

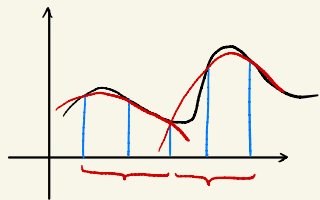
3a. $K_T = \text{maximum of } |f''(x)| \text{ on } [a, b]$, here $f(x) = \frac{3}{x}$, so $f''(x) = \frac{6}{x^3}$ ↓ decrease
 $a=1, b=2$

$$\text{so } K_T = |f''(1)| = \left| \frac{6}{1^3} \right| = 6$$

$$E_n^T \leq \frac{6}{12n^2} (2-1)^3 = \frac{1}{2n^2} \leq \frac{1}{100} \Rightarrow n^2 \geq \frac{100}{2} = 50 \Rightarrow n \geq \sqrt{50} = 7.07 \dots \Rightarrow n \text{ is at least } 8$$

$$\begin{aligned} 3b. \int_0^3 e^{t^2} dt &= \frac{3-0}{3 \times 6} \left[e^{0^2} + 4e^{(\frac{1}{2})^2} + 2e^{1^2} + 4e^{(\frac{3}{2})^2} + 2e^{2^2} + 4e^{(\frac{5}{2})^2} + e^{3^2} \right] \\ &= \frac{1}{6} \left(1 + 4e^{\frac{1}{4}} + 2e + 4e^{\frac{9}{4}} + 2e^4 + 4e^{\frac{25}{4}} + e^9 \right) \end{aligned}$$

3c. It is necessary that n be even for Simpson's Rule, since it uses parabolas, each of which is defined by 3 distinct points, yielding 2 intervals on the x axis



For each parabola, you need 3 points, that's 2 intervals hence you need even number of intervals for Simpson's Rule, BUT there is no need for trapezoidal Rule

$$4a. \int_1^\infty \frac{x^{\frac{3}{2}}}{\pi + x^3} dx \stackrel{\substack{\uparrow \\ \text{comparison property}}}{\leq} \int_1^\infty \frac{x^{\frac{3}{2}}}{x^3} dx = \int_1^\infty x^{-\frac{3}{2}} dx = -2x^{-\frac{1}{2}} \Big|_1^\infty = 0 - (-2) = 2, \text{ so convergent}$$

4b. Trapezoidal Rule yields a bigger number than $\int_a^b f(x) dx$. Graph:

4c

n	0	1	2	3	4	...	n
$f^{(n)}(x)$	e^{-x}	$-e^{-x}$	$(-1)^2 e^{-x}$	$(-1)^3 e^{-x}$	$(-1)^4 e^{-x}$...	$(-1)^n e^{-x}$
$f^{(n)}(0)$	1	-1	$(-1)^2$	$(-1)^3$	$(-1)^4$...	$(-1)^n$

$$\text{Hence } P_n(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n x^n}{n!}$$