Estimating Long-term Rewards by Off-policy Reinforcement Learning

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Amazon

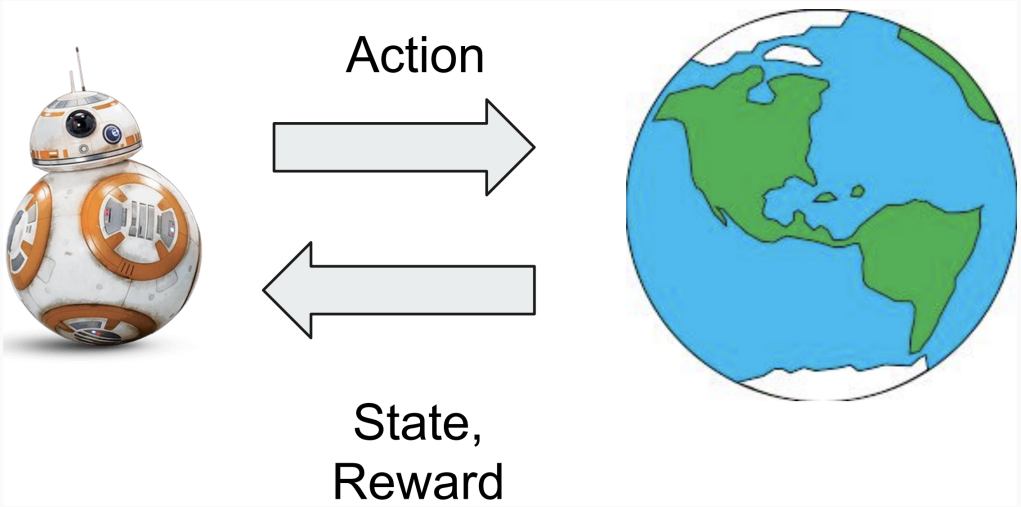
December 15th, 2020

AI for Economics Seminar

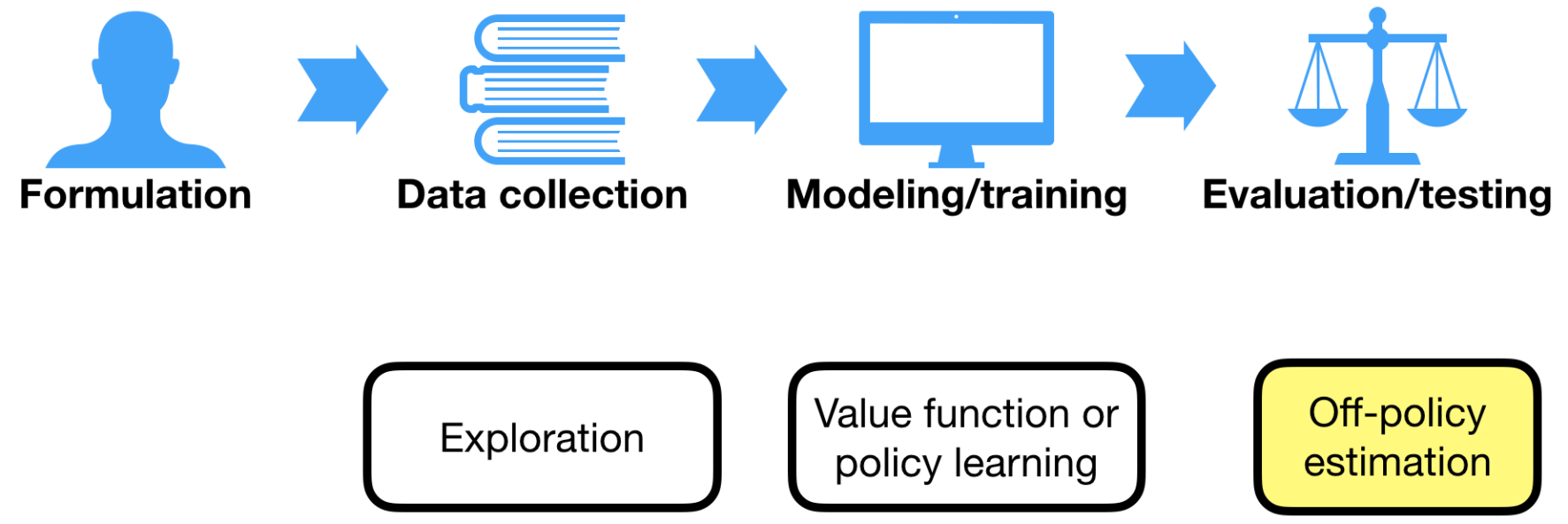
Thanks to Andrew Bennett, Yinlam Chow, Bo Dai, Nathan Kallus, Qiang Liu, Ali Mousavi, Ofir Nachum, Dale Schuurmans, Csaba Szepesvári

RL: A General Learning Paradigm

- Games
- Robotics and control
- Recommendation
- Inventory management
- Medical treatment
- Education systems
- Economic decisions (tax policy, job training, etc.)
- ...



