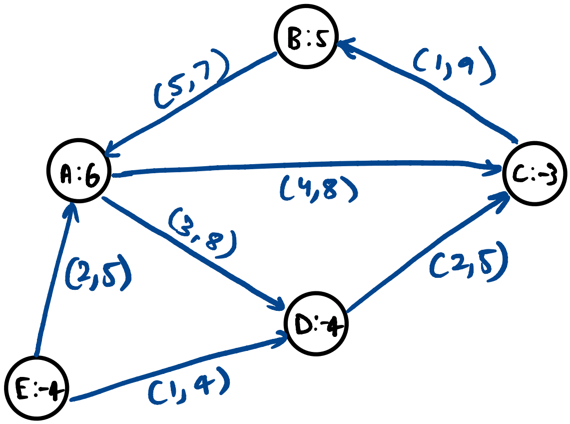


First convert the given circulation w/ demands & LB to a circulation w/ demands

$d'(A) = 6-5-2+4+3 = 6$

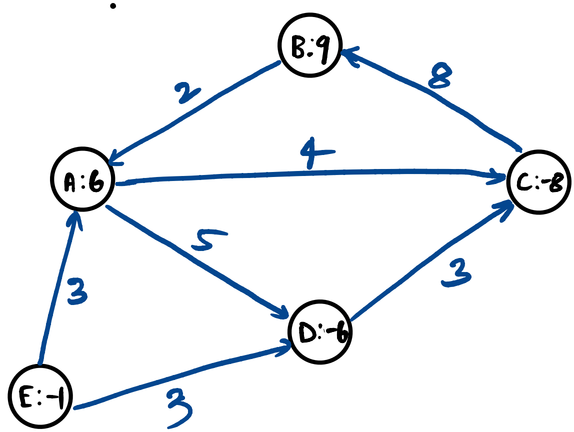
$d'(B) = 5-1+5 = 9$

$d'(C) = -3-2-4+1 = -8$

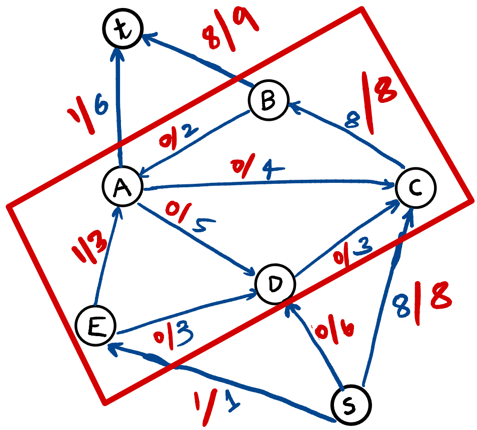
$d'(D) = -4-3-1+2 = -6$

$d'(E) = -4+2+1 = -1$

The new circulation w/ demands & updated edges is .



Now we need to convert the new circulation w/ demands into



A : 6
B : 9
C : -8
D : -6
E : -1

s → E → A → t
s → c → b → t

2. The organizers of the 2022 US Open Tennis Championships are working on making the draws for the first round and scheduling the matches. They are faced with several constraints in doing so.

   There are $2N$ players, seeded (ranked) from 1 to $2N$. The players are divided into two halves - the top half containing players seeded 1 to N, and the bottom half containing seeds $N+1$ to $2N$. The first round consists of N matches among the $2N$ players, each being one where a top-half player plays a bottom-half player.

   ~~Additionally, to keep them a bit more competitive, each match must be between players whose seeds differ by at most N' (a given constant $> N$).~~

   The sports facility has courts $C_1, ..., C_m$. The matches will be scheduled in timeslots $T_1, ..., T_k$. Matches can run in parallel on different courts in a given timeslot. Assume all courts are available in all timeslots, however a single court can have at most p matches to preserve surface quality. For broadcasting reasons, in any given timeslot, there should be at least one match being played and at most r of them.

   The bottom-half players were given the option to make requests if they preferred to play in certain timeslots, and each of them has specified k' of the k timeslots that they are okay to play in. Seeded players on the other hand were given the option to make requests to avoid playing on certain courts, and each of them has specified m' of the m courts that they do not want to play on.

