stats 406 hw8

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```
Lab section 002
UM ID: 49109112
(1)
library(tidyverse)
## -- Attaching packages -
                                                                       ---- tidyverse 1.2.1 --
## v ggplot2 3.2.1
                     v purrr
                                0.3.3
## v tibble 2.1.3 v dplyr 0.8.3
## v tidyr 1.0.0 v stringr 1.4.0
## v readr
           1.3.1 v forcats 0.4.0
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                    masks stats::lag()
library(ISLR)
library(pls)
##
## Attaching package: 'pls'
## The following object is masked from 'package:stats':
##
##
       loadings
library(MASS)
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##
       select
reuse the code in hw7.
set.seed(234)
college_raw = College
#create new response variable
college = college_raw %>% mutate(accept_rate = Accept / Apps)
#split train/test dataset
RNGkind(sample.kind = "Rejection")
nrows_college = nrow(college)
train_id = sample(nrows_college, floor(0.7*nrows_college))
college_train = college[train_id,]
college_test = college[-train_id,]
```

```
#predictors
X = model.matrix(accept_rate ~ ., college)[, -1]
#response
Y <- college$accept_rate</pre>
```

(a)

Based on the summary of college dataset, I chose to standardize the predictors first as they seem to measure on the different scales.

Based on the summary of PCA, we need at least 11 eigenvalues/components in order to explain 95% of the variance in this dataset.

It seems that the first PC mostly represents the average of variable Top10perc, Top25perc, Outstate, Room.Board, PhD, Terminal, Expand and Grad.Rate.

For the second PC, it seems that it represents the average of variable PrivateSchool, Apps, Accept, Enroll, and F.Undergrad.

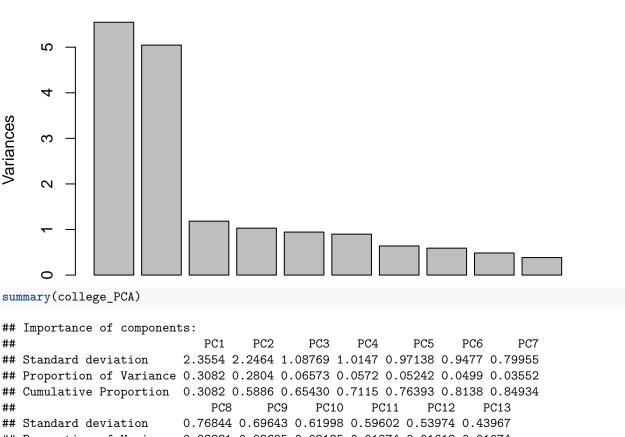
summary(college)