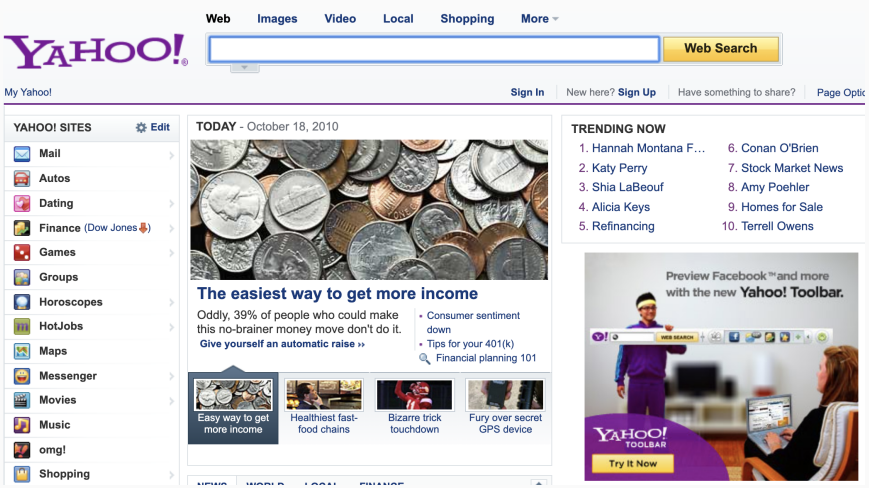
A Simplified View of RL



Informally, off-policy estimation (OPE) asks the question:

How good is a given policy without actually running it?

OPE Example: Personalized News Recommendation



(www.yahoo.com on 2010/10/18)

Naturally an RL problem:

- state: user information
- action: news article
- reward: $\{0, 1\}$ (click or not)
- policy: state $\rightarrow$ action

A typical evaluation dilemma in practice:

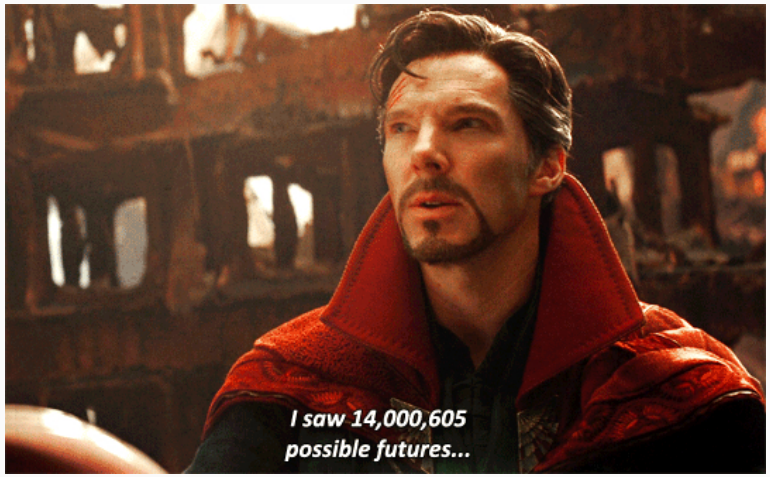
- policy deployment requires performance evaluation
- performance evaluation requires policy deployment

OPE: Can we evaluate performance without policy deployment?

Fundamentally a counterfactual estimation problem.

OPE with Simulators

- Games: (almost) perfect simulation
- Robotics: physical dynamics
- Recommendation: user click model
- Auto. driving: all traffic conditions
- Dialogue: human conversation model
- Healthcare: patient's medical model
- Economy: consumer behavior model
- ...



Reliable simulators are often hard to build.

This talk: a complementary approach without building a simulator.

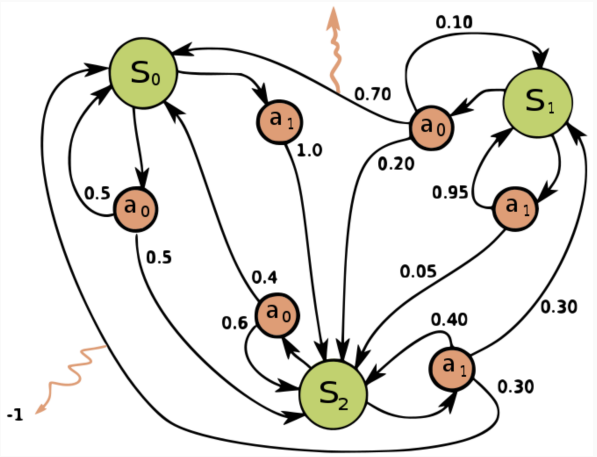
Outline

- The OPE problem
- Existing techniques
- OPE for long-term rewards
  - DualDICE: efficient point estimation
  - CoinDICE: confidence interval estimation
  - Dealing with unmeasured confounding
- Conclusions

Markov Decision Process

MDP $M = \langle \mathcal{S}, \mathcal{A}, T, R, \mu_0, \gamma \rangle$

- $\mathcal{S}$: finite set of states
- $\mathcal{A}$: finite set of actions
- $T : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$: transition kernel
- $R : \mathcal{S} \times \mathcal{A} \to [0, 1]$: reward function
- $\mu_0 \in \Delta(\mathcal{S})$: initial state distribution
- $\gamma \in [0, 1)$: discount factor



(from Wikipedia)

See papers for treatments of infinite state-actions, and of the $\gamma = 1$ case.

Definitions

- Trajectory $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, s_2, \ldots)$, with $s_0 \sim \mu_0$ controlled by policy π: $\quad \forall t,\ a_t \sim \pi(\cdot|s_t),\ s_{t+1} \sim T(\cdot|s_t, a_t)$
- Return: $U(\tau) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t r_t$
- Policy value: $v(\pi) := \mathbb{E}_{\tau \sim \pi}[U(\tau)]$ $\quad \Longleftarrow$ **subject of study in this talk**

Estimating $v(\pi)$

On-policy (Monte Carlo) estimation

- Run π directly to generate trajectories
- Estimate $v(\pi)$ (by averaging rewards over trajectories)

E.g., randomized clinical trials, A/B testing, ...

Correct, and straightforward

Often costly, risky, or even impossible to run π

Examples: healthcare, autonomous driving, ...

