Project 1 - Linear Programming

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INTRODUCTION

Our company needs to find the most optimal budget allocation strategy for its marketing portfolio that comprises of 10 marketing channels: *Print, TV, SEO, AdWords, Facebook, LinkedIn, Instagram, Snapchat, Twitter, and Email*. We use linear programming to optimize the allocation of the budget to these platforms.

Our department reached out two consulting firms to assess the return on investments for these promotion channels. The ROI calculations from both the firms are tabulated below:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

Table 1: ROI estimates provided by consulting firm -1

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

Table 2: ROI estimates provided by consulting firm -2

PROBLEM FORMULATION

We have $10 million in budget that could be used for the listed platforms. In each medium however, the allocation should not exceed more than $3 million. We were also tasked with investing no more than the amount of Facebook and Email in Print and TV. Moreover, the total amount used in social media spaces should be twice as much investment of SEO and AdWords. Below is the mathematical formulation of the problem:

$$Let\ \{x_1, x_2, x_3, x_4 \dots\dots\dots x_{10}\}\ represent\ the\ allocations\ in\ each\ of\ the\ 10\ marketing\ channels$$

$$Objective\ Function:\ Max(\sum_{n=1}^{10} x_i * r_i)\ where\ r_i\ represents\ ROI\ \text{for}\ r_{\text{th}}\ \text{marketing channel}$$

$$Overall\ Budget\ Constraint: \sum_{n=1}^{10} x_i \leq 10$$

$$Individual\ Budget\ Constraint: x_i \leq 3$$

$$Allocation\ Constraint\ 1: Allocation(TV + Print) \leq Allocation(FB + Email)$$

$$Allocation\ Constraint\ 2: Allocation(\text{Social Media}) \geq\ 2 *\ Allocation\ (SEO + AdWords)$$

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ANALYSIS

#3

Using Gurobi, we calculated the optimal budget allocation for each of the platforms, as shown in Figure 1 below. As indicated by the output from our model, TV, Instagram, and Email are allocated with $3 million each while AdWords is $1 million budget. It can be assumed that Instagram is the most profitable social media channel while TV and Email are good channels to communicate with potential customers as well. This is no surprise as these three platforms provide the top three return of investments with TV at 4.9%, Instagram at 4.6%, and Email at 4.4%. Since AdWords has the fourth-highest return of investment, it will not be as effective as TV, Instagram, and Email and therefore will have the least amount of money allocated to this platform. The allocation of these four channels earns our firm a total amount of $456,000. Figure 1 illustrates how the $10 million dollar budget is distributed among the various marketing mediums.

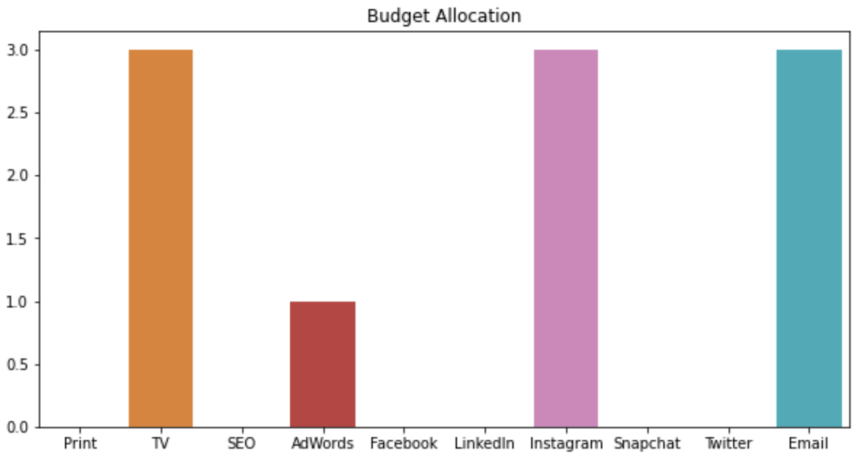

Figure 1. Budget Allocation.
The x-axis shows each platform, and the y-axis shows allocation in millions.

#4

The second estimation of return of investments for each channel was provided to our team to check the robustness of our allocation strategy derived from the first set of ROIs.

Here, we observe that the Top 3 platforms based on ROI have changed from TV, Instagram, and Email to Print, LinkedIn and Facebook. Meanwhile, the fourth highest ROI remains to be AdWords. With ROI values reported at 4.9%, 4.6%, 4.4%, and 3.9% respectively, the comparisons of the first and second budget allocations is provided by the graph in Figure 2. We allocated the top three platforms like our first allocation strategy with $3 million each, and the AdWords with $1 million. Despite change in allocation across promotion channels, our optimal value of profit generated remains the same at $456,000 as shown in Figure 3.

