Risk Modeling for Private Assets

Two-step approach

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Introduction

Asset allocation, performance evaluation and risk modeling are the important components in a portfolio management process.[1] For public and liquid assets, asset returns (prices) are marked-to-market, and these three components are well-studied in the literature and used by practitioners. However, this is not the case for private assets, such as private equity and real estate, where asset returns are usually appraised rather than marked-to-market. Appraisals tend to lag and are smoother than the current market condition because appraised valuation is usually made based on prior sales of comparables. In principle, lagging and smoothing would lead to a stale-pricing bias,[2] and the finance literature has indicated that a stale-pricing bias would cause the underestimation of the overall and systematic risk. In this paper, we review and discuss some current practices of risk modeling for private assets.

The structure of the paper is summarized as follows. In Section 2, we first review how returns on private assets are appraised (smoothed) and discuss how to retrieve economic returns that correct the stale-pricing bias by reversing the appraisal scheme. Section 3 summarizes the current practices on how to measure systematic risk from appraisal returns. Finally, in Section 4 we dicuss how to calculate the idiosyncratic risk from aggregate (appraisal) indices.

[1] Maginn, Tuttle, McLeavey, and Pinto (2007), "Managing Investment Portfolios–A Dynamic Process", John Wiley & Sons, Inc.

[2] Alternatively, the valuation of private assets may be established based on a cash flow analysis. For example, we could use the existing rents contained in the current lease agreements to evaluate the value of a property. However, this method also has its "stale effect" because the leases associated with a property may have been negotiated several years in arrears and do not reflect the current state of the rental markets.

Review of appraisal smoothing in private assets

For simplicity, we consider two of the most popular appraisal smoothing schemes in the literature. For a comprehensive literature review about appraisal smoothing, see Geltner et al. (2003) for more details.[3] To facilitate our subsequent discussion, let's first define three symbols here

- y_t denotes the (observed) appraisal (log) return on private assets at time t;
- r_t denotes the (unobserved) economic return on private assets at time t that mimics the marked-to-market return;
- f_t denotes the (observed) systematic (risk) factor at time t.

Geltner (1989)[4]

Geltner (1989) assumes that the appraisers appraise a property using the following formula

$$y_t = \phi y_{t-1} + (1-\phi) r_t, \quad (1)$$

where $\phi \in [0,1]$ denotes the confidence factor that appraisers have about their last appraisal.[5] Using recursive substitution, we can express Eqn. (1) as

$$y_t = \sum_{i=0}^{\infty} (1-\phi)\phi^i r_{t-i}. \quad (2)$$

Eqn. (2) expresses appraisal returns as 'smoothed' economic returns.

Under the assumption that $\{r_t\}$ behaves like a white noise[6] and Eqn. (2) becomes

$$y_t = \phi y_{t-1} + a_t, \quad (3)$$

where $a_t = (1-\phi) r_t$, and Eqn. (3) follows an autoregressive (AR) process of order one.

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[3] Geltner D., B. D. MacGregor and G. M. Schwann (2003), "Appraisal Smoothing and Price Discovery in Real Estate Markets", *Urban Studies* **40**, 1047-1064.

[4] Geltner (1989),"Estimating Real Estate's Systematic Risk from Aggregate Level Appraisal-Based Returns", *AREUEA Journal* **17**, 463-481.

[5] ϕ is close to 1 if appraisers believe that their last appraisal is closer to the current market price than prior sales of comparables.

[6] This assumption is widely used in the unsmoothing literature and will be adopted in this document.

Getmansky, Lo, and Markarov (2004)[7]

Getmansky et al. (2004) assume that the relationship between the appraisal returns and the economic returns follows the econometric model of Dimson (1979)[8] that generalizes the smoothing scheme in Eqn. (2). Specifically, we have

$$y_t = w_0 r_t + w_1 r_{t-1} + \cdots + w_Q r_{t-Q}, \quad (4)$$

where $w_i \geq 0, i = 0,1, \dots, Q$, and $\sum_{i=0}^{Q} w_i = 1$.[9]

Under the assumption that r_t satisfies EMH, Dimson's model can be expressed as an moving average (MA) process of order Q

$$y_t = \theta_0 a_t + \theta_1 a_{t-1} + \cdots + \theta_Q a_{t-Q}, \quad \theta_0 = 1, \quad (5)$$

where

$$a_t = \frac{r_t}{\sum_{i=0}^{Q} \theta_i}, \quad (6)$$

and

$$w_i = \frac{\theta_i}{\sum_{i=0}^{Q} \theta_i}, \quad i = 0,1, \dots, Q. \quad (7)$$

Using Eqn (6) and (7), we can calculate the economic returns and understand the implied smoothing scheme that converts economic returns to appraisal returns.

