Insight
Centre for Data Analytics

DEEP
LEARNING
WORKSHOP

Dublin City University
27-28 April 2017

DCU

#InsightDL2017

Day 2 Lecture 8

Recurrent Neural Networks

Xavier Giro-i-Nieto
xavier.giro@upc.edu

Associate Professor
Universitat Politecnica de Catalunya
Technical University of Catalonia

UPC

Acknowledgments

Santiago Pascual

Outline

1. The importance of context
2. Where is the memory?
3. Vanilla RNN
4. Problems
5. Gating methodology
    a. LSTM
    b. GRU

The importance of context

- Recall the 5th digit of your phone number
- Sing your favourite song beginning at third sentence
- Recall 10th character of the alphabet

Probably you went straight from the beginning of the stream in each case…

because in sequences order matters!

Idea: retain the information preserving the importance of order

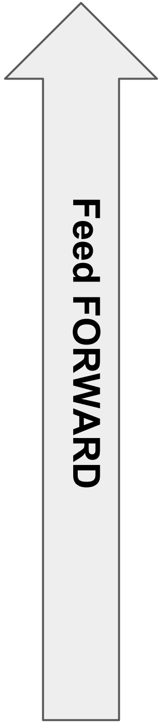
Recall from Day 1 Lecture 4...

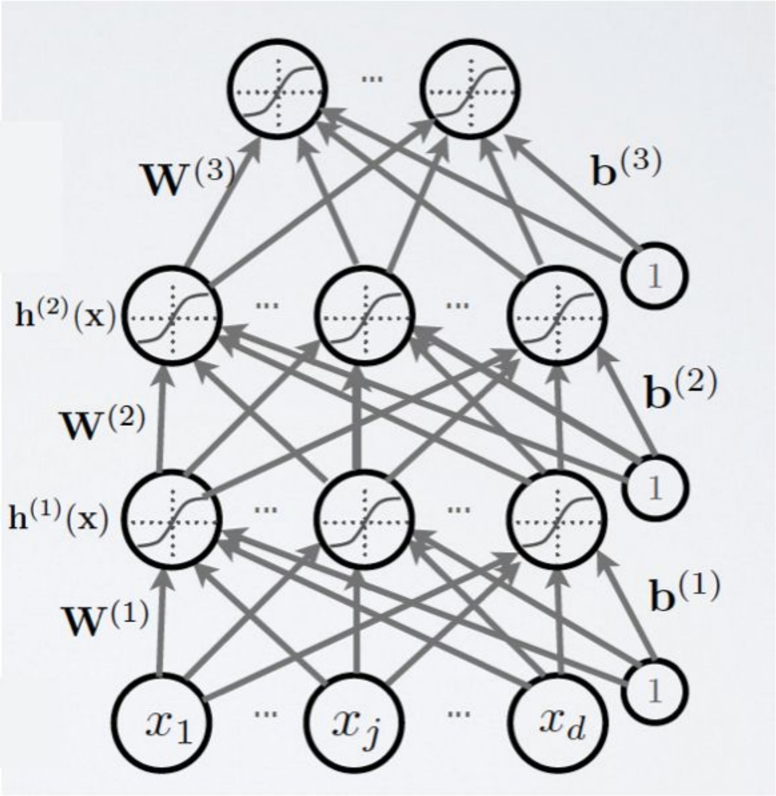
Every **y/hi** is computed from the sequence of forward activations out of input **x**.

$$y = f(W_3 \cdot h_2 + b_3)$$

$$h_2 = f(W_2 \cdot h_1 + b_2)$$

$$h_1 = f(W_1 \cdot x + b_1)$$

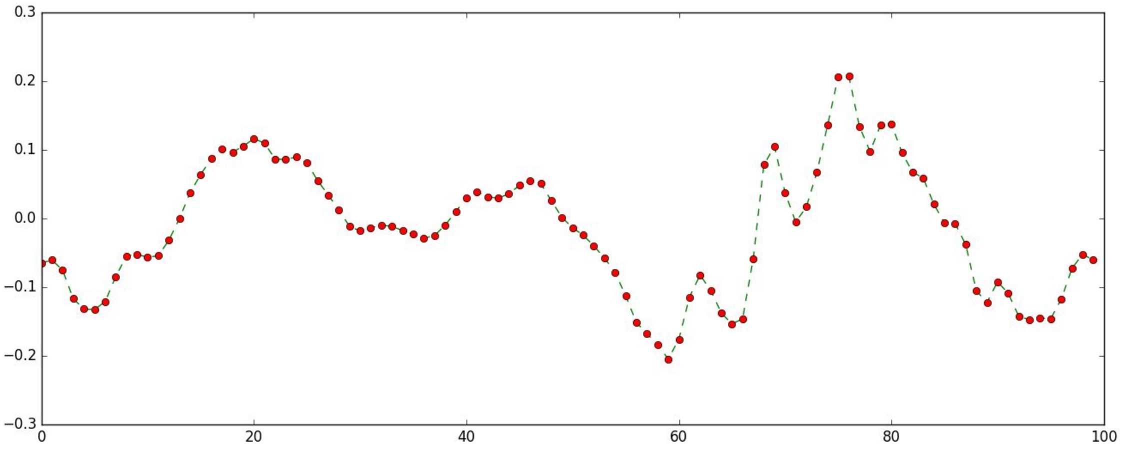




Slide Credit: Hugo Laroche NN course

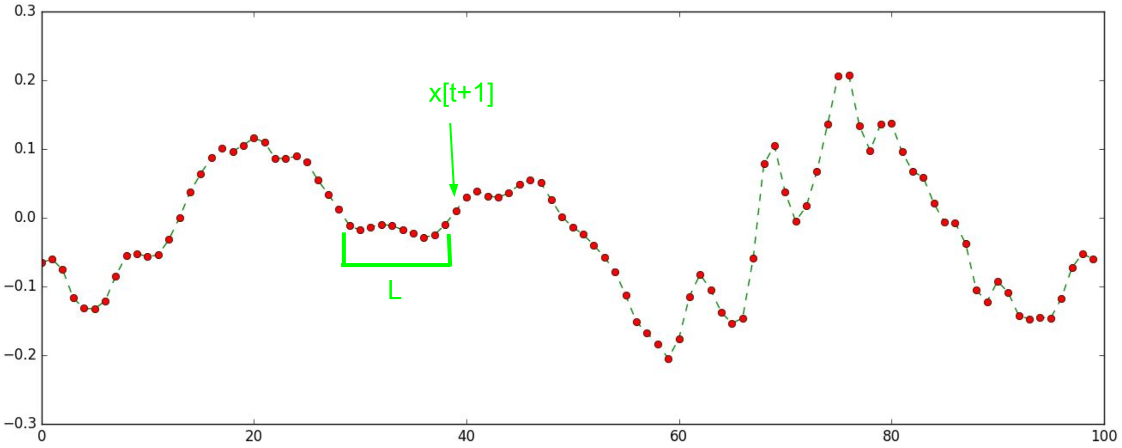
Where is the Memory?

If we have a sequence of samples...



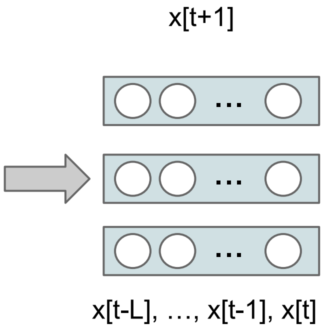
predict sample x[t+1] knowing previous values {x[t], x[t-1], x[t-2], …, x[t-τ]}

Where is the Memory?

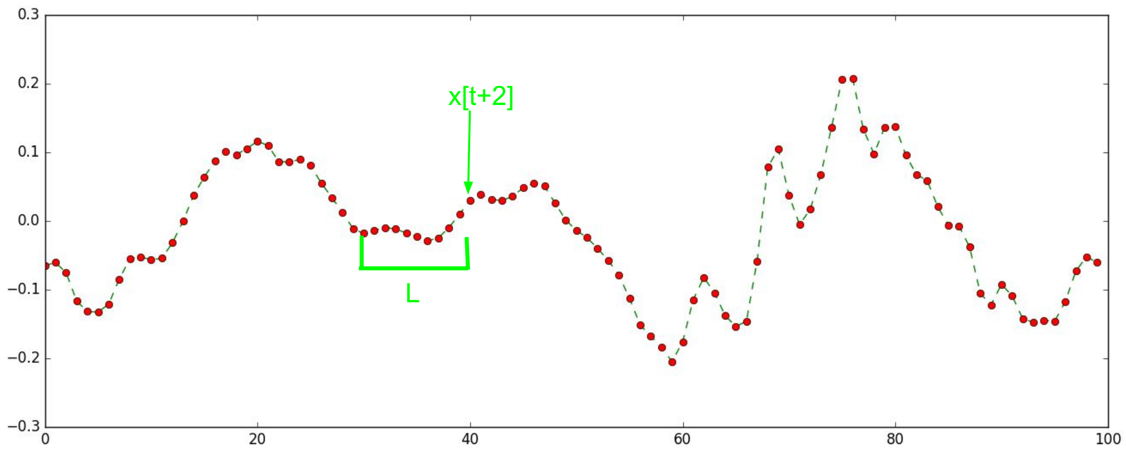


Feed Forward approach:

- static window of size L
- slide the window time-step wise



Where is the Memory?