   Describe a Network Flow/Circulation based algorithm to determine if it is possible to come up with a feasible schedule of matches based on the above constraints.

3)

3. A company is currently trying to fill a large order of steel, brass, and pewter, measured in tons. Manufacturing each ton of material requires a certain amount of time, and a certain amount of special substance (SS), both of which are limited and are given in below table. Note that it is acceptable to manufacture a fraction of a ton (e.g. 0.5t) of material. Specifically, the company currently has 8 hours of time, and 20 units of special substance (SS). Manufacturing each ton of the three products requires:

Product	Time (hours)	SS (units)
Steel	3	3
Brass	1	10
Pewter	2	5

Figure 2: Table describing the amount of time taken and special substance (SS) required for each product.

(a) Write down a linear program that determines the maximum amount of products (in tons) that the company can make.

(b) Due to the potential danger posed by the special substance (SS), the company would like to use up as much of its supply of special substance (SS) as possible, while:

i. spending at most 8 hours

ii. manufacturing a total of at least 2 tons of steel plus pewter.

a) s, b, p

Obj f^n: $\max(s+b+p)$

subject to: $3s+b+2p \leq 8$

$3s+10b+5p \leq 20$

b) Obj f^n: $\max(3s+10b+5p)$

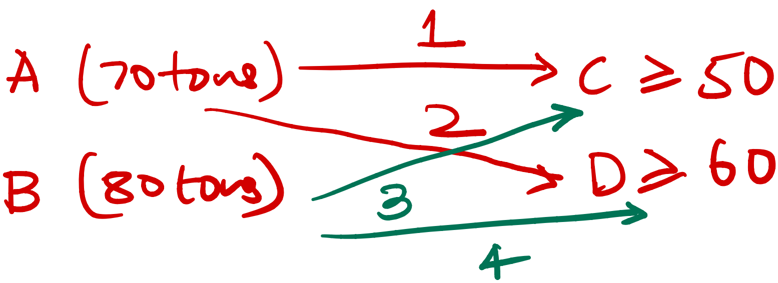
Subject to: $3s+b+2p \leq 8$

$s+p \geq 2$

$s \geq 0, b \geq 0, p \geq 0$

4. Suppose that a cement supplier has two warehouses, one located in city A and another in city B. The supplier receives orders from two customers, one in city C and another in city D. The customer in city C needs at least 50 tons of cement, and the customer in city D needs at least 60 tons of cement. The amount of cement at the warehouse in city A is 70 tons, and the number of units at the warehouse in city B is 80 tons. The cost of shipping each ton of cement from A to C is 1, from A to D is 2, from B to C is 3, and from B to D is 4.

   Formulate the problem of deciding how many tons of cement from each warehouse should be shipped to each customer to minimize the total shipping cost as a linear programming. You can assume that the values of units to be shipped are real numbers.



Obj f^n: $\min(X_{AC} + 2X_{AD} + 3X_{BC} + 4X_{BD})$

Subject:

$$X_{AC} + X_{BC} \geq 50$$

$$X_{AD} + X_{BD} \geq 60$$

$$X_{AC} + X_{AD} \leq 70$$

$$X_{BC} + X_{BD} \leq 80$$

$$X_{AC} \geq 0,\ X_{AD} \geq 0,\ X_{BC} \geq 0,\ X_{BD} \geq 0$$

5. $\max(x_1 - 3x_2 + 4x_3 - x_4)$

$$x_1 - x_2 - 3x_3 \leq -1$$

$$x_2 + 3x_3 \leq 5$$

$$x_3 \leq 1$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ -3 \\ 4 \\ -1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}$$

<u>Dual Nature</u>

$$\begin{matrix} \max(c^T x) \\ Ax \leq b \\ x \geq 0 \end{matrix} \quad \Rightarrow \quad \begin{matrix} \min(b^T y) \\ A^T y \geq c \\ y \geq 0 \end{matrix}$$

$$b^T = \begin{bmatrix} -1 & 5 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\min \left(-y_1 + 5y_2 + y_3 \right)$$

$$y_1 \geq 1$$

$$-y_1 + y_2 \geq -3$$

$$-3y_1 + 3y_2 + y_3 \geq 4$$