Discussion:

- The estimated (or unsmoothed) economic returns can be used for performance evaluation as well as measuring the overall/systematic risk associated with private assets. For example, Marcato and Key (2007)[10] use unsmoothed economic returns for asset allocations and Getmansky et al. (2004). use unsmoothed returns for performance evaluation.

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[7] Getmansky, Lo, and Karkarov (2004), "An econometric model of serial correlation and illiquidity in hedge fund returns", *Journal of Financial Economics* **74**, 529-609.

[8] Dimson (1979), "RISK MEASUREMENT WHEN SHARES ARE SUBJECT TO INFREQUENT TRADING", *Journal of Financial Economics* **7**, 197-226.

[9] The Geltner model in Eqn. (2) is a special case of Dimson's model with $w_i = (1-\phi)\phi^i, i = 0,1, \dots$.

[10] Marcato and Key (2007), "Smoothing and Implications for Asset Allocation Choices", *Journal of Portfolio Management*, 85-99.

- The economic returns in Geltner (1989) can be estimated as

$$r_t = \frac{y_t - \hat{\phi}\, y_{t-1}}{1 - \hat{\phi}}, \quad (8)$$

- where the autoregressive coefficient estimate $\hat{\phi}$ can be estimated using least squares or other estimation methods.
- The economic returns in Getmansky et al. (2004) can be estimated as

$$r_t = \hat{a}_t \sum_{i=1}^{Q} \hat{\theta}_i,$$

- where $\hat{a}_t$ and $\{\hat{\theta}_i\}, i = 1, \dots, Q$ can be estimated using the maximum likelihood estimation, or innovation algorithm. See Brockwell and Davis (1992) for more details.

Measurement of systematic risk

For simplicity and without loss of generality, let's consider a single factor model

$$R_t = a + bf_t + e_t, \quad (9)$$

where R_t denotes asset returns, a and b are constant, and e_t denotes the idiosyncratic risk term. For public and liquid assets, systematic risk is given by $b\sigma_f$ with $\sigma_f^2 = var(f_t)$. Note that σ_f is known if f_t is predetermined. In this case, the measurement of systematic risk is equivalent to the estimation of b, and therefore the estimation of factor loadings plays an important role in measuring systematic risk. However, the estimation of factor loadings for private assets is difficult because of the presence of the stale-pricing bias. Many researchers have made attempt to resolve this issue. According to the number of steps in model estimation, we can categorize these studies into the following streams:

1. *Two-step estimation*:
   This approach starts with unsmoothing economic returns from appraisals (as described in Section 2), and then regresses the unsmoothed economic returns against a given set of systematic factors using the conventional multiple regression.[11] Alternatively, Pedersen et al. (2014)[12] propose a new way to carry out two-step estimation. Their approach also starts with unsmoothing appraisals (using the technique in Section 2.3) but suggests that we regress appraisal returns against

[11] The two-step estimation approach introduces the error in variable (EIV) problem on the L.H.S. which would lead to a bigger estimation error. Additionally, this approach usually unsmooths the economic returns only using lagged appraisal returns, which may introduces model misspecification or higher model risk.

[12] Pedersen, Page, and He (2014), "Asset Allocation: Risk Models for Alternative Investments", *Financial Analysts Journal* **70**, 1-12.

smoothed systematic factors, where the factors are smoothed based on the estimated smoothing weights.[13]

2. *One-step Estimation*:
Instead of estimating factor loadings from unsmoothed economic returns, the one-step estimation infers factor loadings from appraisal returns directly. For example, substituting Eqn. (9) into Eqn. (4), we have

$$y_t = a + b \sum_{i=0}^{Q} w_i \, f_{t-i} + \xi_t, \quad (10)$$

3. where $\xi_t = \sum_{i=0}^{Q} w_i \, e_{t-i}$. Equation (10) is called a distributed lag model (DLM) in econometrics or a (constrained) transfer function noise model (TFN) in time series analysis. Many econometricians have contributed to the development of the 'single input' DLM but the research about 'multiple input' DLM is rarely found in the literature. The lack of development in 'multiple input' DLM impedes the progress of the one-step estimation approach. Consequently, practitioners currently rely on ad hocery to carry out the one-step estimation.

