Week 2: Tutorial Handout

Monotone sequence

1. D&D 2.6.H: Let $x_n := \sqrt{1+\sqrt{2+\sqrt{3+...\sqrt{n}}}}$, we will show that $\lim_{n\to\infty} x_n$ exists.
   (a) Show that $x_n < x_{n+1}$
   (b) $x_{n+1}^2 < 1 + \sqrt{2}x_n$ Hint: square x_{n+1}^2 and factor a 2 out of the square root.
   (c) Conclude that $x_n \leq 2$ and thus $\lim_{n\to\infty} x_n$ exists.

Bolzano-Weierstrass

2. D&D 2.7. H: Let $\{x_n\}_{n\geq}$ be a sequence in $\mathbb{R}$. Suppose there is a number L such that every subsequence $\{x_{n_k}\}_{k\geq}$ has a subsubsequence $\{x_{n_{k_m}}\}_{m\geq}$ with $\lim_{m\to\infty} x_{n_{k_m}} = L$. Show that the whole sequence converges to L. HINT: If not, you could find a subsequence bounded away from L.

Cauchy

3. D& D 2.8.C: Let $\{a_n\}$ satisfy $\lim_{n\to\infty} \sum_{n=1}^{N} |a_n - a_{n+1}| = 0$. Show that $\{a_n\}$ is Cauchy.