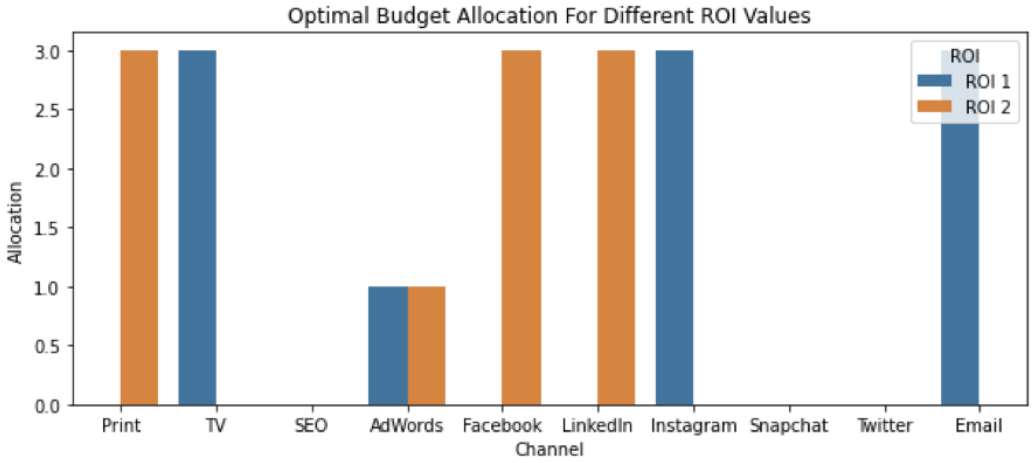

Figure 2. Budget Allocations Comparison.
The x-axis shows each platform, and the y-axis shows allocation in millions. The parts shown in blue denote the first allocation strategy based on the first ROI values, and the parts shown in orange denote the second allocation strategy based on the second ROI values.

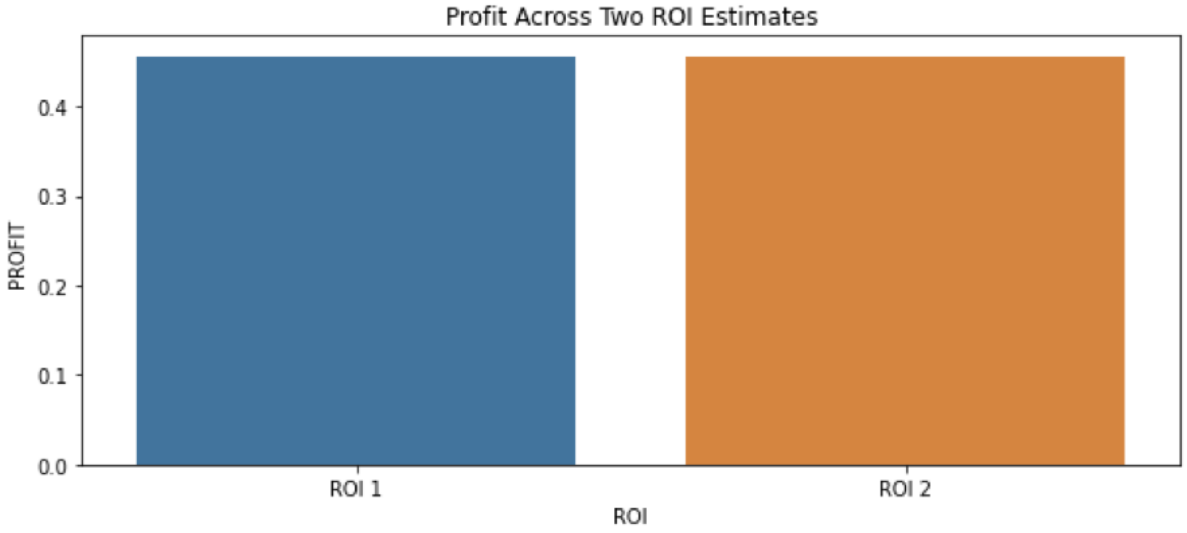

Figure 3. Profits Based on the Two Allocation Strategies.
The x-axis shows the ROI values used, and the y-axis shows profit in millions.

#5

With optimal profit values remaining unchanged between the two allocation strategies, we analyzed how much it would change if we switched ROI values and platform allocation.

