Subspace: a subset *W* of R^n is a subspace of R^n if

$$\vec{0} \in W, \qquad \forall x, y \in W, x + y \in W, \qquad \forall x \in W, c \in R, cx \in W$$

For *W* in R^2, only a linear line through the origin will be a subspace of R^2

$e.x.$ Is $W = \left\{\begin{pmatrix} x1 \\ x2 \end{pmatrix} \middle| \; x_1^2 = x_2^2\right\}$ a subspace of R^2.

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \; not\ in\ W, so\ not\ a\ subspace$$

$e.x.$ Is $W = \{x \in R^n | \, Ax = b\}$ a subspace of R^n.

No, because $\vec{0} \notin W$

$e.x.$ Is $W = \{x \in R^n | \, Ax = 0\}$ a subspace of R^n.

Yes, *W* is a subspace of R^n, and *W* is the subspace of *A*

Define $null(A) = \{x \in R^n \mid Ax = 0\}$ is a subspace of R^n.$A\ m \times n$

Definition 2: $v1v2 \ldots vk \in R^n, W = span\{v1, v2, \ldots, vk\}$ is the subspace of R^n

$e.x.\ A\ m \times n, c1, c2, \ldots, cn\ be\ columns\ of\ A,$ is $W = \{b \in R^m \; | b = Ax\}$ a subspace of R^m

$$b = Ax = (c1c2 \ldots cn)\begin{pmatrix} x1 \\ x2 \\ \ldots \\ xn \end{pmatrix} = c1x1 + c2x2 + \cdots cnxn = span\{c1, c2, \ldots, cn\}$$

$$x1, x2, \ldots, xn \in R, c1, c2, \ldots, cn \in R^m$$

Define $col(A) = \{b \in R^m \; | b = Ax\} = span\{c1, c2, \ldots, cn\}$ is subspace of R^m

Finding the smallest set that $span\{col(A)\}$, it's equal to finding all the columns of *A* that are linear independent.

Basis: *W* is a subspace of R^n, set of vectors *S* from R^n is basis for *W*, if $W = span\{S\}$ & S is linear independent.

Let $A\ m \times n, R\ rref\ of\ A, a1, a2, \ldots, an$ be columns of A, $r1, r2, \ldots, rn$ be columns of *R*

$$\forall x \in R^n, Ax = 0 \to Rx = 0, x1r1 + x2r2 + \cdots + xnrn = 0$$

Whatever you can say about the independence or dependence of R, you can say about A

$$e.x. A = \begin{pmatrix} 1 & 1 & 1 & 1 & 5 \\ 2 & 3 & 1 & 2 & 11 \\ 1 & 1 & 1 & 3 & 7 \\ 1 & 2 & 0 & -1 & 4 \end{pmatrix} \gg R = \begin{pmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Basis\ of\ A = \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = null(A) = \begin{pmatrix} -2s - 3t \\ s - t \\ s \\ -t \\ t \end{pmatrix} = s\begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} = span\left\{\begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}\right\}$$

columns in span = # free variables in *A*