

# ML Homework 4

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## 1

When  $w_0$  increases, then the probability of predicting class 1 becomes higher, and vice versa.

$$h(x) = \frac{1}{1 + e^{-(w_0 + \dots)}}$$

This is because as  $w_0$  gets bigger, the result of exponentiation gets smaller. As the denominator gets smaller, the quotient gets larger.  $h(x)$  is used as the probability we will predict a certain class. The higher  $h(x)$  is, the more likely we will predict class 1 (as opposed to class 0).

## 2

The logistic function is

$$\frac{1}{1 + e^{-\mathbf{w}^T \mathbf{x}}}$$

If we were to double  $\mathbf{w}$ , the result wouldn't actually change, as long as the threshold is 0.5.

The geometric interpretation is because  $\mathbf{w}^T \mathbf{x}$  represents a (hyper)plane. Multiplying the equation of a plane doesn't affect any points that are on the plane (threshold = 0.5  $\implies$  on hyperplane), but it makes the points that are off it seem further away, increasing the certainty with which we pick a class. For example, if  $\mathbf{w}^T \mathbf{x} = 0$ , then there is no change ( $0 \cdot 2 = 0$ ). But if it's positive, it becomes a larger positive number, and vice versa.

### 3

(a)

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

(b) :

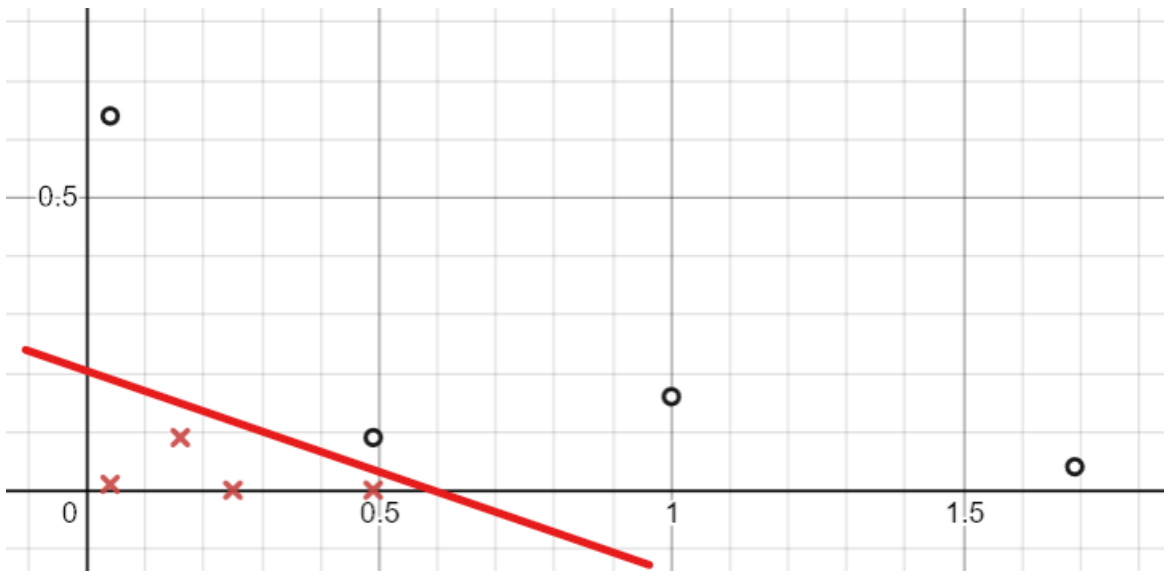


Figure 1: o: class 0, x: class 1

(c)  $\text{TPR} = \frac{1}{3+1} = 1/4$

(d)  $\text{FPR} = \frac{3}{3+1} = 3/4$

(e)  $\text{accuracy} = \frac{3+3}{3+1+3+1} = 3/4$

(f)  $\text{recall} = \frac{3}{3+1} = 3/4$

(g)  $\text{precision} = \frac{3}{3+1} = 3/4$

(h)

$$-[0 \cdot \ln(0.389) + (1-0) \ln(1-0.389)] - \dots - [1 \cdot \ln(0.638) + (1-1) \ln(1-0.638)] = 4.25205106 \dots$$

(i)

$$-\left[0 \cdot \ln\left(\frac{1}{1 + \exp(-\mathbf{w}'^T \mathbf{X}^{(1)})}\right)\right] + \dots - \left[0 \cdot \ln\left(\frac{1}{1 + \exp(-\mathbf{w}'^T \mathbf{X}^{(8)})}\right)\right] = 3.0158793 \dots$$

Since  $\mathbf{w}'$  has a lower cross-entropy, it is a better fit for this data.