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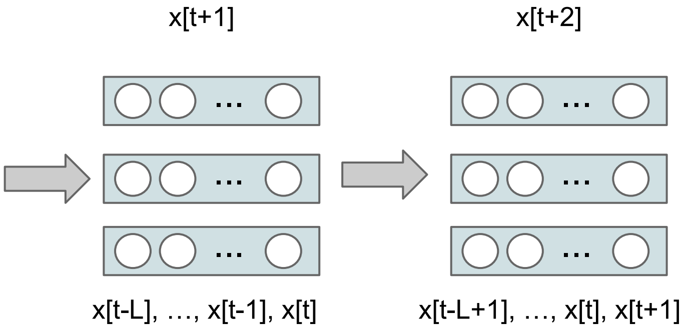
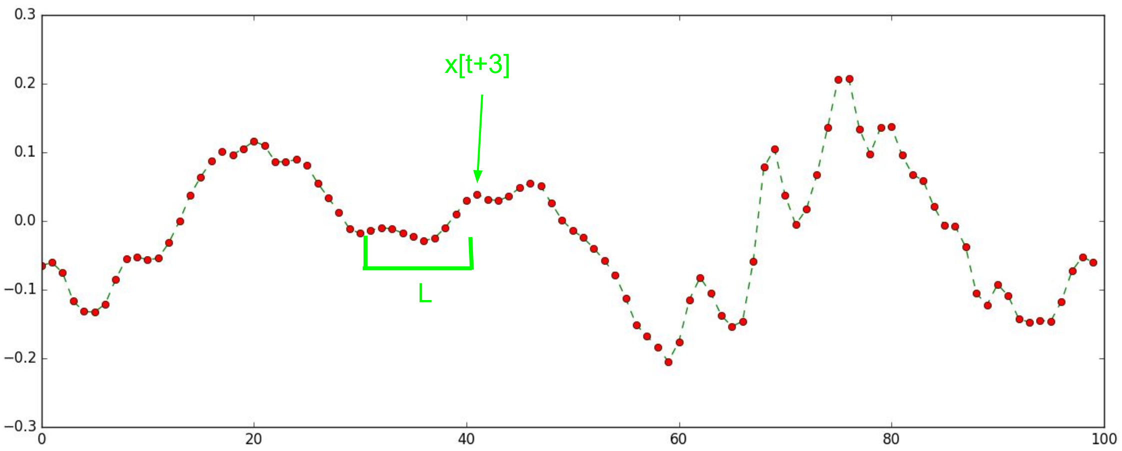


Where is the Memory?



Feed Forward approach:

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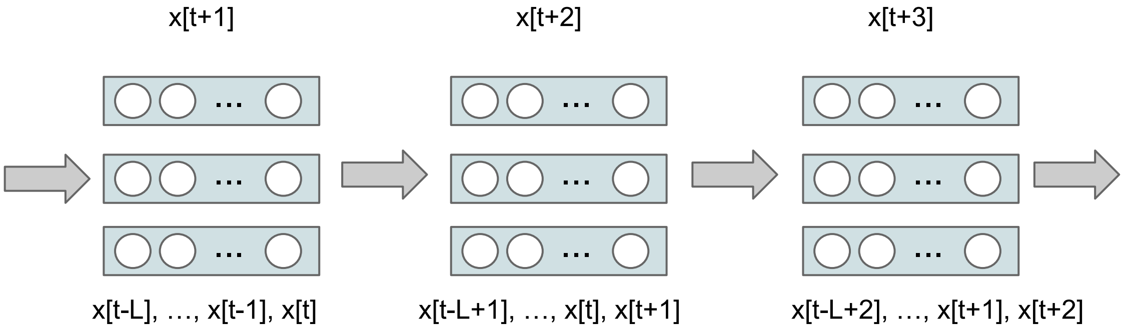
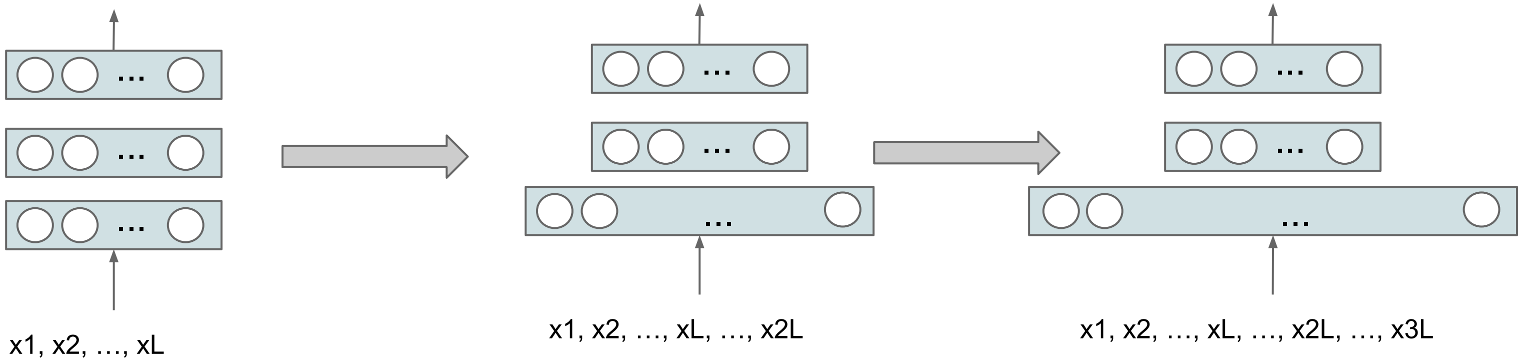


Where is the Memory?

Problems for the feed forward + static window approach:

- What's the matter increasing L? → Fast growth of num of parameters!
- Decisions are independent between time-steps!
  - The network doesn't care about what happened at previous time-step, only present window matters → doesn't look good
- Cumbersome padding when there are not enough samples to fill L size
  - Can't work with variable sequence lengths

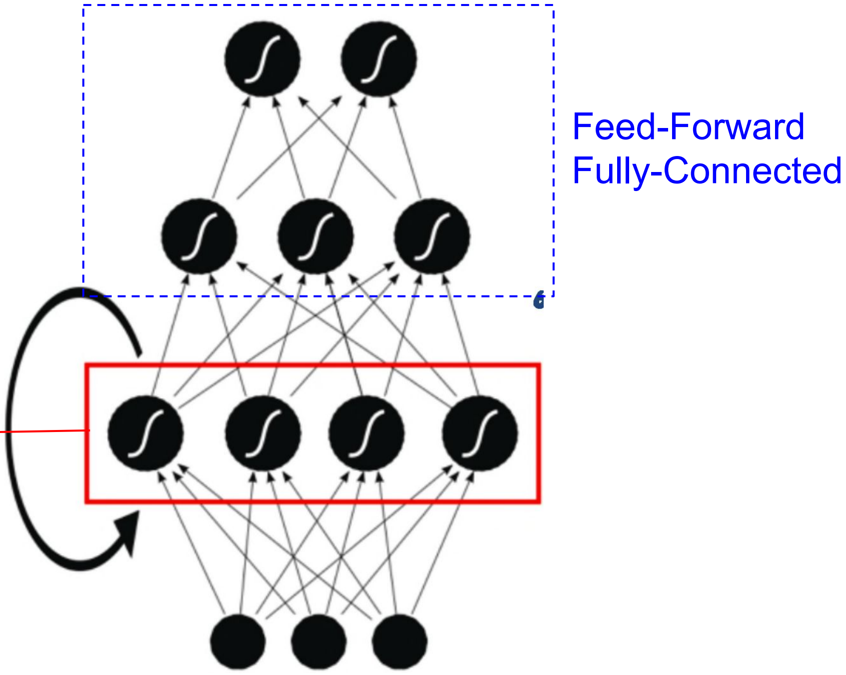


Recurrent Neural Network

Solution: Build specific connections capturing the temporal evolution → **Shared weights in time**

