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Matrix A (a)
$$\lambda_1 = 4, \lambda_2 = -2, v_1 = (1,1), v_2 = (-1,1)$$

(b)
$$1 + 1 = 1 + 1 = 2$$

(c)
$$4 \cdot -2 = -8$$

(d)
$$\arccos\left(\frac{\langle(1,1),(-1,1)\rangle}{\sqrt{1+1}\cdot\sqrt{1+1}}\right) = \arccos(0) = \pi/2$$

(e) The eigenvalues are real. The matrix is symmetric.

Matrix B (a)
$$\lambda_1 = 37, \lambda_2 = -15, v_1 = (3,2), v_2 = (-2,3)$$

(b)
$$37 - 15 = 21 + 1 = 22$$

(c)
$$37 \cdot -15 = -555$$

(d)
$$\arccos\left(\frac{\langle(3,2),(-2,3)\rangle}{\sqrt{3^2+2^2}\cdot\sqrt{3^2+2^2}}\right) = \arccos(0) = \pi/2$$

(e) The eigenvalues are real. The matrix is symmetric.

Matrix C (a)
$$\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 1, v_1 = (-1,0,1), v_2 = (1,0,1), v_3 = (0,1,0)$$

(b)
$$-1 + 1 + 1 = 0 + 1 + 0 = 1$$

(c)
$$-1 \cdot 1 \cdot 1 = -1$$

(d) Find the inner products. $\langle v_1, v_2 \rangle = \langle v_2, v_3 \rangle = \langle v_1, v_3 \rangle = 0$. Since the inner products are all 0, then the angle is $\pi/2$

(e) The eigenvalues are real. The matrix is symmetric.

(f) Given that X has all real values, $X = X^T \implies \mathrm{Im}(v_i) = 0$

2

$$\theta = \arccos\left(\frac{\langle v, w\rangle}{\|v\|\|w\|}\right)$$

$$PQ = (2,1,0,3) - (1,2,0,1) = (1,-1,0,2)$$
$$PR = (0,1,1,0) - (1,2,0,1) = (-1,-1,1,-1)$$
$$RQ = (2,1,0,3) - (0,1,1,0) = (2,0,-1,3)$$

As long as we take the acute angle, it doesn't matter if a vector is backwards: PQ, QP are equivalent.

$$P = \arccos\left(\frac{\langle PQ, PR\rangle}{\|PQ\|\|PR\|}\right) = 114.095^\circ$$

$$Q = \arccos\left(\frac{\langle PQ, RQ\rangle}{\|PQ\|\|RQ\|}\right) = 29.206^\circ$$

$$R = 180 - \arccos\left(\frac{\langle PR, RQ\rangle}{\|PR\|\|RQ\|}\right) = 36.700^\circ$$

The 3 sum to 180°.

3

$$\begin{aligned}
\langle \vec{N}, \vec{X} - P \rangle &= 0 \\
\langle \vec{N}, \vec{X} \rangle - \langle \vec{N}, P \rangle &= 0 \\
(a_1 x_1 + \cdots + a_n x_n) - (r_1 x_1 + \cdots + c_n x_n) &= 0 && \text{(solves means evals to } b\text{)} \\
b - b &= 0
\end{aligned}$$

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$$\theta = \arccos\left(\frac{\langle(1,1,1,1),(2,1,3,7)\rangle}{\sqrt{1+1+1+1}\cdot\sqrt{2^2+1+3^2+7^2}}\right) = 35.023^\circ$$

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$$\theta = \arccos\left(\frac{\langle(1,1,1,1),(2,3,0,5)\rangle}{\sqrt{1+1+1+1}\cdot\sqrt{2^2+3^2+0+5^2}}\right) = 35.800^\circ$$

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(a)