Off-policy estimation

- Assume historical data $\bar{\mathcal{D}}$ generated by behavior policy π_0
- Estimate $v(\pi)$ from $\bar{\mathcal{D}}$

Cheap and safe

Biased: simple average over $\bar{\mathcal{D}}$ estimates $v(\pi_0)$, not $v(\pi)$

Connection to Causal Effect Estimation

A Special Case

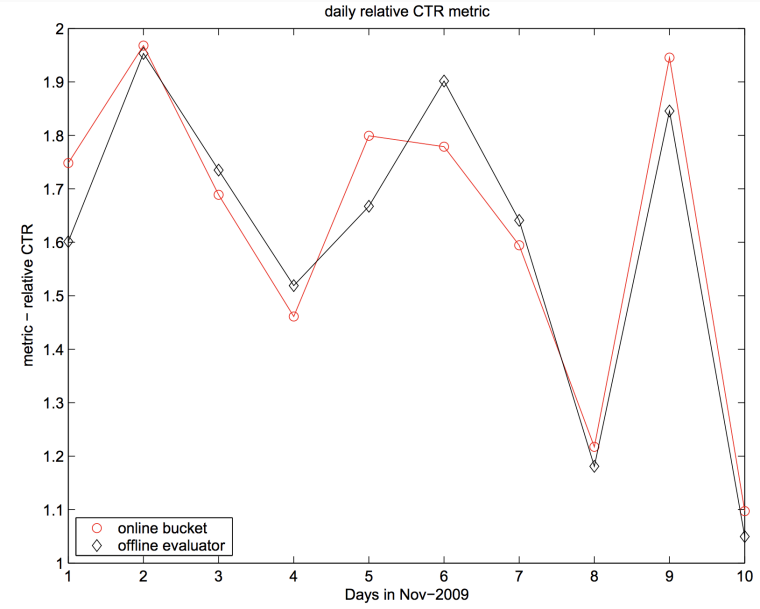
- $\mathcal{S}$ is set of patients
- $\mathcal{A} = \{\texttt{NewDrug}, \texttt{Placebo}\}$
- Reward is $\{0, 1\}$ (cured or not)
- Discount $\gamma = 0$
- $\pi_{\texttt{treatment}}(s) \equiv \texttt{NewDrug}$,
  $\pi_{\texttt{control}}(s) \equiv \texttt{Placebo}$
- Want to estimate $v(\pi_{\texttt{treatment}}) - v(\pi_{\texttt{control}})$

Same as the average treatment effect.

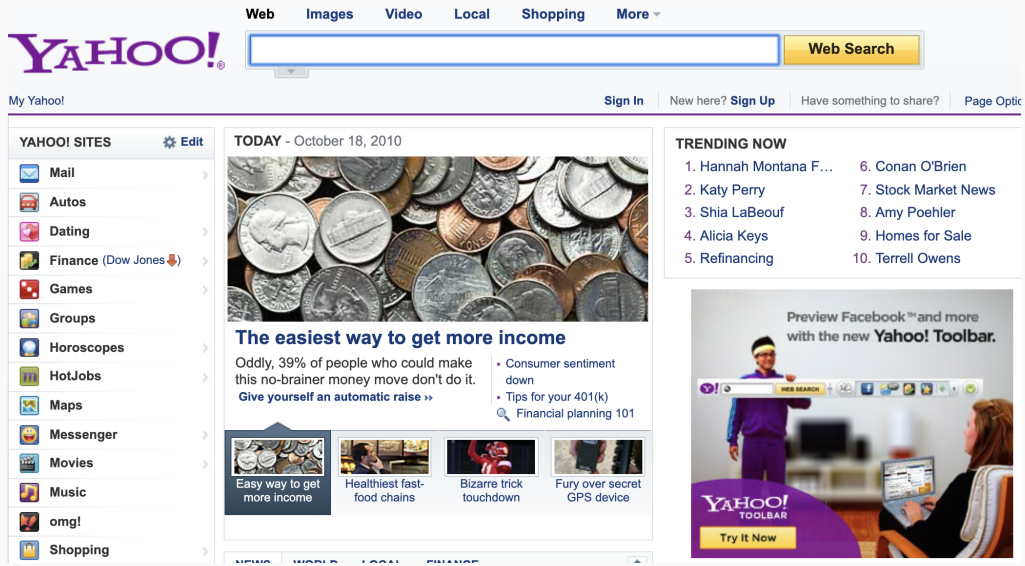
Also closely related to covariate shift in machine learning.

Personalized News Recommendation

- $\mathcal{S}$ is all users
- $\mathcal{A}$ is news articles
- Reward is $\{0, 1\}$ (click or not)
- $\gamma = 0$ (i.e., contextual bandits)
- $v(\pi)$ is click-through rate



[LCLW'11]



Survey on more applications on the Web [HLR'16]

Job Training Programs

Impact of job training on long-term employment rates [Athey+'19]

- $\mathcal{S}$: certain workforce population
- $\mathcal{A}$: participation in training or not
- Reward: employment rate 9 years from now

Like a contextual bandit with long-delayed reward.

Policy value $v(\pi)$ measures "long-term rewards":

- accumulated over a long period of time
- allows repeated exposure to treatment
- more general

Family Functioning

Fast Track prevention program [Murphy'03]

- Goal: prevent/reduce conduct disorder of children
- $\mathcal{S}$: child's behavioral condition at the end of semester
- $\mathcal{A}$: number of home visits scheduled in the next semester
- Reward: academic achievement in 10 semesters
- $\gamma = 0$

Each step $t \in \{0, 1, \ldots, 9\}$ corresponds to 1 semester.

