STA457 practice questions

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Introduction

1. Define weak stationarity of a time series.
2. Define classical decomposition.
3. State the steps of modeling time series data.
4. Consider

$$x_t = \frac{1}{3}a_t + \frac{1}{3}a_{t-1}, \qquad a_t \sim NID(0, \sigma_a^2)$$

and

$$y_t = \frac{1}{2}x_t - \frac{1}{2}x_{t-2}.$$

   a. Calculate the autocovariance function for x_t and y_t;
   b. Calculate the cross-correlation function between x_t and y_t.

5. Suppose that

$$X_t = a_t + \theta a_{t-1}, \qquad a_t \sim NID(0, \sigma_a^2),$$

$$Y_t = \sum_{i=1}^{m} w_i X_{t+1-i},$$

and

$$Z_t = Y_{t-1} X_t.$$

   a. Calculate $E(Y_t)$, $E(X_t)$, and $E(Z_t)$.
   b. Calculate $\gamma_Y(h) = cov(Y_t, Y_{t+h}), h = 0, \pm 1, \pm 2, \ldots$
   c. Calculate $\gamma_X(h) = cov(X_t, X_{t+h}), h = 0, \pm 1, \pm 2, \ldots$
   d. Calculate $\gamma_Z(h) = cov(Z_t, Z_{t+h}), h = 0, \pm 1, \pm 2, \ldots$
   e. Calculate PACF of Y_t, X_t, and Z_t.

ARMA models

1. State Wold decomposition. Discuss the reason why Wold decomposition supports the use of ARMA models for modeling time series.

2. Consider an $ARMA(2,2)$ model

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} = a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}, \qquad a_t \sim NID(0,1).$$

   a. Under what condition, the above $ARMA(2,2)$ model is causal/stationary.

   b. Under what condition, the above $ARMA(2,2)$ model is invertible. State the reason for us to consider an invertible ARMA model.

   c. Suppose that x_t is causal, i.e.

$$x_t = a_t + \sum_{j=1}^{\infty} \psi_j a_{t-j}.$$

   Calculate $\psi_j,\ j = 1,2,3,4,5,6.$

   d. Suppose that x_t is invertible, i.e.

$$x_t - \sum_{j=1}^{\infty} \pi_j x_{t-j} = a_t.$$

   Calculate $\pi_j,\ j = 1,2,3,4,5,6.$

3. Consider a stationary time series

$$x_t = \theta_0 a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}, \qquad a_t \sim NID(0,1).$$

   a. Calculate the autocorrelation functions of x_t for lags 1, 2, and 3.

   b. Calculate the partial autocorrelation functions (PACF) for lags 1, 2, and 3.

4. Consider a stationary $AR(2)$ model

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} = a_t, \qquad a_t \sim NID(0,1).$$

   a. Calculate the autocorrelation functions of x_t for lags 1, 2, and 3.

   b. Calculate the partial autocorrelation functions (PACF) for lags 1, 2, and 3.

   Remark: Calculate PACF using Cramer's rule and Durbin-Levinson algorithm (optional).

5. Define two portmanteau tests taught in class for checking model adequacy.

6. Define AIC and BIC criteria for model selection. Discuss which of them tends to select a more parsimonious model.

Box-Jenkins approach and Unit root tests

1. Suppose that $x_t = \alpha + \beta t^2 + y_t$ and $y_t = a_t + \theta_1 a_{t-1},\ a_t \sim NID(0,1)$. Show that we can make x_t a stationary time series by differencing $\{x_t\}$.
2. Suppose that $x_t = \mu + x_{t-1} + a_t + \theta a_{t-1},\ a_t \sim NID(0,1)$, and $x_0 \sim NID(\mu_0, \sigma_0^2),\ a_\tau = x_\tau = 0,\ \forall \tau < 0$. Show that

$$x_t = \mu \cdot t + x_0 + \sum_{j=0}^{t} \theta_j^\star a_{t-j}$$

   and express θ_j^* in terms of θ.
3. Define a $SARIMA\ (p,d,q)(P,D,Q)_s$ model.
4. Define a Holt-Winter linear and seasonal model.
5. Write down a Periodic AR(3) and MA(3) model.
6. Define $ARIMA(p,d,q)$ models. Use the definition of ARIMA model to explain the idea of the unit root test and discuss the definition of an $I(d)$ process.
7. State/define the basic Dickey-Fuller test? What are the issues related to the basic Dickey-Fuller test taught in class?
8. State/define the augmented Dickey-Fuller (ADF) test. Discuss how to select the length of lag for the ADF test.
9. State how to deal with nonstationarity in variable using Box-Cox transformation.