First, we assumed that the ROI values provided by first consulting firm are correct, and allocated budget amounts suggested by second consulting firm ROI estimates. It yielded us ~$252,000 profit, which is a negative value difference of $204,000. This is because the second

allocation strategy utilizes platforms that are not within the top 3 highest ROI values reported by first consulting firm. Instead, we allocated our $10 million budget on Print, LinkedIn, and Facebook with respective ROI values at 3.1%, 2.4%, and 1.6%. These are relatively underwhelming compared to the actual top 3 ROI from the first list of values.

On the other hand, using the second ROI values with the first allocation amounts only yielded us $264,000, a negative value difference of $192,000. While the difference is not as wide as the first variety - using the first ROI values and second allocation amounts - this difference is again because we are allocating based on ROI values that are not the highest in the second report. Our budget of $10 million is going towards TV, Instagram, and Email, which will only give us an ROI percentage of 2.3%, 2.6%, and 2.6%, respectively.

One silver lining between these two comparisons is that AdWords is the fourth highest ROI value at 3.9% for both reported estimates. AdWords as the common denominator for both revised strategies shows that these mixed allocations are more similar in optimal values with a shear difference of $12,000 than their original counterparts.

With that being said, we believe that a max allocation of $3 million for each platform is useful because it gives us the ability to diversify our marketing channels. Without this constraint, we would only be allocating most of our $10 million budget in the channels with highest ROI values.

```
## Using ROI-1 based attribution on ROI-2
profit_a = sum(roi_1* np.array(list(budget_dict_roi2.values())))

print("When using ROI for first case and attribution for second, the change in obj function is {:.4f}".format(profit_a-
## Using ROI-2 based attribution on ROI-1
profit_b = sum(roi_2* np.array(list(budget_dict_roi1.values())))
#print(np.round(profit_b-ojModel.objVal,2))
print("When using ROI for Second case and attribution for First, the change in obj function is {:.4f}".format(np.round(
```

```
When using ROI for first case and attribution for second, the change in obj function is -0.2040
When using ROI for Second case and attribution for First, the change in obj function is -0.1900
```

Figure 4. Code Chunk of Optimal Value Differences

The ROI difference between 1st ROI values & 1st Allocation vs 1st ROI values & 2nd allocation is - $204,000 in rounded decimal form. The ROI difference between 2nd ROI values & 2nd Allocation vs 2nd ROI values & 1st allocation is - $192,000 in rounded decimal form.

#6

Since the ROI values reported by both the consulting firms are estimates of the true performance and lead to optimal allocation in different channels. We need to perform sensitivity analysis and determine by how much our ROI rates can change before the optimal allocation of budget changes. Figure 5 displays the lower and upper bound ROI for each marketing channel to maintain the optimal allocation obtained from first ROI values.

For Print, SEO, Facebook, LinkedIn, Snapchat, and Twitter, these channels will never become non-binding on the lower bound. This means that from their respective starting points, the ROI can infinitely decrease if other channels remain the same. For Instagram and Email, the opposite can be said that the ROI values can infinitely increase from their starting points. In both cases, the optimal allocation will never change as long as the ROI points are in the ranges specified by the table below.

In other channels like TV and AdWords, these are more confined in the sense that they have lower and upper bounds. If TV's ROI does not decrease to 3.9% or increase to 6.2%, the optimal value of allocation stays the same (assuming other channels' ROI values do not change). The same can be said for AdWords' ROI values where it can vary between 3.3% and 4.6% with no impact to the optimal allocation.

