STAT400

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1 Practice problems

1.1 Practice1 - 9/3/2020

Exercise 1.1 ([1] Section 2.1, Exercise 1.).
Four universities (1, 2, 3, and 4) are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4)

a. List all outcomes in $\mathcal{S}$

b. Let A denote the event that 1 wins the tournament. List outcomes in A

c. Let B denote the event that 2 gets into the championship game. List outcomes in B

d. What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A'?

Solution. The first two digits represent the two winners of the first round, the last two digits represent the two losers of the first round, the order of the first two digits determine the first and second place from the second round, the order of the last two digits determine the third and fourth place from the second round. Note that what the first two digits are automatically determine what the last two digits are and vice versa

a. The first two digits must consist one from $\{1, 2\}$ and the other from $\{3, 4\}$

$$\mathcal{S} = \begin{Bmatrix} 1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, \\ 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231 \end{Bmatrix}$$

b. The first digit must be 1

$$A = \{1324, 1342, 1423, 1432\}$$

c. The first two digits must contain 2

$$B = \{2314, 2341, 3214, 3241, 2413, 2431, 4213, 4231\}$$

d. Note that if 1 wons the first place is equivalent of saying 2 doesn't make into the championship game, hence A, B are disjoint, therefore $A \cap B = \varnothing$

$$A \cup B = \{1324, 1342, 1423, 1432, 2314, 2341, 3214, 3241, 2413, 2431, 4213, 4231\}$$

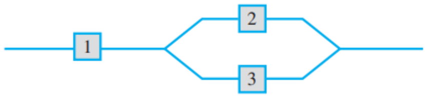
and

$$A' = \{2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231\}$$

□

Exercise 1.2 ([1] Section 2.1, Exercise 3.).
Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2-3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2-3 subsystem. The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component]

a. Which outcomes are contained in the event A that exactly two out of the three components function?



b. Which outcomes are contained in the event B that at least two of the components function?

c. Which outcomes are contained in the event C that the system functions?

d. List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$

Solution. Write $T_1T_2T_3$ for the outcome, where $T_i \in \{S, F\}$ stands for whether component i is functioning successfully or not, thus the sample space is

$$\mathcal{S} = \{SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF\}$$

a.

$$A = \{SSF, SFS, FSS\}$$

b.

$$B = \{SSF, SFS, FSS, SSS\}$$

c. Since component 1 and subsystem 2-3 are connected in series, the whole system functions if both component 1 and subsystem function, and since component 2,3 are connected in parallel, subsystem 2-3 functions if at least one of components 2,3 works

$$C = \{SSF, SFS, SSS\}$$

d.

$$C' = \{FSS, SFF, FFS, FSF, FFF\}$$
$$A \cup C = \{SSSF, SFS, FSS, SSS\}$$
$$A \cap C = \{SSF, SFS\}$$
$$B \cup C = \{SSF, SFS, FSS, SSS\}$$
$$B \cap C = \{SSF, SFS, SSS\}$$

□

1.2 Practice2 - 9/10/2020

Exercise 1.3. Suppose that vehicles taking a particular highway exit can either turn right (denoted as R), turn left (denoted as L), or go straight (denoted as S). Consider observing three successive cars who takes this exit

(a) Let A denote the event that only one car turns right. List all the outcomes in the event A. (An event is represented as a union of outcomes in sample space occurrence of which makes this event occur)

(b) List all outcomes for the event that all three cars take different directions. Call this event as B

(c) Describe the elements in the sample space and and compute the size of the sample space

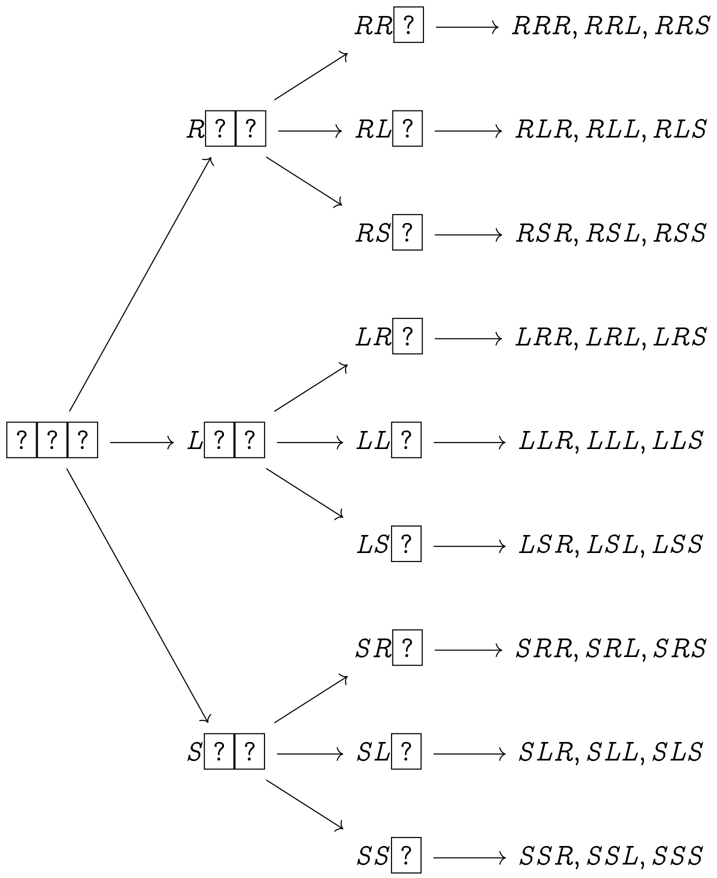
(d) List the outcomes in $A \cap B$, $A \cap B'$