```
Enroll
                                                                    Top10perc
##
    Private
                    Apps
                                    Accept
##
                                            72
    No :212
               Min.
                           81
                                Min.
                                                 Min.
                                                            35
                                                                 Min.
                                                                         : 1.00
    Yes:565
                                1st Qu.:
                                                 1st Qu.: 242
                                                                  1st Qu.:15.00
               1st Qu.:
                         776
                                          604
               Median: 1558
                                                                 Median :23.00
##
                                Median: 1110
                                                 Median: 434
                                                         : 780
##
               Mean
                      : 3002
                                Mean
                                        : 2019
                                                 Mean
                                                                 Mean
                                                                         :27.56
##
                                3rd Qu.: 2424
               3rd Qu.: 3624
                                                 3rd Qu.: 902
                                                                  3rd Qu.:35.00
                                        :26330
##
               Max.
                      :48094
                                Max.
                                                 Max.
                                                         :6392
                                                                 Max.
                                                                         :96.00
      Top25perc
##
                      F. Undergrad
                                       P. Undergrad
                                                             Outstate
##
    Min.
           : 9.0
                     Min.
                             :
                                139
                                              :
                                                   1.0
                                                                  : 2340
                                      Min.
                                                          Min.
##
    1st Qu.: 41.0
                     1st Qu.:
                                992
                                       1st Qu.:
                                                  95.0
                                                          1st Qu.: 7320
##
    Median: 54.0
                     Median: 1707
                                      Median :
                                                 353.0
                                                          Median: 9990
##
    Mean
            : 55.8
                     Mean
                             : 3700
                                      Mean
                                                 855.3
                                                          Mean
                                                                  :10441
                                      3rd Qu.:
##
    3rd Qu.: 69.0
                     3rd Qu.: 4005
                                                 967.0
                                                          3rd Qu.:12925
##
    Max.
            :100.0
                     Max.
                             :31643
                                      Max.
                                              :21836.0
                                                          Max.
                                                                  :21700
##
                                                            PhD
      Room.Board
                        Books
                                          Personal
##
    Min.
            :1780
                               96.0
                                              : 250
                                                                 8.00
                    Min.
                                      Min.
                                                       Min.
                    1st Qu.: 470.0
                                                       1st Qu.: 62.00
##
    1st Qu.:3597
                                       1st Qu.: 850
##
    Median:4200
                    Median : 500.0
                                      Median:1200
                                                       Median: 75.00
                                              :1341
##
            :4358
                            : 549.4
                                                              : 72.66
    Mean
                    Mean
                                      Mean
                                                       Mean
##
    3rd Qu.:5050
                    3rd Qu.: 600.0
                                       3rd Qu.:1700
                                                       3rd Qu.: 85.00
                                              :6800
##
    Max.
            :8124
                    Max.
                            :2340.0
                                      Max.
                                                       Max.
                                                              :103.00
##
       Terminal
                       S.F.Ratio
                                       perc.alumni
                                                            Expend
                             : 2.50
                                              : 0.00
                                                               : 3186
##
           : 24.0
    Min.
                     Min.
                                      Min.
                                                        Min.
    1st Qu.: 71.0
                     1st Qu.:11.50
                                      1st Qu.:13.00
                                                        1st Qu.: 6751
##
##
    Median: 82.0
                     Median :13.60
                                      Median :21.00
                                                        Median: 8377
##
    Mean
            : 79.7
                     Mean
                             :14.09
                                      Mean
                                              :22.74
                                                        Mean
                                                                : 9660
    3rd Qu.: 92.0
                     3rd Qu.:16.50
##
                                      3rd Qu.:31.00
                                                        3rd Qu.:10830
##
    Max.
            :100.0
                             :39.80
                                      Max.
                                              :64.00
                     Max.
                                                        Max.
                                                                :56233
##
      Grad.Rate
                       accept rate
##
           : 10.00
                              :0.1545
   Min.
                      Min.
##
    1st Qu.: 53.00
                      1st Qu.:0.6756
    Median : 65.00
                      Median :0.7788
```

```
## Mean : 65.46 Mean :0.7469
## 3rd Qu.: 78.00 3rd Qu.:0.8485
## Max. :118.00 Max. :1.0000

college_PCA = prcomp(x = X[train_id,],center = TRUE,scale. = TRUE)
plot(college_PCA,main = "scree plot of the eigenvalues")
```

scree plot of the eigenvalues



```
## Standard deviation 2.3554 2.2464 1.08769 1.0147 0.97138 0.9477 0.79955 ## Proportion of Variance 0.3082 0.2804 0.06573 0.0572 0.05242 0.0499 0.03552 ## Cumulative Proportion 0.3082 0.5886 0.65430 0.7115 0.76393 0.8138 0.84934 ## Proportion 0.76844 0.69643 0.61998 0.59602 0.53974 0.43967 ## Proportion of Variance 0.03281 0.02695 0.02135 0.01974 0.01618 0.01074 ## Cumulative Proportion 0.88215 0.90909 0.93045 0.95018 0.96637 0.97711 ## PC14 PC15 PC16 PC17 PC18 ## Standard deviation 0.38491 0.32996 0.28547 0.21288 0.16805 ## Proportion of Variance 0.00823 0.00605 0.00453 0.00252 0.00157 ## Cumulative Proportion 0.98534 0.99139 0.99591 0.99843 1.00000 ** cumsum((college_PCA$sdev)^2)/sum((college_PCA$sdev)^2)
```

```
## [1] 0.3082229 0.5885783 0.6543047 0.7115077 0.7639292 0.8138265 0.8493422
```

[8] 0.8821475 0.9090928 0.9304471 0.9501829 0.9663673 0.9771068 0.9853376

[15] 0.9913861 0.9959136 0.9984311 1.0000000

Below is the ouput for the first and the second components.

