2D Raindrop Simulation report

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June 4, 2021

Abstract

This is a summary of the simulation of raindrop falling & collisions in 2D scheme with OpenFOAM's interFOAM solver. The result shows raindrop terminal velocity increases with diameter, bounded at 5m/s when $D \geq 3mm$. The raindrop exhibits oscillation in both horizontal and vertical direction. With free falling simulation, collision cases are constructed by superimposing falling raindrop profile (water fraction, velocity and pressure) with each other. Various collision outcome types are found in the computational experiment, i.e. coalesce, neck/filament breakup, disk breakup, crown breakup and sheet breakup. The collision outcome regime of contact angle, Weber number and diameter ratio is investigated, which agrees with tower experiment qualitatively. However, both terminal velocity Webb number deviate from real experiment quantitatively, which can be attributed to the difference between 2D model and 3D reality.

1 Introduction

1.1 Governing Equations

The constant-density continuity equation reads:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

The momentum equation reads:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j u_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} + \tau_{t_{ij}}) + \rho g_i + f_{\sigma i} \tag{2}$$

u represent the velocity, g_i the gravitational acceleration, p the pressure and τ_{ij} and $\tau_{t_{ij}}$ are the viscous and turbulent stresses, $f_{\sigma i}$ is the surface tension.

The density ρ is defined as follows:

$$\rho = \alpha\rho_1 + (1-\alpha)\rho_2 \tag{3}$$

α is 1 inside fluid 1 with the density ρ_1 and 0 inside fluid 2 with the density ρ_2. At the interphase between the two fluids α varies between 0 and 1. The surface tension $f_{\sigma i}$, is modelled as continuum surface force (CSF). It is calculated as follows:

$$f_{\sigma i} - \sigma_\kappa \frac{\partial \alpha}{\partial x_i} \tag{4}$$

σ is the surface tension constant and κ the curvature. The curvature can be approximated as follows:

$$\kappa = -\frac{\partial n_i}{\partial x_i} = -\frac{\partial}{\partial x_i}(\frac{\partial \alpha / \partial x_i}{|\partial \alpha / \partial x_i|}) \tag{5}$$

In order to know where the interphase between the two fluids is, an additional equation for α has to be solved.

$$\frac{\partial \alpha}{\partial t} + \frac{\partial (\alpha u_j)}{\partial x_j} = 0 \tag{6}$$

The equation can be seen as the conservation of the mixture components along the path of a fluid parcel.

1.2 Sliding Mesh

Generally, the domain width is ten times of raindrop diameter and domain height is twice of domain width.

To simulate the raindrop falling towards the equilibrium, a long falling distance is required. To save the computational effort, a sliding mesh is implemented. At the end of several time steps, the output is analyzed to find the location of raindrop. If the raindrop is too close to the boundary ($distance < \frac{1}{4}$, the bottom half domain is lifted to the top half, while the remaining is filled with initial conditions. The approach assumes half of domain height is long enough that has no effect on the raindrop.

1.3 Physical condition

The simulation condition is provided in the table below:

Pameter	**Value**	**Unit**
Pressure	80000	Pa
Temperature	15	$^\circ C$
Water Density	999.13	kg/m^3
Air Density	0.96718	kg/m^3
Water Kinematic Viscosity	1.1384	m^2/s
Air Kinematic Viscosity	1.48	m^2/s
Surface Tension Coefficient	0.7315	N/m

2 Velocity Profile

The free falling of different-diameter raindrops is simulated. After reaching terminal velocity, the raindrop's speed, accelerated time, path distance, horizontal oscillation, vertical oscillation and b/a ratio are measured. The results are summarized in the table below:

Diameter (mm)	Terminal Velocity (m/s)	Accelerated Time (s)	Path Distance (m)	Horizontal Oscillation Period (s)	Vertical Oscillation Period (s)	b/a ratio
0.1	0	NA	NA	NA	NA	0.98-1.02
0.5	0.25	0.15	0.07	0.003	0.05	1.00-1.03
1	3.5	0.25	0.55	0.15	0.1	0.5-1.3
1.5	4.4	0.4	0.95	0.1	0.1	0.5-1.4
2	4.4	1.5	5.42	0.04	0.04	0.74-0.79
2.5	4.8	1.5	4.66	0.05	0.025	0.705-0.728
3	5	1.5	3.92	0.15	0.1	0.645-0.668
3.5	5	1.5	4.95	0.1	0.1	0.677-0.706
4	5.1	1	1.8	0.15	0.15	0.63-0.68

The terminal velocities are plotted below:

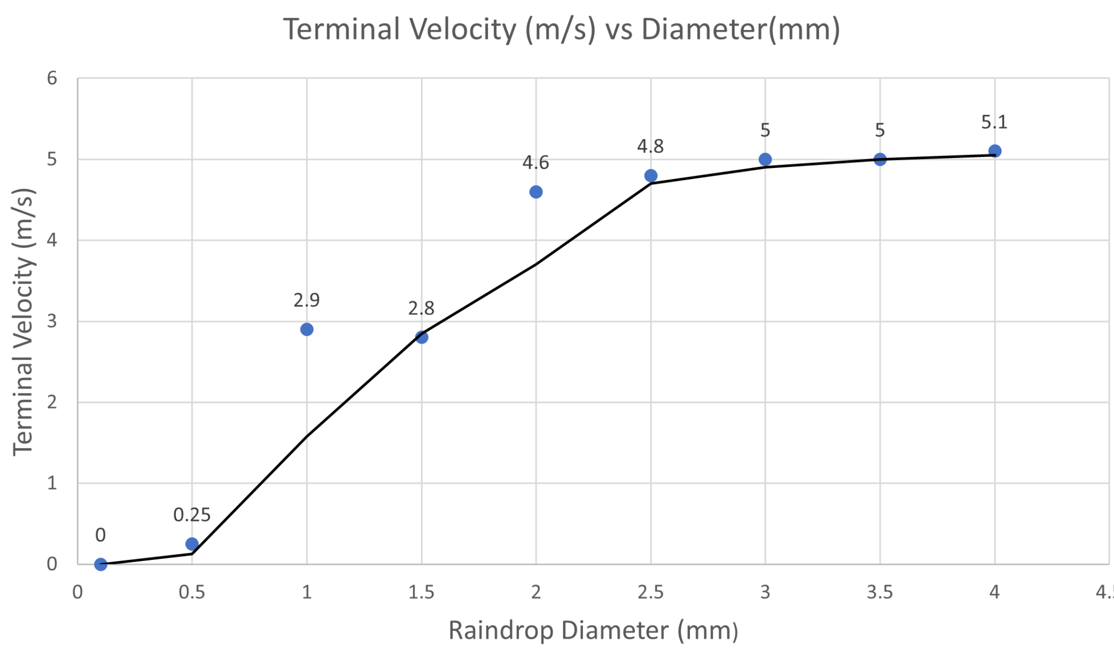


Figure 1: Terminal Velocity vs Drop Diameter

The terminal velocity simulated is smaller than field experiments (Hasan and Barros), which is shown below. The reason can be explained by 2D-3D difference in section 4.

