Integration by parts

Since $d(fg) = fdg + gdf$, $\quad fdg = d(fg) - gdf$

$$\int \ln x\, dx = \int \left(d(x\ln x) - x\, d\ln x\right) = \int d(x\ln x) - \int x\, d\ln x$$
$$= x\ln x - \int x \cdot \tfrac{1}{x} dx = x\ln x - \int dx$$
$$= x\ln x - x + C$$

$$\int x\cos x\, dx = \int x\, d\sin x = \int \left(d(x\sin x) - \sin x\, dx\right)$$
$$= x\sin x - \int \sin x\, dx = x\sin x + \cos x + C$$

$$\int x e^x dx = \int x\, de^x = \int \left(d(xe^x) - e^x dx\right) = xe^x - \int e^x dx$$
$$= xe^x - e^x + C$$

$$\int x^2 e^x dx = \int x^2 de^x = \int \left(d(x^2e^x) - e^x dx^2\right) = x^2e^x - \int e^x dx^2$$
$$= x^2 e^x - \int 2xe^x dx = x^2e^x - 2(xe^x - e^x) + C$$
$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\int x^3 e^{x^2} dx = \frac{1}{2}\int x^2 e^{x^2} 2x\, dx = \frac{1}{2}\int x^2 e^{x^2} dx^2 \overset{u=x^2}{=\!=\!=} \frac{1}{2}\int ue^u du$$
$$= \frac{1}{2}(ue^u - e^u) + C = \frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C$$

$$\int x\sin(2x)dx = \frac{1}{4}\int (2x)\sin(2x)\, d(2x) \overset{u=2x}{=\!=\!=} \frac{1}{4}\int u\sin u\, du$$
$$= -\frac{1}{4}\int u(-\sin u\, du) = -\frac{1}{4}\int u\, d\cos u = -\frac{1}{4}\int \left(d(u\cos u - \cos u\, du)\right)$$
$$= -\frac{1}{4}\left(u\cos u - \int \cos u\, du\right) = -\frac{1}{4}(u\cos u - \sin u) + C$$
$$= -\frac{1}{4}(2x\cos(2x) - \sin(2x)) + C = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x)$$

$\int x\sqrt{2-x}\, dx$, let $u = \sqrt{2-x}$, then $u^2 = 2-x \Rightarrow x = 2-u^2$, hence

$$\int (2-u^2)u\, d(2-u^2) = \int (2u - u^3)(-2u\, du) = \int (2u^4 - 4u^2)\, du$$
$$= \frac{2}{5}u^5 - \frac{4}{3}u^3 + C = \frac{2}{5}(2-x)^{\frac{5}{2}} - \frac{4}{3}(2-x)^{\frac{3}{2}} + C$$