Induction Template

Simple induction

$P(0)$

Let $n \in \mathbb{N}$ be arbitrary

Assume $P(n)$

…

$P(n+1)$

$P(n)\ IMPLIES\ P(n+1)$ direct proof

$\forall n \in \mathbb{N}. P(n)\ IMPLIES\ P(n+1)$ generalization

$\forall n \in \mathbb{N}. P(n)$ induction

To prove $\forall n \in \mathbb{N}. \big(n \geq b\ IMPLIES\ P(n)\big)$ by induction, it's sufficient to prove

$P(b)$

$\forall n \in \mathbb{N}. \big(n \geq b\ AND\ P(n)\big) IMPLIES\ P(n+1)$

To prove $\forall n \in \mathbb{N}. even(n)\ IMPLIES\ q(n)$

Method 1

Let $p(k) = q(2k)$

$\forall k \in \mathbb{N}. p(k)\ IFF\ q(2k)$

$p(0)$

$\forall k \in \mathbb{N}. p(k)\ IMPLIES\ p(k+1)$

$\forall k \in \mathbb{N}. p(k)$

$\forall n \in \mathbb{N}. even(n)\ IMPLIES\ q(n)$ substitution

Method 2

$q(0)$

$\forall n \in \mathbb{N}. \big(even(n)\ AND\ q(n)\big) IMPLIES\ q(n+2)$

Theorem: consider any square chessboard whose sizes have length which is a power of 2. If any 1 square is removed, the resulting shape can be tiled using only 3 square L-shaped tiles.

Proof:

$\forall n \in \mathbb{N}.$ Let C_n denote the set of all $2^n * 2^n$ chessboard with 1 square removed.

Let $P(n)$: $\forall c \in C_n . c$ can be tiled using only L-tiles. (L-tiles means square L-shaped tiles)

Define predicate

Let $n = 0$,

c contains only the zero square chessboard, which can be tiled by 0 L-tiles.

$P(0)$

Base case

Let $n \in \mathbb{N}$ be arbitrary

Assume $P(n)$

Let $c \in C_{n+1}$ be arbitrary

Divide *c* into four $2^n * 2^n$ chessboards.

There is one $2^n * 2^n$ chessboard with one square removed, by induction hypothesis, this chessboard can be tiled using L-tiles.

For the rest three chessboard, remove one square that is closest to *c*'s center in each $2^n * 2^n$ chessboard, which can form a L-tile. For the rest pieces, each section has one square removed and side-length is 2^n, by induction hypothesis, they can be tiled using L-tiles.

$P(n+1)$

$P(n) IMPLIES\ P(n+1)$ direct proof

$\forall n \in \mathbb{N}. P(n)\ IMPLIES\ P(n+1)$ generalization

$\forall n \in \mathbb{N}. P(n+1)$ induction