(j) After one iteration,

$$\begin{aligned}
 \mathbf{w} &= \mathbf{w} + \frac{\alpha}{N} \mathbf{x}^T (y - \sigma(\mathbf{xw})) \\
 &\quad \begin{bmatrix} 0.66 \\ -2.24 \\ -0.18 \end{bmatrix} \\
 &+ \frac{0.01}{N} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.49 & 1.69 & 0.04 & 1 & 0.16 & 0.25 & 0.49 & 0.04 \\ 0.09 & 0.04 & 0.64 & 0.16 & 0.09 & 0 & 0 & 0.01 \end{bmatrix} \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \sigma \left( \begin{bmatrix} 1 & 0.49 & 0.09 \\ 1 & 1.69 & 0.04 \\ 1 & 0.04 & 0.64 \\ 1 & 1 & 0.16 \\ 1 & 0.16 & 0.09 \\ 1 & 0.25 & 0 \\ 1 & 0.49 & 0 \\ 1 & 0.04 & 0.01 \end{bmatrix} \begin{bmatrix} 0.66 \\ -2.24 \\ -0.18 \end{bmatrix} \right) \end{pmatrix} \\
 &= \begin{bmatrix} 0.661 \dots \\ -2.239 \dots \\ -0.181 \dots \end{bmatrix}
 \end{aligned}$$

(k) These data points would contribute a medium amount to the new  $\mathbf{w}$ .

(l) These data points would contribute a low amount to the new  $\mathbf{w}$ .

(m) These data points would contribute a high amount to the new  $\mathbf{w}$ .

(n)

$$- \left[ 0 \cdot \ln \left( \frac{1}{1 + \exp(-\mathbf{w}_g^T \mathbf{X}^{(1)})} \right) \right] + \dots - \left[ 0 \cdot \ln \left( \frac{1}{1 + \exp(-\mathbf{w}_g^T \mathbf{X}^{(8)})} \right) \right] = 4.2519 \dots$$

This is a very minor decrease from 4.2520. This makes sense because gradient ascent is supposed to make the model fit better and have less error.

## 4

- lasso

$$-\lambda (|w_1| + \dots + |w_d|) + \sum_{i=1}^N y^{(i)} \ln(h(x)) + (1 - y^{(i)}) \ln(1 - h(x))$$

- ridge

$$-\lambda (w_1^2 + \dots + w_d^2) + \sum_{i=1}^N y^{(i)} \ln(h(x)) + (1 - y^{(i)}) \ln(1 - h(x))$$

- gradient

$$\mathbf{x}^T(y - \sigma(\mathbf{x}\mathbf{w})) - 2\lambda I'\mathbf{w}$$

- The optimal  $\lambda = 0$ . It didn't help because we got the best results when we multiplied the penalty by 0. The training error and the validation error increased.

```
# TODO Q09
# Write the gradient ascent function
def Gradient_Ascent(X_train_1, y_2d_train, learning_rate, num_iters, L=0):
    # Number of training examples.
    N = X_train_1.shape[0]
    # Initialize w(<np.ndarray>). Zeros vector of shape X_train.shape[1],1
    w = np.zeros((X_train_1.shape[1], 1))
    # Initiating list to store values of likelihood(<list>) after few iterations.
    likelihood_values = []
    for i in range(num_iters):
        y_hat = hypothesis(X_train_1, w)
        error = y_2d_train - y_hat

        gradient = np.dot(X_train_1.T, error) - 2*L*(w-w[0,0])
        # Updating Parameters
        w = w + learning_rate / N * gradient
        if (i % 100) == 0:
            likelihood_values.append(likelihood(X_train_1,y_2d_train,w,N))

    return w, likelihood_values
```