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Precise Formulation

Generation of data

$$\begin{aligned} \bar{\mathcal{D}} &= \left\{\tau^{(i)}\right\}_{1\le i\le m} \\ \tau^{(i)} &= \left(s_0^{(i)}, a_0^{(i)}, r_0^{(i)}, \ldots, s_{H-1}^{(i)}, a_{H-1}^{(i)}, r_{H-1}^{(i)}\right), \end{aligned}$$

for some large enough horizon H and behavior policy π_0, where $a_t \sim \pi_0(\cdot|s_t)$, $r_t = R(s_t, a_t)$, $s_{t+1} \sim T(\cdot|s_t, a_t)$.

OPE: Off-policy estimation

Estimate $\hat{v}(\bar{\mathcal{D}}, \pi) \approx v(\pi)$ **without** knowing $\{T, R\}$.

When π_0 is unknown, "behavior-agnostic OPE".

Bellman Equation

Value functions

Generalizing policy value:

$$Q_\pi(s,a) = \mathbb{E}_{\tau\sim\pi}[\sum_{t=0}^{\infty}\gamma^t r_t | s_0 = s, a_0 = a]$$

So $v(\pi) = (1-\gamma)\mathbb{E}_{s_0\sim\mu}[Q_\pi(s_0, \pi(a_0))]$.

Bellman equation

$$Q_\pi(s,a) \;=\; R(s,a) + \gamma\mathbb{E}_{s'\sim T(\cdot|s,a), a'\sim\pi(\cdot|s)}[Q_\pi(s',a')]$$

Compute Q_π from known (T, R): dynamic programming, ...

Regression Estimator

A plug-in approach

- Use $\bar{\mathcal{D}}$ to estimate (T, R) of MDP by $(\hat{T}, \hat{R})$
- Compute $\hat{Q}_\pi$ from $(\hat{T}, \hat{R})$, and estimate $v(\pi)$ by

$$\begin{aligned}\hat{v}_{\text{Reg}} &= (1-\gamma)\hat{\mathbb{E}}_{s_0}[\hat{Q}_\pi(s_0, \pi(s_0))] \\ &= \frac{1-\gamma}{m}\sum_{\tau \in \bar{\mathcal{D}}} \hat{Q}_\pi(s_0, \pi(s_0))\end{aligned}$$

Pros: simple and low variance.

Cons: uncontrolled bias due to model misspecification of $(\hat{T}, \hat{R})$.

Inverse Propensity Score (IPS)

Key idea: change of measure

$$
\begin{aligned}
v(\pi) &= \mathbb{E}_{\tau\sim\pi}[U(\tau)] \\
&= \mathbb{E}_{\tau\sim\pi_0}\left[\frac{\Pr_\pi(\tau)}{\Pr_{\pi_0}(\tau)}U(\tau)\right] \\
&= \mathbb{E}_{\tau\sim\pi_0}\left[\frac{\prod_{t=0}^{H-1} T(s_t|s_{t-1},a_{t-1})\pi(a_t|s_t)}{\prod_{t=0}^{H-1} T(s_t|s_{t-1},a_{t-1})\pi_0(a_t|s_t)}U(\tau)\right] \\
&\approx \frac{1}{|\bar{\mathcal{D}}|}\sum_{\tau\in\bar{\mathcal{D}}}\left[\underbrace{\prod_{t=0}^{H-1}\frac{\pi(a_t|s_t)}{\pi_0(a_t|s_t)}}_{\text{“propensity score”}}U(\tau)\right] =: \hat{v}_{\text{IPS}}\,.
\end{aligned}
$$

Pros: unbiased.

Cons: high variance (more discussion soon).

Many Variants of IPS/DR

- Per-decision IPS [Precup+'01]
- Self-normalized IPS [Precup+'01, Swaminathan+'15]
- Doubly robust estimators [JL'16, Thomas+'16]
- More robust doubly robust estimators [Farajtabar+'18]
- Use of temporal abstraction to reduce variance [Guo+'17]
- ...

Unfortunately, not widely used in practice unless $\gamma \ll 1$.

Curse of Horizon

- For $\tau = (s_0, a_0, r_0, \ldots, s_{H-1}, a_{H-1}, r_{H-1})$,

$$\frac{\Pr_\pi(\tau)}{\Pr_{\pi_0}(\tau)} = \prod_{t=0}^{H-1} \frac{\pi(a_t|s_t)}{\pi_0(a_t|s_t)}$$

  whose variance easily blows up (exponentially in horizon H).
- Exponential blow-up is unavoidable in *minimax* rate [LMS'15]
- Similar for doubly robust estimator [JL'16]
- Can truncate ratios when they grow too large, at the cost of introducing large bias (e.g. ReTrace [Munos+'16])

Can we break the Curse of Horizon (at least in "benign" cases)?
Many recent developments [LLTZ'18, NCDL'19, DNCLSS'20, ...].

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Behavior-agnostic OPE

Now consider a more general setting where data consists of n transition tuples: $\mathcal{D} = \{(s_i, a_i, r_i, s_i')\}_{1 \le i \le n}$

Assumptions:

- $(s_i, a_i) \sim d_{\mathcal{D}}$ (unknown behavior distribution)
- $\mathbb{E}[r_i] = R(s_i, a_i)$
- $s_i' \sim T(s_i, a_i)$

Otherwise, no assumption on π_0 (other than regularity conditions).
Data do not even have to be in the form of trajectories.