```
college_PCA$rotation[,1:2]
```

```
## PrivateYes -0.062546079 0.35219990
## Apps -0.238959092 -0.32381712
## Accept -0.180543932 -0.36363068
```

```
## Enroll
               -0.143951433 -0.39380001
               -0.356438459 0.05002215
## Top10perc
               -0.339809756 0.01910038
## Top25perc
## F.Undergrad -0.118244306 -0.40445653
## P.Undergrad -0.004632796 -0.29641340
## Outstate
              -0.318273811 0.20409325
## Room.Board -0.259916643 0.09907049
## Books
               -0.066513963 -0.03568954
               0.055940733 -0.18346507
## Personal
## PhD
               -0.306400129 -0.09693786
## Terminal
               -0.308558424 -0.08811304
## S.F.Ratio
                0.204033322 -0.22085352
## perc.alumni -0.219895650 0.21913203
## Expend
               -0.324959248 0.09340372
## Grad.Rate
               -0.260895172 0.14281641
```

(b)

Based on the output below, I choose 18 components as it yields the lowest cv error.

The test error rate is 0.01334912.

accept_rate

40.13

46.65

```
college_PCR = pcr(accept_rate~.,data = college,subset = train_id,scale = TRUE,validation = "CV")
summary(college_PCR)
## Data:
            X dimension: 543 18
## Y dimension: 543 1
## Fit method: svdpc
## Number of components considered: 18
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##
          (Intercept) 1 comps 2 comps 3 comps 4 comps 5 comps
                                                                     6 comps
## CV
               0.1478
                        0.1277
                                 0.1273
                                           0.1272
                                                    0.1272
                                                             0.1269
                                                                      0.1268
  adjCV
               0.1478
                        0.1275
                                 0.1273
                                           0.1272
                                                    0.1271
                                                             0.1268
##
                                                                      0.1267
##
          7 comps 8 comps 9 comps 10 comps 11 comps 12 comps 13 comps
## CV
           0.1269
                    0.1272
                             0.1264
                                       0.1263
                                                  0.1178
                                                            0.1184
                                                                      0.1182
## adjCV
           0.1268
                    0.1272
                             0.1263
                                        0.1265
                                                  0.1176
                                                            0.1182
                                                                      0.1180
##
          14 comps
                    15 comps 16 comps 17 comps 18 comps
## CV
            0.1184
                      0.1130
                                0.1134
                                         0.09982
                                                    0.09511
## adjCV
            0.1182
                      0.1128
                                0.1132
                                         0.09952
                                                    0.09485
##
## TRAINING: % variance explained
##
                1 comps
                         2 comps
                                 3 comps
                                          4 comps
                                                    5 comps
                                                              6 comps
                                                                       7 comps
## X
                  30.82
                           58.86
                                    65.43
                                              71.15
                                                       76.39
                                                                81.38
                                                                          84.93
                                                                          28.46
                  25.66
                           26.13
                                    26.44
                                              27.03
                                                       28.08
                                                                28.34
## accept_rate
                8 comps 9 comps
                                 10 comps 11 comps 12 comps
                                                                 13 comps
##
                                      93.04
## X
                  88.21
                           90.91
                                                95.02
                                                          96.64
                                                                    97.71
                                                                    39.90
## accept_rate
                  28.53
                           29.99
                                      30.55
                                                39.13
                                                          39.24
##
                14 comps 15 comps
                                    16 comps
                                               17 comps 18 comps
## X
                   98.53
                             99.14
                                        99.59
                                                  99.84
                                                           100.00
```

58.07

62.58

47.05