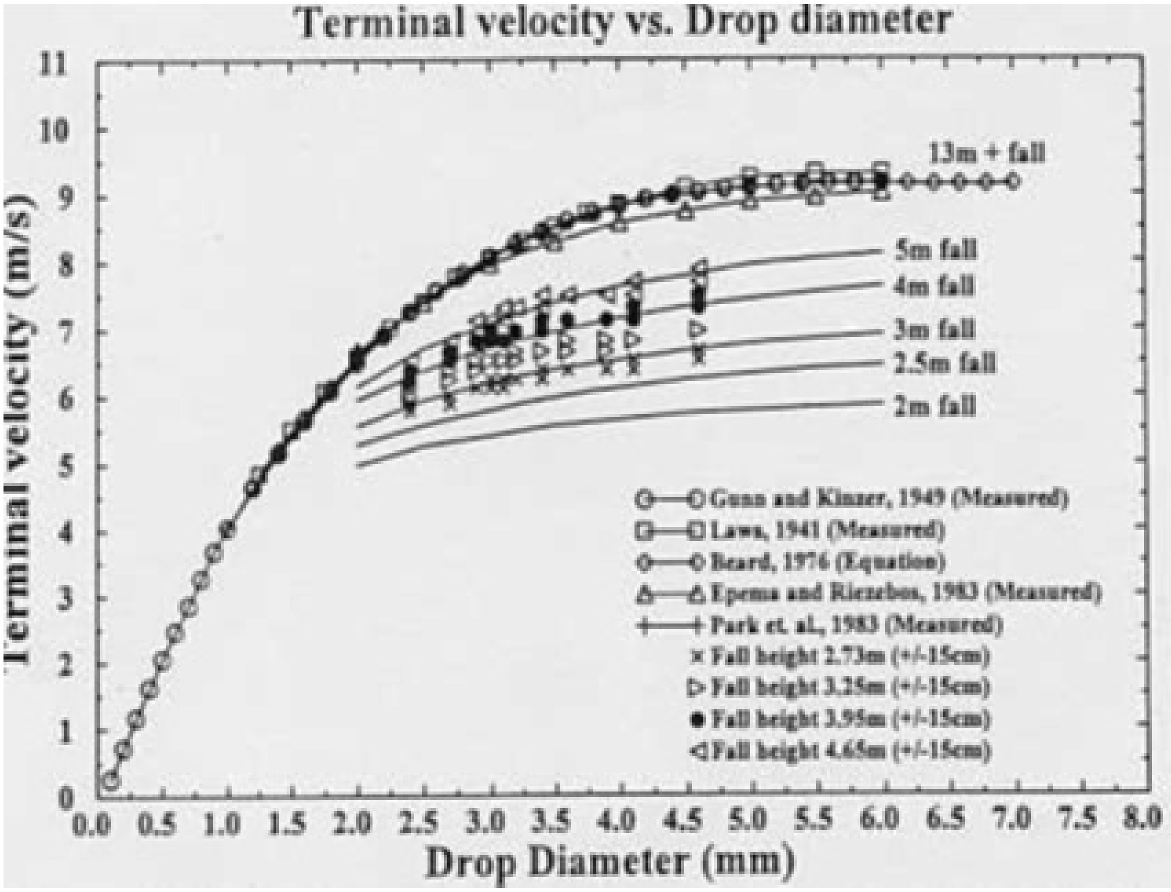


Figure 2: Field Measurements of Terminal Velocity

3 Collisions

3.1 Model Setup

After running the free falling simulation of single raindrop, raindrop profiles with velocity between 0 and terminal velocity can be obtained. To build the collision case, first an empty domain is constructed. Then the domain is filled by slices of two raindrops, cut from original profiles according to the drop location. The empty grids are filled with initial values. The initial profile is pre-run for a short time (10^{-3} sec) to eliminate the discontinuity at slice boundaries. The details are described in the attachment "collision model description".

3.2 Collision Outcomes

By experiment in wind tunnel, Testic and Barros identified five collision outcomes, bounce, coalescence, filament (neck) breakup, disk breakup, sheet breakup and crown breakup. In the simulation, all cases are found in collisions.

When the relative is small (not enough for breakup) and angle is large (not enough for coalescence), the two raindrops separate and reshape back to ellipses, which is characterized as bounce.

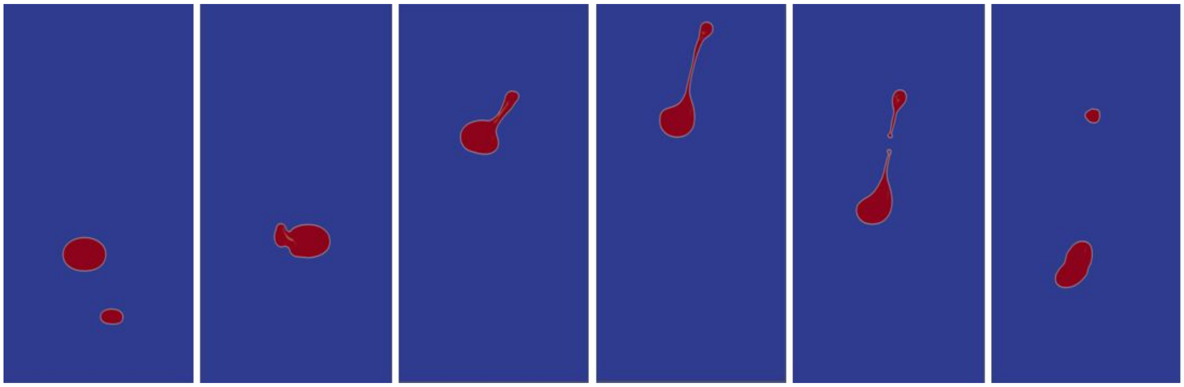


Figure 3: Collision Outcome: Bounce

A typical neck (filament) breakup is shown below. A series of small raindrops are observed, which is due to high velocity difference (higher momentum). Two drops produce extra seven drops (two are relatively large and five are small). In the last two pictures, two separated raindrops merge again, which involves a secondary collision. The outcome of secondary collision is coalescence.

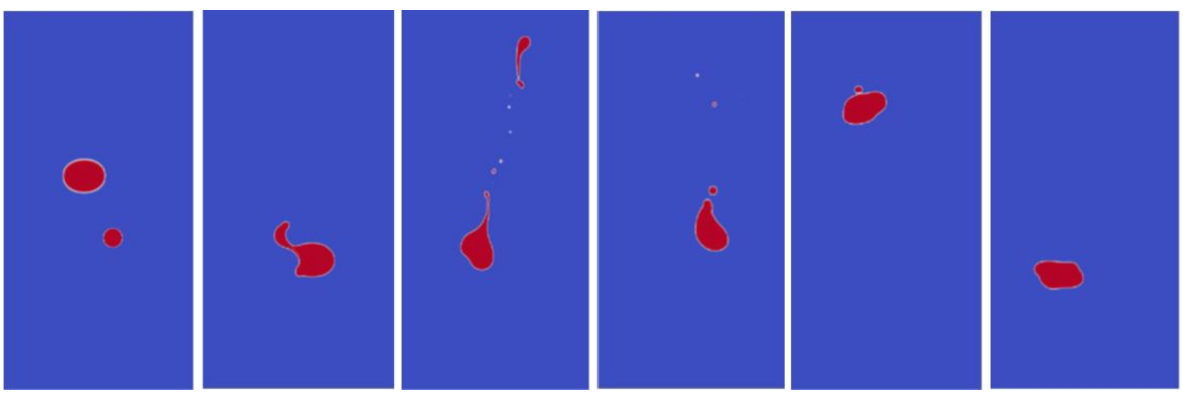


Figure 4: Collision Outcome: Neck (Filament) Breakup and Coalescence

Disk breakup happens when the two raindrop diameters are similar, or momentum is large enough to deform shape instantly. In 3D, there will be extension on z axis. The cross-section taken by camera should be a “disk”.

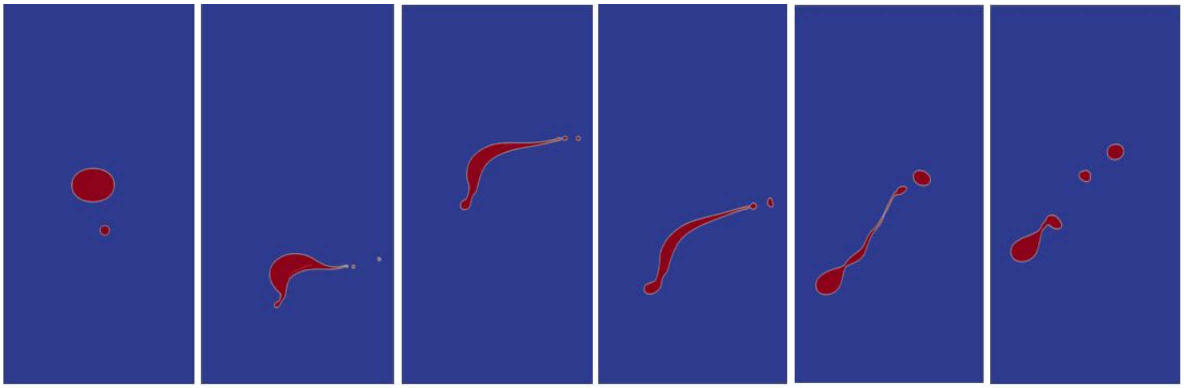


Figure 5: Collision Outcome: Disk Breakup

Crown Breakup occurs when a small drop hits a larger one with medium momentum (not strong enough to deform shapes but not weak enough to form coalescence). With higher resolution, crown breakup can be observed more clearly.

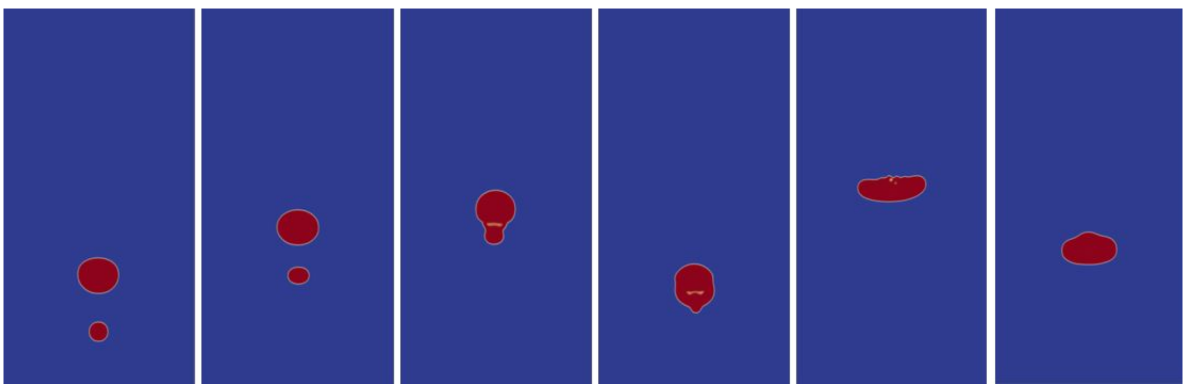


Figure 6: Collision Outcome: Crown Breakup

The sheet type breakup occurs when top drop has lower velocity, then increases velocity and catch up with the bottom raindrop. The momentum energy is betwen the disk breakup limit and coalescence limit.