Linear Program Characterizations of Policy Value

Two equivalent LPs for the policy value [Puterman'94]:

Primal LP

$$\begin{aligned} v(\pi) = \quad & \min_{Q:\mathcal{S}\times\mathcal{A}\to\mathbb{R}} \quad (1-\gamma)\,\mathbb{E}_{(s_0,a_0)\sim\mu_0\pi}[Q(s_0,a_0)] \\ & \text{s.t.} \qquad\qquad Q \geq R + \gamma\mathcal{P}_\pi Q \end{aligned}$$

Dual LP

$$\begin{aligned} v(\pi) = \quad & \max_{d\in\Delta(\mathcal{S}\times\mathcal{A})} \quad \mathbb{E}_{(s,a)\sim d}[R(s,a)] \\ & \text{s.t.} \qquad\qquad d = (1-\gamma)\mu_0\pi + \gamma\mathcal{P}_\pi^* d \end{aligned}$$

where

$$\mathcal{P}_\pi Q(s,a) := \sum_{s',a'} T(s'|s,a)\pi(a'|s')Q(s',a')$$

$$\mathcal{P}_\pi^* d(s,a) := \sum_{\bar{s},\bar{a}} \pi(a|s)T(s|\bar{s},\bar{a})d(\bar{s},\bar{a})$$

OPE based on Dual LP

$$
\begin{aligned}
v(\pi) = \quad & \max_{d \in \Delta(\mathcal{S} \times \mathcal{A})} \quad \mathbb{E}_{(s,a) \sim d}[R(s,a)] \\
& \text{s.t.} \qquad\qquad d = (1-\gamma)\mu_0 \pi + \gamma \mathcal{P}_\pi^* d \qquad (*)
\end{aligned}
$$

Observations

- There is a unique solution $d = d_\pi$ that satisfies (*).
  Known as stationary distribution, steady-state distribution, ...
- A dual approach to obtaining policy value:

$$
v(\pi) = \mathbb{E}_{d_\pi}[R(s,a)] = \mathbb{E}_{d_\mathcal{D}}[\zeta_\pi(s,a) R(s,a)]
$$

  where $\zeta_\pi(s,a) = \frac{d_\pi(s,a)}{d_\mathcal{D}(s,a)}$ is Radon-Nikodym derivative (RND)
  (under absolute continuity condition $d_\pi \ll d_\mathcal{D}$)
- OPE is now reduced to estimating $\zeta(s,a)$

DualDICE Approach to OPE

Observation

ζ_π is the solution to a linear system:

$$\zeta d_{\mathcal{D}} = (1-\gamma)\mu_0\pi + \gamma\mathcal{P}_\pi^*\zeta d_{\mathcal{D}}$$

But solving it is nontrivial:

- The dimension ($|\mathcal{S}\times\mathcal{A}|$) is large or even ∞
- We need to approximate $d_{\mathcal{D}}$ and $\mathcal{P}_\pi^*$ with empirical data (and proper regularization is critical)

Solution

DualDICE (Dual DIstribution Correction Estimation) formulates it into an optimization problem to solve for ζ_π [NCDL'19].

More on DualDICE

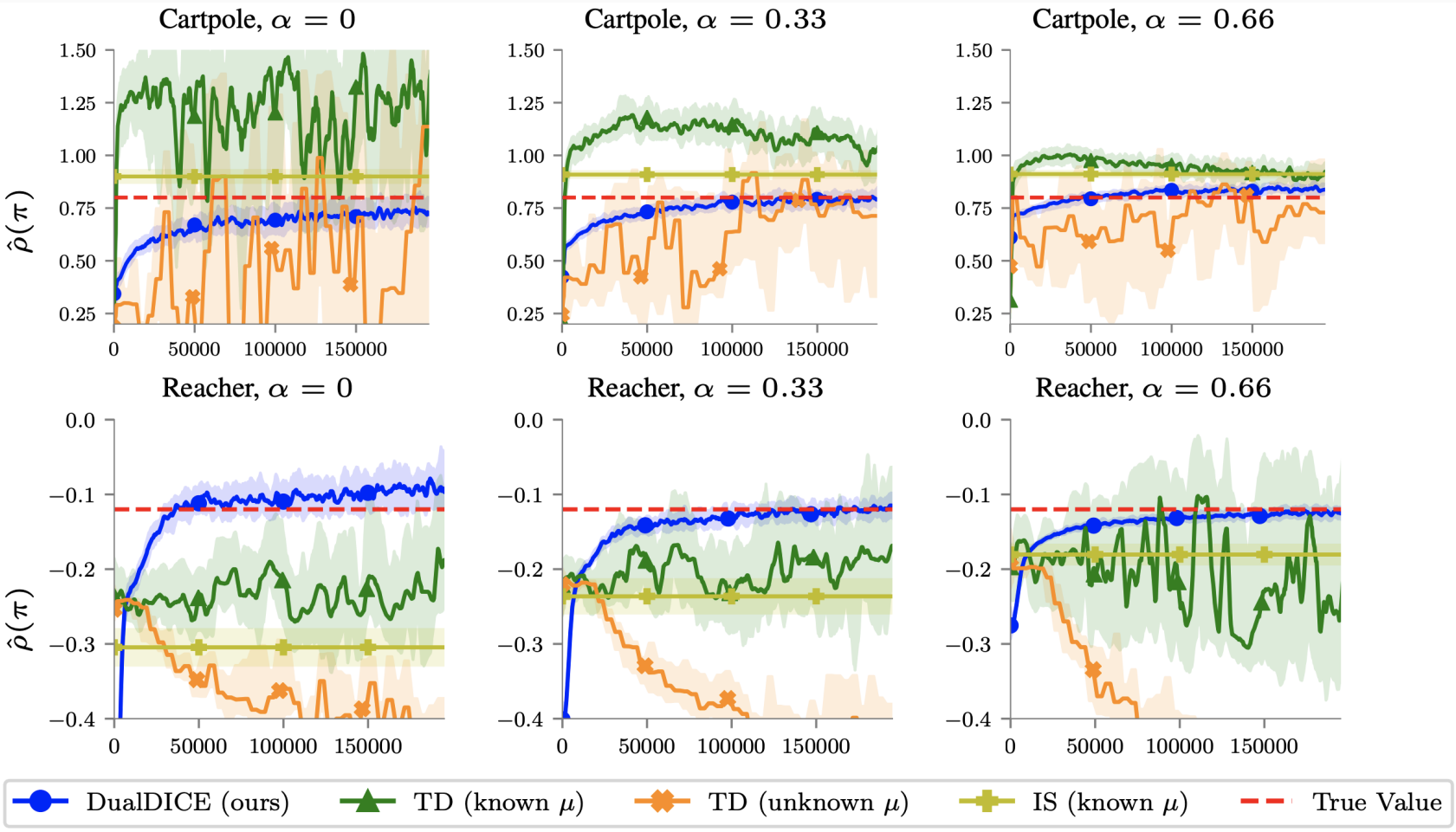
Convergence [NCDL'19]
Under proper assumptions, the mean squared error