- Give volatile memory to the network

$$h_t = f(W \cdot x_t + U \cdot h_{t-1} + b)$$



Recurrent Neural Network

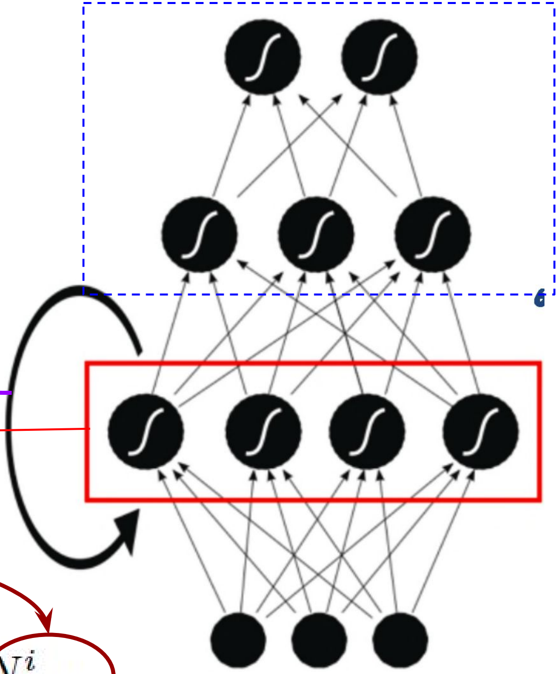
Solution: Build specific connections capturing the temporal evolution → **Shared weights in time**

- Give volatile memory to the network

Fully connected in time:
recurrent matrix

$$\boxed{h_t = f(W \cdot x_t + U \cdot h_{t-1} + b)}$$

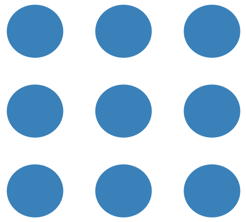
$$N^i_{params} = N^i_{inputs} \times N^i_{units} + N^i_{units} \times N^i_{units} + N^i_{units}$$

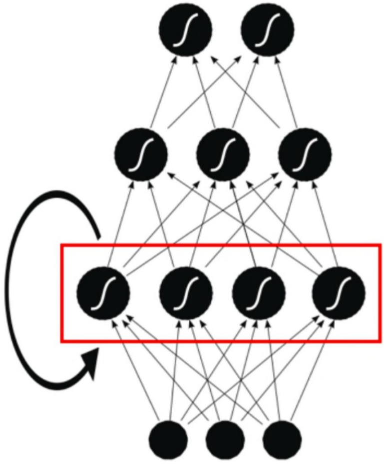


Feed-Forward
Fully-Connected

Recurrent Neural Network

Front View



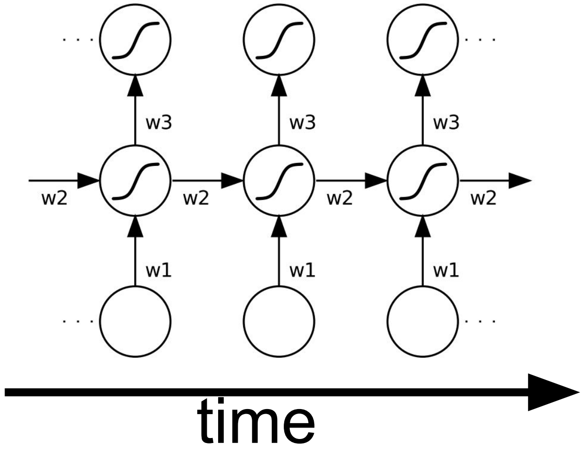


time

Rotation 90º

Rotation 90º

Side View

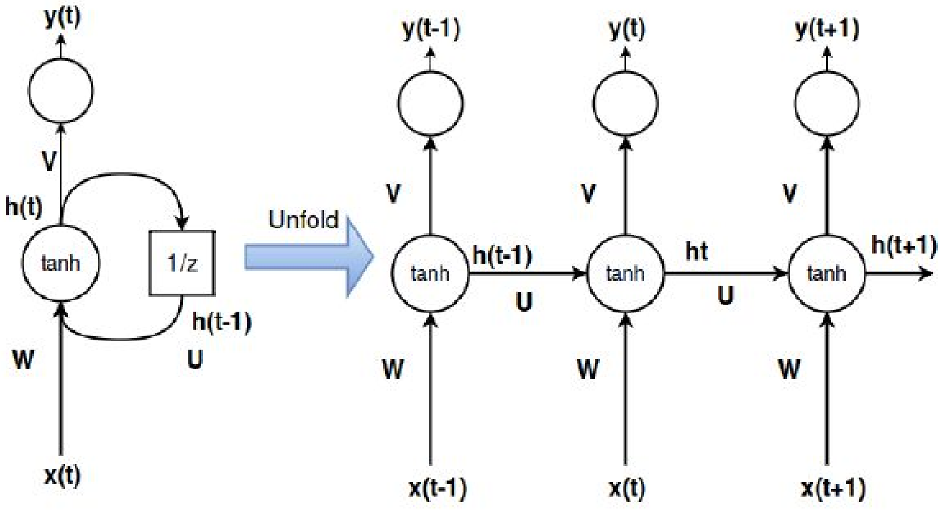


Slide credit: Xavi Giro

Recurrent Neural Network

Hence we have two data flows: **Forward in layers + time** propagation

BEWARE: We have extra depth now! Every time-step is an extra level of depth (as a deeper stack of layers in a feed-forward fashion!)



Recurrent Neural Network

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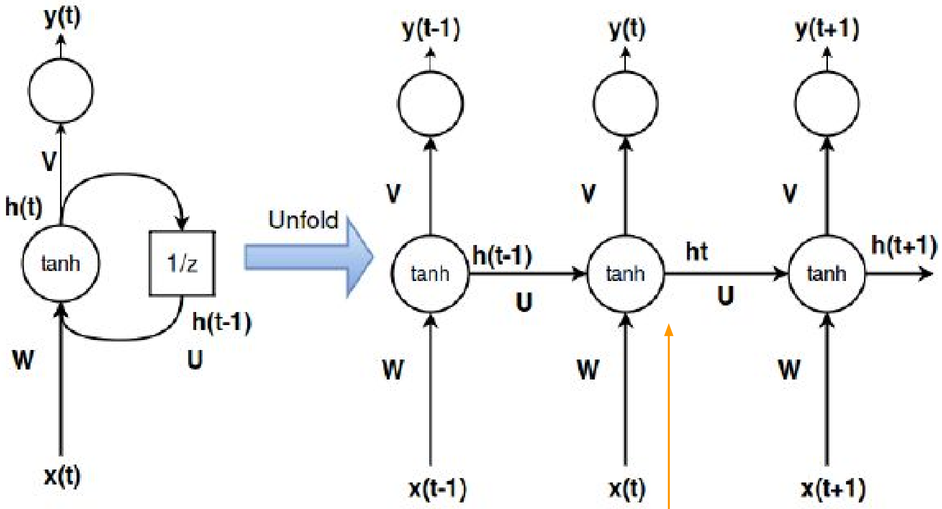


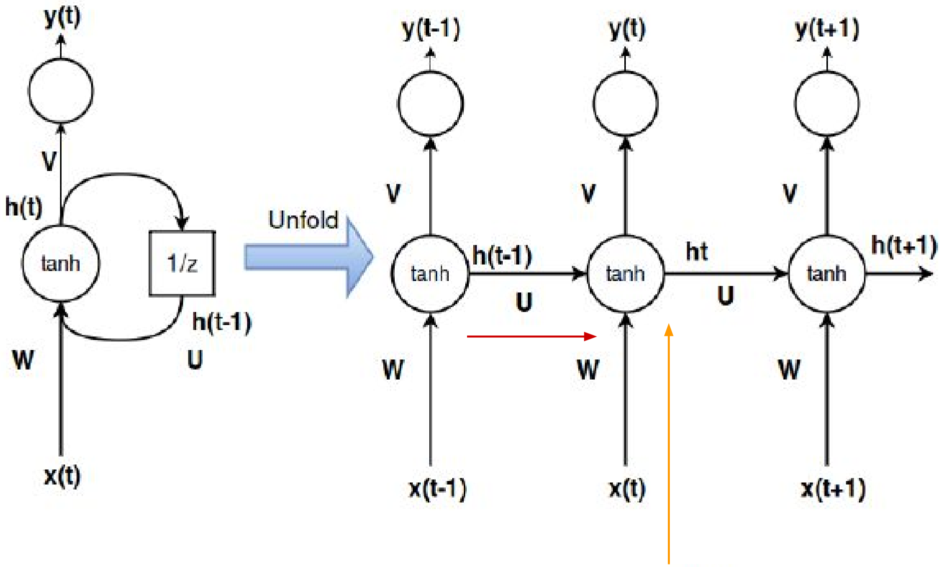
$$\mathbf{x}_t \in \mathbb{R}^n$$

$$\mathbf{h}_t = g(\mathbf{W} \cdot \mathbf{x}_t + \mathbf{U} \cdot \mathbf{h}_{t-1} + \mathbf{b}_h)$$

