① $\int (1-2x)^5 dx = \frac{1}{-2}\int (1-2x)^5 d(-2x) = -\frac{1}{2}\int (1-2x)^5 d(1-2x)$

$\overset{u=1-2x}{=\!=} -\frac{1}{2}\int u^5 du = -\frac{1}{2}\cdot\frac{1}{6}u^6 + C = -\frac{1}{12}u^6 + C$

$= -\frac{1}{12}(1-2x)^6 + C$

② $\int \frac{e^{\frac{3}{x}}}{x^2} dx = -\int e^{\frac{3}{x}}\cdot\left(-\frac{1}{x^2}dx\right) = -\int e^{\frac{3}{x}} d\left(\frac{1}{x}\right)$

$= -\frac{1}{3}\int e^{\frac{3}{x}} d\left(\frac{3}{x}\right) \overset{u=\frac{3}{x}}{=\!=} -\frac{1}{3}\int e^u du = -\frac{1}{3}e^u + C = -\frac{1}{3}e^{\frac{3}{x}} + C$

③ $\int \frac{8x}{e^{x^2}} dx = 4\int \frac{2x\,dx}{e^{x^2}} = 4\int \frac{1}{e^{x^2}} dx^2 = 4\int e^{-x^2} dx^2 = -4\int e^{-x^2} d(-x^2)$

$\overset{u=-x^2}{=\!=} -4\int e^u du = -4e^u + C = -4e^{-x^2} + C$

④ $\int \frac{1}{x\ln x^2} dx = \int \frac{1}{\ln x^2} d\ln x = \int \frac{1}{2\ln x} d\ln x$

$\overset{u=\ln x}{=\!=} \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln u = \frac{1}{2}\ln(\ln x)$

⑤ $\int \frac{\ln\sqrt{x}}{x} dx = \int \ln x^{\frac{1}{2}} d\ln x = \frac{1}{2}\int \ln x\, d\ln x \overset{u=\ln x}{=\!=} \frac{1}{2}\int u\, du$

$= \frac{1}{2}\cdot\frac{1}{2}u^2 + C = \frac{1}{4}\ln^2 x + C$

⑥ $\int \frac{e^x}{1+e^x} dx = \int \frac{1}{1+e^x} de^x \overset{u=e^x}{=\!=} \int \frac{1}{1+u} du = \int \frac{1}{1+u} d(u+1)$

$\overset{v=u+1}{=\!=} \int \frac{1}{v} dv = \ln v + C = \ln(u+1) + C = \ln(e^x+1) + C$

⑦ $\int \frac{e^{-x}}{1+e^{-x}} dx = -\int \frac{e^{-x}}{1+e^{-x}} d(-x) \overset{y=-x}{=\!=} -\int \frac{e^y}{1+e^y} dy \overset{\text{by ⑥}}{=\!=}$

$-\ln(e^y+1) + C = -\ln(e^{-x}+1) + C$

⑧ $\int \frac{1}{1+e^x} dx = \int \frac{\frac{1}{e^x}}{\frac{1+e^x}{e^x}} dx = \int \frac{e^{-x}}{1+e^{-x}} dx \overset{\text{by ⑦}}{=\!=} -\ln(e^{-x}+1) + C$

Or $\int \frac{1}{1+e^x}dx = \int \frac{1+e^x-e^x}{1+e^x}dx = \int \left(\frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}\right)dx$

$= \int \left(1 - \frac{e^x}{1+e^x}\right)dx = \int dx - \int \frac{e^x}{1+e^x}dx \;\underset{=\!=\!=}{\overset{\text{by ⑥}}{=\!=\!=}}$

$x - \ln(e^x+1) + C$

⑨ $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$, notice $d(e^x+e^{-x}) = de^x + de^{-x}$

$= e^x dx + (-e^{-x})dx = e^x dx - e^{-x}dx = (e^x - e^{-x})dx$

hence $\int \frac{e^x - e^{-x}}{e^x+e^{-x}}dx = \int \frac{1}{e^x+e^{-x}} d(e^x - e^{-x}) \;\underset{=\!=\!=}{\overset{u=e^x+e^{-x}}{=\!=\!=}}$

$\int \frac{1}{u}du = \ln u + C = \ln(e^x+e^{-x}) + C$

⑩ $\int \frac{\sin x - \cos x}{\sin x + \cos x}dx$, notice $d(\sin x + \cos x) = d\sin x + d\cos x$

$= \cos x\,dx + (-\sin x)dx = (\cos x - \sin x)dx$

hence $\int \frac{\sin x - \cos x}{\sin x + \cos x}dx = -\int \frac{1}{\sin x + \cos x} d(\sin x + \cos x)$

$\underset{=\!=\!=}{\overset{u=\sin x+\cos x}{=\!=\!=}} -\int \frac{1}{u}du = -\ln u + C = -\ln(\sin x + \cos x) + C$