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Matrix A (a)

$$\lambda_1 = 4, \lambda_2 = -2, v_1 = (1, 1), v_2 = (-1, 1)$$

(b)

$$1 + 1 = 1 + 1 = 2$$

(c)

$$4 \cdot -2 = -8$$

(d)

$$\arccos \left(\frac{\langle (1, 1), (-1, 1) \rangle}{\sqrt{1+1} \cdot \sqrt{1+1}} \right) = \arccos(0) = \pi/2$$

(e) The eigenvalues are real. The matrix is symmetric.

Matrix B (a)

$$\lambda_1 = 37, \lambda_2 = -15, v_1 = (3, 2), v_2 = (-2, 3)$$

(b)

$$37 - 15 = 21 + 1 = 22$$

(c)

$$37 \cdot -15 = -555$$

(d)

$$\arccos \left(\frac{\langle (3, 2), (-2, 3) \rangle}{\sqrt{3^2+2^2} \cdot \sqrt{3^2+2^2}} \right) = \arccos(0) = \pi/2$$

(e) The eigenvalues are real. The matrix is symmetric.

Matrix C (a)

$$\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 1, v_1 = (-1, 0, 1), v_2 = (1, 0, 1), v_3 = (0, 1, 0)$$

(b)

$$-1 + 1 + 1 = 0 + 1 + 0 = 1$$

(c)

$$-1 \cdot 1 \cdot 1 = -1$$

- (d) Find the inner products. $\langle v_1, v_2 \rangle = \langle v_2, v_3 \rangle = \langle v_1, v_3 \rangle = 0$. Since the inner products are all 0, then the angle is $\pi/2$
- (e) The eigenvalues are real. The matrix is symmetric.
- (f) Given that X has all real values, $X = X^T \implies \text{Im}(v_i) = 0$

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$$\theta = \arccos \left(\frac{\langle v, w \rangle}{\|v\| \|w\|} \right)$$

$$PQ = (2, 1, 0, 3) - (1, 2, 0, 1) = (1, -1, 0, 2)$$

$$PR = (0, 1, 1, 0) - (1, 2, 0, 1) = (-1, -1, 1, -1)$$

$$RQ = (2, 1, 0, 3) - (0, 1, 1, 0) = (2, 0, -1, 3)$$

As long as we take the acute angle, it doesn't matter if a vector is backwards: PQ , QP are equivalent.

$$P = \arccos \left(\frac{\langle PQ, PR \rangle}{\|PQ\| \|PR\|} \right) = 114.095^\circ$$

$$Q = \arccos \left(\frac{\langle PQ, RQ \rangle}{\|PQ\| \|RQ\|} \right) = 29.206^\circ$$

$$R = 180 - \arccos \left(\frac{\langle PR, RQ \rangle}{\|PR\| \|RQ\|} \right) = 36.700^\circ$$

The 3 sum to 180° .

3

$$\langle \vec{N}, \vec{X} - P \rangle = 0$$

$$\langle \vec{N}, \vec{X} \rangle - \langle \vec{N}, P \rangle = 0$$

$$(a_1x_1 + \cdots + a_nx_n) - (r_1x_1 + \cdots + c_nx_n) = 0$$

$$b - b = 0$$

(solves means evals to b)

4

$$\theta = \arccos \left(\frac{\langle (1, 1, 1, 1), (2, 1, 3, 7) \rangle}{\sqrt{1+1+1+1} \cdot \sqrt{2^2+1+3^2+7^2}} \right) = 35.023^\circ$$

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$$\theta = \arccos \left(\frac{\langle (1, 1, 1, 1), (2, 3, 0, 5) \rangle}{\sqrt{1+1+1+1} \cdot \sqrt{2^2+3^2+0+5^2}} \right) = 35.800^\circ$$

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(a)

