

Outline: Week 1 R

Least upper bound

1. We prove the least upper bounded principle (D&D 2.3)
2. we prove uniqueness of supremum
3. We study exercise 1.3.5 from Abbott: Proving $\sup(cA) = c\sup(A)$ for $c > 0$. WTS: $\forall \varepsilon > 0$ find $s_\varepsilon \in cA$ s.t. $c\sup(A) - \varepsilon \leq s_\varepsilon$. We pick $\tilde{\varepsilon} = \frac{\varepsilon}{c}$ to get for $v := \sup(A)$

$$v - \frac{\varepsilon}{c} \leq a_{\varepsilon/c} \Rightarrow cv - \varepsilon \leq a_{\varepsilon/c}c$$

and thus let $s_\varepsilon := a_{\varepsilon/c}c$. By having $\sup(S) = -\inf(-S)$ we have $\sup(cA) = \sup((-c)(-A)) = (-c)\sup(-A) = c\inf(A)$ for $c < 0$.