Strong induction

$$\frac{\forall i \in \mathbb{N}.\left(\forall j \in \mathbb{N}\left(j < i \rightarrow Q(j)\right)\right) \rightarrow Q(i)}{\forall i \in \mathbb{N}. Q(i)}$$

$$R: \mathbb{N} \rightarrow \{T, F\}.\ R(i)\ \ IFF\ \ \forall j \in \mathbb{N}.\left(j < i \rightarrow Q(j)\right)$$
$$\forall i \in \mathbb{N}. R(i) \rightarrow Q(i)$$

WTP $\forall \mathrm{i} \in \mathbb{N}. \mathrm{Q(i)}$

1. $\forall i \in \mathbb{N}. (\forall j \in \mathbb{N}. \left(j < i\ IMPLIES\ Q(j)\right) IMPLIES \left(Q(i)\right)$
2. $\forall i \in \mathbb{N}. R(i) IMPLIES\ Q(i)$
3. Let $j \in \mathbb{N}$ be arbitrary
4. $NOT(j < 0)$ axiom of natural numbers
5. $NOT(A) \rightarrow (A \rightarrow B)$ tautology
6. $NOT(j < 0) \rightarrow \left(j < 0 \rightarrow Q(j)\right)$ substitution
7. $j < 0 \rightarrow Q(j)$ modus ponens
8. $\forall j \in \mathbb{N}. j < 0 \rightarrow Q(j)$ generalization
9. $R(0)$
10. Let $n \in \mathbb{N}$ be arbitrary
11. Suppose $R(n)$
12. $\forall j \in \mathbb{N}. j < n \rightarrow Q(j)$ by definition of R
13. Let $k \in \mathbb{N}$ be arbitrary
14. Assume $k \leq n$
15. $k < n\ OR\ k = n$ property of natural numbers
16. Suppose $k < n$
17. $k < n\ IMPLIES\ Q(k)$
18. $Q(k)$ modus ponens
19. Suppose $k = n$
20. $R(n) IMPLIES\ Q(n)$ specialization 2
21. $Q(n)$ modus ponens
22. $Q(k)$ substitution
23. $k \leq n\ IMPLIES Q(k)$
24. $\forall k \in \mathbb{N}. k \leq n\ IMPLIES\ \ Q(k)$
25. $\forall n \in \mathbb{N}. R(n) IMPLIES R(n+1)$
26. $\forall n \in \mathbb{N}. R(n)$
27. Let $i \in \mathbb{N}$ be arbitrary
28. $R(i)$ specialization
29. $R(i) \rightarrow Q(i)$
30. $Q(i)$ modus ponens
31. $\forall i \in \mathbb{N}. Q(i)$