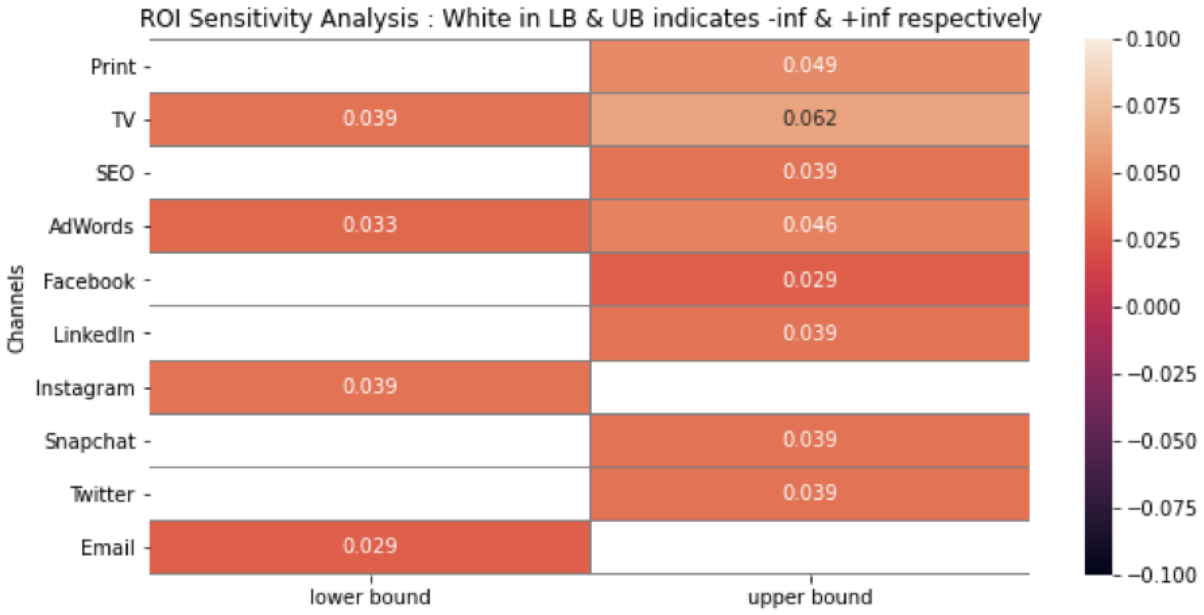

Figure 5. Sensitivity Analysis of ROI Value by Channel
ROI values within the range will not change optimal Allocation.

#7

Next, we use the *forecasted ROI*, as provided by consulting firm -1, to calculate the optimal allocation. We use *actual ROI* to calculate the profit generated by each channel every month. 50% of this profit is reinvested in the following month. The optimal allocation for each month and promotion channel are shown in Figure 6 below. TV, Instagram, and Email has an allocation budget of $3 million across all months starting from January to December. AdWords starts off with an allocation of $1 million from January and then grows to $1.5 million in April until finally growing an allocation budget of $2 million in December. For the Twitter channel, there is

$0 in allocation of the budget until April where it starts to grow at a steady rate until December where it reaches a final allocation budget of ~$1 million.

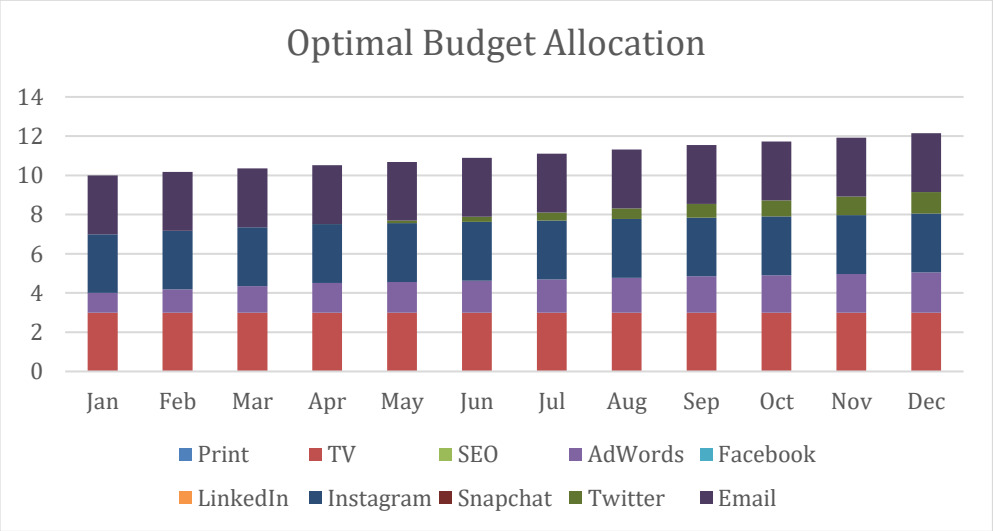

Figure 6 Monthly budget trend across promotion channels

#8

The allocation we found in our optimization model can be defined as a stable budget. This means that for each platform, the monthly change in spending is no more than $1 million dollars. As shown in Figure 6, all the platforms remain consistent month over month. In promotional channels such as AdWords, Twitter there is a gradual increase of less than $1 Million month over month. If it was an unstable budget, then we would add one more additional constraint to the model for each marketing channel to constraint the difference in the consecutive months to less than or equal to $1 million for each channel.