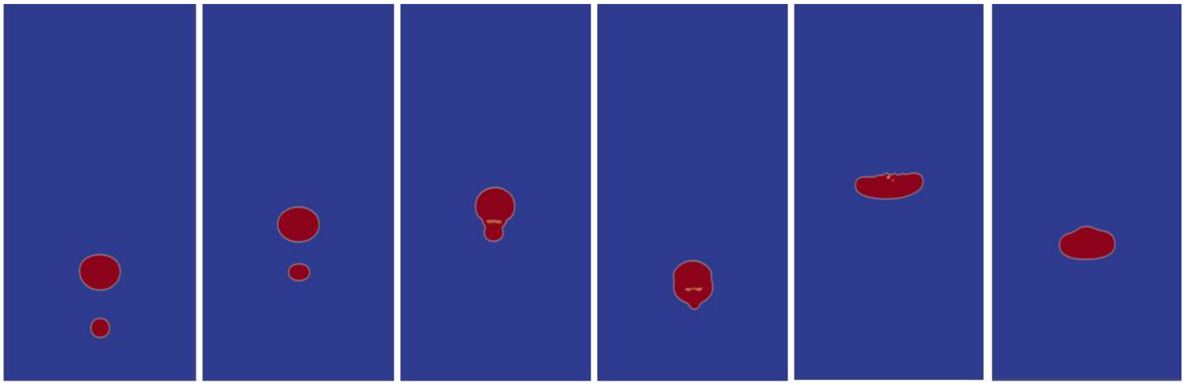


Figure 7: Collision Outcome: Sheet Breakup

3.3 Weber number, collision angle and diameter ratio regime

4 2D-3D difference

4.1 Stokes Law

4.1.1 derivation

Stocks law applies to low Re number, derived from Stokes equation by neglecting advection term.

The Stokes equation reads:

$$\begin{aligned} \nabla p = \mu \nabla^2 \mathbf{u} = -\mu \nabla \times \boldsymbol{\omega} \\ \nabla \cdot \mathbf{u} = \mathbf{0} \end{aligned} \tag{7}$$

By using some vector calculus identities, we can get

$$\begin{aligned} \nabla^2 \boldsymbol{\omega} = 0 \\ \nabla^2 p = 0 \end{aligned} \tag{8}$$

Solve above equations in cylindrical coordinates, the z direction forces are:

$$\begin{aligned} F_{pressure} = 2\pi a \mu u \\ F_{shear} = 4\pi a \mu u \end{aligned} \tag{9}$$

The Stokes law reads:

$$F_d = 6\pi \mu R u \tag{10}$$

4.1.2 Reynolds Number

$$Re = \frac{InertiaForce}{ViscousForce} = \frac{advectionterm}{diffusionterm} = \frac{\rho u du/dx}{\mu d^2u/dx^2} = \frac{\rho u L}{\mu} = \frac{uL}{\nu} \tag{11}$$

As the derivation of Stokes Law above, the advection term is neglected. So it only applies to creeping flow.

4.1.3 Mapping with Stokes Law

Here we assmume the Reynolds number is very small and the raindrop is a rigid sphere. If integrate 2D drag force along y axis, we could get 3D drag force, i.e.

$$\int_{-R}^{R} F_y dx = 6\pi\mu R\nu \tag{12}$$

Therefore, F_r does not depend on R, i.e. shape independent.

$$F_r = k\mu u, \quad k \text{ } is \text{ } constant \tag{13}$$

In 2D:

$$F_r = k\mu u = \pi R^2 \rho g \tag{14}$$

solve and get

$$u_{2D} = \frac{\pi R^2 \rho g}{k\mu} \tag{15}$$

In 3D:

$$\int_{-R}^{R} F_x dx + \int_{-R}^{R} F_y dy = 4Rk\mu u = \frac{4}{3}\pi R^3 \rho g \tag{16}$$

solve and get

$$u_{3D} = \frac{4}{3}\frac{\rho\pi R^2 g}{k} \tag{17}$$

Based on Stokes Law, the 3D terminal velocity is $\frac{4}{3}$ of velocity in 2D result.

4.2 Drag Equation

4.2.1 Introduction to Drag Equation

For high Reynolds number, the Stokes Law is not appropriate. Instead, we use the drag equation, i.e.

$$F_D = \frac{1}{2}\rho u^2 C_D A \tag{18}$$

Where F_D is the drag force, ρ is the mass density of fluid, u is the flow velocity relative to the object, A is the reference area, and C_D is the drag coefficient. The only derivation I can found is based on dimensional analysis (I don't think it is strict).

4.2.2 Mapping with Drag Equation

Here we assume the raindrop is rigid sphere. In 2D, the object is treated as a cylinder:

$$F_{D2} = \frac{1}{2}\rho u^2 C_D 2RL = \rho\pi R^2 Lg \tag{19}$$

solve and get:

$$u2D = \sqrt{\pi Rg/\rho C_{2D}} \tag{20}$$

In 3D, the object is treated as a sphere:

$$F_{D3} = \frac{1}{2}\rho u^2 C_D \pi R^2 = \rho \frac{4}{3}\pi R^3 g \tag{21}$$

solve and get:

$$u_{3D} = \sqrt{\frac{8}{3}Rg/C_{3D}} \tag{22}$$

Therefore, the ratio between 3D velocity and 2D velocity is:

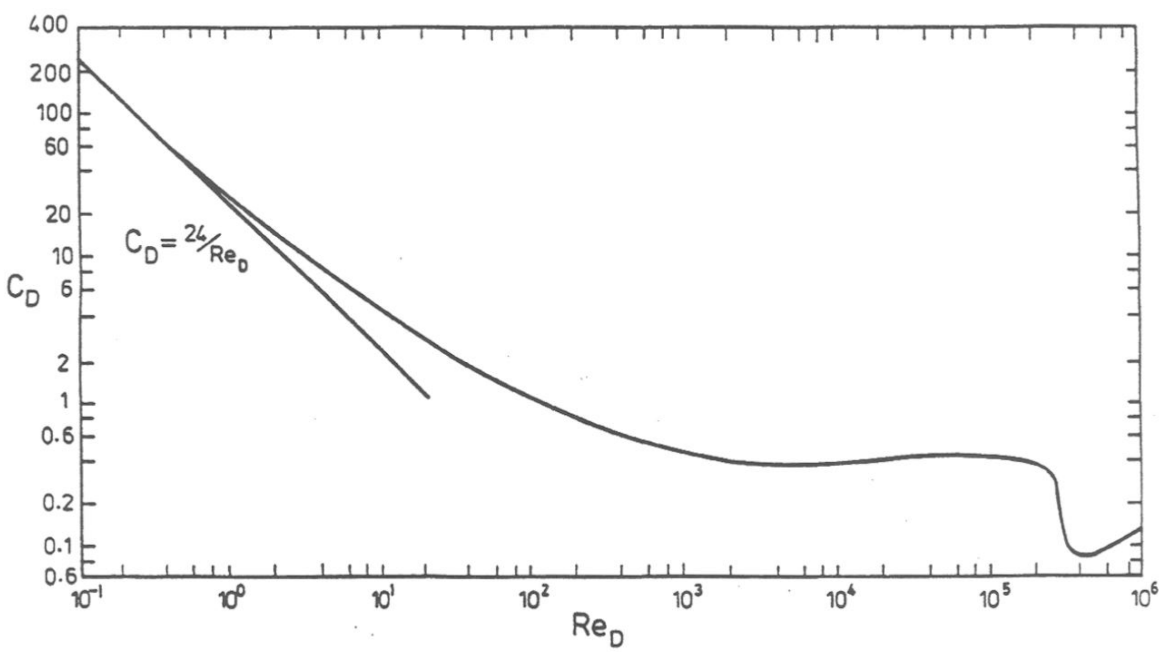
$$\frac{u_{3D}}{u_{2D}} = \sqrt{\frac{8}{3\pi}\frac{C_{2D}}{C_{3D}}} \tag{23}$$

Hoernner (1958) provided the drag coefficient for sphere and cylinder. For sphere, $C_{3D} = 0.47$ and for cylinder, $C_{2D} = 1.20$, then

$$\frac{u_{3D}}{u_{2D}} = \sqrt{\frac{8}{3\pi}\frac{1.20}{0.47}} = 1.47 \tag{24}$$

4.2.3 Drag Coefficient with Reynolds Number

In Matthew P. Juniper Fluid Mechanics book:



Drag coefficient of spheres as a function of Reynolds number.