Recurrent Neural Network

Hence we have two data flows: **Forward in layers + time** propagation

- Last time-step includes the context of our decisions recursively



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Recurrent Neural Network

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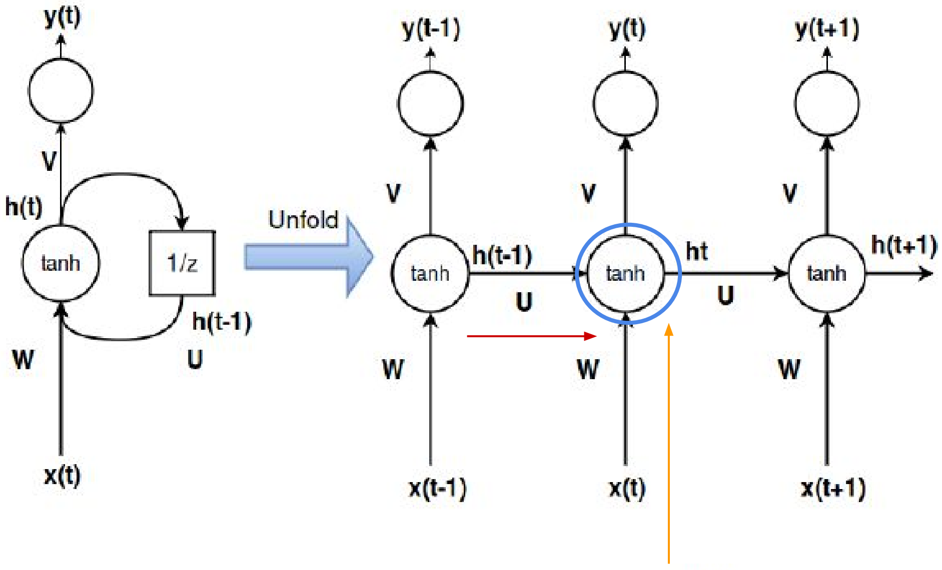


$$\mathbf{h}_t = g(\mathbf{W} \cdot \mathbf{x}_t + \mathbf{U} \cdot \mathbf{h}_{t-1} + \mathbf{b}_h)$$

$$\mathbf{x}_t \in \mathbb{R}^n$$

Recurrent Neural Network

Back Propagation Through Time (BPTT): The training method has to take into account the time operations → a cost function ***E*** is defined to train our RNN, and in this case the total error at the output of the network is the sum of the errors at each time-step:

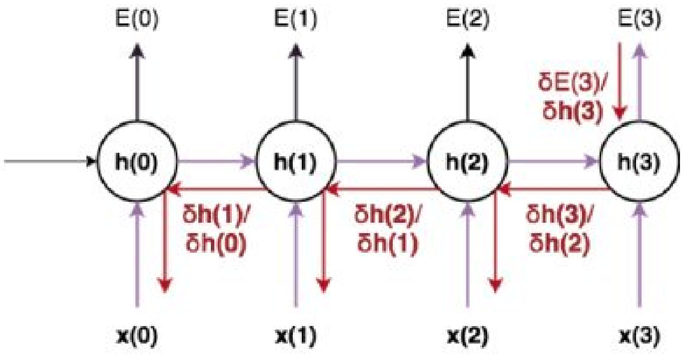
$$E(\mathbf{y}, \hat{\mathbf{y}}) = \sum_{t=1}^{T} E_t(\mathbf{y}_t, \hat{\mathbf{y}}_t)$$

$$\frac{\partial E}{\partial \mathbf{W}} = \sum_{t=0}^{T-1} \frac{\partial E_t}{\partial \mathbf{W}}$$

T: **max amount of time-steps to do back-prop**. In Keras this is specified when defining the “input shape” to the RNN layer, by means of:
(batch size, sequence length (T), input_dim)

Input shape

3D tensor with shape `(nb_samples, timesteps, input_dim)`.



Example back-prop in time with 3 time-steps

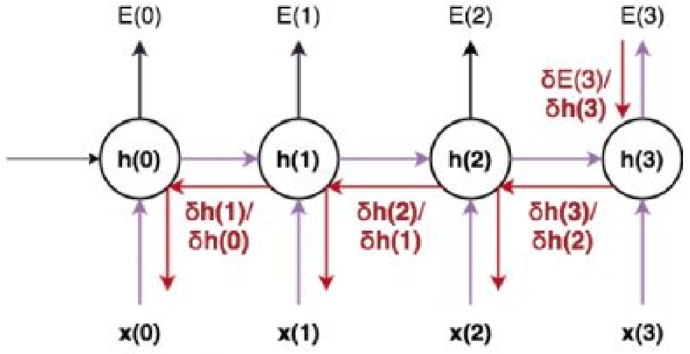
Recurrent Neural Network

Main problems:

- **Long-term memory** (remembering quite far time-steps) **vanishes quickly** because of the recursive operation with **U**

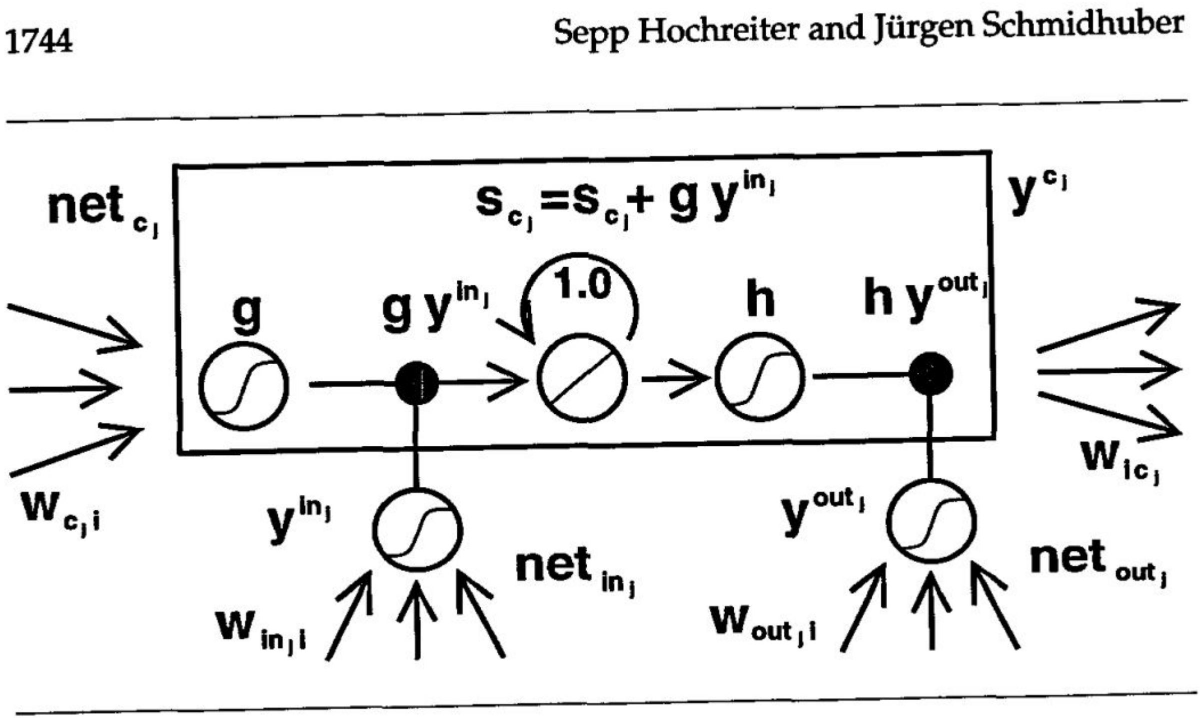
$$\mathbf{h}_t = g(\mathbf{W} \cdot \mathbf{x}_t + \mathbf{U} \cdot g(\cdots g(\mathbf{W} \cdot \mathbf{x}_{t-T} + \mathbf{U} \cdot \mathbf{h}_{t-T} + \mathbf{b}_h) \cdots) + \mathbf{b}_h)$$

- **During training gradients explode/vanish easily because of depth-in-time** → Exploding/Vanishing gradients!



