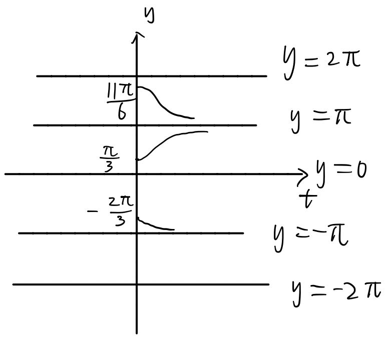
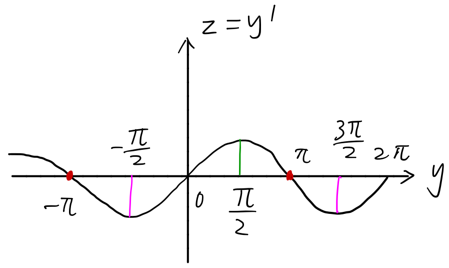
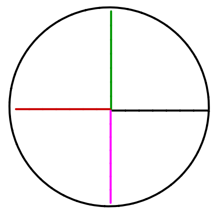
$y' = \sin y$, $0 = \sin y \Rightarrow y = k\pi, k \in \mathbb{Z}$, $y(0) = -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{11\pi}{6}$







10.6 Logistic DE: $\frac{dN}{dt} = rN\frac{K-N}{K} = \frac{r}{K}N(K-N) = -\frac{r}{K}N^2 + rN$

K: carrying capacity , r: intrinsic rate of growth

zeros $\Leftrightarrow$ constant solutions $\Leftrightarrow$ $N=0$ or $N=K$

If $\frac{r}{K} > 0$, the parabola open downwards

symmetric axis is $N = \frac{K}{2}$

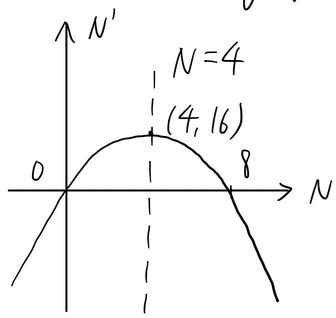
vertex is $\left(\frac{K}{2}, \frac{rK}{4}\right)$

Example: $\frac{dN}{dt} = N(8-N)$ with IC: $N(0) = 3.5$

(a) determine carrying capacity and intrinsic rate

notice $N(8-N) = \frac{8}{8}N(8-N)$, thus $r = K = 8$

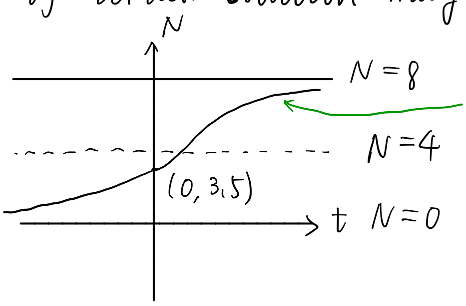
(b) Sketch the graph of $N' = \frac{dN}{dt}$ versus N in the Nz-plane



hence constant solutions are $N=0$ and $N=8$

$N=4$ is when N grows the fastest

(c) plot the constant solution and place a dashed line where the concavity of certain solution may change



this kind of curve is called a logistic curve where it grows faster and then slows down to settle down to the constant solution $N=K$

(d) Sketch the solution curve corresponding to the given IC

Ex: The fish population in a pond with capacity of 200 fish is modeled by the logistic equation $\frac{dN}{dt} = \frac{0.2}{2000}N(2000-N)$. Here N denotes the number of fish at time t in years. When the number of fish reached 500, the owner of the pond decided to remove 61 fish per year

(a) Modify the DE to model the fish population from the time it reached 500

$$\frac{dN}{dt} = \frac{0.2}{2000}N(2000-N) - 61$$

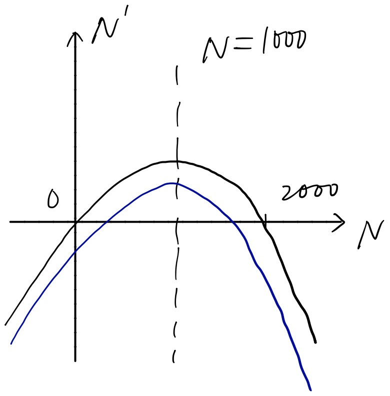
[rate of change in number of fish per year] = [rate of change due to the system itself (the number of fish)] − [rate of change due to outer interference (artificial interference)]

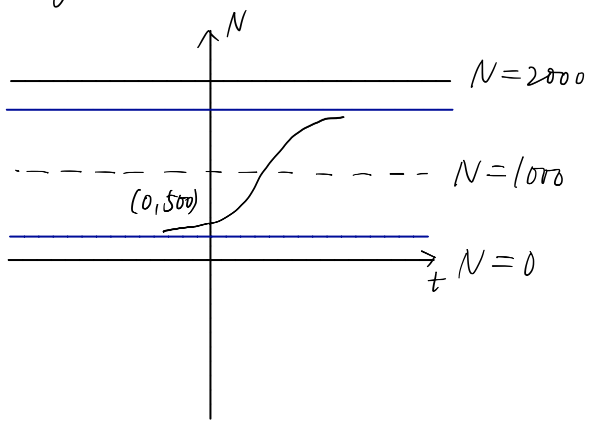
$N(0) = 500$

(b) Plot the solution curve of the new equation with $N(0)=500$

First: $\frac{0.2}{2000}N(2000-N)-61 = -0.0001N^2 + 0.2N - 61$

you can always compute the zero, symmetry axis and vertex by using quadratic formula, but here let's do it in another way, notice





−61 has the effect of moving down the graph by 61, in order to if the solution curve is increasing, you only need to plug in $N(0)=500$, get $N'(0) = \frac{0.2}{2000}500(2000-500) - 61 = 75 - 61 = 14 > 0$, you get a logistic curve

(d) Is the practice of catching 61 fish per year sustainable, or will it deplete the fish population in the pond? YES!!

Remark: as you K, r change (due to natural causes), or IC $N(0)$ changes, or the pratice changes (artificial interference, put in or take out) see how you can "control" the system

Challenge: What is the maximal profit you can have annually if you can change the IC and practice such that it is still sustainable