Stress Analysis of a HyperLoop Pod

ME 362 Final Project

Adrien Aubrun, Harish Chockalingam, Zachary Fenske, and Nikhil Marri

June 2, 2017

Introduction:

The Hyperloop is a new transportation concept developed by renowned inventor Elon Musk to enable incredibly fast land travel. The Hyperloop Alpha is intended to connect cities that are less than 900 miles apart, for example the first Hyperloop should traverse the 350 mile distance from Los Angeles to San Francisco in 35 minutes. The design incorporates pressurized passenger capsules that are supported by an air cushion using air bearings, and accelerates using linear induction motors and air compressors. A successful Hyperloop will be a fast, safe, cheap, and environmentally sound method of transportation.

Objective: The high-level goal of this project is to determine the stresses in a passenger pod traveling through the Hyperloop tunnel. The pod itself is subject to multiple forces resulting from the low-pressure conditions in the tube. There is the atmospheric pressure pushing outwards on the walls of the passenger compartment, the force of gravity acting downwards on the vessel, and the cushion of air the holds the vessel off of the ground.

Requirements: Due to the unique nature of Hyperloop travel, there are some specific design constraints that must be met. As the Hyperloop is a pod passing through a tube filled with air, it is subject to Kantrowitz’s limit. This is the tube-to-pod area ratio which limits the top speed a pod can reach. As the pod must be held off the ground by a column of air, there is a non-trivial amount of air in the tube. Thus, Kantrowitz’s limit must be addressed. To do this, the tube pressure is set to 100 pascals. At this pressure, a pump at the front of the pod can easily relieve pressure and transfer it to the back of the pod where it will not limit the speed. Also, the design pressure of 100 pascals in the tube reduces the drag coefficient of the pod by 1000 times as opposed to being in air [1], therefore pressure due these loads are negligible. At the base of the pod there are 28 air bearing suspensions that lift the pod and each bearing is supported by an air pressure load 9.4kPa, thus the entire capsule is base has a load of 263.2 kPa. The internal cabin pressure will be maintained at atmospheric pressure (101.325kPa). These loads (and geometry constraints) must produce stresses that are less than the yield strength of the structure.

Proposed Design: For the geometry of our Hyperloop passenger pod, we decided on a cylinder with a semi circle face (see Appendix for sketch) and approximated dimensions based on a similar passenger vehicle, the CTA train cars. The CTA train cars have a width of 9 feet and 4 inches [2]. However, the design of the CTA cars is intended to fit more passengers and is less concerned with drag forces. Thus, our design is narrower at 8 feet in width. As the hyperloop system is much longer than it is wide, the plane strain approximation can be used in our analysis to save computation time. Thus strain, or displacement, along the z-axis of the pod is negligible. The only relevant stresses and strains will be along the x and y axes. As our pod is a semi circle in design, the max height is 4ft and is approximately is in line with the height requirement set for the hyperloop pod in the Hyperloop alpha document of 3.63 ft .[1].

The wall thickness sealing the passenger compartment from the outside low-pressure environment has a thickness of 0.5 feet. This estimated value ensures the interior of the pod remains pressurized and passengers are safely insulated from the outside, other estimates of thickness resulted in a safety factor of less than 1.

The hyperloop pod's weight of 33,000 lbs has been chosen based on data provided by SpaceX. This accounts for the weight of the various components of the pod. These are the weight of the interior and seats, the propulsion system, compressor, and other devices crucial to the operation of the hyperloop. Factoring in the weight of passengers and luggage, the total weight of the pod is 15,000 kg or 33,000 pounds. However, the weight of the pod was not used in the analysis. The breakdown of components by weight can be seen in the table below:

Vehicle Component	Cost ($)	Weight (kg)
Capsule Structure & Doors:	$ 245,000	3100
Interior & Seats:	$ 255,000	2500
Propulsion System:	$ 75,000	700
Suspension & Air Bearings:	$ 200,000	1000
Batteries, Motor & Coolant:	$ 150,000	2500
Air Compressor:	$ 275,000	1800
Emergency Braking:	$ 50,000	600
General Assembly:	$ 100,000	N/A
Passengers & Luggage:	N/A	2800
Total/Capsule:	$ 1,350,000	15000
Total for Hyperloop:	**$ 54,000,000**	

Table 1: Estimated breakdown of weights of components that make the entirety of the pod's weight and associated cost

The material chosen for our design is an aircraft grade 6061 Aluminum alloy (Young's Modulus is 69 GPa and Poisson's ratio is 0.33, and yield strength of 276 MPa and 310 MPa for ultimate tensile strength for 6061T6 Aluminum Alloy [3]). This alloy was chosen because it is easily worked, resists corrosion even when abraded, and exhibits good weldability. Furthermore, the Hyperloop is designed to travel at speeds similar to planes, therefore we chose an alloy that is successfully utilized in plane body design.

