NF $(G, s, t, C \in N^+)$

- Find Aug path $= O(V+E)$
- Aug flow $= O(V)$
- Find Bottleneck $= O(V)$
- FORD FULKERSON $= O(|f| \cdot (V+E))$
- EDMUND KARP $= O(VE^2)$
- $\exists$ circulation, if maxflow $= D$
- Edmund Karp 1 $= O(E^2 \cdot \log V \cdot \log |f|)$ – augmenting largest capacity path
- Edmund Karp 2 $= O(E^2 \cdot V)$ – augmenting in shortest path wrt edges.

LP

- $\max(c^T x)$, $Ax \le b$ $\iff$ $\min(b^T y)$, $A^T y \ge c$
- Standard Form:
  - $=$ $\Rightarrow$ $-$version $\le$, $+$version $\le$
  - $\ge$ $\Rightarrow$ $-$version $\le$
  - $x \ge 1 \Rightarrow$ new $z_1 = x - 1 \ge 0$
  - free variable $\Rightarrow$ new $z_5 - z_6 \mid z_5, z_6 \ge 0$
- Assignment problems

$x_{ij} \begin{cases} 0 \\ 1 \end{cases}$

P\D	F.B.	F.U.	I.
F.B.	Yes cor. 1	NO cor. 1	? NO
F.U.	NO cor. 1	? NO	Yes cor. 2
I.	? NO	Yes cor. 2	? Yes

DP

- SOBST formula:

Expected cost $= \sum_{i=1}^{n} p_i \times \text{depth}(k_i)$

Woody

$OPT[i,j] = \min_{i \le k \le j} (OPT[i,k] + OPT[k,j] + d[j] - d[i])$ | $OPT[i,i] = 0$, $OPT[i,i+1] = 0$

Final answer: $OPT[0, n+1]$

- Till k approach (Burst balloons)

$OPT[i,j] = \max \{ OPT[i,k-1] + OPT[k+1,j] + nums[i-1] + nums[k] + nums[j+1] \}$

Car fueling: $OPT[i] = \min_{1 \le k < i} (f(k,i) + t[i] + OPT[k])$

Rod problem: $OPT[i] = \max_{1 \le j \le i} (P(j) + OPT[i-j])$

Factory worker: $OPT[m,j] = \max_{0 \le k \le j} (P(m,k) + OPT[m-1, j-k])$

- Bellman ford

$D(V,K) = MIN(D[V,K-1], \min_u (D[u,K-1] + w(u,v))) \Rightarrow \underline{O(V \cdot E)}$

$D(V,0) = \infty$, $D(S,K) = 0$ } Basecase

Stock: $OPT[i,j] = \max(OPT[i-1,j], \max_{m=0}^{i-1}(OPT[m,j-1] + P[i] - P[m]))$

Street Lamp $= \max(OPT[i-1,T], P[i] + OPT[i-1-S[i], T-T_i])$ $\forall t_i < T$

- River: $OPT[i,j] = \min_{j \le k \le i} (C[i,k] + OPT[k,j])$

from i to j

- River (n to 1): $OPT[i] = \min_{j \le k \le 1} (C[i,k] + OPT[k])$
- LCS: $LCS[i,j] = 1 + LCS[i-1,j-1]$ if $s[i] = j[i]$

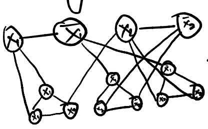
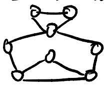
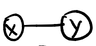
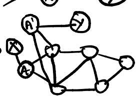
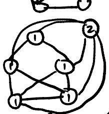
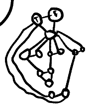
$LCS[i,j] = MAX[LCS[i-1,j], LCS[i,j-1]]$ if $s[i] \ne s[j]$

- Palindrome: $OPT[i,j] = OPT[i+1, j-1] + 2$ if $s[i] = s[j]$

$OPT[i,j] = MAX[OPT[i+1,j], OPT[i,j-1]]$ if $s[i] \ne s[j]$

CMM: $C[i,j] = \min_{i \le k < j} \{ C[i,k] + C[k+1,j] + d_{i-1} \times d_k + d_j \}$

NP

- $3SAT \le_p IS \Rightarrow$ Easy, connect x to $\bar{x}$ & so on.
- $3SAT \le_p VC \Rightarrow$  $\Rightarrow$ $l + 2m$ clauses (literal, triangle) $\Rightarrow$ Total graph has $2l + 3m$
- $VC \le_p$ Event management $\Rightarrow$ Applicant (vertices) + Days (edges) $\Rightarrow$ One applicant satisfies requirement for one day
- MaxClique $\le$ MaxIS $\Rightarrow$ Given $G(V,E)$, G' such that $(u,v) \in E'$ & $(u,v) \notin E$
- $VC \le VCE \Rightarrow$  $\Rightarrow$ G has a VC of K iff G' has a VC of $K+2$ (at most)
- $IS \le_p VC \Rightarrow$  $\Rightarrow$ For $G(V,E)$, S is an IS iff $V-S$ is a VC $\Rightarrow x \in IS, y \in IS, (x,y) \in IS \Leftarrow$ Pick $\forall x,y$ such that $x \notin VC$, $y \notin VC$
- $HC \le_p HP \Rightarrow$  $\Rightarrow$ G has a HC $\iff$ G' has a HP
- $HP \le_p$ Pebbles $\Rightarrow$  $\Rightarrow$ G has a HP $\iff$ G' has a winning sequence
- $3SAT \le_p$ 3-colourable $\Rightarrow$  $\Rightarrow$ 3SAT is satisfiable $\iff$ G is 3 colorable
- $3SAT \le_p ILP \Rightarrow$ In each of them clauses convert $x_1 + \bar{x}_2 + x_3 \ge 1$ into $-z_1 - z_2 - z_3 \le -1$ where $x_1 = z_1$ & $\bar{x}_1 = 1 - z_1$ [3SAT is satisfiable iff ILP has a solution]
- $IS \le_p$ HALF-IS $\Rightarrow$ If $K = \frac{|V|}{2}$ $IS \le_p$ HalfIS, if $K < \frac{|V|}{2}$ add $m = |V| - 2k$ nodes (dont connect), if $K > \frac{|V|}{2}$ add $m = |V| + 2k$ (connect to all $|V| + m - 1$ nodes)