hw4

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1

All 4 of the integrals are equal to $\boxed{\text{zero}}$. None of them have singularities inside the curve of integration, and the Cauchy-Goursat Theorem tells us that an integral on a simple closed path is equal to zero if the interior is continuously differentiable (no singularities).

2

a) $z = -1$

b) $z = 0, 1$

c) $z = 0$

d) $z = \pm i$

3

a)

$$H(z) = z^2 \implies \frac{z^2}{z-(-1)} \implies z^2\bigg|_{z=-1} = \boxed{1}$$

b)

$$H(z) = \frac{z^2+3z+1}{z} \implies \frac{\frac{z^2+3z+1}{z}}{z-(-1)} \implies \frac{z^2+3z+1}{z}\bigg|_{z=-1} = \boxed{1}$$

$$H(z) = \frac{z^2+3z+1}{z-1} \implies \frac{\frac{z^2+3z+1}{z-1}}{z-0} \implies \frac{z^2+3z+1}{z-1}\bigg|_{z=0} = \boxed{-1}$$

c)

$$H(z) = \cos(z) \implies \frac{\cos(z)}{z} \implies \cos(z)\bigg|_{z=0} = \boxed{1}$$

d)

$$H(z) = \frac{e^z}{z-i} \implies \frac{\frac{e^z}{z-i}}{z-(-i)} \implies \frac{e^z}{z-i}\bigg|_{z=-i} = \boxed{\frac{e^{-i}}{-2i}}$$

$$H(z) = \frac{e^z}{z+i} \implies \frac{\frac{e^z}{z+i}}{z-i} \implies \frac{e^z}{z+i}\bigg|_{z=i} = \boxed{\frac{e^{i}}{2i}}$$

4

The singularities are at $z = 0, 1, 2$. Let's find all the residues first.

$$\frac{\frac{z+1}{(z-1)(z-2)}}{z} \implies \left.\frac{z+1}{(z-1)(z-2)}\right|_{z=0} \implies 1/2$$

$$\frac{\frac{z+1}{z(z-2)}}{z-1} \implies \left.\frac{z+1}{z(z-2)}\right|_{z=1} \implies -2$$

$$\frac{\frac{z+1}{z(z-1)}}{z-2} \implies \left.\frac{z+1}{z(z-1)}\right|_{z=2} \implies 3/2$$

4.1 Case I

We only need to worry about the singularity at $z = 0$, since $z = 1, 2 \notin |z| = 1/2$ (the singularities at $z = 1, 2$ are not inside the current circle). Since the integral is about a circle of radius R (simple closed curve), the answer is just the residue at $z = 0$. The answer is $\boxed{1/2}$.

4.2 Case II

Sum up the residue at $z = 0, 1$. The answer is $1/2 - 2 = \boxed{-3/2}$.

4.3 Case III

Sum up the residue at $z = 0, 1, 2$. The answer is $1/2 - 2 + 3/2 = \boxed{0}$.

5

The Cauchy-Integral Theorem is

$$\oint_{\gamma} f(z)\,dz = 0$$

if γ is a simple closed path.

6

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