Homework 5

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1

(a) Rewrite as

$$\frac{z^2+2}{z-1}$$

The singularity is at $z=1$, the order is 1, and the residue is $1^2+2=3$.

(b) Rewrite as

$$\frac{z^3/2^3}{(z+1/2)^3}$$

The singularity is at $z=-1/2$, the order is 3, and the residue is $\left(\frac{\mathrm{d}^2\left(z^3/8\right)}{\mathrm{d}z^2}\right)_{-1/2}\cdot 1/2! = 6\cdot -1/2/8\cdot 1/2 = -3/16$

(c) Rewrite as

$$\frac{\exp\left(z\right)/(z+i\pi)}{z-i\pi}, \frac{\exp\left(z\right)/(z-i\pi)}{z+i\pi}$$

The poles are at $z=\pm i\pi$, the orders are both 1, and the residues are $\frac{\exp\left(i\pi\right)}{2i\pi}, \frac{\exp\left(-i\pi\right)}{-2i\pi} = \frac{1}{2i\pi}, \frac{1}{-2i\pi} = \frac{-i}{2\pi}, \frac{i}{2\pi} = \mp\frac{i}{2\pi}$

(d) Rewrite as

$$\frac{f(z)}{g(z)} = \frac{1}{\sin\left(z\right)} \longrightarrow \frac{f(z)}{g'(z)} = \frac{1}{\cos\left(z\right)}$$

The singularities are located at $\pi n, n\in\mathbb{Z}$, the order is 1, and the residue is $(-1)^n$

2

2.1 f(z)

(a)

2.2 g(z)

(a) a

3

(a) $R = |a|$ because the point a is $|a|$ away from $z = 0$.

(b)