Course Syllabus

Jump to Today

Logistics

Welcome to MAT337!

- Course coordinator: Tomas Kojar tkojar@math.toronto.edu (mailto:tkojar@math.toronto.edu)
- Office hours: Mondays 12-1(PG101), Tuesday 1-2 (Classroom) and Thursdays 2-3 (PG101)
- TA in tutorials: Jamal Kawach Jamal.Kawach@mail.utoronto.ca (mailto:Jamal.Kawach@mail.utoronto.ca)
- TA marking assignments: Christopher Kennedy christopherpa.kennedy@mail.utoronto.ca (mailto:christopherpa.kennedy@mail.utoronto.ca)
- (mailto:christopherpa.kennedy@mail.utoronto.ca) Email policy: Admin questions will be sent to the coordinator. Math questions can be posted on Piazza but do not sent them over email.
- Class schedule:Section LEC0101 and Times T11-13 and R11-12 at BA1170 (Bahen)
- Texbooks and sources:The main textbook for this course is Kenneth R. Davidson and Allan P. Donsig "Real Analysis and Applications: Theory in Practice". But we will draw problems and material from other sources as well, here is an ongoing list of them:
  - Abbott S. "Understanding analysis" (http://cms.dm.uba.ar/academico/materias/verano2012/taller_de_calculo_avanzado/Libros/Abbott%5C%20-%5C%20Understanding%5C%20Analysis.pdf)
  - Pugh C. "Real Mathematical Analysis" (https://www.springer.com/gp/book/9783319177700)
  - Folland G. B. Real Analysis: Modern Techniques and Their Applications (https://www.amazon.ca/Real-Analysis-Modern-Techniques-Applications/dp/0471317160)
  - Tao T. "Analysis I,II" (https://www.springer.com/gp/book/9789811017896)
  - (https://www.springer.com/gp/book/9783319177700) Rudin W. Principles of Mathematical Analysis 3ed (https://notendur.hi.is/vae11/%C3%9Eekking/principles_of_mathematical_analysis_walter_rudin.pdf)
  - Gelbaum B.R, Olmsted J.M.H. "Counterexamples in Analysis " (https://fac.ksu.edu.sa/sites/default/files/_olmsted_1.pdf)
- Our Platforms:
  - The official course site is the quercus homepage.
  - There is an online forum for this course on Piazza (https://piazza.com/class/jv8ldoglpjh4kn) . This group is a resource for students to meet other MAT337 students, ask questions, discuss problems, make study groups, and in general help each other.
  - Post any questions related to practice problems that are not in the HW
  - For HW questions, ask general math questions:
    - Unclear steps from proofs in class or book eg. for problems similar to the HW problem
    - Definitions
- Deadline to drop this course: July 15 Last day to drop Y section code courses from academic record and GPA.
- Prerequisites:MAT224H1/MATB24H3/MAT224H5/ MAT247H1/MAT247H5, MAT235Y1/MAT235Y5/(MATB41H3, MATB42H3)/MAT237Y1/(MATB41H3, MATB42H3, MATB43H3)/MAT237Y5, MAT246H1; NOTE: These Prerequisites will be waived for students who have MAT257Y1. The course is a continuation of MAT137 and MAT237. Therefore, a lot of their basic material will be quickly skimmed the first few weeks. Ignoring this material or letting it go past the second week will make it very difficult to succeed in this course.
  - graphing and integrating functions
  - familiarity with epsilon-delta arguments and good understanding of the limit definition
  - mathematical notation such as sigma sum notation, sup and inf and set notation.
- Accesibility Needs: If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom, or course materials, please contact Accessibility Services: \url{http://www.accessibility.utoronto.ca/index.html }

Course objectives

The goal of this course is to explain key concepts of Real Analysis with the view at applications. The course is about the same level as MAT357, but while MAT357 deals mainly with theory, the present course aims at developing interesting applications.

- Getting a good foundation on the properties of real numbers and metric spaces, including some topological concepts. In any science field you will often hear the words "metric" or "distance" and so you will be able to formulate their ideas in rigorous self-consistent language.
- Understand the nuances of function properties such as continuous but not differentiable. The modern study of numerical methods and partial differential equations (pde) is the degree to which a pde solution is smooth. This is because we can then extract numerous conclusions about their qualitative behaviour: how does it decay in time? is the energy of the system blowing up? can we approximate the solution with smoother ones?