Solution. Represent the outcomes as three letter sequences, where the first letter indicates the first car's choice, the second indicates the second car's choice and the third letter indicates the third car's choice. For example, RLS means the first car turns right, the second car turns left and the third car goes straight

(a)
$$A = \{RLS, RSL, RSS, RLL, SRL, LRS, SRS, LRL, SLR, LSR, SSR, LLR\}$$

(b)
$$B = \{RSL, RLS, SRL, SLR, LRS, LSR\}$$

(c) Suppose the outcome is covered with cards, and you try to reveal what they are one by one



You can see there are in total $3 \times 3 \times 3 = 27$ different outcomes

(d) $A \cap B = B$ as B is contained in A. $A \cup B = (A \setminus B) \cup B$, since $A \setminus B$ and B are disjoint

□

Exercise 1.4. Let P denote the probability function which is defined as a function that satisfies Axiom1, Axiom 2, Axiom 3 in Section 2.2 of Devore's book (these are the axioms we talked about in class). Let A, B, C denote three events defined on a sample space, justify that the following two expressions are correct. Note: Using Venn diagrams to display the sets is fine, please make sure to provide enough explanations and refer to the right axioms

(a) $P(A \cup B) = P(A \cap B') + P(B)$

(b) $P(A) = P(A - (B \cup C)) + P(A \cap C) + P(A \cap B) - P(A \cap B \cap C)$

Solution.

(a) $A \cup B = (A \setminus B) \cup B$, since $A \setminus B$ and B are disjoint, thus by Axiom 3

$$P(A \cup B) = P(A \setminus B) + P(B) = P(A \cap B') + P(B)$$

(b)

$$P(A) = P(1 \cup 2 \cup 3 \cup 4) = P(1) + P(2) + P(3) + P(4) \tag{1.1}$$

where

$$P(1) = P(A \setminus (B \cup C))$$
$$P(2) = P(A \cap B) - P(A \cap B \cap C)$$
$$P(3) = P(A \cap C) - P(A \cap B \cap C)$$
$$P(4) = P(A \cap B \cap C)$$

Then equation (1.1) gives the result

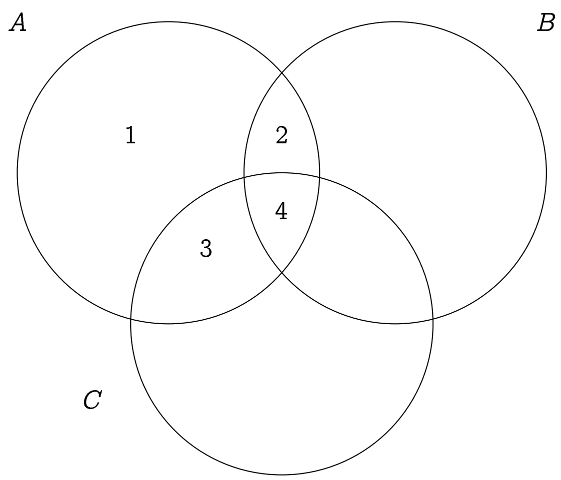


Figure 1.1: The Venn diagram

□

Exercise 1.5. A purse contains five 10\$, four 5\$, and eight 1\$ bills. The bills are selected one by one -without replacement- in random order, Let A denote the event that at least two bills are selected to obtain the first 10\$ bill. Describe this event as the complement of another event which is much a smaller set and easier to comprehend. Next week you will compute the probability of the event A

Solution. The complement event would be the event that a 10$ bill is selected at the first drawal □

Exercise 1.6 ([1] Section 2.2, Exercise 26.).
A certain system can experience three different types of defects. Let $A_i (i = 1, 2, 3)$ denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

a What is the probability that the system does not have a type 1 defect?

b What is the probability that the system has both type 1 and type 2 defects?

c What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

d What is the probability that the system has at most two of these defects?

Solution.

a $P(A_1') = 1 - P(A_1) = 1 - .12 = .88$

b

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06$$

c According to a.

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3') &= P(A_1 \cap A_2 - A_3) \\ &= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) \\ &= .06 - .01 = .05 \end{aligned}$$

d Denote the event that the system has at most two of these defects as E, then E' will be the event that all three types of defects occurs at the same time, i.e. $A_1 \cap A_2 \cap A_3$, hence

$$P(E) = 1 - P(E') = 1 - P(A_1 \cap A_2 \cap A_3) = 1 - .01 = .99$$

□

References

[1] *Probability and Statistics for Engineering and the Sciences (9th edition)* - Jay Devore