Figure 8: C_D vs Re_D

References

[1] Santiago Márquez Damián, *Description and utilization of interFoam multiphase solver*

[2] Hasan Guzel and Ana P. Barros, *Using Acoustic Emission Testing to Monitor Kinetic Energy Of Raindrop and Rainsplash Erosion*

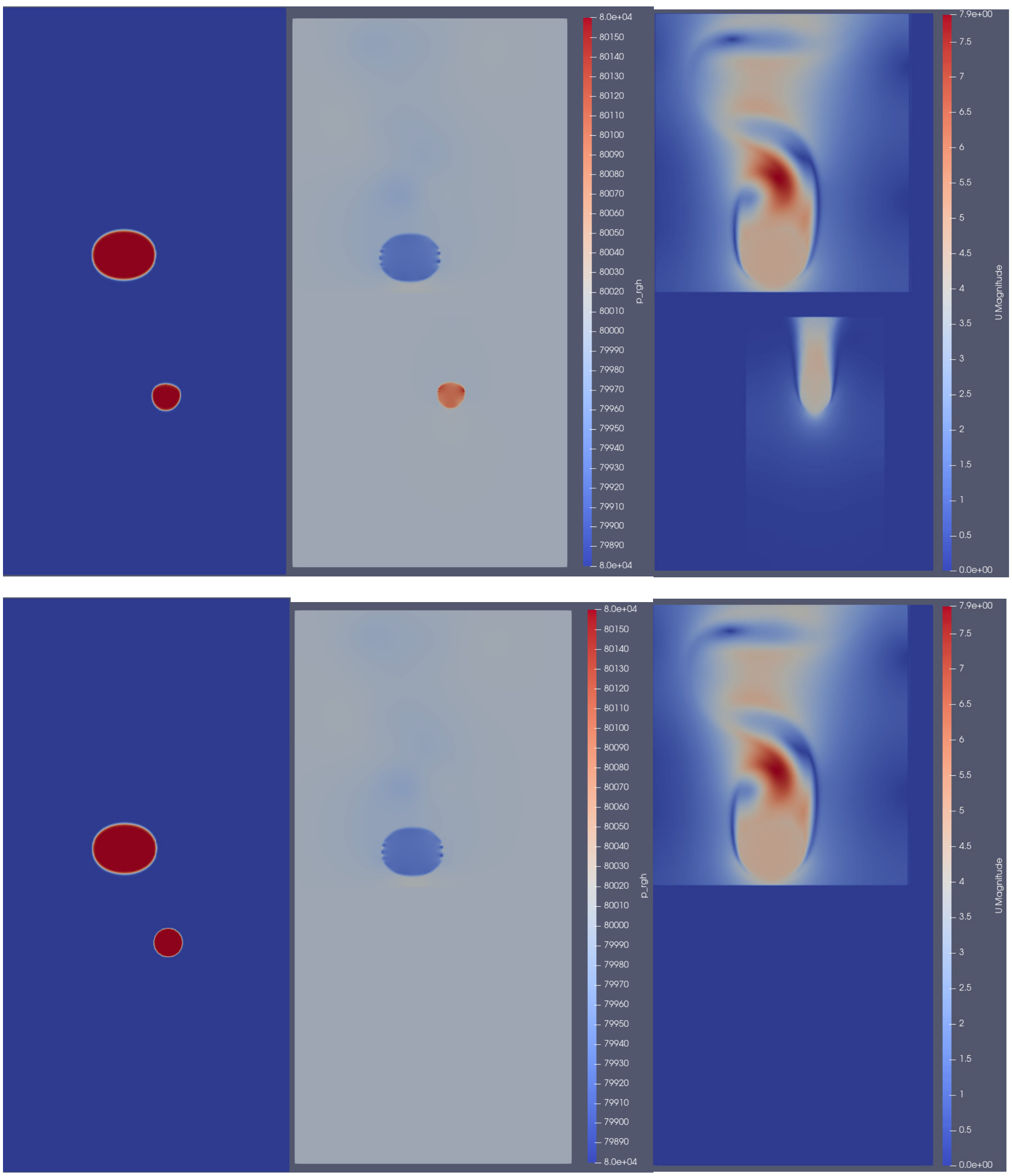
[3] F.Y.Testic, A.P.Barros, L.F.Bliven, *Toward a Physical Characterization of Raindrop Collision Outcome Regimes*

Attachment

Model Description:

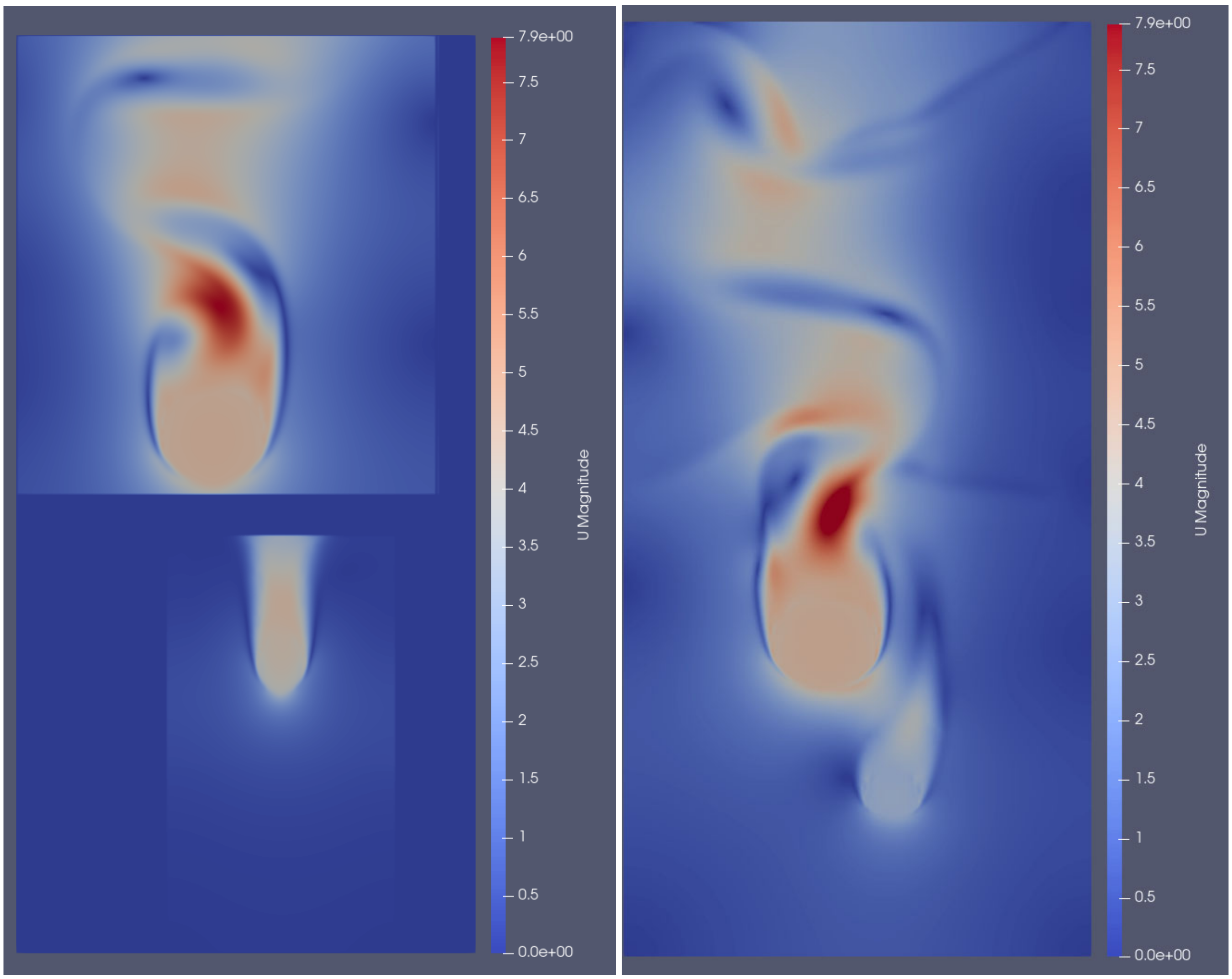
The domain set up: For example, this a side collision case. The top three graphs show the initial fields of the collision between two moving raindrops and bot three graphs show the initial fields of the collision between a moving large drop and static small drop (round shape, pressure=80hPa, U=0 so the bottom raindrop looks same with environment).

The idea is to cut the main part of output fields of terminal velocity simulation of a raindrop and arrange it in the new domain with concern of raindrop locations. For this case, the side collision, the top raindrop is moved left, and bot raindrop is moved right slightly to ensure the two drops are in center horizontally. Then the field that has not been filled with output data, is filled with initial conditions (water fraction=0, pressure= atmosphere pressure 80hPa and U=0)



The assumption is unfilled field is far enough from raindrop center such that is it is not affected. The assumption works great in water fraction (the left) and pressure (the right) because there's a shape change of field. For velocity, the assumption works fine because air velocity difference of 1m/s is observed, comparing highest velocity of 8m/s. And after a short time, the unfilled field velocity adjusts quickly correspondingly.

The bottom figure shows the initial velocity field (t=0) and velocity field after short time (t=0.001s). The momentum diffuses quickly and after 0.001s a velocity special gap cannot be observed anymore.



The parameters setting:

For the pressure and temperature, they are not involved directly in the model. But they affect simulation by changing viscosity, density, and surface tension coefficients.

