DFA & NFA

Every DFA can be re-defined as a NFA by replacing $\delta: Q \times S \to Q$ by $\delta': Q \times S \to \mathcal{P}(Q) \coloneqq \delta'(q,a) = \{\delta(q,a)\}\ \forall a \in \mathbb{Z}. \forall q \in Q$

Theorem $\forall \text{NFA } M = (Q, \Sigma, \delta, q_0, F). \exists \text{DFA } \widehat{M} = (\widehat{Q}, \Sigma, \hat{\delta}, \widehat{q_0}, \widehat{F})$ s.t. $\mathcal{L}(M) = \mathcal{L}(\widehat{M})$

Proof Let $M = (Q, \Sigma, \delta, q_0, F)$ be an arbitrary NFA.

Let $\widehat{M} = (\widehat{Q}, \Sigma, \hat{\delta}, \widehat{q_0}, \widehat{F})$ where

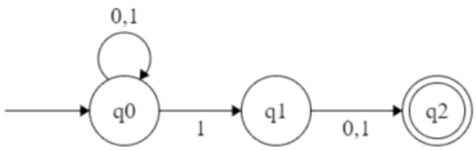
$\widehat{Q} = \mathcal{P}(Q)$,

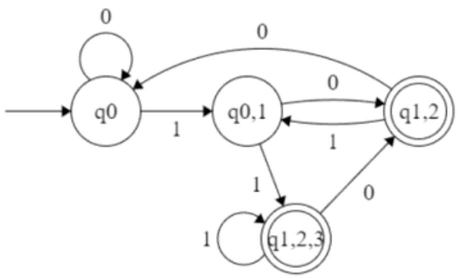
$\widehat{q_0} = \{q_0\}$,

$\hat{\delta}(Q', a) = \cup_{\{\delta(q,a) | q \in Q'\}}\ \ \forall Q' \in \widehat{Q}.\ \forall a \in \Sigma$

$\widehat{F} = \{ Q' \in \widehat{Q} \mid Q' \cap F \neq \emptyset \}$

Example $L = \mathcal{L}((0+1)^* 1 (0+1))$





Proof $\forall w \in \Sigma^*$. Let $P(w) \coloneqq \forall q \in Q. \hat{\delta}^*(q_0, w) = \delta^*(q_0, w)$

Base case: $\hat{\delta}^*(q_0, \lambda) = q_0 = \delta^*(q_0, \lambda)$

Constructor case: Let $w \in \Sigma^*$ be arbitrary, assume $P(w)$

Consider $wx \in \Sigma^*$, where $x \in \Sigma$

$\hat{\delta}^*(q_0, wx) = \hat{\delta}(\hat{\delta}^*(q_0, w), a) = \cup_{\{\delta(q,a) | q \in \hat{\delta}^*(q_0, w)\}}$

By induction hypothesis, $\hat{\delta}^*(q_0, w) = \delta^*(q_0, w)$

$\cup_{\{\delta(q,a) | q \in \hat{\delta}^*(q_0, w)\}} = \cup_{\{\delta(q,a) | q \in \delta^*(q_0, w)\}}$

Hence $\delta^*(q_0, wa) = \delta(\delta^*(q_0, w), w) = \cup_{\{\delta(q,a) | q \in \delta^*(q_0, w)\}} = \hat{\delta}^*(q_0, wx)$

Variants of NFA

1. NFA with multiple start states $M = (Q, \Sigma, \delta, Q_0, f)$ $Q_0 \subseteq Q$ instead of $q_0 \in Q$
   $x \in \mathcal{L}(A)$ IFF $\exists$ path from some start state to some final state s.t. x represents the path
2. NFA with λ transitions $\delta: Q \times \{\Sigma \cup \{\lambda\}\} \to \mathcal{P}(Q)$
   $\forall q \in Q$. Let $E(q) = \{ q' \mid q' \text{can be reached from } q \text{ by a path labelled } \lambda \}. \forall q \in Q. q \in E(q)$
   $M' = (Q, \Sigma, \delta', Q_0, f)$ where $\delta'(q, \lambda) = \cup_{\{E(q') | q' \in \delta(q,a)\}}. \forall q \in Q. \forall a \in \Sigma$
   Example