Week 3: Tutorial Handout

Closed and open

1. If a subset $C \subset \mathbb{R}$ is closed, then it is complete.

2. Decide whether the following sets are open, closed, or neither. If a set is not open, find a point in the set for which there is no epsilon-neighborhood contained in the set. If a set is not closed, find a limit point that is not contained in the set.

   - the cartesian product $(0,1) \times (0,1)$
   - the cartesian product $[0,1] \times [0,1]$
   - the interval $(0,1]$.
   - Ab-3.2.3: $\{\sum_{k=1}^{n} \frac{1}{k^2} : n \in \mathbb{N}^+\}$.

Compact and Heine Borel

3. An arbitrary intersection $\bigcap_{n\geq 1} C_n$ of compact sets C_n is compact.

4. Which of the following sets is compact? For those that are compact use Heine-Borel theorem or open cover definition. For those that are not compact give an example of a sequence contained in the given set that does not possess a subsequence converging to a limit in the set.

   - the discrete set $\mathbb{Q} \cap [0, \infty)$
   - the discrete set $\mathbb{Q} \cap [0, 1]$
   - the discrete set $\mathbb{N}^+$
   - the discrete set $\mathbb{N}^+ \cap [0, M]$ for $M \geq 2$.
   - $\{x : x \geq 0, x \in \mathbb{R}\}$.
   - $\{(x, y) \in \mathbb{R}^2 : 2x^2 - y^2 \leq 1\}$.