Analysis Approach

Using ANSYS (FEA), we modeled the pod using a plane strain assumption because the pod is relatively long, and we do not expect to see much strain along the length of the pod. We chose a Solid Quad 8 183 element to better approximate the curved aspect of the body rather than a Quad 4 182 element. The geometry of the structure was modeled using a partial annulus with theta set from 0 to 90 and radius set to 1.0668 (3.5 ft) m and 1.2192 m (4ft) and a rectangle of 1.2192 m and 0.1524 m (0.5ft) glued to the bottom of partial annulus, and due to symmetry the entire width (8ft) was not modeled. The material properties were set to linear elastic isotropic and the Young's modulus and poisson ratio of Aluminum for was then inputted. The pressure loads as stated in the requirements were then applied. For the inside a pressure of 1 atmosphere or 101.325 kPa, the outside has negligible pressure due to the fact that the pod is placed in a low pressure tube and was thus set to 0. The pressure load of 9.4kPa from an air bearing suspension on the bottom surface due to the air cushion required to levitate the pod was added. Two point loads, of 667 N (150 lbs) were applied to simulate the load due to two passengers. The displacement boundary condition was set to

0 in the x direction at the edges of the bottom beam and partial annulus, and the corner node in the inside was also constrained in all directions.

Results:

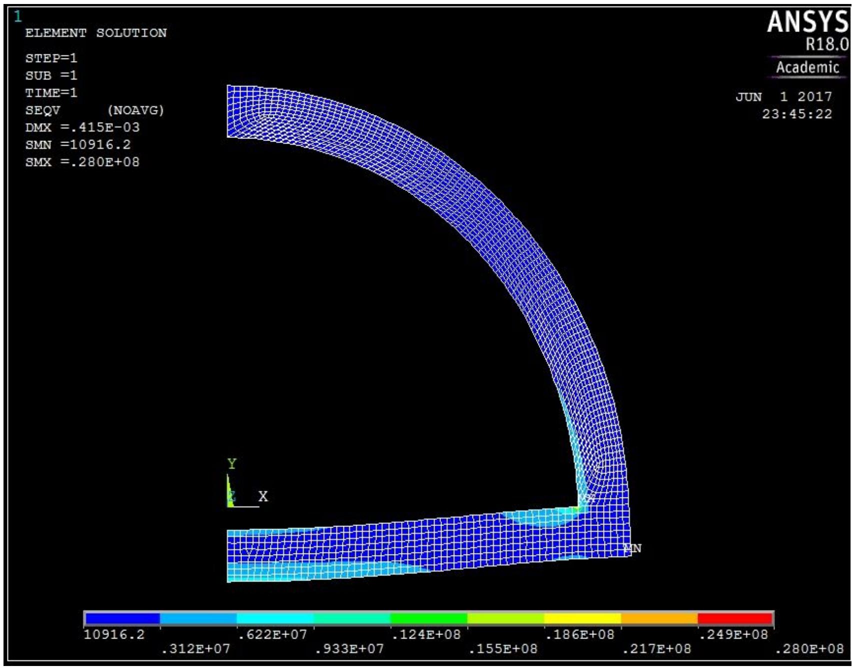

FIgure 1: The above is a vonMises contour plot of the body. With a max vonMises stress between 246e6 and 28e6 Pa on the inside corner