13

Specifically, Pedersen et al. (2014) first assume that economic returns satisfy

$$r_t = \alpha + \sum_{j}^{K} \beta_j \, f_{j,t} + u_t,$$

where $f_{j,t}$ denotes the *j*-th factor realization at time *t*. Inserting the above equation into Eqn. (4), we have

$$y_t = \alpha + \sum_{i}^{Q} \sum_{j}^{K} w_i \, \beta_j f_{j,t-i} + \xi_t,$$

where ξ_t is serially correlated. Let $X_{j,t} = \sum_{i}^{Q} w_i \, f_{j,t-i}$. The above equation becomes

$$y_t = \alpha + \sum_{j}^{K} \beta_j \, X_{j,t} + \xi_t.$$

As shown above, factor loadings $\{\beta_j\}$ can be estimated by regressing appraisal returns y_t against smoothed factor returns $X_{j,t}$.

Discussion:

Ang et al. (2013)[14] suggest a new approach to retrieve a common 'real estate' factor from public and private real estate indices. By design, the common 'real estate' factor follows an AR(1) process and therefore contains the stale-pricing bias. The detailed description of the approach is summarized in the Appendix.

Measurement of idiosyncratic risk

Appraisal returns are usually only available at the aggregate level, such as location or property types. This practice introduces additional work for measuring the idiosyncratic risk of a given property (or private investment deal).

Suppose that the economic returns on the i-th property follow

$$R_{it} = a_i + b_i f_t + e_{it}, i = 1, \dots, n, \quad (11)$$

where $var(e_i) = \sigma_i^2$ denotes the idiosyncratic risk on the i-th property. The returns on the aggregate index are given by

$$\overline{R}_t = \overline{a} + \overline{b} f_t + \overline{e}_t, \quad (12)$$

where $\overline{R}_t = \sum v_i R_{it}$, $\overline{a} = \sum_i v_i\, a_i$, $\overline{b} = \sum_i v_i\, b_i$, $\overline{e}_t = \sum_i v_i\, e_{it}$, and $v_i \in [0,1]$, $\sum_i v_i = 1$, $i = 1, \dots, n$ deontes the index weight on the i-th property.

Note that The index level idiosyncratic risk $var(\overline{e}_t) = \sigma_{\overline{e}}^2$ can be obtained by regressing the index level economic returns against systematic factor. In theory, $\sigma_{\overline{e}}^2$ tends to underestimate the idiosyncratio risk for a given property, which is demonstrated in the following example. Let's assume that the idiosyncratic errors $e_{it}, \forall i$ are homogeneous and independent. Under this assumption, we have $\sigma_i^2 = \sigma^2$ and

$$\sigma_{\overline{e}}^2 = \sigma^2 \sum_i v_i^2 . \quad (13)$$

Therefore, the idyocyncratic risk on the i-th property can be calculated as

$$\sigma^2 = \frac{\sigma_{\overline{e}}^2}{\sum_i v_i^2} . \quad (14)$$

Using Eqn. (14), we have $\sigma_{\overline{e}}^2 \le \sigma^2$ since $\sum_i v_i^2 \le 1$.

[14] Ang, Nabar, and Wald (2013), "Searching for a Common Factor in Public and Private Real Estate Returns", *Journal of Portfolio Management*, Special Real Estate Issue, 120-133.

Appendix

Ang, Nabar and Wald (2013)

Ang et al. (2013) assume that returns on public and private real estate indices can be explained by a common 'real estate' factor f_t and an index specific component, $g_{s,t}$. Mathematically, we have

$$r_{s,t} = \gamma_s f_t + g_{s,t}, \quad (A1)$$

where γ_s represents the loading of the real estate investment vehicle on the common 'real estate' factor, and $g_{s,t}$ is orthogonal to f_t. Moreover, the data generating processes (DGP) of $\{f_t\}$ and $\{g_{s,t}\}$ are given by

$$f_t = c_f + \phi_f f_{t-1} + \sigma_f \varepsilon_t, \quad (A2)$$

and

$$g_{s,t} = c_s + \phi_s g_{s,t-1} + \sigma_s \nu_{s,t}, \quad (A3)$$

where both ε_t and $\nu_{s,t}$ follow an IID standard normal random variable, and parameters ϕ_f and ϕ_s are used to specify the time series characteristics of f_t and $g_{s,t}$, respectively. According to Eqn. (A2), the common 'real estate' factor f_t follows an AR(1) process and therefore contains the stale-pricing bias.[15]

[15] Ang et al. (2013) also shows that the autocorrelation function of $\{r_{s,t}\}$ is given by $cor(r_{s,t}, r_{s,t-k}) = \phi_f^k$. Therefore, $\{r_{s,t}\}$ also behaves like Geltner's smoothing scheme and contains the stale-pricing bias.