligned}\sin(75°) &= \sin(30° + 45°) \\ &= \sin 30° \cos 45° + \cos 30° \sin 45° \\ &= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{4}(1+\sqrt{3})\end{aligned}$$

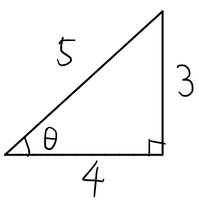
Connection with right triangle:



$a^2 + b^2 = c^2$ (Pythagorean identity)

$\sin t = \frac{b}{c}$, $\cos t = \frac{a}{c}$, $\tan t = \frac{b}{c}$

Example:



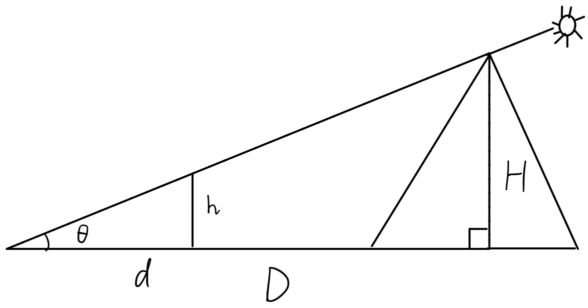
$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

Real life applications:

The king want to measure the height of Pyramid

a clever man put a pole and wait the shows of them

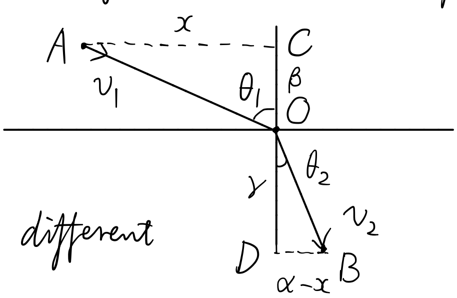
coincide



then measure h, d, D, but $\tan\theta = \frac{h}{d} = \frac{H}{D} \Rightarrow D = \frac{Hd}{h}$

Refraction: Férmat's Principle: light travels the path

which takes least time



A, B are two fixed points on different

medium which light travels at speeds v_1, v_2

where should O be such that it takes least time

to get B from A, which will take time

$$T = \frac{\sqrt{x^2+\beta^2}}{v_1} + \frac{\sqrt{(\alpha-x)^2+\gamma^2}}{v_2}$$

$$\frac{dT}{dx} = \frac{x}{v_1\sqrt{x^2+\beta^2}} - \frac{\alpha-x}{v_2\sqrt{(\alpha-x)^2+\gamma^2}}$$

$$\frac{dT}{dx} = 0 \Rightarrow \frac{v_1}{v_2} = \frac{x}{\sqrt{x^2+\beta^2}} \Big/ \frac{\alpha-x}{\sqrt{(\alpha-x)^2+\gamma^2}} = \frac{\sin\theta_1}{\sin\theta_2}$$