Pressure (Pa)	80000
Temperature (°C)	15
Water Density (kg/m^3)	999.13
Air Density (kg/m^3)	0.96718
Water Kinematic Viscosity (m^2/s)	1.1384e-6
Air Kinematic Viscosity (m^2/s)	1.48e-5
Surface Tension Coefficient (N/m)	0.7315

Boundary Conditions:

- Water fraction: Inletoutlet
  (i.e. Flow out of the domain assigns a zero gradient condition & Flow into the domain: assigns a velocity based on the flux in the patch-normal direction)
  https://www.openfoam.com/documentation/guides/latest/doc/guide-bcs-outlet-inlet-outlet.html
- Velocity: pressureInletOutletVelocity
  (i.e. Flow out of the domain: assigns a zero gradient condition & Flow into the domain: assigns a velocity based on the flux in the patch-normal direction
  https://www.openfoam.com/documentation/guides/latest/doc/guide-bcs-outlet-pressure-inlet-outlet.html
- P_rgh: totalPressure (P_0) uniform 80000

  $$p_p = p_0 - \frac{1}{2}|\mathbf{u}|^2$$

- P: Calculated from P_rgh (total pressure)
  https://www.openfoam.com/documentation/guides/latest/doc/guide-bcs-inlet-outlet-total-pressure.html

Fall velocities with diameters

Domain width is five times of raindrop diameter, mesh sizes are 200 (width)×400(height), initial velocities are zero.

Raindrop diameters are 0.1mm, 0.5mm, 1mm, 1.5mm, 2mm, 2.5mm, 3mm, 3.5mm,4mm

Diameter (mm)	Terminal Velocity (m/s)	Time to Terminal Velocity (s)	Distance to Terminal Velocity (m)	Horizontal Velocity Oscillation Period (s)	Vertical Velocity Oscillation Period (s)	b/a ratio
0.1	≈0	NA	NA	NA	NA	0.98-1.02
0.5	0.25	0.15	0.07	0.003	0.05	1.00-1.03
1	2.9	0.25	0.55	0.15	0.1	0.5-1.3
1.5	2.8	0.4	0.95	0.1	0.1	0.5-1.4
2	4.6	1.5	5.42	0.04	0.04	0.74-0.79
2.5	4.8	1.5	4.66	0.05	0.025	0.705-0.728
3	5.0	1.5	3.92	0.15	0.1	0.645-0.668
3.5	5.0	1.5	4.95	0.1	0.1	0.677-0.706
4	5.1	1	1.80	0.15	0.15	0.63-0.68

