03 June, 2019

- ***Administration items***
    - *Pick up midterm papers during TA office hours tomorrow*
    - *Exact Time (TBA)*
- ***Application of ARMA models in investments***
- Alternative assets modeling: : $y_t$ and $r_t$ denote observable appraisal and latent economic returns, respectively
    - Issues: persistent and smooth ➔ stale-pricing bias
- Goal: To infer unobservable economic returns using appraisal returns

  Two popular approaches to model appraisal returns:

  1) Geltner method: Commercial real estate

$$y_t = \phi y_{t-1} + \underbrace{(1-\phi) r_t}_{a_t} = \sum_{j=0}^{\infty} \phi^j \underbrace{(1-\phi) r_{t-j}}_{a_{t-j}} = \sum_{j=0}^{\infty} w_j r_{t-j} \,,$$

$$\phi \in (0,1)$$

- $w_j$: weight on $r_{t-j}$

$$y_t = \hat{\phi} y_{t-1} + \hat{a}_t, \qquad \hat{r}_t = \frac{\hat{a}_t}{1-\hat{\phi}}$$

2) Getmansky, Low, & Markorov (2005, Journal of financial economics):

$$y_t = \sum_{i=0}^{q} w_i r_{t-i} \,, w_i \in (0,1), \sum w_i = 1.$$

$$y_t = \theta_0 a_t + \theta_1 a_{t-1} + \cdots + \theta_q a_{t-q}, \qquad \theta_0 = 1$$

$$= \frac{\sum_{i=0}^{q} \theta_i}{\sum_{i=0}^{q} \theta_i} \left( \theta_0 a_t + \theta_1 a_{t-1} + \cdots + \theta_q a_{t-q} \right)$$

$$
= \overbrace{\frac{\theta_0}{\sum_{i=0}^{q} \theta_i}}^{w_0} \overbrace{(\sum_{i=0}^{q} \theta_i \cdot a_t)}^{r_t} + \overbrace{\frac{\theta_1}{\sum_{i=0}^{q} \theta_i}}^{w_1} \overbrace{(\sum_{i=0}^{q} \theta_i \cdot a_{t-1})}^{r_{t-1}} + \cdots
$$

$$
+ \overbrace{\frac{\theta_q}{\sum_{i=0}^{q} \theta_i}}^{w_q} \overbrace{(\sum_{i=0}^{q} \theta_i \cdot a_{t-q})}^{r_{t-q}}
$$

- Factor modeling: $r_t = \alpha + \beta r_{Mt} + e_t$

$$y_t = \sum_{i=0}^{q} w_i\left(\alpha + \beta r_{M,t-i} + e_{t-i}\right)$$

$$= \sum w_i \alpha + \sum_{i=0}^{q} w_i\, \beta r_{t-i} + \sum_{i=0}^{q} w_i e_{t-i}$$

$$= \alpha + \sum_{i=0}^{q} \underbrace{\beta_i}_{w_i \beta}\ r_{M,t-i} + \sum_{i=0}^{q} w_i e_{t-i}$$

- Transfer function noise model
    - Distributed lag model

$$y_t = \sum_{i=0}^{\infty} v_i x_{t-i} + a_t, \qquad a_t \sim NID(0, \sigma^2)$$

$$x_t \sim ARMA(p, q)\ model$$

$$v_i = \phi^i (1 - \phi)$$

$$y_t = \sum_{i=0}^{m} v_i x_{t-i} + a_t$$

- Pre-whitening:

$$y_t = \sum_{i=0}^{m} v_i x_{t-i} + a_t,$$

$$\phi(B)x_t = \theta(B)e_t, \qquad cov(a_t, e_s) = 0, \forall a_t, e_s$$

$$\underbrace{\frac{\phi(B)}{\theta(B)}}_{\sum_{i=0}^{\infty} \pi_i B^i} x_t = e_t$$

$$\underbrace{\frac{\phi(B)}{\theta(B)}y_t}_{\tau_t} = v(B)\underbrace{\frac{\phi(B)}{\theta(B)}x_t}_{e_t} + \underbrace{\frac{\phi(B)}{\theta(B)}a_t}_{\tilde{a}_t=\sum \pi_i a_{t-i}}, \qquad v(B) = \sum_{i=0}^{m} v_i B^i$$

$$\tau_t = v(B)e_t + \tilde{a}_t$$

$$\underbrace{E(e_t\tau_t)}_{cov(e_t,\tau_t)=\gamma_{e\tau}(0)} = \underbrace{E[(v_0 e_t + v_1 e_{t-1} + \cdots)e_t]}_{v_0\sigma_e^2} + \underbrace{E(e_t\tilde{a}_t)}_{=0}$$

$$\underbrace{E(e_{t-1}\tau_t)}_{\gamma_{e\tau}(1)} = \underbrace{E[(v_0 e_t + v_1 e_{t-1} + \cdots)e_{t-1}]}_{v_1\sigma_e^2} + \underbrace{E(e_{t-1}\tilde{a}_t)}_{=0}$$

$$\gamma_{e\tau}(k) = v_k\sigma_e^2$$