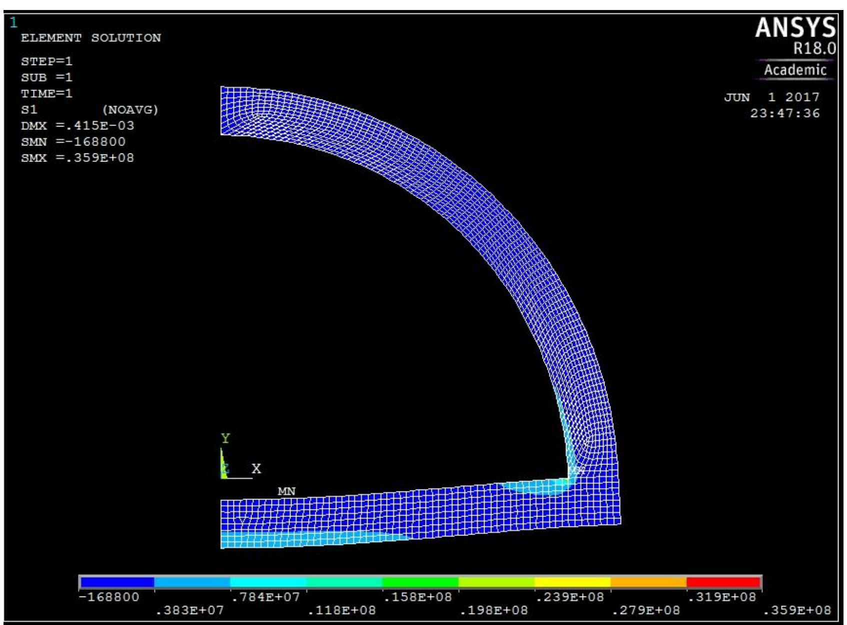

FIgure 2: The above is a contour plot of principal stress on the body. With a max principal stress of 31.9e6 to 35.9e6 Pa on the inside corner

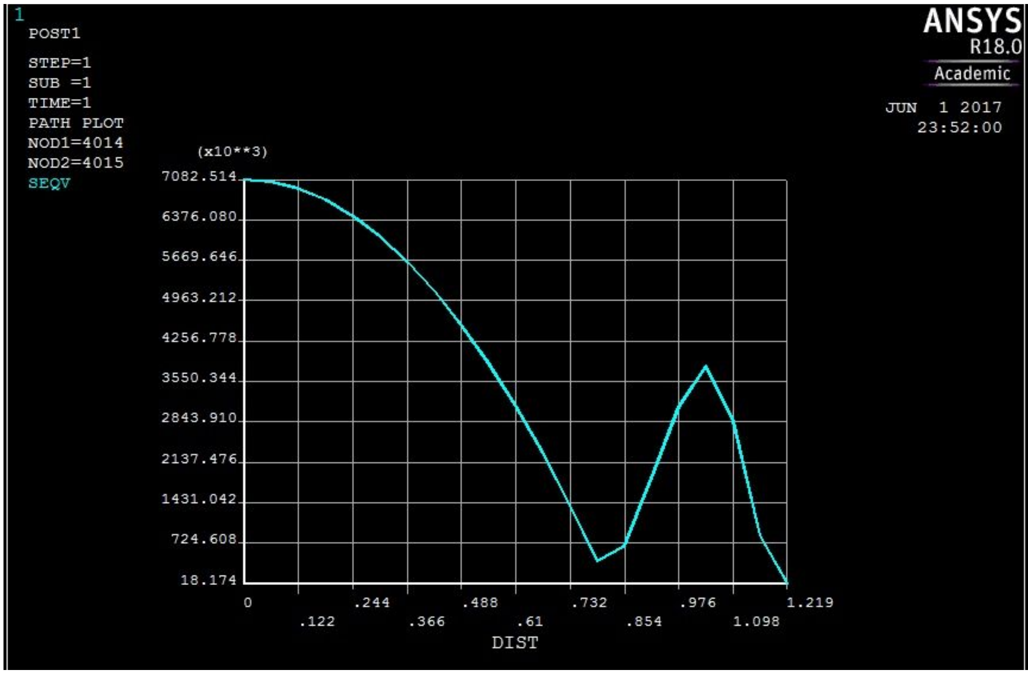

Figure 3: A plot of the bottom edge of the outer bottom part of the pod. The second peak is due to the increase in stress concentration due to the inner/outer corners. (vonMises stress on the y axis and length along the bottom edge on x axis). Plot of principal stress added to appendix