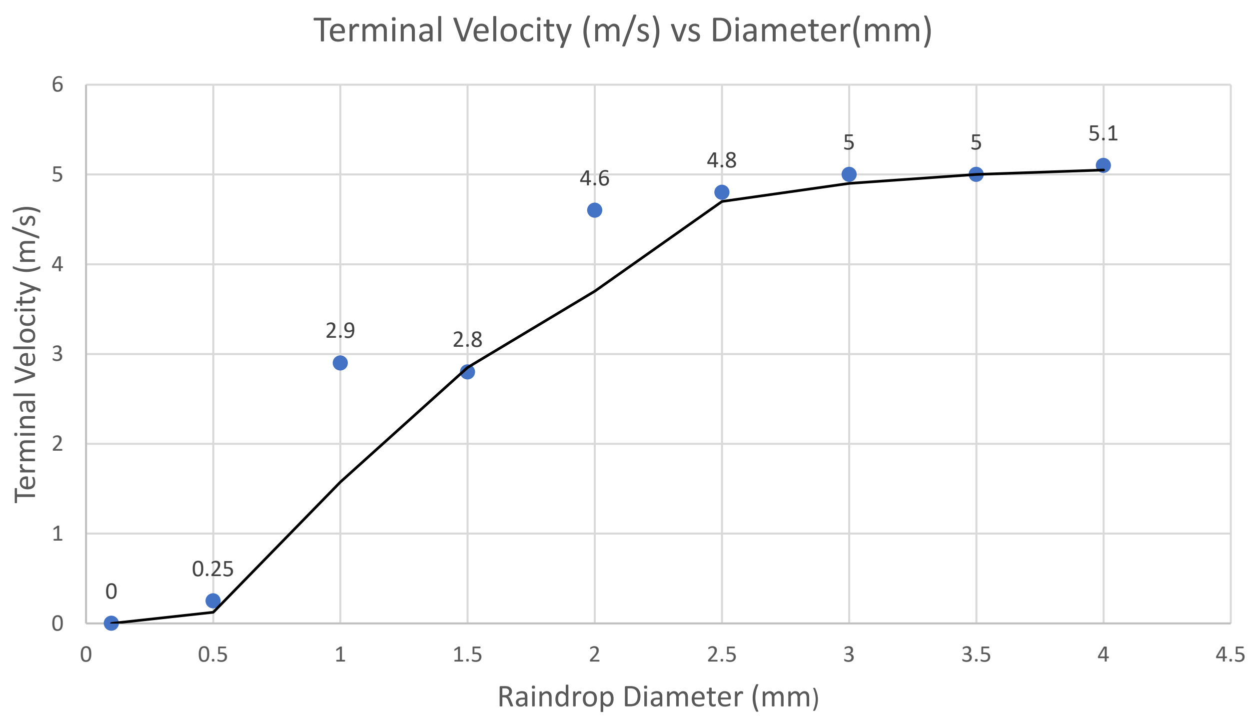


Figure D=0.1mm

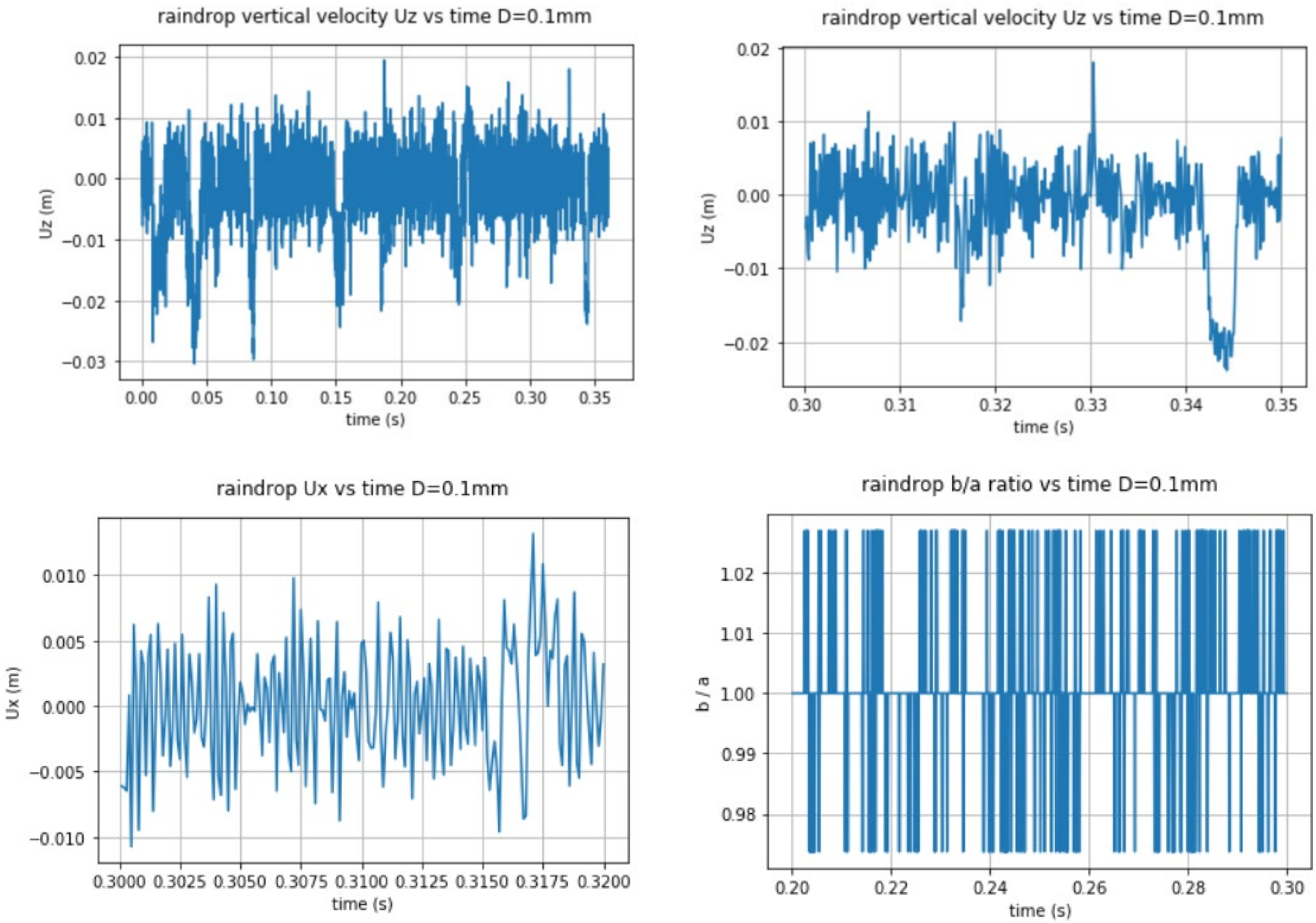


Figure D=0.5mm

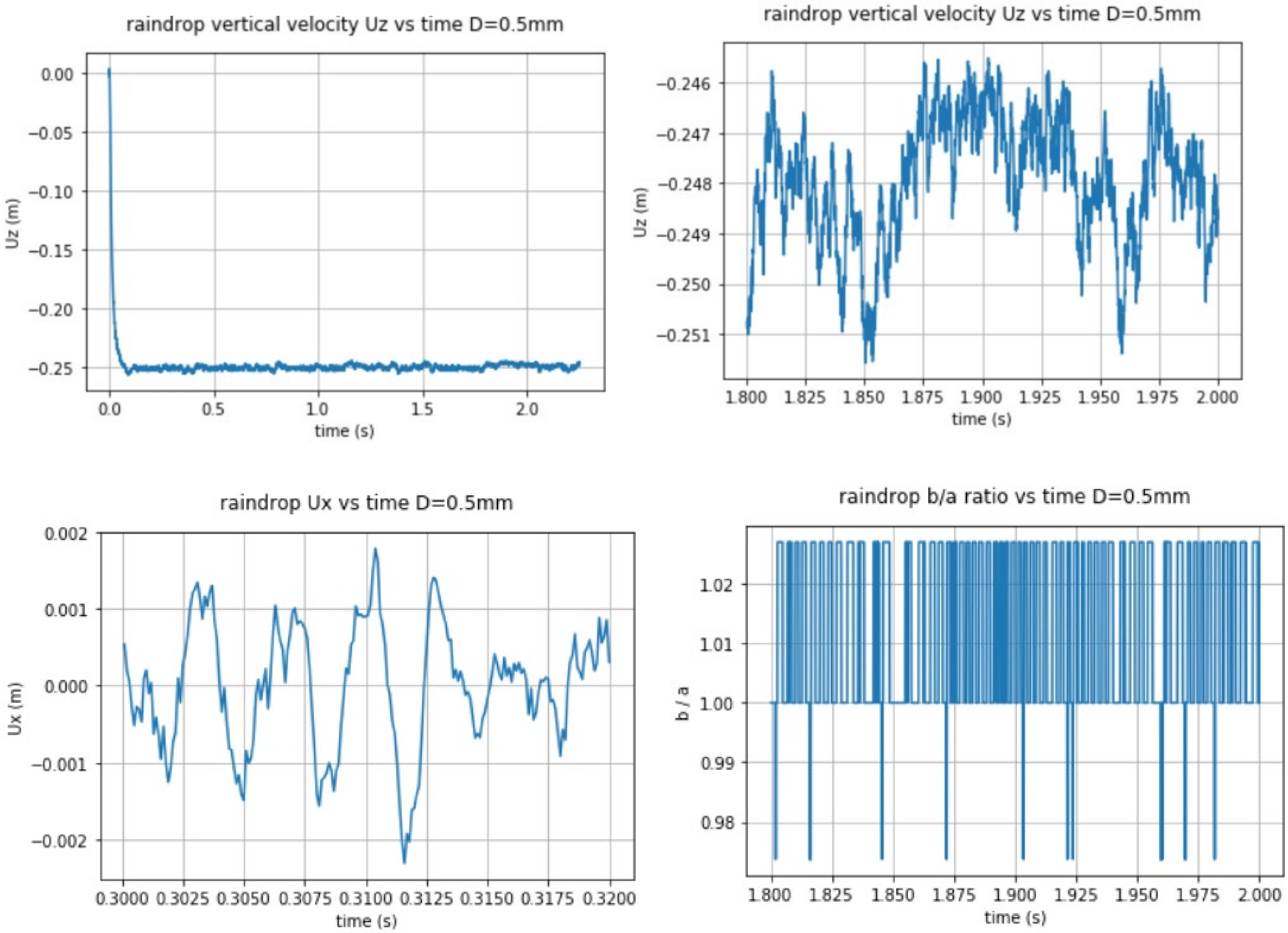


Figure D=1mm

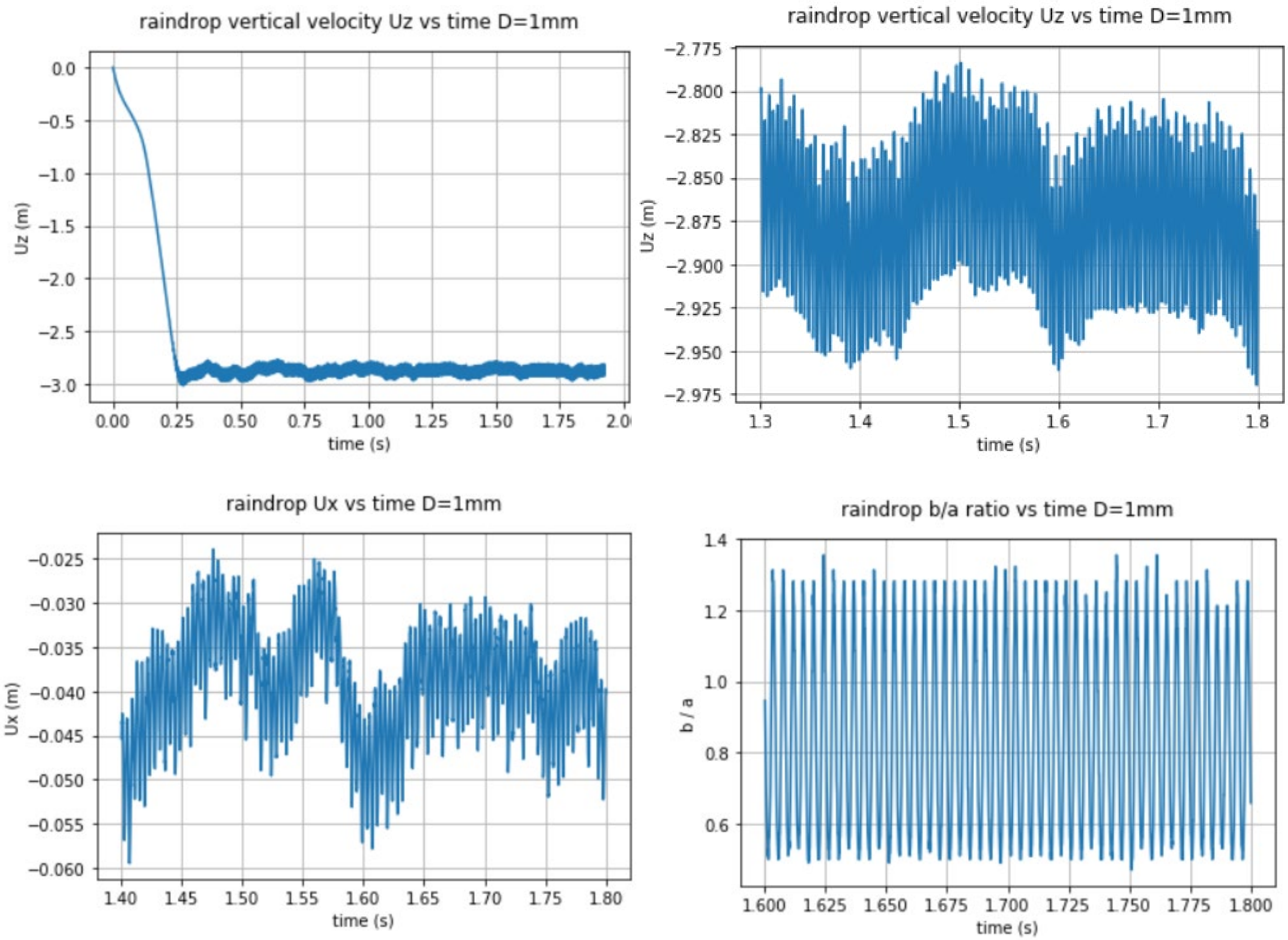

raindrop vertical velocity Uz vs time D=1mm
Uz (m)
time (s)
raindrop vertical velocity Uz vs time D=1mm
Uz (m)
time (s)
raindrop Ux vs time D=1mm
Ux (m)
time (s)
raindrop b/a ratio vs time D=1mm
b / a
time (s)

