Euler's Method:

The endpoints $(t_0, y_0), \cdots, (t_n, y_n)$ of line segments approximating the solution of $y' = g(t,y)$, $y(a) = y_0$ on the time interval $a \le t \le b$ are given by the following formulas, $h = (b-a)/n$

$t_0 = a$ (given) $\qquad\qquad$ y_0 (given)

$t_1 = t_0 + h$ $\qquad\qquad$ $y_1 = y_0 + g(t_0, y_0) \cdot h$

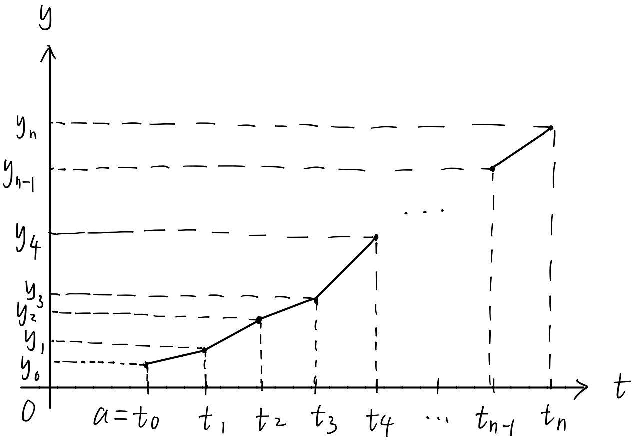
$t_2 = t_1 + h$ $\qquad\qquad$ $y_2 = y_1 + g(t_1, y_1) \cdot h$

$t_3 = t_2 + h$ $\qquad\qquad$ $y_3 = y_2 + g(t_2, y_2) \cdot h$

$t_4 = t_3 + h$ $\qquad\qquad$ $y_4 = y_3 + g(t_3, y_3) \cdot h$

$\vdots$ $\qquad\qquad$ $\vdots$

$t_n = t_{n-1} + h$ $\qquad\qquad$ $y_n = y_{n-1} + g(t_{n-1}, y_{n-1}) \cdot h$



Use Euler's method with $n=4$ to approximate the solution $f(t)$ to $y' = 6t - y + 7$, $y(1) = 9$ for $1 \le t \le 3$. Estimate $f(3)$

$a = 1$, $b = 3$, $n = 4$, $h = (b-a)/n = (3-1)/4 = 0.5$, $y_0 = 9$, $g(t,y) = 6t - y + 7$

$y_1 = y_0 + g(t_0, y_0) \cdot h = 9 + (6 \times 1 - 9 + 7) \times 0.5 = 9 + 4 \times 0.5 = 11$, $t_1 = t_0 + h = 1 + 0.5 = 1.5$

$y_2 = y_1 + g(t_1, y_1) \cdot h = 11 + (6 \times 1.5 - 11 + 7) \times 0.5 = 11 + 5 \times 0.5 = 13.5$, $t_2 = t_1 + h = 1.5 + 0.5 = 2$

$y_3 = y_2 + g(t_2, y_2) \cdot h = 13.5 + (6 \times 2 - 13.5 + 7) \times 0.5 = 13.5 + 2.75 = 16.25$, $t_3 = t_2 + h = 2 + 0.5 = 2.5$

$y_4 = y_3 + g(t_3, y_3) \cdot h = 16.25 + (6 \times 2.5 - 16.25 + 7) \times 0.5 = 16.25 + 3.75 \times 0.5 = 16.25 + 1.875 = 18.125$

Thus $f(3) \approx y_4 = 18.125$