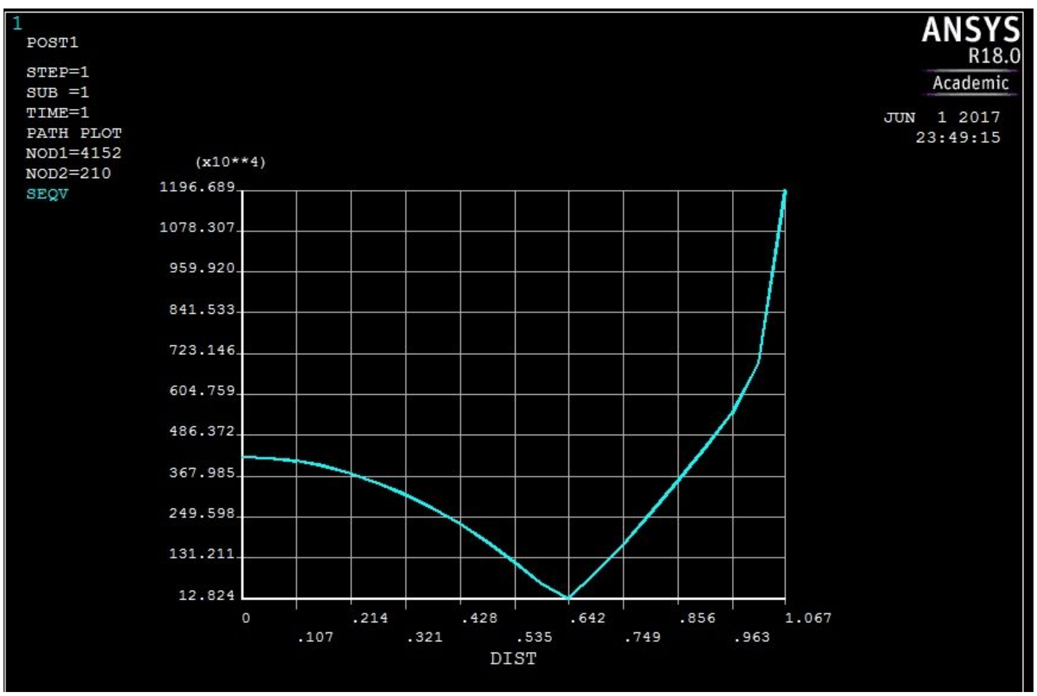

Figure 4: A plot of the top edge of the inner bottom part of the pod. The sudden rise can be is due to the stress concentration at the corner. (vonMises stress on the y axis and length along the top edge on x axis). Plot of principal stress vs distance added to appendix

Discussion:

Based on our analytical results the stress at the inner radius in the θθ direction was 763.3 kPa the numerical Ansys result at r=3.5ft is 168.8kPa for principal stress. The great difference is due to the fact the analytical calculation was based on an Airy stress function of a pipe under pressure which used a full circle, whereas the numerical solution is based on a semi-circle with a rectangular base. The analytical solution for max stress in the xx direction was found to be 0.215E8 Pa and the max principal stress (ignoring corner) from the numerical result was found to be 7.9E6 Pa (Figure 6 Appendix) at x=0 on the bottom edge. Since the base was taken to be a plate in the analytical calculation and the numerical calculation a beam the numerical solution is lower, however in both cases the max stress happens in the middle (midpoint for beam and midline along z for plate). The max principal stress at the corner, from Figure 2, is 35.9 MPa, compared to the ultimate tensile strength of 310 MPa of Aluminum that is 8.6 times smaller thus the design is intact without fracture. From Figure 1, it can be seen that the max vonMises stress occurs at the inner corner, which is a stress concentrator where the base of the curved body and flat base meet. The vonMises max stress at the corner is 28 MPa (although from figure 4 its 0.119e8, the discrepancy is probably due to selecting the nodes without great precision), this stress concentration can be reduced by designing the pod with a rounded corner rather than a sharp corner to reduce the stress concentration. But the stress concentration at this point is less than the yield strength of Aluminum and thus has a safety factor of (276/28) 9.86 before yielding, thus the design constraints are met and feasible to make the pod with Aluminum. However, this safety factor is too high, in the simulation, two force loads were applied along the top edge of the base to simulate two passengers along a single cross section of the pod (plane strain) the numerical model should be improved to account for more loads, however, even with the increased load this hyperloop pod design would still have a safety factor greater than 1 as the most relevant stresses as stated in the requirements above have been accounted.

References:

[1] Musk, Elon (August 12, 2013). "Hyperloop Alpha"(PDF). *SpaceX*. Retrieved August 13, 2013.