$$\mathbb{E}\left[\hat{v}_{\text{DualDICE}}(\pi) - v(\pi)\right]^2 = \tilde{O}\left(\varepsilon_{\text{approx}} + \varepsilon_{\text{opt}} + \frac{1}{\sqrt{n}}\right)$$

Variations and Extensions

- Extension to $\gamma = 1$ by GenDICE [ZDLS'20]
- Doubly robustness [ZFLZL'20,Uehara+'20]
- A unification [YNDLS'20] (DualDICE, GenDICE, LSTDQ [Lagoudakis&Parr'03], MQL/MWL [Uehara+'20], ...)
- Varitional power iteration for easier optimization [PDLS'20]
- Consistent policy gradient estimation (for policy optimization) from off-policy data [NDKCLS'19]

DualDICE Numerical Experiments



- Mujoco: simulated control
- α controls the degree of "off-policy-ness" ($d_\pi = d_\mathcal{D}$ if $\alpha = 1$)

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Uncertainty Estimation

Point estimate alone (like DualDICE) has limited use.
Variance matters in most applications!

Popular methods

- Normal approximation based on CLT [Bottou+'13, LCKG'15]
- Concentration inequalities [Thomas+'15]
- Bootstrap [Thomas+'15; Hanna+'17]

CoinDICE

Our approach: CoinDICE [DNCLSS'20]

- Based on empirical likelihood method
- Computationally tractable
- Avoids the curse of horizon

Basic ideas

- Perturb the linear system with noise w within some range:

$$\zeta d_{\mathcal{D}} = (1-\gamma)\mu_0\pi + \gamma\mathcal{P}_\pi^*\zeta d_{\mathcal{D}}$$

- Solve for ζ while allowing w to vary
- The range of solved ζ induces an (approximate) confidence interval for $v(\pi)$

CoinDICE: Statistical Coverage

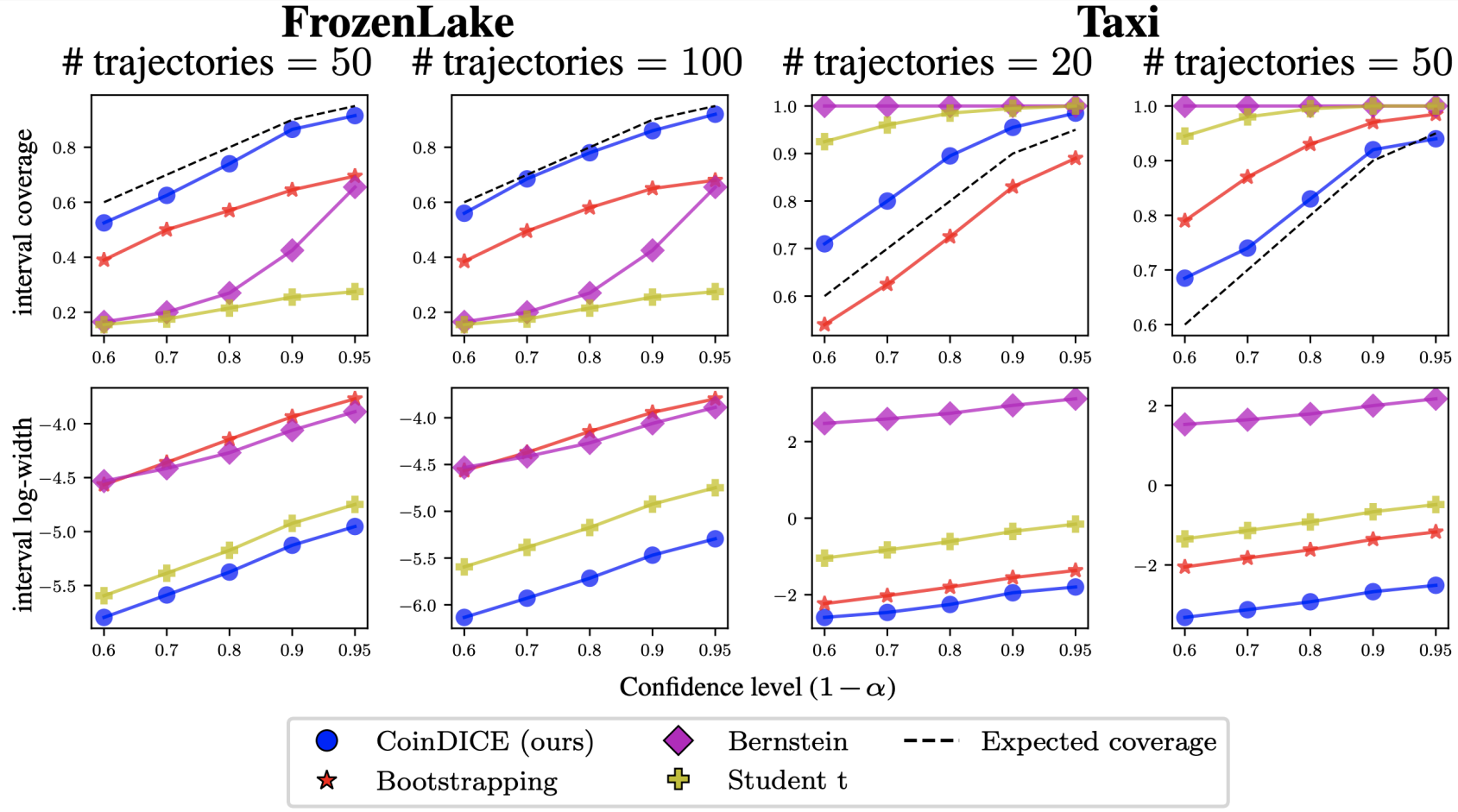
Asymptotic coverage
Under proper assumptions:

$$\lim_{n\to\infty} \Pr\left(\tilde{v}(\pi) \in C_{n,\xi}\right) = \Pr\left(\chi^2_{(1)} \leq \xi\right)$$

Non-asymptotic coverage
$C_{n,\xi}$ is a valid confidence interval, up to error of order $O(n^{-1})$.

Word of caution
The CI considers statistical error, but not approximation error.

CoinDICE Numerical Experiments



- Two standard benchmark problems
- α is the intended confidence interval coverage

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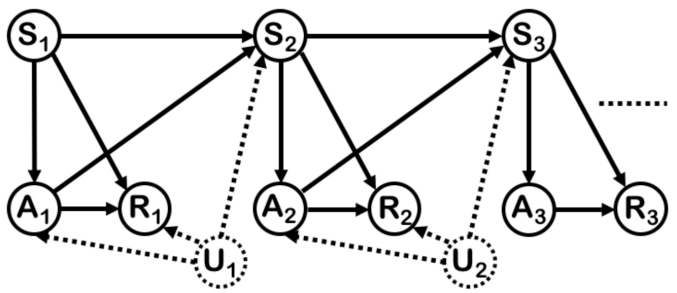
MDPs with Unmeasured Confounding



- Described by tuple $\langle \mathcal{S}, \mathcal{A}, ..., \mathcal{U} \rangle$
- $(u_1, u_2, u_3, \cdots)$ are IID
- Same as MDP except for $\Pr(s'|s, a, u)$, $R(s, a, u)$, $\pi_0(a|s, u)$ and $\pi(a|s, u)$
- MDPs $\subset$ MDPUCs $\subset$ partially observable MDPs
- Simplified example: insulin injection for type-1 diabetes patients
  - $\mathcal{S}$: blood glucose levels
  - $\mathcal{U}$: exogenous unmeasured events (e.g., food intake, exercise)

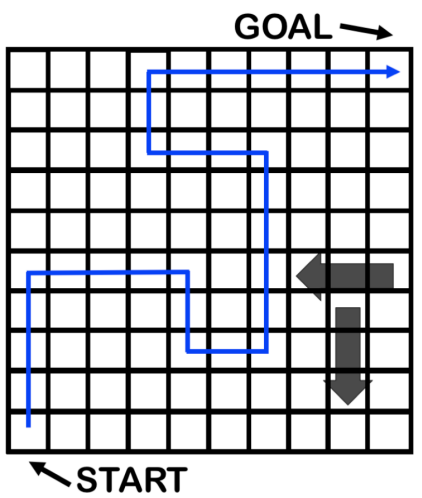
OPE in MDPUC

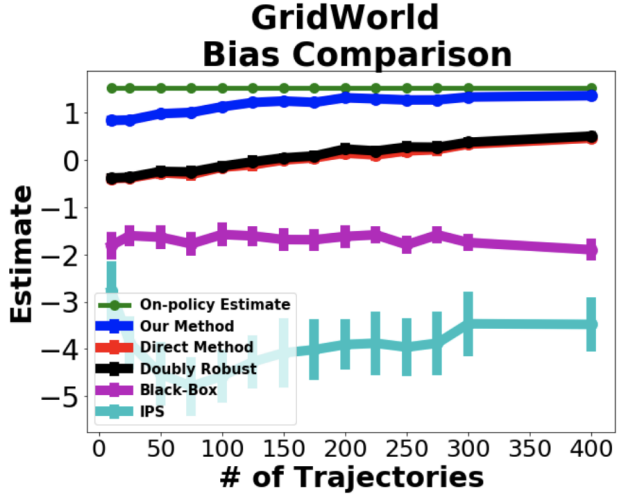
- Same form of estimator:

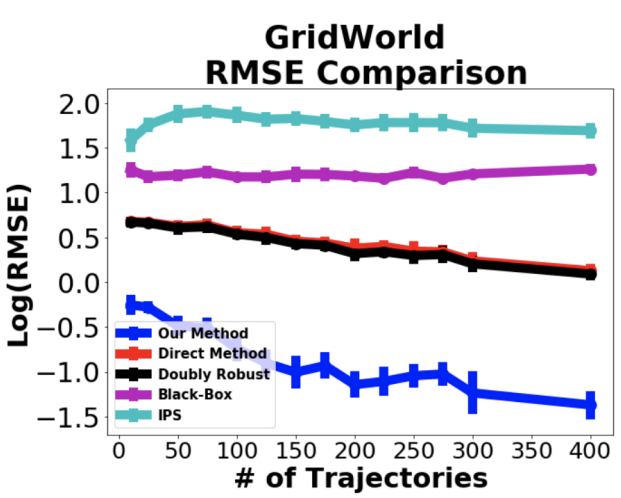
$$\hat{v}_{\text{MDPUC}} = \frac{1}{n} \sum_{i=1}^{n} \zeta_i r_i$$

  with ζ_i is the importance ratio as before (u gets canceled thanks to IID assumption)
- Previous method can be extended (under certain technical assumptions such as ergodicity), to achieve $1/n^{-1/2}$ convergence rate.
- See paper for details [BKLM'20]