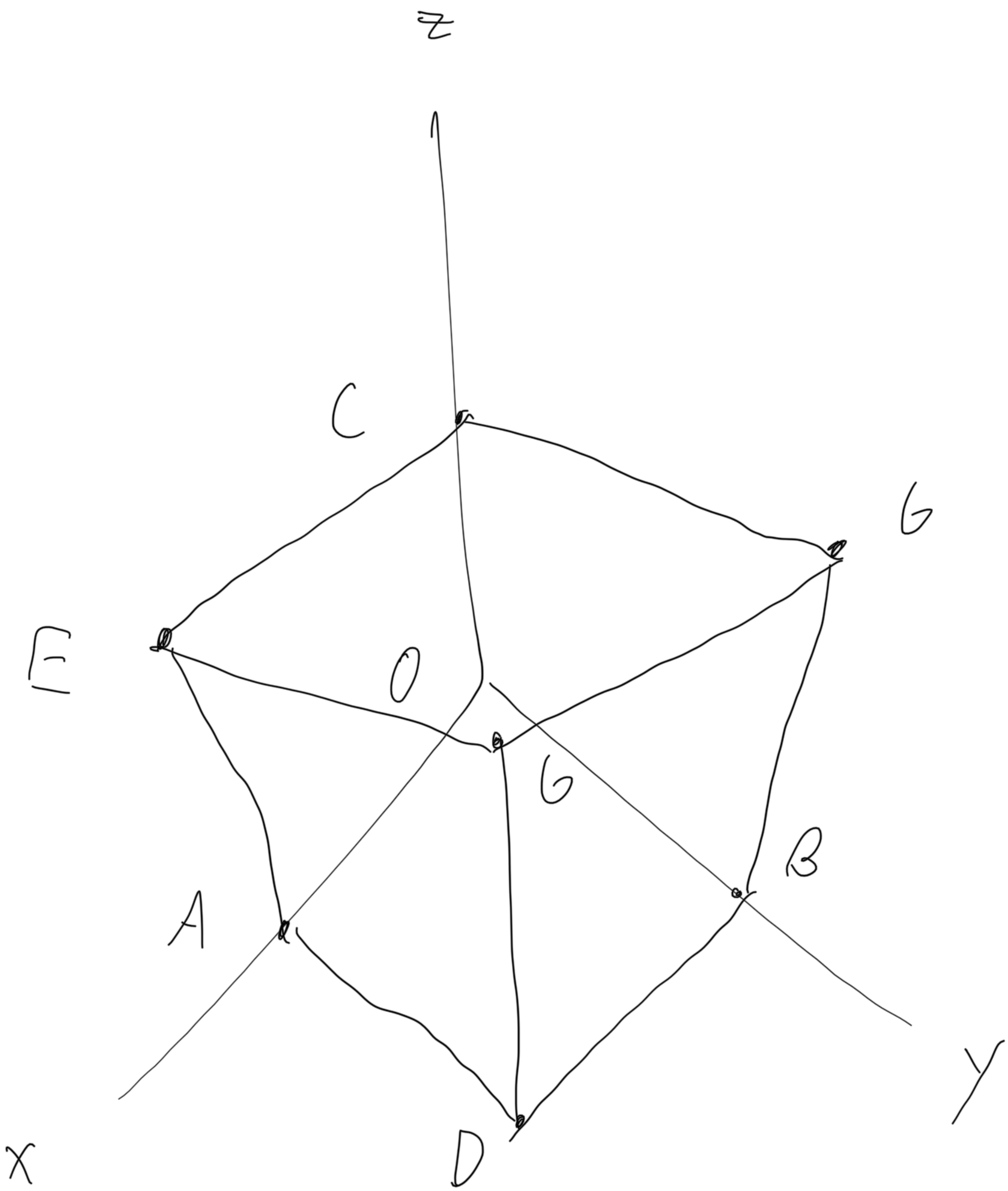

Figure 1: rough sketch

(b)

$$\sqrt{1+1+1} = \sqrt{3}$$

(c)

$$\arccos\left(\frac{\langle(1,0,0),(1,1,1)\rangle}{\sqrt{1}\cdot\sqrt{3}}\right) = 54.736^\circ$$

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(a)

$$\|x^2\| = \sqrt{\langle x^2, x^2\rangle} = \sqrt{\int_0^1 x^2 \cdot x^2 \, dx} = \sqrt{1/5}$$

$$\|x^3\| = \sqrt{\langle x^3, x^3\rangle} = \sqrt{\int_0^1 x^3 \cdot x^3 \, dx} = \sqrt{1/7}$$

$$\theta = \arccos\left(\frac{\langle x^2, x^3\rangle}{\sqrt{1/35}}\right) = \arccos\left(\frac{\int_0^1 x^2 x^3 \, dx}{\sqrt{1/35}}\right) = 9.594^\circ$$

(b)

$$\|\sin(m\pi x)\| = \sqrt{\langle \sin(m\pi x), \sin(m\pi x)\rangle} = \sqrt{\int_0^1 \sin(m\pi x)\sin(m\pi x)\, dx} = \frac{1}{2} - \frac{\sin(2m\pi)}{4\pi^2}$$

$$\|\sin(n\pi x)\|$$