[2] Chicago "L".org: Car Roster - 5000-Series http://www.chicago-l.org/trains/roster/5000mkII.html

[3]*Aluminum 6061-T6; 6061-T651*. N.p., n.d. Web. 02 June 2017.
http://www.matweb.com/search/datasheet_print.aspx?matguid=1b8c06d0ca7c456694c7777d9e10be5b

Appendix

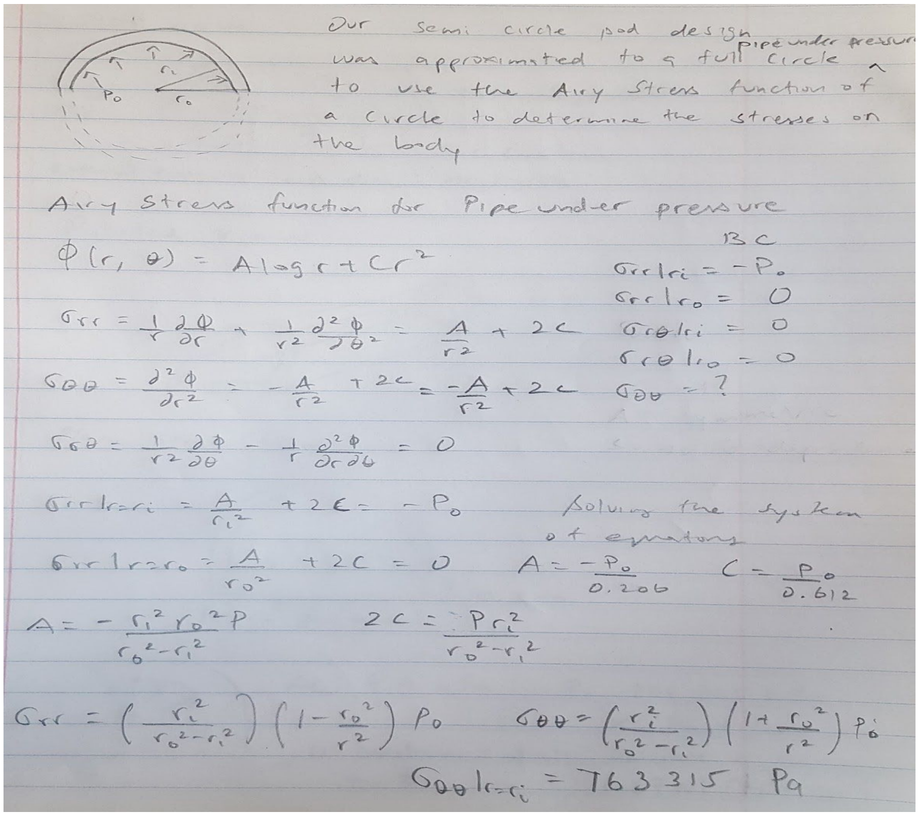

Figure 1: Calculation of stress at inner radius using Airy stress function. Po is 101.325 kPa

