Definition 0.0.1. G is a **topological group** if it is a group and a topological space so that the group multiplication $G \times G \to G$ and the inverse map $G \to G$ are continuous maps

Definition 0.0.2. $f : G \to \mathbb{R}/\mathbb{C}$ is a continuous function, $L_y f(x) = f(y^{-1}x)$, $R_y f(x) = f(xy)$, $L_{yz} = L_y L_z$, $R_{yz} = R_y R_z$, f is called left/right uniformly continuous if $\forall \varepsilon > 0$, $\exists V \ni e$ such that $\|L_y f - f\| < \varepsilon / \|R_y f - f\| < \varepsilon$, $\forall y \in V$, $\| \cdot \|$ is the supremum norm

Proposition 0.0.3. If $f \in C_c(G)$, then f is both left and right uniformly continuous

Proof. Easy proof by a very standard analysis argument $\square$

Definition 0.0.4. If f is a Borel measurable function on G, then f factor through G/H, otherwise suppose $f(y) \neq f(z)$, $y, z \in xH$, $f^{-1}(f(y)) \cap xH \subsetneq xH$ is a Borel set which is impossible, because then $x^{-1}f^{-1}(f(y)) \cap H \subsetneq H$ will also be a Borel set, consider $\Gamma = \{S \in \mathscr{P} | H \subseteq H \text{ or } H \cap S = \emptyset\}$, then Γ is a sigma algebra containing all open sets hence Borel algebra, we reached a contradiction
Thus for most purposes one may as well work with G/H which is Hausdorff(L^p spaces for instance, mod almost everywhere vanishing function)
For a locally compact Hausdorff group, A Borel measure μ on G is called left/right invariant if $\mu(xE) = \mu(E)/\mu(Ex) = \mu(E)$, $x \in G, E \in \mathscr{B}(G)$
A linear functional I is left/right invariant if $I(L_x f) = I(f)/I(R_x f) = I(f)$
A left/right Haar measure on G is a left/right invariant Radon measure μ on G, for example, Lebesgue measure on $\mathbb{R}^n$, counting measure on G with discrete topology

Example 0.0.5. Continuous bijective group homomorphism doesn't imply homeomorphism, which is really obvious, by taking the identity map and a discrete topology on the topological group G

Definition 0.0.6. Let G be a topological group, then a 1-parameter subgroup means a continuous group homomorphism $\varphi : \mathbb{R} \to G$, $\varphi(s+t) = \varphi(s)\varphi(t)$, in the case of a Lie group, φ is required to be smooth

Definition 0.0.7. Suppose G is a connected, locally pathconnected and (semi-)locally simply connected topological space, then it has a universal cover $\tilde{G}$ which is unique up to an isomorphism, a connected Lie group certain satisfies this

Proposition 0.0.8. Denote $\pi : \tilde{G} \to G$ as the covering map, let $\bar{G}$ be the set of maps $T : \tilde{G} \to \tilde{G}$, such that $\pi(Tx) = g(\pi x)$ for some $g \in G$, i.e. the following diagram commutes

$$\begin{array}{ccc} \tilde{G} & \xrightarrow{T} & \tilde{G} \\ \downarrow{\scriptstyle\pi} & & \downarrow{\scriptstyle\pi} \\ G & \xrightarrow{g} & G \end{array}$$

Then $\bar{G}$ which is a group acts transitively and freely on $\tilde{G}$, thus we can think of the universal cover $\tilde{G}$ also as a topological group

Proof. Given $x, y \in \tilde{G}$, there is a unique $g \in G$ such that $g(\pi x) = \pi y$, since $\tilde{G}$ is the universal cover, there is a unique lift such that $T(x) = y$, thus the action is free and transitive $\square$