Figure 2: Adding Ridge regression

```

from sklearn.model_selection import KFold
def cross(L):
    nfold = 5
    kf = KFold(n_splits=nfold)
    X_unscaled = cancer.data
    X_unscaled = np.hstack((np.ones((X_unscaled.shape[0], 1)), X_unscaled))
    y_orig = cancer.target
    likelihoods = []
    for train, test in kf.split(X_unscaled):
        X_tr = X_unscaled[train,:]
        y_tr = y_orig[train]
        X_ts = X_unscaled[test, :]
        y_ts = y_orig[test]

        scaler_kFold = preprocessing.StandardScaler()
        X_tr = scaler_kFold.fit_transform(X_tr)
        X_ts = scaler_kFold.transform(X_ts)

        y_2d_ts = y_ts.reshape((y_ts.shape[0], 1))
        y_2d_tr = y_tr.reshape((y_tr.shape[0], 1))

        w_kfold, w_likelihood = Gradient_Ascent(X_tr, y_2d_tr, 0.01, 1000, L)
        likelihoods.append(likelihood(X_ts, y_2d_ts, w_kfold, X_ts.shape[0]))

    return max(likelihoods)

lambdas = []
for lambda_ in range(0, 100):
    lambdas.append(cross(lambda_/100000))
    print(lambda_/100000, lambdas[lambda_])

print(f"index: {lambdas.index(max(lambdas))}, value:{max(lambdas)}")

```

Figure 3: 5 fold

For k-fold, I started off using `range(0,100)`, but without doing the `lambda_/100000`. This was to see which integer  $\lambda$  in the interval  $[0, 100)$  the best one was closest to. I then started dividing, to decrease the distance between tested  $\lambda$ s, all the way until `1/100000`. The best  $\lambda$  I got was 0.

## 5

- (a) Predictors: average pitch; response: gender
- (b) Predictors: matrix of where the strokes were; response: letter or number written

## 6

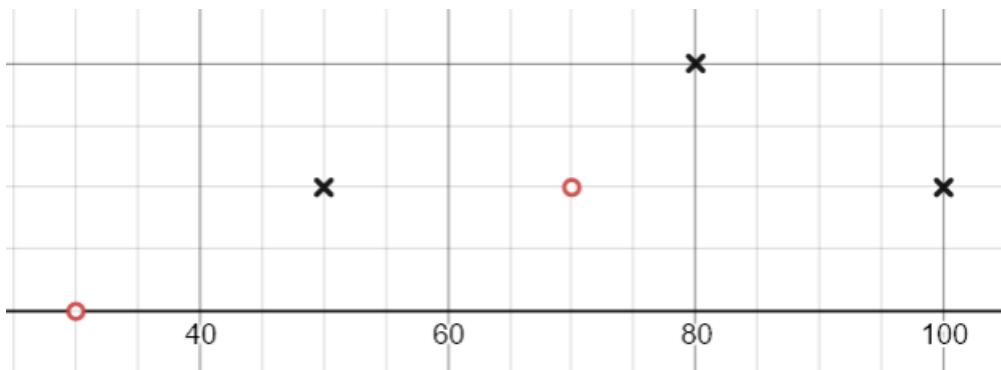


Figure 4: o: no donation, x: donation

(a)

(b) I will choose the horizontal line  $x_2 = 0.5 \implies 0.5 + 0 \cdot x_1 + w_2$ 

$$\mathbf{w}^T = [-0.5, 0, 1]$$

- (c) The least likely would be the one that is incorrect. In this case, it's same 3, or the one that earns 70k, follows 1 site, and did not donate. No calculator was needed for this!
- (d) They would not change the values of  $\hat{y}$ , but they would change the likelihoods. If the prediction was correct, then the likelihood would increase. If the predictions were incorrect, then the likelihood would decrease.