```
pcr_test_predict = predict(college_PCR,college[-train_id,names(college)!="accept_rate"],ncomp = 18)
mean((pcr_test_predict - college[-train_id, "accept_rate"])^2)
## [1] 0.01334912
(c)
Based on the output below, I choose 14 components as it yields the lowest cv error.
The train error is 0.008144985. The test error is 0.01340262.
college_PLS = plsr(accept_rate~.,data = college, subset = train_id,scale = TRUE,validation = "CV")
summary(college_PLS)
            X dimension: 543 18
## Data:
  Y dimension: 543 1
## Fit method: kernelpls
## Number of components considered: 18
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##
          (Intercept)
                        1 comps
                                 2 comps 3 comps 4 comps 5 comps
## CV
               0.1478
                         0.1253
                                  0.1157
                                           0.1121
                                                     0.1071
                                                              0.1011
                                                                       0.09816
               0.1478
                         0.1253
                                  0.1153
                                            0.1122
                                                     0.1069
                                                              0.1008
  adjCV
##
          7 comps 8 comps
                             9 comps
                                      10 comps 11 comps
                                                           12 comps
                                                                      13 comps
          0.09717
                   0.09612
                            0.09535
                                       0.09514
                                                  0.09506
                                                            0.09503
                                                                       0.09489
## CV
  adjCV
          0.09687
                   0.09591 0.09508
                                       0.09488
                                                  0.09480
                                                            0.09477
                                                                       0.09463
##
##
          14 comps
                    15 comps
                               16 comps 17 comps
                                                    18 comps
                                                     0.09486
## CV
           0.09485
                     0.09487
                                0.09486
                                          0.09487
           0.09459
                      0.09462
                                0.09460
                                          0.09461
## adjCV
                                                     0.09460
##
## TRAINING: % variance explained
##
                1 comps
                         2 comps
                                   3 comps
                                            4 comps
                                                      5 comps
                                                               6 comps
                                                                         7 comps
## X
                  30.65
                            35.36
                                     58.74
                                               66.32
                                                        69.38
                                                                  73.22
                                                                           76.26
                                                                           60.59
## accept_rate
                  28.90
                            42.76
                                     45.29
                                               50.44
                                                        56.11
                                                                  59.33
                8 comps
##
                         9 comps
                                  10 comps
                                             11 comps
                                                        12 comps
                                                                  13 comps
                            82.24
## X
                  79.75
                                      85.28
                                                 87.95
                                                           91.25
                                                                      92.53
                                      62.38
                            62.23
                                                 62.46
                                                           62.50
                                                                      62.55
## accept_rate
                  61.48
##
                           15 comps
                                     16 comps
                                                17 comps
                                                          18 comps
                14 comps
## X
                   94.58
                              96.73
                                        98.06
                                                   99.29
                                                            100.00
## accept rate
                   62.57
                              62.57
                                        62.58
                                                   62.58
                                                             62.58
pls_train_predict = predict(college_PLS,college[train_id,names(college) != "accept_rate"],ncomp = 14)
pls_train_mse = mean((pls_train_predict - college[train_id, "accept_rate"])^2) %>% print()
## [1] 0.008144985
pls_test_predict = predict(college_PLS,college[-train_id,names(college) != "accept_rate"],ncomp = 14)
pls_test_mse = mean((pls_test_predict - college[-train_id,"accept_rate"])^2) %>% print()
## [1] 0.01340262
(d)
```

The methods used in hw6 and hw7 yield testing error around 0.008 to 0.010, with the smallest testing error (0.0083) coming from the forward selection model. The testing error of PCR and PLS are both around 0.013.

Since the forward selection model has the best performance and taking into account the fact PCR and PLS models are relatively difficult to interpret, I recommend using forward selection model with this dataset.

(2)