- Know when to use approximation results such as Taylor series theorem or polynomial approximation. Given a graph of function or a solution to your equation, the goal is that you will be able to know if and how to approximate them by "nicer" objects.

- Applications in signal processing (Fourier series) and differential equations. The first goal is that you will be able to understand the relation between a graph of a (periodic) function and its Fourier series. The second goal is that you will get a taste of modern ode problems.

  The path is long, but the reward at the end of the way is well worth it.

Course structure

- Book chapters:We will mostly cover material from Chapters 2–10,12. Occasionally I will introduce some material not in the book. If this is significant, I will state so.
- Classroom time: All four hours of class time are equally important. Do not think of them as three hours of lecture plus one hour of tutorial.

One of the objectives of the course is to continue your transformation from students to theoretically-able researchers. You will accomplish this not by merely watching me do mathematics, but by doing lots of math yourselves. Hence there will be important parts of the course, including big theorems, that you will be responsible for proving yourself. This includes some class time.
The textbook has more space than we have class meeting time. I prefer to spend class time motivating difficult concepts or elaborating on harder examples or guiding you to construct your own proofs rather than writing the details of an easy proof that is found in the book and that you could reconstruct on your own.

- Reading: There will be roughly weekly reading assignments from the textbook, corresponding to the material that we are covering. Even if we do not complete all details in class, I still expect you to read the entirety of each chapter and to understand it, in addition to class notes. In other words, everything is equally important, and everything prepares you equally well for the tests. I am happy to go over any details with you during office hours.
- Homework: There will be roughly biweekly homework assignments that I ask you to complete.
  - The assignments are long, but you are only required to turn in a small number of problems every two weeks, which will be marked. I choose problems that will help solidify the concepts and that will help you understand the subsequent lectures. Because of that, it is important that you do all of the assigned problems, including the ones not to be turned it, and I will assume you have. This is particularly relevant for exams. I will not accept late assignments. This course is unforgiving if you fall behind, so I do not want to encourage it.
  - You will be receiving a notification email from Crowdmark. If you are typing your assignment, simply upload the pdf. If you are handwriting it, take a picture or scan it. Read the **crowdmark instructions from MAT137 (http://www.math.toronto.edu/~alfonso/137/PS/137_CM.html)** or contact us.
  - We will most likely grade only a subset of the questions.
  - We will post sample solutions for all the questions so that you can compare with your answers.
  - We will post the homework assignments and grades on the course website.
  - For remarking requests come talk to me, Tomas Kojar. Do not bother the marking TA, his hours are already quite limited.
- Quizzes:There will be weekly online quizzes that I ask you to complete.
  - For remarkings or typos please contact Tomas directly.
- Midterms: There will be two midterms. I will announce the exact dates and the test structure in class and on the course website. If you have an academic conflict for one of the tests (for example, a tutorial or a lab for a different course), then we will offer you an alternative time. For this, you will need to let us know at least one week before the date of the test. We will post more information on the course site.

- The 8/10 test questions will be modifications of problems from problem sets, tutorial handouts, quizzes and the practice problems listed below. The first question will be stating theorems and/or definitions. The last question will be one you haven't seen before.

- Project: There will be a project due in August,. I will design the canditate projects to give you a taste of what research is like. As a warning, past students in MAT244 who completed these projects spent a very large number of hours working on then, but many claimed it was their favourite part of the course and a great learning experience. You will need to and produce a written paper in LATEX or Word with your results. I will post more information, including help to learn Latex.
  - Up to 4 people can work together.

- This is an ongoing list for bonus work:
  - Piazza participation:
    - 1%-3% for having asked at least 14 questions in total (eg. 2 per week). If you go above that: 2%-4% for asking at least 21 questions.
    - 1%-3% for having answered at least 7 questions in total (eg. 1 per week) .If you go above that: 2%-4% for answering at least 14 questions. Multiple distinct answers to the same question are welcomed.
  - Bonus questions in assignments or tests.
  - The Final project.