Example back-prop in time with 3 time-steps

Long Short-Term Memory (LSTM)

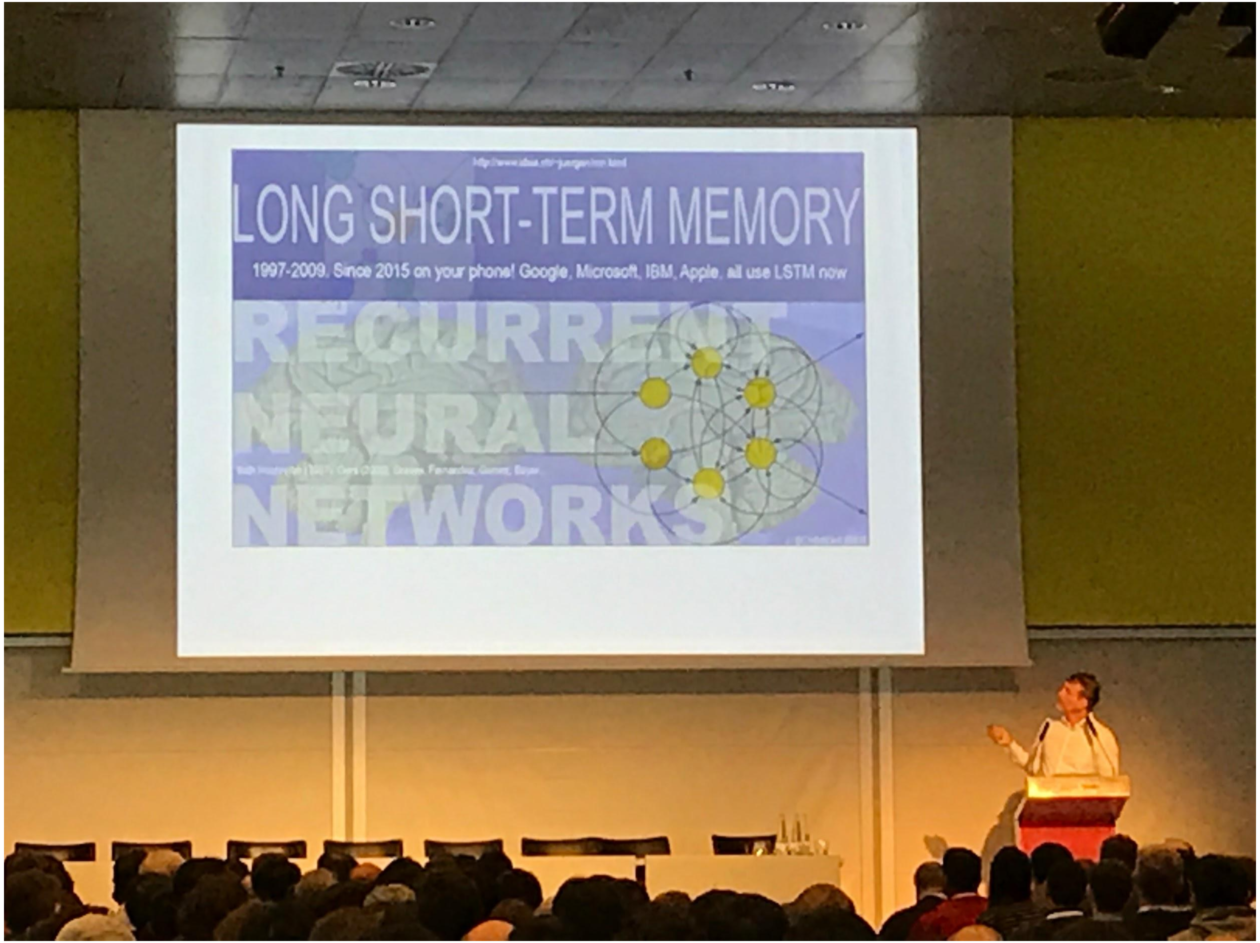


Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." Neural computation 9, no. 8 (1997): 1735-1780.

Long Short-Term Memory (LSTM)

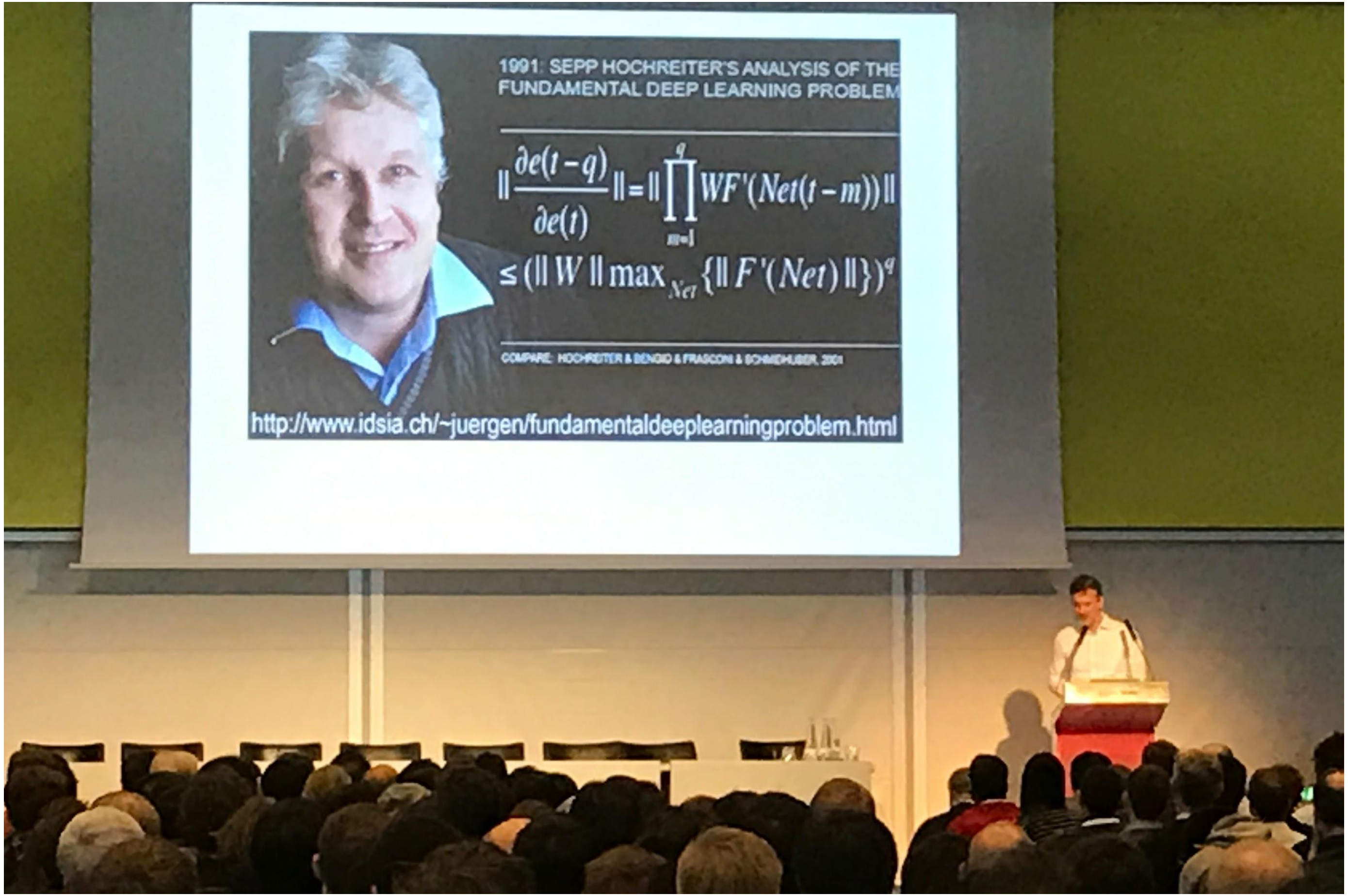
The New York Times, [“When A.I. Matures, It May Call Jürgen Schmidhuber ‘Dad’”]
(November 2016)

Long Short-Term Memory (LSTM)



[Jürgen Schmidhuber]() @ NIPS 2016 Barcelona

Long Short-Term Memory (LSTM)



Jürgen Schmidhuber @
NIPS 2016 Barcelona

Long Short-Term Memory (LSTM)