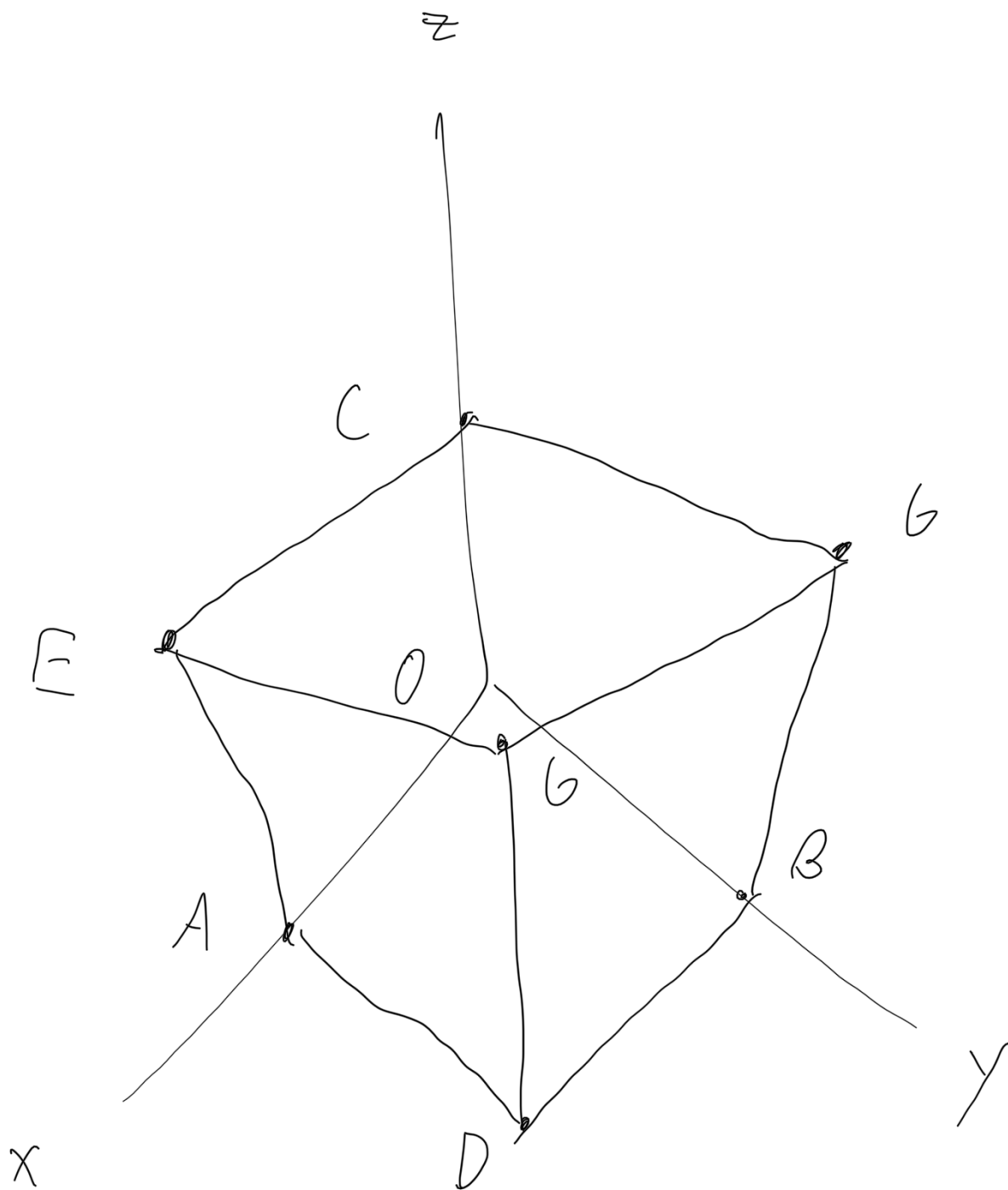


Figure 1: rough sketch

(b)

$$\sqrt{1+1+1} = \sqrt{3}$$

(c)

$$\arccos\left(\frac{\langle(1,0,0), (1,1,1)\rangle}{\sqrt{1} \cdot \sqrt{3}}\right) = 54.736^\circ$$

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(a)

$$\|x^2\| = \sqrt{\langle x^2, x^2 \rangle} = \sqrt{\int_0^1 x^2 \cdot x^2 dx} = \sqrt{1/5}$$

$$\|x^3\| = \sqrt{\langle x^3, x^3 \rangle} = \sqrt{\int_0^1 x^3 \cdot x^3 dx} = \sqrt{1/7}$$

$$\theta = \arccos\left(\frac{\langle x^2, x^3 \rangle}{\sqrt{1/35}}\right) = \arccos\left(\frac{\int_0^1 x^2 x^3 dx}{\sqrt{1/35}}\right) = 9.594^\circ$$

(b)

$$\|\sin(m\pi x)\| = \sqrt{\langle \sin(m\pi x), \sin(m\pi x) \rangle} = \sqrt{\int_0^1 \sin(m\pi x) \sin(m\pi x) dx} = \frac{1}{2} - \frac{\sin(2m\pi)}{4\pi^2}$$

$$\|\sin(n\pi x)\|$$