Figure D=1.5mm

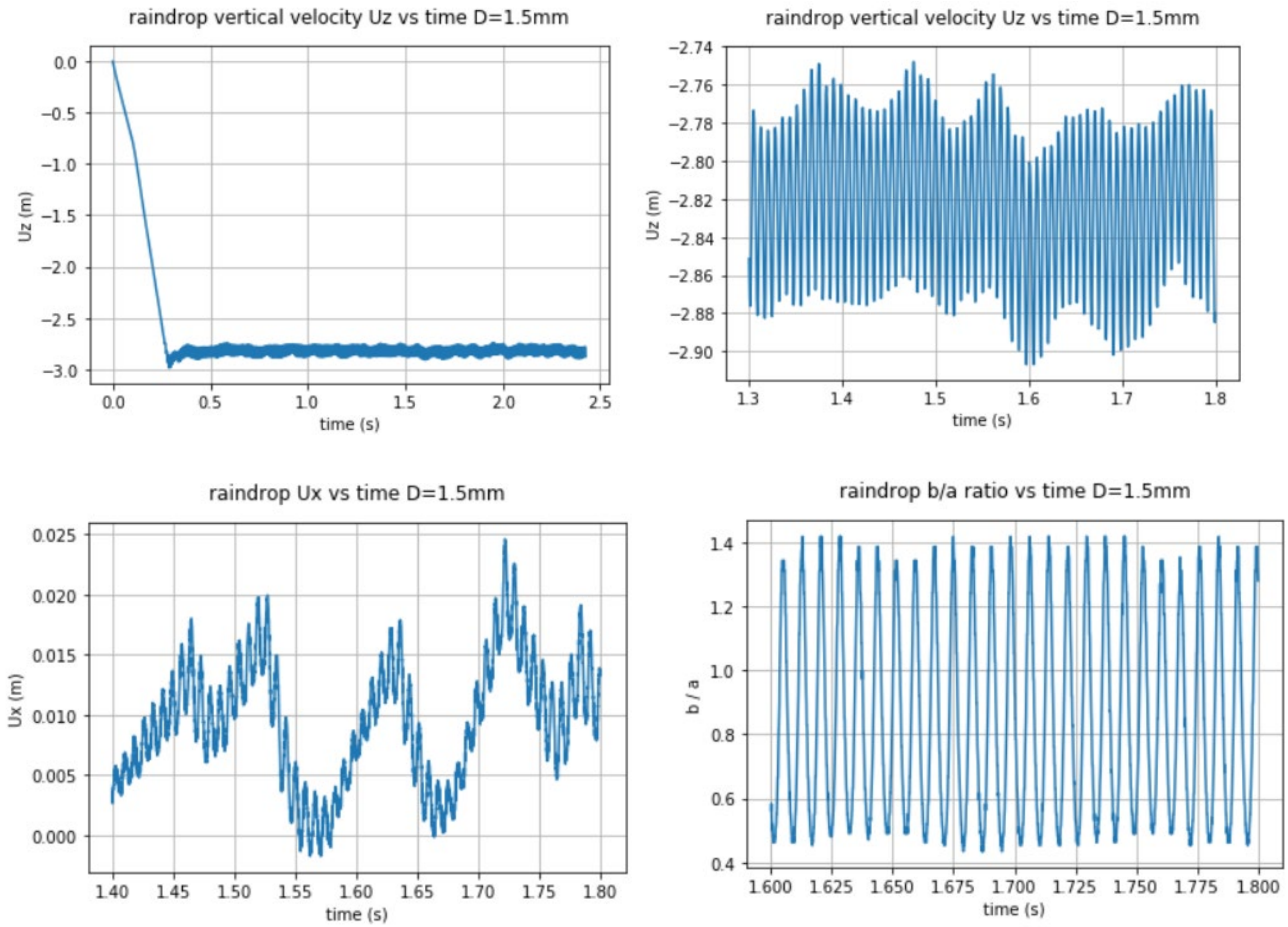


Figure D=2mm

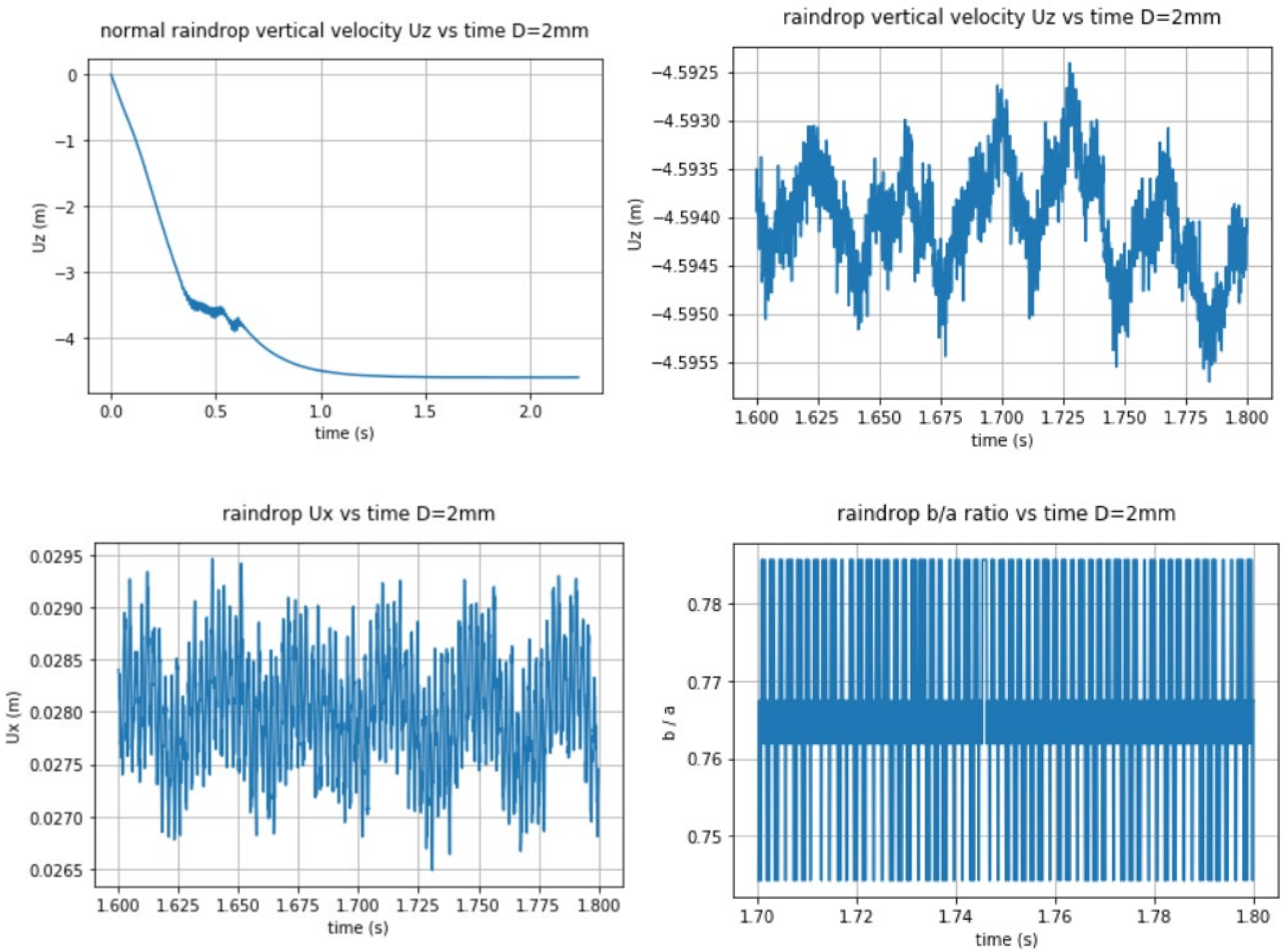

normal raindrop vertical velocity Uz vs time D=2mm
Uz (m)
time (s)
raindrop vertical velocity Uz vs time D=2mm
Uz (m)
time (s)
raindrop Ux vs time D=2mm
Ux (m)
time (s)
raindrop b/a ratio vs time D=2mm
b / a
time (s)