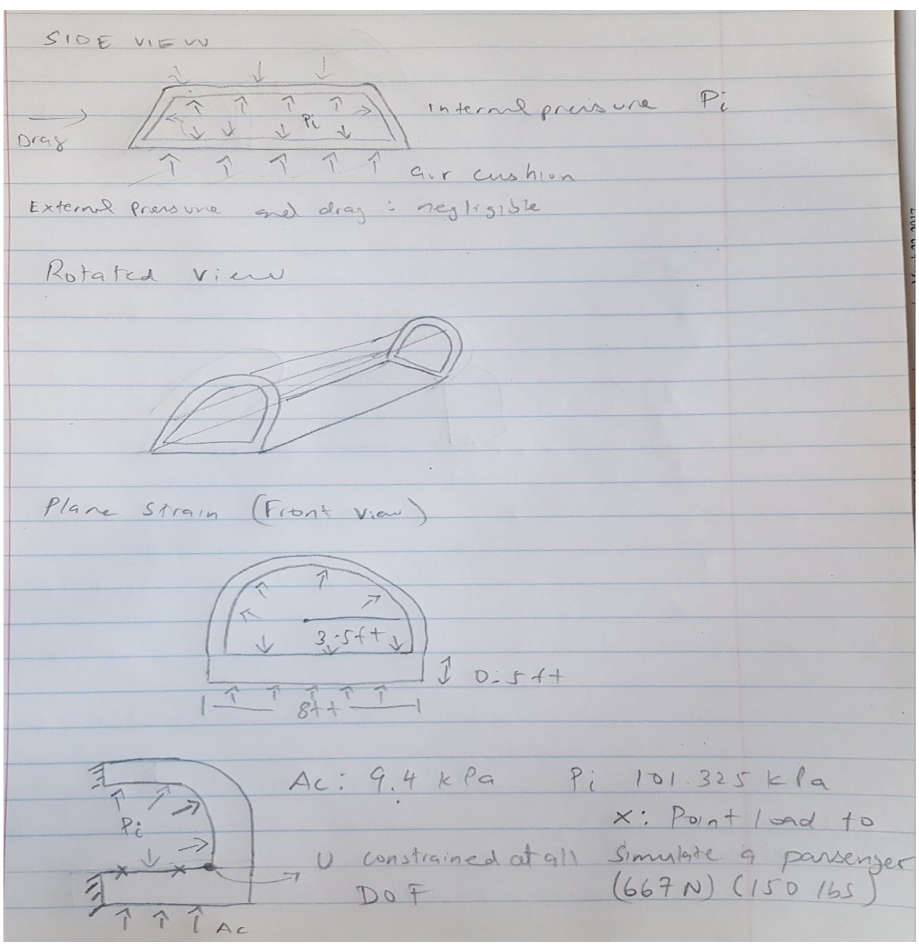

Figure 2: Sketch of proposed designed and free body diagram at the bottom.

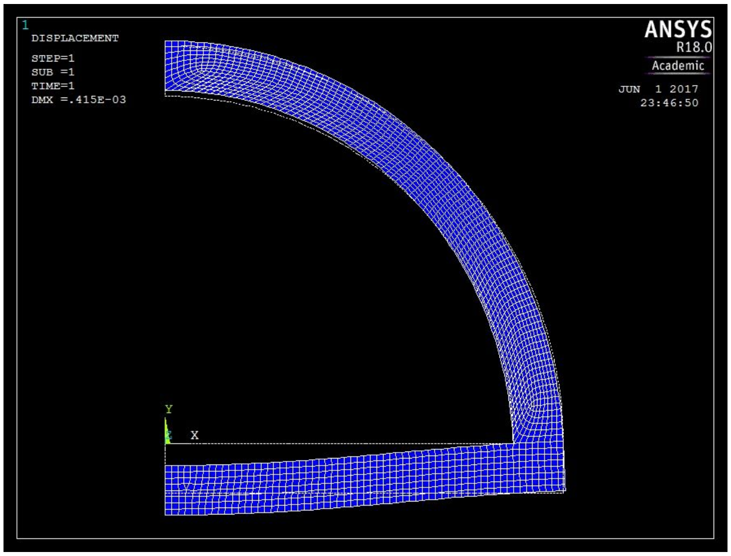

Figure 3: Exaggerated deformed shape of half the pod. A downward deformation, which is expected as the pressure is greater inside.