- A word of caution:Even though it may seem like we start slowly, we will pick up the pace, and I guarantee you you will be doomed if you fall behind. That would be a pity as the final part of this course is pure beauty

Marking scheme

Your course mark will be

- $11\% \cdot PS + 50\% \cdot F + 18\% \cdot T_A + 14\% \cdot T_B + 7\% \cdot Q$ or
- $13\% \cdot PS + 36\% \cdot F + 23\% \cdot T_A + 19\% \cdot T_B + 9\% \cdot Q$
- $\textbf{In case of justified missed test: } 11\,\% \cdot \mathrm{P\,S} + 60\,\% \cdot \mathrm{F} + 22\,\% \cdot \mathrm{T\,A} + 7\,\% \cdot \mathrm{Q}$

whichever is higher. In the formulas above:

- PS may be 0 if you have committed an academic offense. Otherwise, it is the average of your best 5 out of the 6 problem set marks.
- F is your final exam mark.
- TA is the grade in your best term test.
- TB is the grade in your second best term test.
- Q is the average of your best 8 out of the 11 online quiz marks.
- FP is your final applied project mark.

We wish you a productive and rewarding summer, and we look forward to working with you. Thanks!

Frequently asked questions

- *Which results are we allowed to use?*
  - You can use without proof any of the results from 1)the main textbook Donsig and Davidson and 2) lecture.
  - You can use any results that were proved in the prerequisite courses.
  - For all other results you have to state the proof. Of course, if a test question is about proving a theorem that is contained in the book or lecture, then you have to repeat that proof.
  - You are not allowed to use results from practice problems (i.e. listed below, tutorial handouts, quizzes, assignments).
  - Some results you can use without the proof that are not in this course
    - divergence of harmonic series
    - All calculus results: derivative optimization results (Rolles, Fermat), convexity results, Mean value theorem (MVT), Intermediate Value theorem (IVT), Fundamental theorem of calculus (FTC), any integral trick.

Timeline

By D&D we mean Davidson & Donsig. By Ab we mean Abbott

Week	Material	Book sections	Practice problems

Week 1: May 7	Intro to reals and Least upper bound principle	D&D: 2.1-2.3 and Ab: 1.3	• D&D-2.2: C,D,G • D&D-2.3: D,E • Ab-1.3: 5,6,9
Week 2: May 14	Monotone convergence, Bolzano-Weierstrass and Cauchy sequence	D&D:2.4-2.8	• D&D-2.6: C,J,M,N • D&D-2.7: F,H • D&D-2.8: A,B,C,D,F,H • Ab-2.4: 2,3,5,6
Week 3: May 21	Completeness, closed, compact and Heine Borel theorem	D&D:4.2-.4.4	• D&D-4.3: Aacd,B,F,J,L • D&D-4.4: A,C,E • Ab-3.3: 1,4,7
Week 4: May 28	Extreme value theorem and Uniform continuity	D&D:5.4, 5.5 Ab: 4.4	• D&D-5.4: A,C,G,H • D&D-5.5:A,D,E,F,H,J,K
Week 5: June 4	Intermediate value theorem and Norms	D&D:5.6, 7.1-7.2	• D&D-5.6: A,B,C,D,E • D&D-7.1:A,F,I,J • D&D-7.2: C,D,E,F,I,L,N
Week 6: June 11	Inner Products and Fourier series	D&D:7.4-7.6	• D&D-7.4: B,F,I,J • D&D-7.6: B,C,E,H,I
Week 7: June 18	Exams Week		
Midterm Test 1 June 26 9am-12pm during June exams week	Covers material from Weeks 1-4 including IVT from week 5		
Week 8: June 25	Reading Week		
Week 9: July 2	Limits of Functions: Interchange of limiting processes	D&D:8.1-8.3	• D&D-8.1: B,C,F,H, I • D&D-8.2: A,D,E,F,G,H • D&D-8.3: B,C,D,E
Week 10: July 9	Limits of Functions: Power series	D&D: 8.4-8.5	• D&D-8.4: C,D,E,F,I • D&D-8.5: A,C, G,I
Week 11: July 16	Limits of Functions: Arzela Ascoli	D&D: 8.6	• D&D-8.6: A,D,E,G,I
July 15	Last day to cancel Y section code courses without academic penalty		
July 29 Monday 12:00-15:00 Midterm Test 2	Covers material from Weeks 5(norms),6,9,10		
Week 12: July 23	Taylor Series and Weierstrass Theorem	D&D: 10.1,10.2,10.3	• D&D-10.1: F(a,b), H • D&D-10.2: B,D,F,J,K
Week 13: July 30	Stone Weierstrass and ODEs fixed point	Pugh 4.4 (proof of SW theorem) D& D: 12.3	• D&D-12.3: A,B,C,D
Week 14: August 6	ODEs existence	D& D: 12.4	• D&D-12.4: A,D
August 12	Deadline to request Late		