Jürgen Schmidhuber @
NIPS 2016 Barcelona

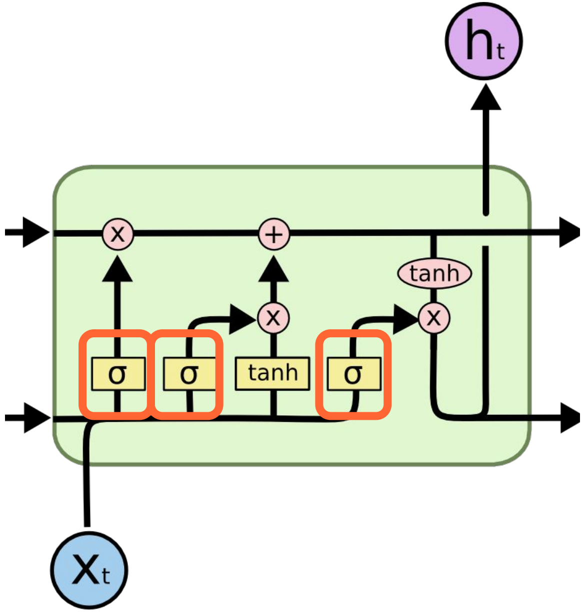
Gating method

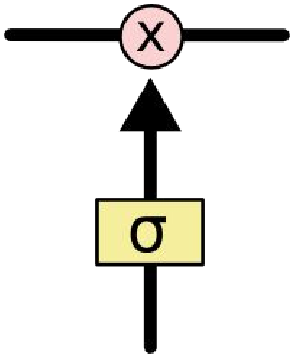
Solutions:

1. Change the way in which past information is kept → create the notion of **cell state**, a memory unit that keeps long-term information in a safer way by protecting it from recursive operations
2. Make every RNN unit able to **forget whatever may not be useful anymore** by clearing that info from the cell state (optimized clearing mechanism)
3. Make every RNN unit able to decide whether **the current time-step information matters or not**, to accept or discard (optimized reading mechanism)
4. Make every RNN unit able to **output the decisions whenever it is ready to do so** (optimized output mechanism)

Long Short-Term Memory (LSTM)

Three **gates** are governed by *sigmoid* units (btw [0,1]) define the control of in & out information..





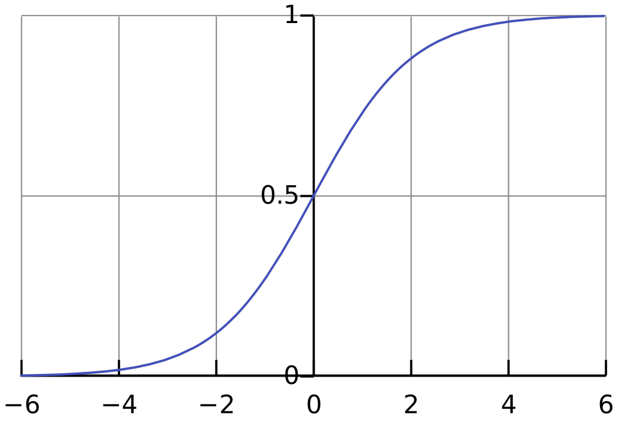
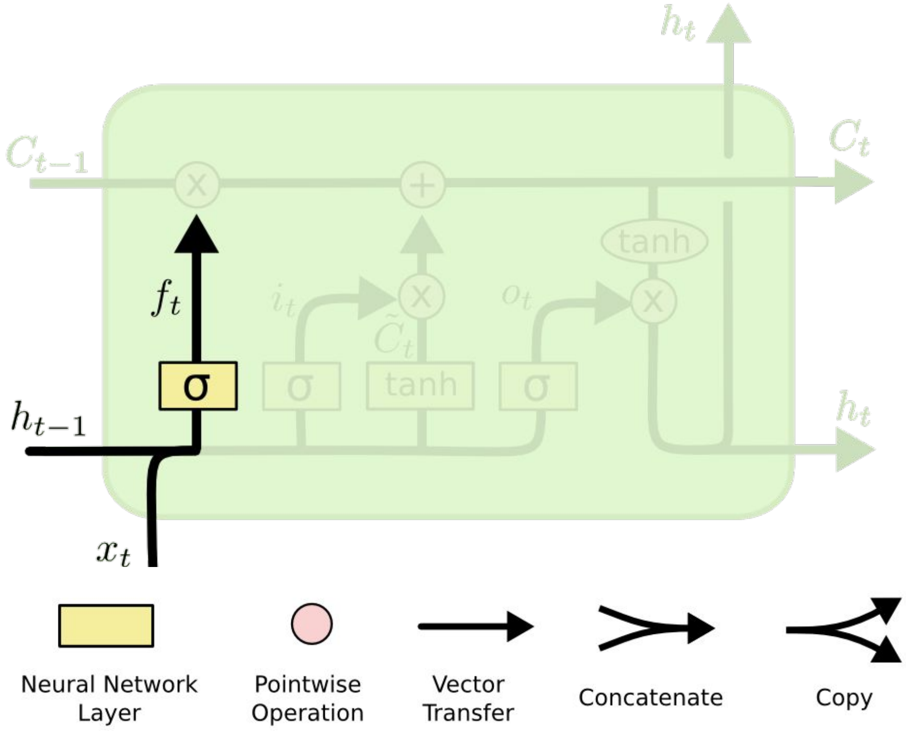


Figure: Cristopher Olah, [“Understanding LSTM Networks”](#) (2015)

slide credit: Xavi Giro

Long Short-Term Memory (LSTM)

Make every RNN unit able to **forget whatever may not be useful anymore** by clearing that info from the cell state (optimized clearing mechanism)



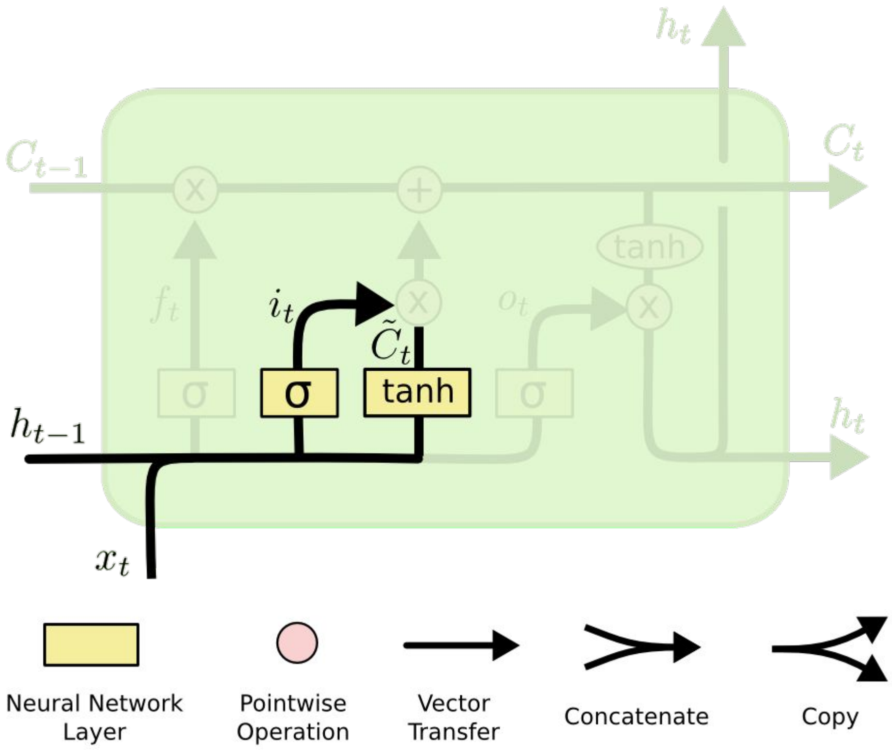
Forget Gate:

$$f_t = \sigma\left(W_f \cdot \underbrace{[h_{t-1}, x_t]}_{\text{Concatenate}} + b_f\right)$$

Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Long Short-Term Memory (LSTM)

Make every RNN unit able to decide whether **the current time-step information matters or not**, to accept or discard (optimized reading mechanism)



Input Gate Layer

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

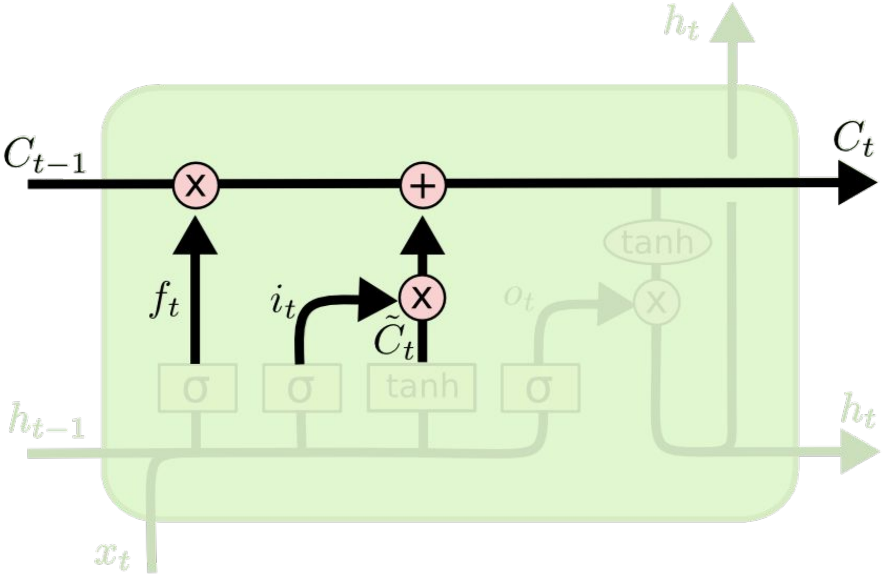
New contribution to cell state

$$\tilde{C}_t = \underbrace{\tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)}_{\text{Classic neuron}}$$

Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Long Short-Term Memory (LSTM)

Make every RNN unit able to decide whether **the current time-step information matters or not**, to accept or discard (optimized reading mechanism)



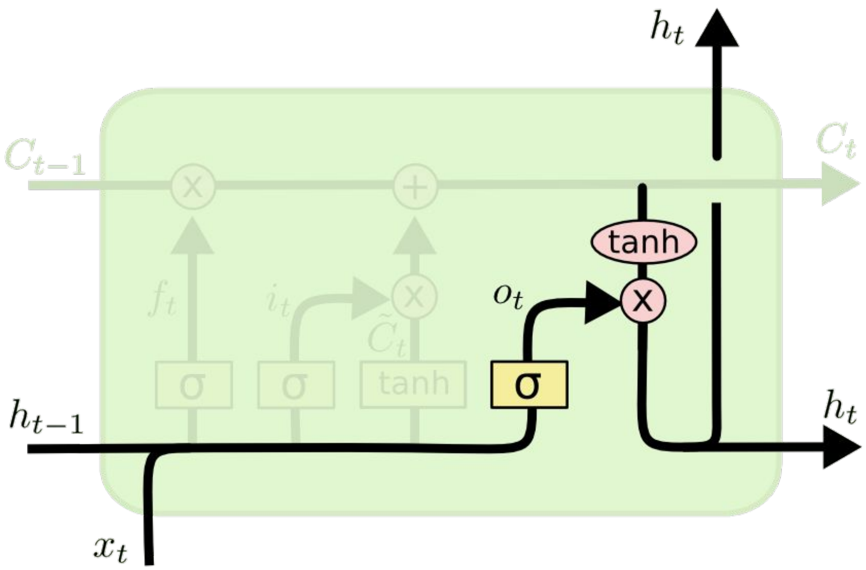
Update Cell State (memory):

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Long Short-Term Memory (LSTM)

Make every RNN unit able to **output the decisions whenever it is ready to do so** (optimized output mechanism)



Output Gate Layer

$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

Output to next layer

$$h_t = o_t * \tanh\left(C_t\right)$$

Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Long Short-Term Memory (LSTM)

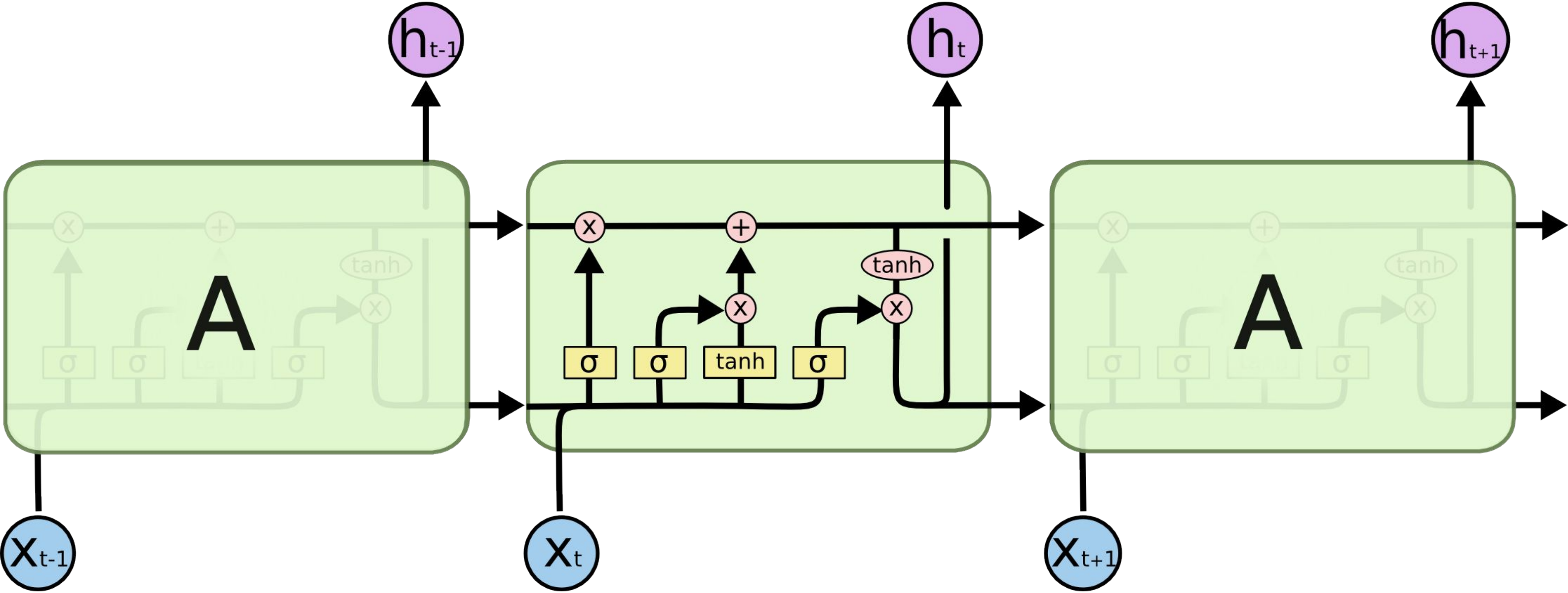
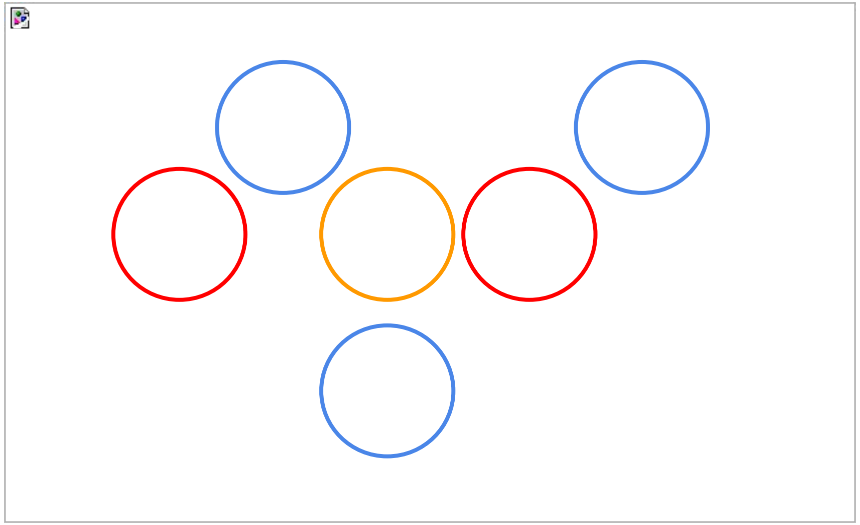


Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Long Short Term Memory (LSTM) cell

An LSTM cell is defined by two groups of neurons plus the cell state (memory unit):

1. Gates
2. Activation units
3. Cell state



$$\mathbf{i}_t = \sigma(\mathbf{W}_i \mathbf{x}_t + \mathbf{U}_i \mathbf{h}_{t-1} + \mathbf{b}_i)$$

$$\hat{\mathbf{C}}_t = \tanh(\mathbf{W}_c \mathbf{x}_t + \mathbf{U}_c \mathbf{h}_{t-1} + \mathbf{b}_c)$$

$$\mathbf{f}_t = \sigma(\mathbf{W}_f \mathbf{x}_t + \mathbf{U}_f \mathbf{h}_{t-1} + \mathbf{b}_f)$$

$$\mathbf{C}_t = \mathbf{i}_t \odot \hat{\mathbf{C}}_t + \mathbf{f}_t \odot \mathbf{C}_{t-1}$$

$$\mathbf{o}_t = \sigma(\mathbf{W}_o \mathbf{x}_t + \mathbf{U}_o \mathbf{h}_{t-1} + \mathbf{b}_o)$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{C}_t)$$