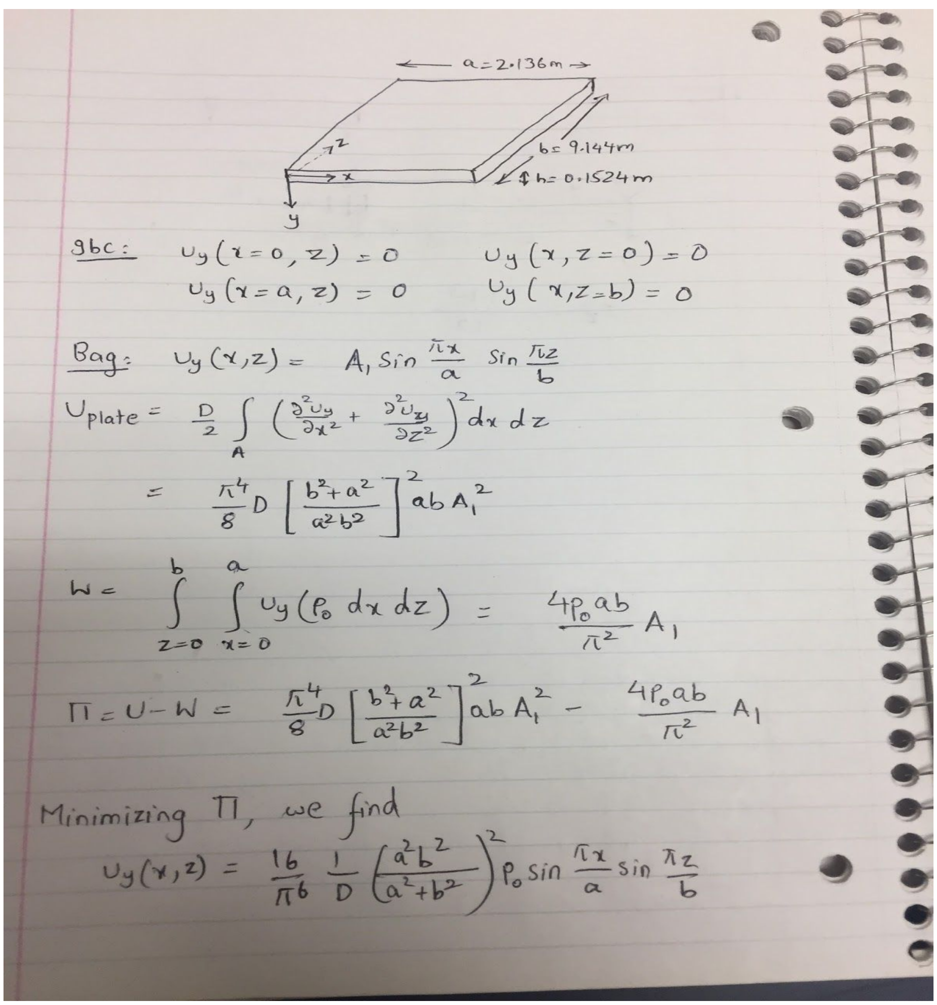

Figure 4: Analytical calculation of bottom base of pod. Assumed to be a plate rather than a beam (in the case of the plane strain assumption). Max stress in x direction was calculated to be 2.15 e 7 Pa

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{7\times10^{10} \times (0.1524)^3}{12(1-0.33^2)}$$

$$= 2.32 \times 10^7$$

P_0 = Net pressure in the y-direction

$$= 101.325 - 9.4 = 91.925 \text{ kpa}$$

$$U_y(x,z) = \frac{16}{\pi^6} \times \frac{1}{2.32\times10^7} \times \left(\frac{381.48}{88.17}\right)^2 \times (91.925 \times 10^3) \sin\left(\frac{\pi x}{2.136}\right) \sin\left(\frac{\pi z}{9.144}\right)$$

$$= \underline{0.00123 \sin\left(\frac{\pi x}{2.136}\right)} \sin\left(\frac{\pi z}{9.144}\right)$$

$$\sigma_{xx} = \frac{-E}{1-\nu^2}\, y \left(\frac{\partial^2 U_y}{\partial x^2} + \nu \frac{\partial^2 U_y}{\partial z^2}\right)$$

$$= (2.83\times10^8)\, y \sin\left(\frac{\pi x}{2.136}\right) \sin\left(\frac{\pi z}{9.144}\right)$$

$$\sigma_{zz} = \sigma_{xx}$$

$$\sigma_{xz} = \frac{-E}{1+\nu}\, y \frac{\partial^2 U_y}{\partial x \partial z} = (-1.41\times10^8)\, y \cos\left(\frac{\pi x}{2.136}\right) \cos\left(\frac{\pi z}{9.144}\right)$$

Principal stresses

$$\sigma_{xx}\Big|_{max} = 2.83 \times 10^8 \times \frac{0.1524}{2}$$

$$= \underline{\underline{2.15 \times 10^7 \text{ Pa}}} = \sigma_{zz}\Big|_{max}$$