Experiments: Windy Gridworld







- Binary confounders that affect behavior policy, reward and transition probabilities
- Our method (blue) consistently estimates the true value, but not other baselines
- Sensitivity analysis and more results in the paper [BKLM'20]

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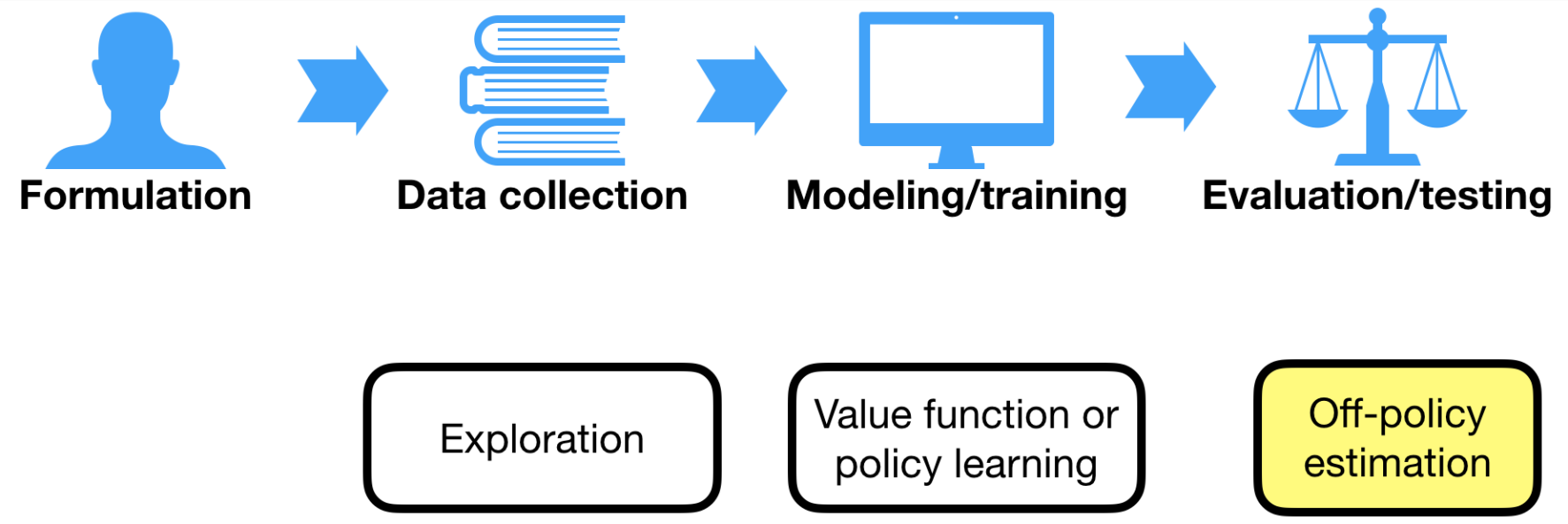


Conclusions

Recap

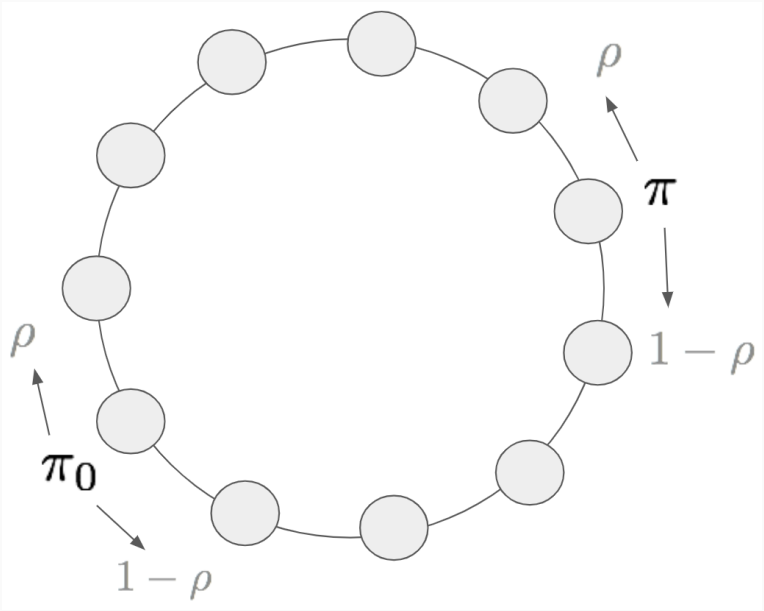
- Off-policy estimation is a critical bottleneck in "real-life" RL
- Many successes, mostly in short-horizon problems
- We break the curse of horizon based on duality structure in RL
- Extensions to confidence interval, and confounded cases

Future work

- OPE in more general confounded cases
- OPE in multiagent systems [Zheng+'20]
- Empirical tests in economic applications
- Connection with related work in econometrics

THANK YOU!

Is Importance-Weighting Trajectories Necessary?



"Circle" MDP

- $n = |\mathcal{S}|$ states arranged on circle
- Two actions:
  clockwise, counterclockwise
- Deterministic transitions

As $H \to \infty$:

- IPS/DR variance goes to ∞
- But both policies visit every state equally often