Week 1: Tutorial Handout on Real number and least upper bound principle

Real numbers properties

1. Prove that if $x \leq y$, then there is a rational number r with a finite decimal expansion and an integer k so that

$$x < r < r + 10^{-k} < y.$$

   - What is r and k for the case $x = x_1...x_n700...$ and $y = y_1...y_n80....$
   - Find r and k for the case $x = x_1...x_n...$ and $y = x_1...x_{n-1}y_nx_{n+1}...$ where $y_n = x_n + 1$.
   - Find r and k for the general case $x = x_1...x_n...$ and $y = y_1...y_n...$ where $x_n + 1 \leq y_n$ and $x_m \leq y_m$ for all $m \neq n$.

2. Use the triangle inequality

$$|a + b| \leq |a| + |b|$$

   to prove the reverse triangle inequality

$$||a| - |b|| \leq |a - b|.$$

   (Hint: split cases i)$|a| > |b|$ and ii)$|a| \leq |b|$).

Least upper bound principle

3. For the following sets, state their supremum and infimum (no proof). Which of them have a max or min (prove or disprove)?

   - $S_0 := (0, 1)$
   - $S_1 := \{\frac{n}{m+n} : n, m \in \mathbb{N}^+\}$
   - $S_2 := \{a + \frac{1}{a} : a \in \mathbb{Q}, a > 0\}$
   - $S_3 := \{e^{-x} : x \in \mathbb{R}\}$

4. Let A be a lower bounded set and $B := \{-a : a \in A\}$, then prove that $\sup(B) = -\inf(A)$.