MASTER THEOREM (Abdul Bari)

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1$
$b > 1$

$$f(n) = \theta(n^k \log^p n)$$

Case 1: If $\log_b a > k$ then $\theta(n^{\log_b a})$

case 2: If $\log_b a = k$ then:

if $p > -1$ $= \theta(n^k \log^{p+1} n)$

if $p = -1$ $= \theta(n^k \log \log n)$

if $p < -1$ $= \theta(n^k)$

case 3: If $\log_b a < k$ then:

if $p \geq 0$ $\theta(n^k \log^p n)$

if $p < 0$ $\theta(n^k)$

eg: $T(n) = 2T(n/2) + 1$

$a = 2$

$b = 2$

$f(n) = \theta(1)$

$= \theta(n^0 \log^0 n)$

$\log_b a = 1$

$1 > 0$

case 1: $\theta(n)$

eg 2: $T(n) = 4T(n/2) + n$

$a = 4$

$b = 2$

$f(n) = \theta(n)$

$= \theta(n^1 \log^0 n)$

$\log_b a = 2$

$2 > 1$

case 1 $\theta(n^2)$

eg 3: $T(n) = 8T(n/2) + n$

case 1 $= \Theta(n^3)$

$\log_2 8 = 3$

$3 > 1$

$T(n) = 9T(n/3) + 1$

$\log_3 9 = 2 > k = 0$

case 1 $\Theta(n^2)$

eg: $8T(n/2) + n \log n$

$\log_2 8 = 3$

$f(n) = \Theta(n^1 \log^1 n)$

$3 > 1$

case 1 $= \Theta(n^3)$

eq 5 : $2T(n/2) + n^1$

$\log_2 2 = 1$

$k = 1$

$\log_a b = k$

case 2 $= \Theta(n^k \log^{p+1} n)$

$= \Theta(n \log n)$

eq 6 : $T(n) = 2T(n/2) + \frac{n}{\log n}$

$k = 1 \quad \log_b a = 1$

case 2 : case $p = -1$

$\Theta(n \log(\log n))$

$\longrightarrow T(n) = 2T(n/2) + \frac{n}{\log^2 n}$

$p < -1$

$\therefore \Theta(n)$