```
crab = crabs
crab %>% head(3)
     sp sex index FL RW
                           CL
               1 8.1 6.7 16.1 19.0 7.0
## 1 B
         М
## 2 B
               2 8.8 7.7 18.1 20.8 7.4
         M
## 3 B
               3 9.2 7.8 19.0 22.4 7.7
str(crab)
## 'data.frame':
                   200 obs. of 8 variables:
         : Factor w/ 2 levels "B", "O": 1 1 1 1 1 1 1 1 1 1 ...
## $ sex : Factor w/ 2 levels "F", "M": 2 2 2 2 2 2 2 2 2 2 ...
## $ index: int 1 2 3 4 5 6 7 8 9 10 ...
## $ FL
          : num 8.1 8.8 9.2 9.6 9.8 10.8 11.1 11.6 11.8 11.8 ...
   $ RW
          : num 6.7 7.7 7.8 7.9 8 9 9.9 9.1 9.6 10.5 ...
         : num 16.1 18.1 19 20.1 20.3 23 23.8 24.5 24.2 25.2 ...
##
  $ CL
## $ CW
         : num 19 20.8 22.4 23.1 23 26.5 27.1 28.4 27.8 29.3 ...
          : num 7 7.4 7.7 8.2 8.2 9.8 9.8 10.4 9.7 10.3 ...
## $ BD
(a)
set.seed(6789)
RNGkind(sample.kind = "Rejection")
bm = which(crabs$sp == "B" & crabs$sex == "M") #blueMale
om = which(crabs$sp == "0" & crabs$sex == "M") #orangeMale
bf = which(crabs$sp == "B" & crabs$sex == "F") #blueFemale
of = which(crabs$sp == "0" & crabs$sex == "F") #orangeFemale
train_id = c(sample(bm, size = floor(0.80 * length(bm))),
sample(om, size = floor(0.80 * length(om))), sample(bf, size = floor(0.80 * length(bf))),
sample(of, size = floor(0.80 * length(of))))
crab_train = crab[train_id,]
crab_test = crab[-train_id,]
```

(b)

Based on the output from the cross-validation and the constraint of no more than 10 splits, the optimal size is 10

"FL", "CW", "BD", "CL" are the variables used in the classification tree.

The train error is 0.05263158, and the test error is 0.2903226.

```
library(tree)
```

Registered S3 method overwritten by 'tree':

```
## method from
## print.tree cli
library(gbm)

## Loaded gbm 2.1.5

crab_tree = tree(sp ~ sex + FL + RW + CL + CW + BD,data = crab,subset = train_id)

cv_crab_tree = cv.tree(crab_tree,FUN = prune.misclass)

plot(cv_crab_tree$size,(cv_crab_tree$dev)/length(train_id),
    ylab = "cross validation error",xlab = "size",type = "b",col = "red")

Output

Out
```

```
cross validation error cross validation error
```

```
pruned_crab = prune.misclass(crab_tree, best=10)
plot(pruned_crab)
text(pruned_crab,pretty = 0)
```

```
FL < 17.45
                            CW ₹ 36.2
                                                     CW < 44.25
                                                           FL < 19.8
                 BD < 11.7
                                                    0
                                             В
                                                            В
                                                                   0
     CW < 28.25
                           BD < 12.75
BD < 10.05
                      CL < 28.45
                                     0
               В
                              В
 В
        0
crab_train_pred = predict(pruned_crab, crab[train_id,],type="class")
table(crab_train_pred, crabs$sp[train_id])
##
## crab_train_pred B O
##
                B 79 7
##
                0 1 73
tree_train_error = 8/(73+79)
tree_train_error
## [1] 0.05263158
crab_train_pred = predict(pruned_crab, crab[-train_id,],type="class")
table(crab_train_pred, crabs$sp[-train_id])
##
## crab_train_pred B 0
##
                B 15 4
##
                0 5 16
tree_test_error = 9/(31)
tree_test_error
```

[1] 0.2903226