$$\sigma_{xz}\Big|_{max} = -1.41 \times 10^8 \times \frac{0.1524}{2}$$

$$= \underline{\underline{-1.07 \times 10^7 \text{ Pa}}}$$

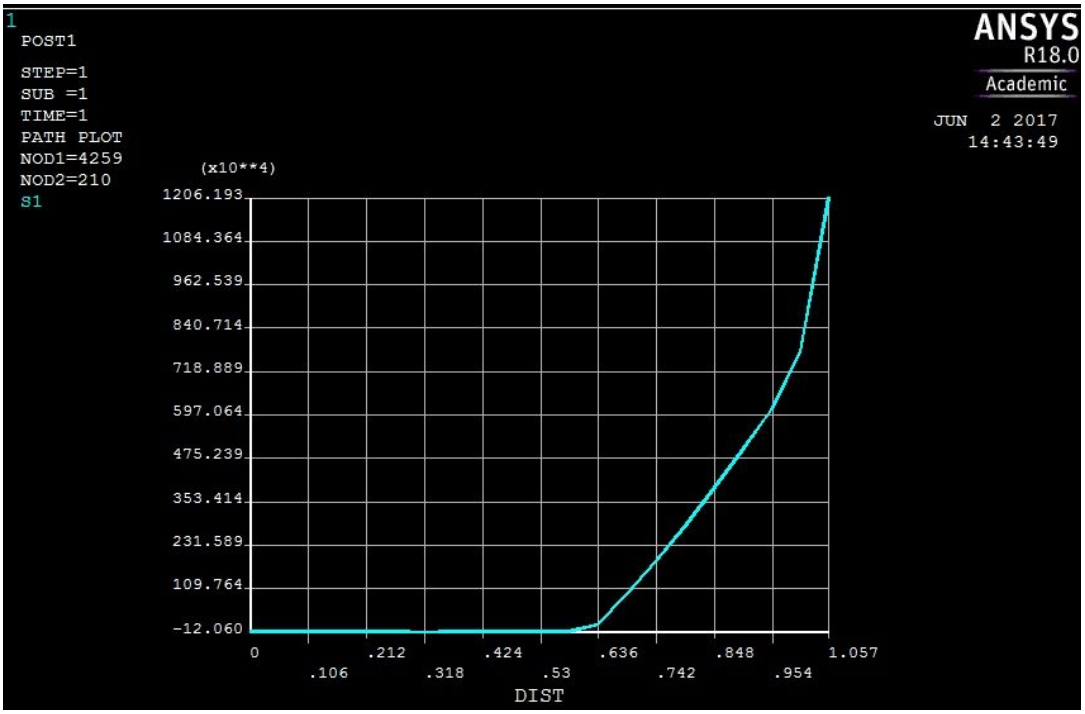

Figure 5: Principal stress vs length along the top edge of the inner part of base. The is greatest at the corner, as a corner is the stress concentration where the base and the curved body meet.

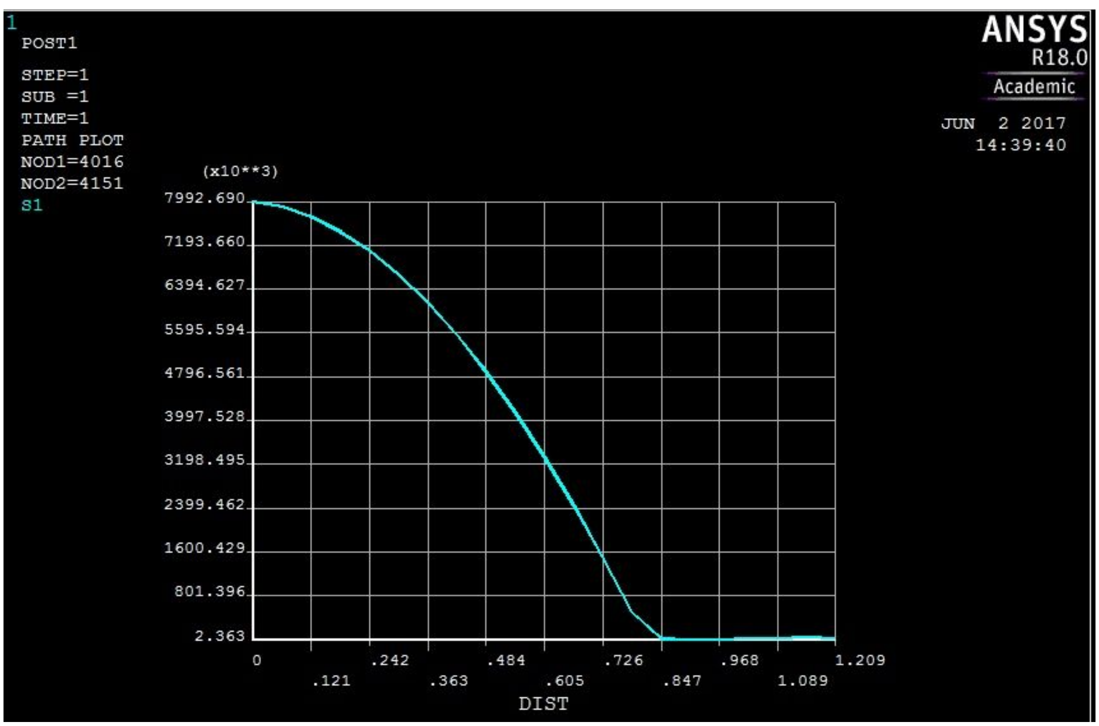

Figure 6: Principal stress vs length along the bottom edge of the outer part of base.