Computation Flow

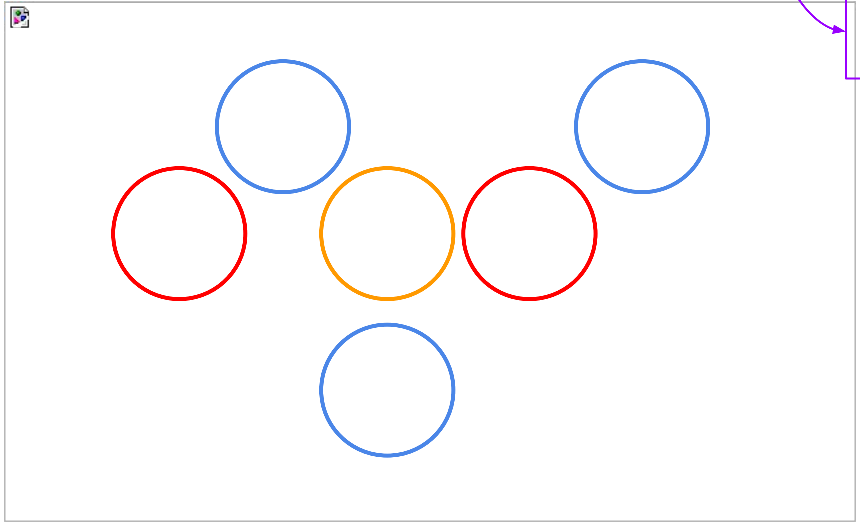
Long Short Term Memory (LSTM) cell

An LSTM cell is defined by two groups of neurons plus the cell state (memory unit):

1. **Gates**
2. **Activation units**
3. **Cell state**

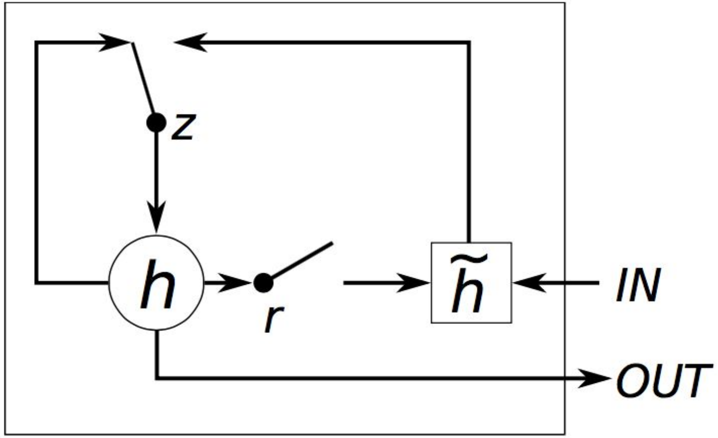
$$N^i_{params} = 4 \times (N^i_{inputs} \times N^i_{units} + N^i_{units} \times N^i_{units} + N^i_{units})$$

3 gates + input activation



Gated Recurrent Unit (GRU)

Similar performance as LSTM with less computation.



$$u_i = \sigma\left(W^{(u)}x_i + U^{(u)}h_{i-1} + b^{(u)}\right) \quad (1)$$

$$r_i = \sigma\left(W^{(r)}x_i + U^{(r)}h_{i-1} + b^{(r)}\right) \quad (2)$$

$$\tilde{h}_i = \tanh\left(Wx_i + r_i \circ Uh_{i-1} + b^{(h)}\right) \quad (3)$$

$$h_i = u_i \circ \tilde{h}_i + (1 - u_i) \circ h_{i-1} \quad (4)$$

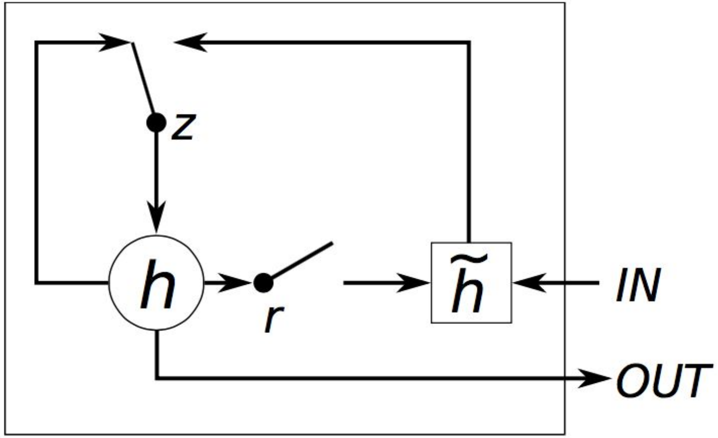
$$N^i_{params} = 3 \times (N^i_{inputs} \times N^i_{units} + N^i_{units} \times N^i_{units} + N^i_{units})$$

Cho, Kyunghyun, Bart Van Merriënboer, Caglar Gulcehre, Dzmitry Bahdanau, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. "Learning phrase representations using RNN encoder-decoder for statistical machine translation." AMNLP 2014.

slide credit: Xavi Giro

Gated Recurrent Unit (GRU)

Similar performance as LSTM with less computation.



$$u_i = \sigma\left(W^{(u)}x_i + U^{(u)}h_{i-1} + b^{(u)}\right) \quad (1)$$

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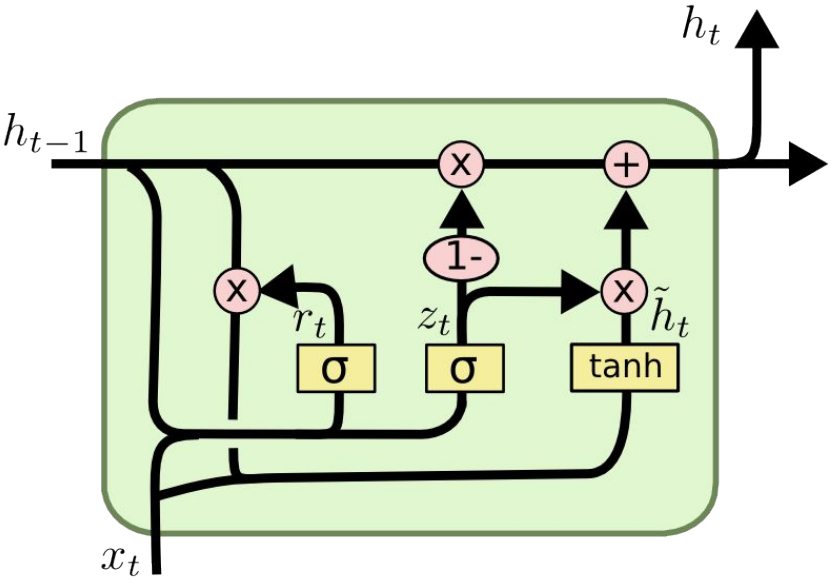


$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

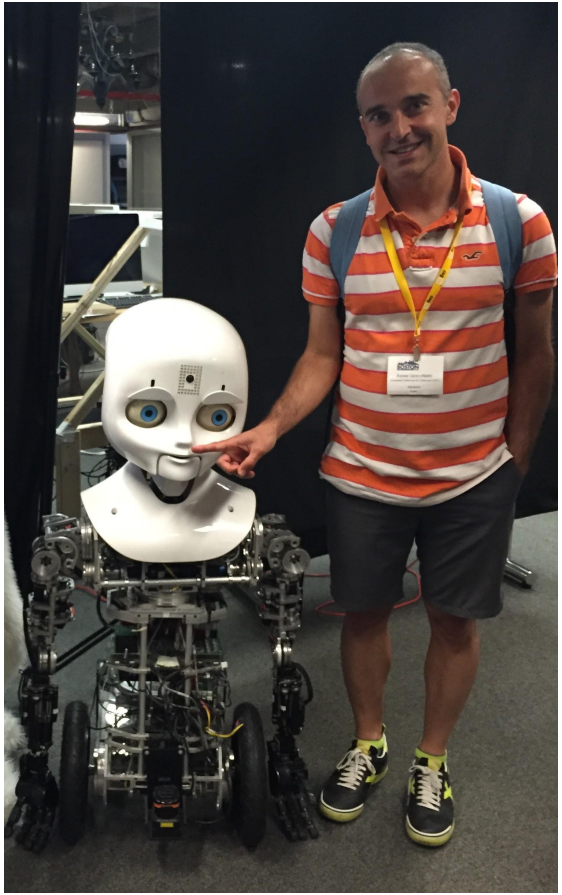
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Figure: Cristopher Olah, "Understanding LSTM Networks" (2015) / Slide: Alberto Montes

Thanks ! Q&A ?



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