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MATH240 Summer 2023

Haoran Li

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1 Lecture 1 - System of linear equations

1.1 Linear systems

Throughout this course, we adopt the following notations

- **Natural numbers:** $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.
- **Integers:** $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.
- **Rational numbers:** $\mathbb{Q} = \{\frac{m}{n} | m, n \in \mathbb{Z}, n \neq 0\}$ is the set of fractions. Here \in means **belong to**.
- **Real numbers:** \mathbb{R} is the set of numbers on the whole real number line. It includes
 - irrational numbers (like $\sqrt{2}$, $\sqrt[3]{3}$)
 - transcendental numbers (like π , e)
- **Complex numbers:** $\mathbb{C} = \{a + bi | a, b \in \mathbb{R}\}$, $i = \sqrt{-1}$ is the imaginary number such that $i^2 = -1$.
- $\mathbb{N} \subsetneq \mathbb{Z} \subsetneq \mathbb{Q} \subsetneq \mathbb{R} \subsetneq \mathbb{C}$.
- $\mathbb{R}^n = \{(r_1, r_2, r_3, \dots, r_n) | r_1, r_2, r_3, \dots, r_n \in \mathbb{R}\}$ is the set of all n -tuples of real numbers. Geometrically, it is the n -dimensional Euclidean space. For example
 - $\mathbb{R}^1 = \mathbb{R}$ is a line.
 - \mathbb{R}^2 is a plane.
 - \mathbb{R}^3 is our usual physical space.

Definition 1.1. A **linear equation** in the variables $x_1, x_2, x_3, \dots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b \quad (1.1)$$

where the coefficients $a_1, a_2, a_3, \dots, a_n$ and b are real or complex numbers, usually known in advance.