May-June 2019

STA457 Practice questions

Date: 06 June 2019

ARMA model forecast

1. Consider the following models:
   i. $(1-0.5B)(Z_t-3)=a_t$
   ii. $(1-B+0.25B^2)(Z_t-1)=a_t$
   iii. $(1-B+0.25B^2)(Z_t+3)=(1+0.25)a_t$

   1) Derive the l-step ahead forecast $\hat{Z}_t(l)$ (or conditional expectation $E_t(X_{t+l})$)
   2) Calculate the variance of the l-step ahead forecast error for $l=1,2,3$
   3) Calculate the 95% confidence interval for the l-step ahead forecast in the above questions.

Transfer function noise model and intervention analysis

2. Consider a dynamic regression model

$$y_t=\sum_{i=0}^{k} v_i x_{t-i}+n_t,$$

   where both x_t and n_t are stationary and invertible ARMA model and given by

$$\phi_x(B)x_t=\theta(B)a_t, \qquad a_t \sim NID(0,\sigma_a^2),$$
$$\phi_n(B)n_t=\theta_n(B)e_t, \qquad e_t \sim NID(0,\sigma_e^2),$$

   and $cov(e_t,a_s)=0, \forall\, t,s$.

   1) State the prewhitening process of how to identify the value of k.
   2) State the steps of using Box-Tiao transformation to estimate v_j, $\forall j$.
   3) Find the l-ahead optimal forecast of y_{t+l}, $\hat{y}_t(l)$, using $\{a_t\}$ and $\{e_t\}$. (See Section 14.4 of Wei's book)
   4) Derive the mean square of the forecast error $E[y_{t+l}-\hat{y}_t(l)]^2$ for the above question.

Vector autoregression, Granger causality, and cointegration

Consider a $VAR(p)$ model

$$\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \sum_{i}^{p} \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} X_{1,t-i} \\ X_{2,t-i} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}. \quad (1)$$

Answer the following questions:

1) State how to check the stationarity of Equation (1);
2) Describe the methods to select the order for Equation (1), i.e. the value of p, taught in class.
3) State how to how to test Granger causality for the case that X_{1t} grander causes X_{2t} but not the other way around. Based on the same condition, express X_{2t} as the transfer function noise model of X_{1t}.
4) Suppose that

$$\phi_1(B)X_{1t} = \theta_1(B)u_{1t}$$

and

$$\phi_2(B)X_{2t} = \theta_2(B)u_{2t},$$

where $\phi_k(B) = 1 - \phi_1^{(k)}B - \cdots - \phi_{p_k}^{(k)}B^{p_k}$ and $\theta_k(B) = 1 + \theta_1^{(k)}B - \cdots - \theta_{q_k}^{(k)}B^{q_k}$ for $k = 1,2$. Describe how to test Granger causality using univariate approach.
5) Suppose that X_{1t} and X_{2t} are not weakly stationary. How do you model the joint dynamics of $\{X_{1t}, X_{2t}\}$? Discuss your decisions based on whether these two series are cointegrated or not.
6) Discuss the reasons why we have to choose different models based the condition of cointegration.
7) Discuss the Engle-Granger approach for modeling cointegrated X_{1t} and X_{2t}.
8) Discuss the implication of Granger representation theorem.

Bootstrap time series

Consider an AR(2) model

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t, \qquad a_t \sim NID(0,1).$$

1) Describe the steps of (unconditional) parametric bootstrap for the above $AR(2)$ process.
2) Describe the steps of carrying out the Sieve bootstrap for the above $AR(2)$ process.
3) Describe the steps of carrying out the block bootstrap method for the above $AR(2)$ process.
4) Discuss the pros and cons for the above methods.

Other important concepts

1. Define Granger causality in terms of a vector autoregression process.
2. Test Granger causality using vector autoregression or univariate approach.
3. State the approaches and procedures for cointegration modeling.
4. Granger's representation theorem and its implication for modeling multivariate time series.
5. State time series bootstrapping methods (in particular for dependent time series and dynamic regression models taught in class).
6. Define a generalized autoregressive heteroscedasticity $GARCH(p,q)$ model.
7. ***MTS_R (27 Nov 2017)*** course notes. Important topics include Check stationarity of a vector autoregression, model selection, and Granger causality test~~, and estimating cointegration models using Johansen's method.~~
8. Private asset modeling: Understanding how ARMA models can be used for inferring unobservable economic returns from observed appraisal returns: Geltner method, and Getmansky, Lo, and Markorov (2005).