Figure D=2.5mm

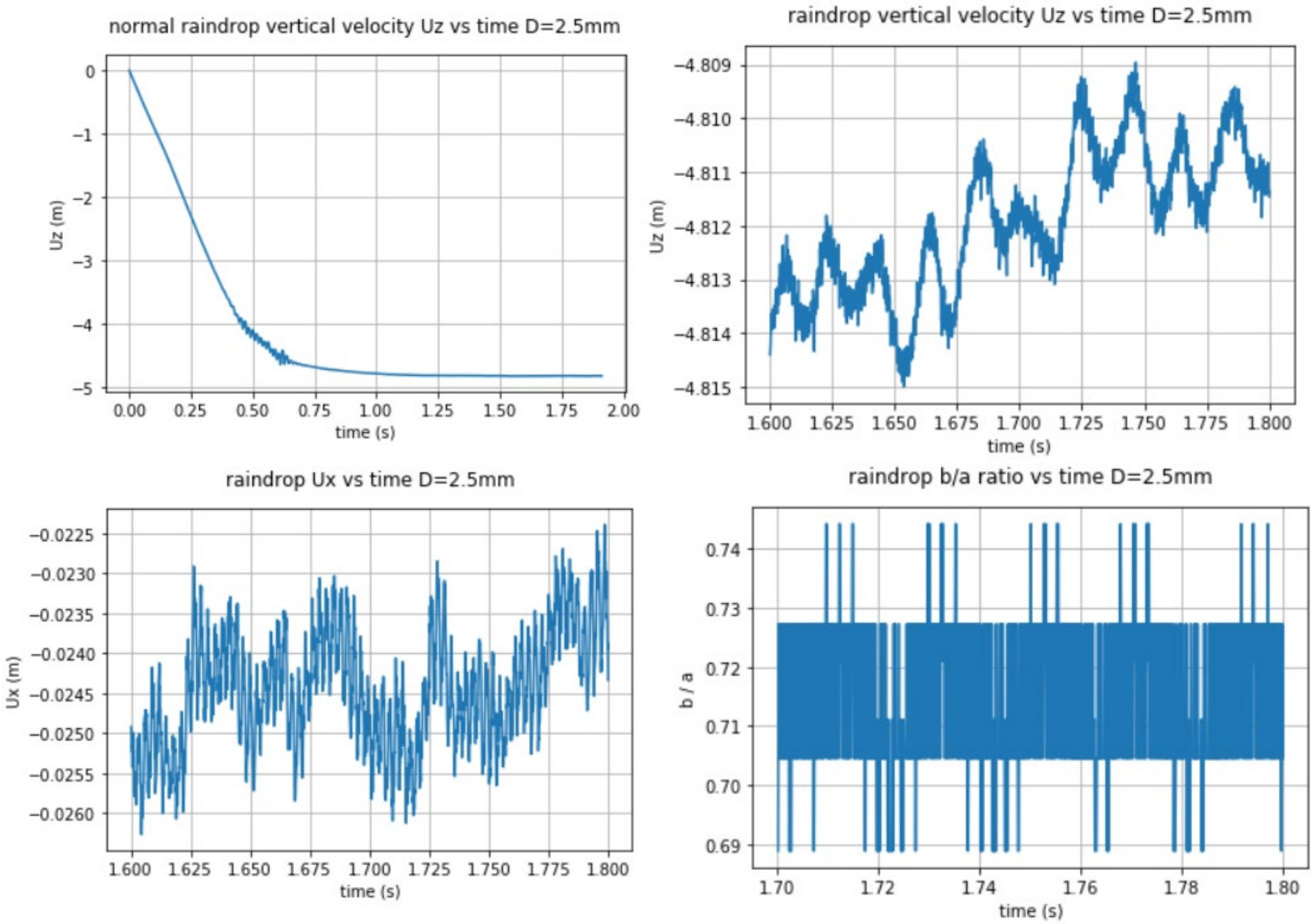


Figure D=3mm

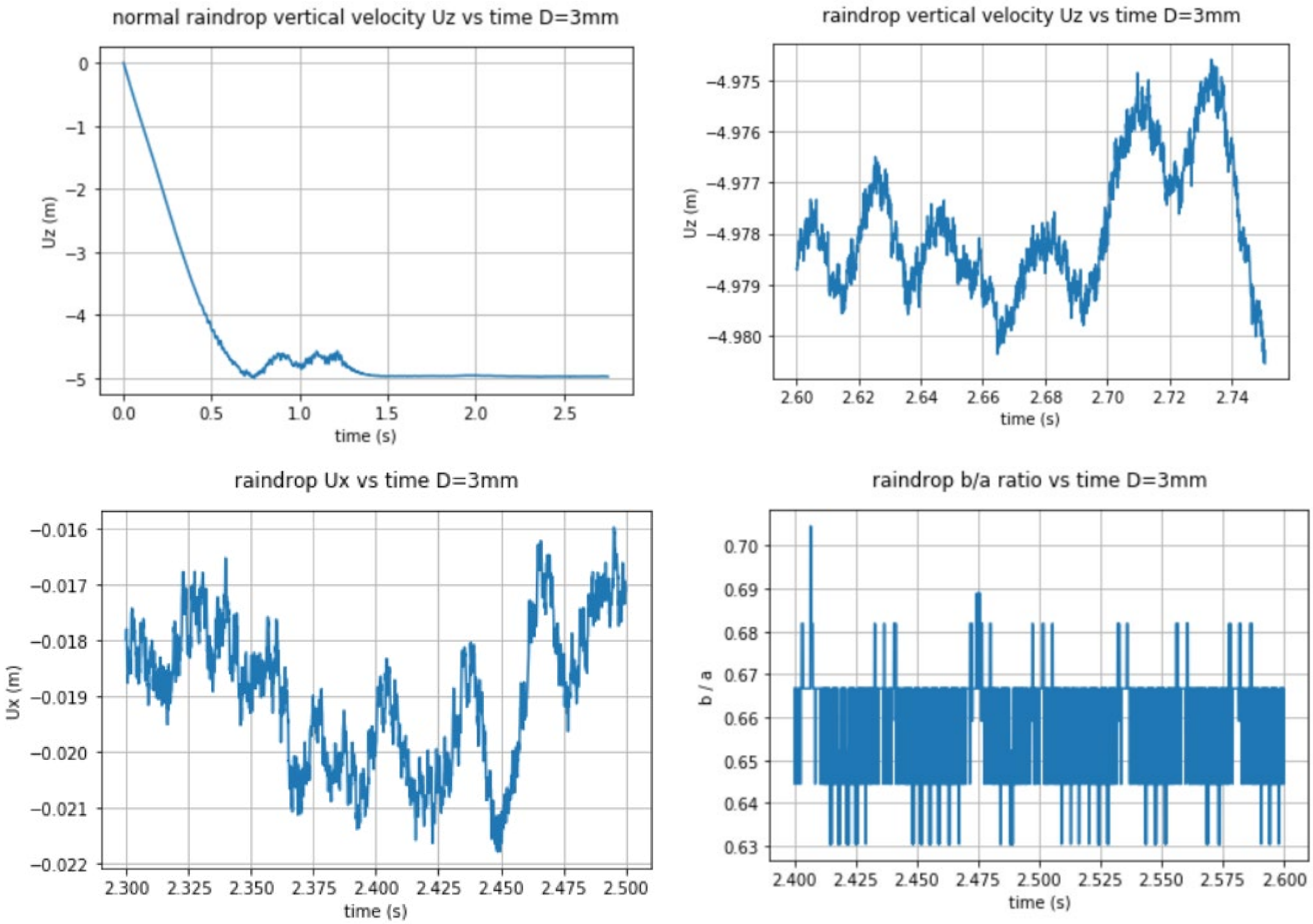

normal raindrop vertical velocity Uz vs time D=3mm
Uz (m)
time (s)
raindrop vertical velocity Uz vs time D=3mm
Uz (m)
time (s)
raindrop Ux vs time D=3mm
Ux (m)
time (s)
raindrop b/a ratio vs time D=3mm
b / a
time (s)

Figure D=3.5mm

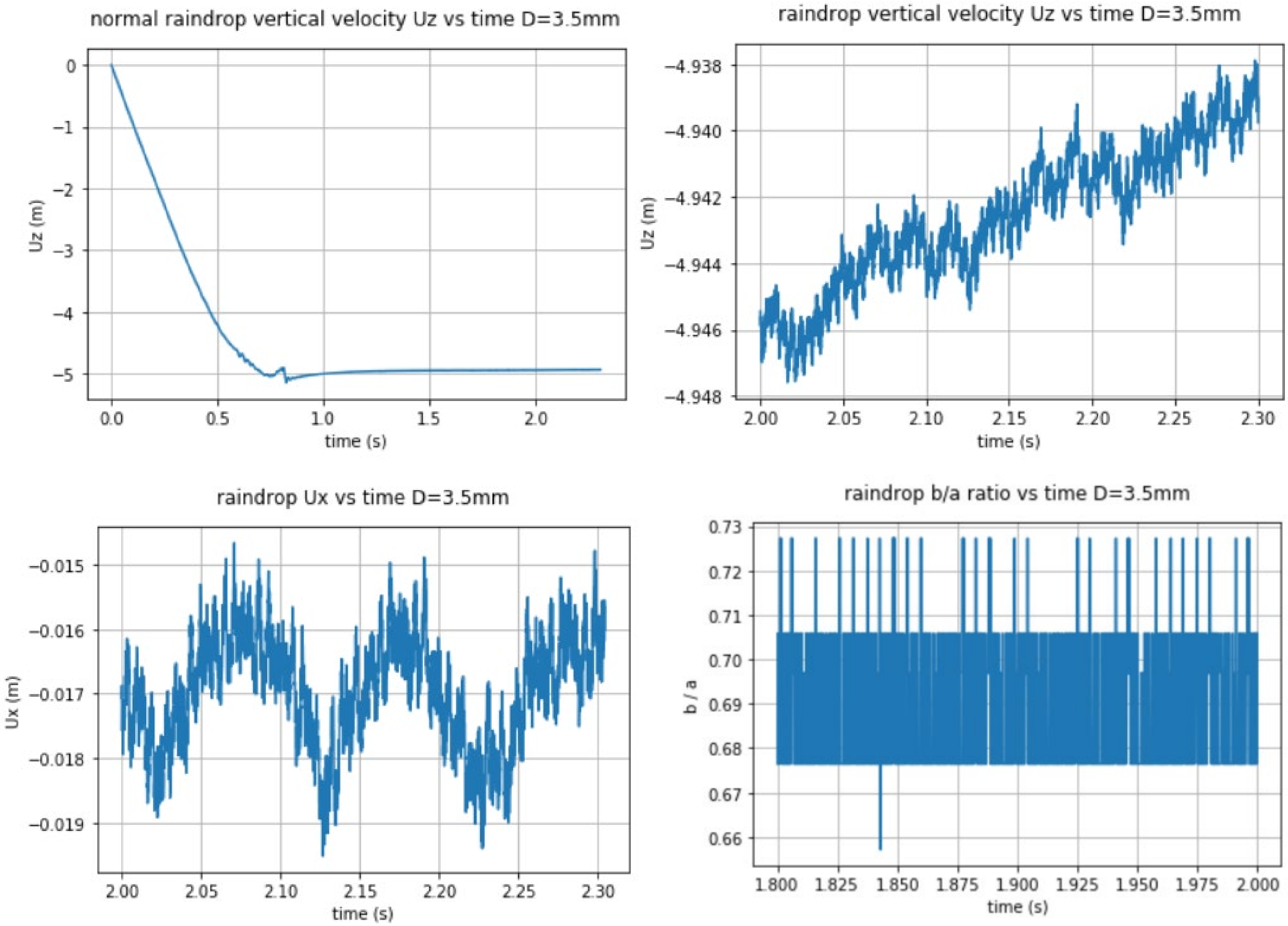


Figure D=4mm

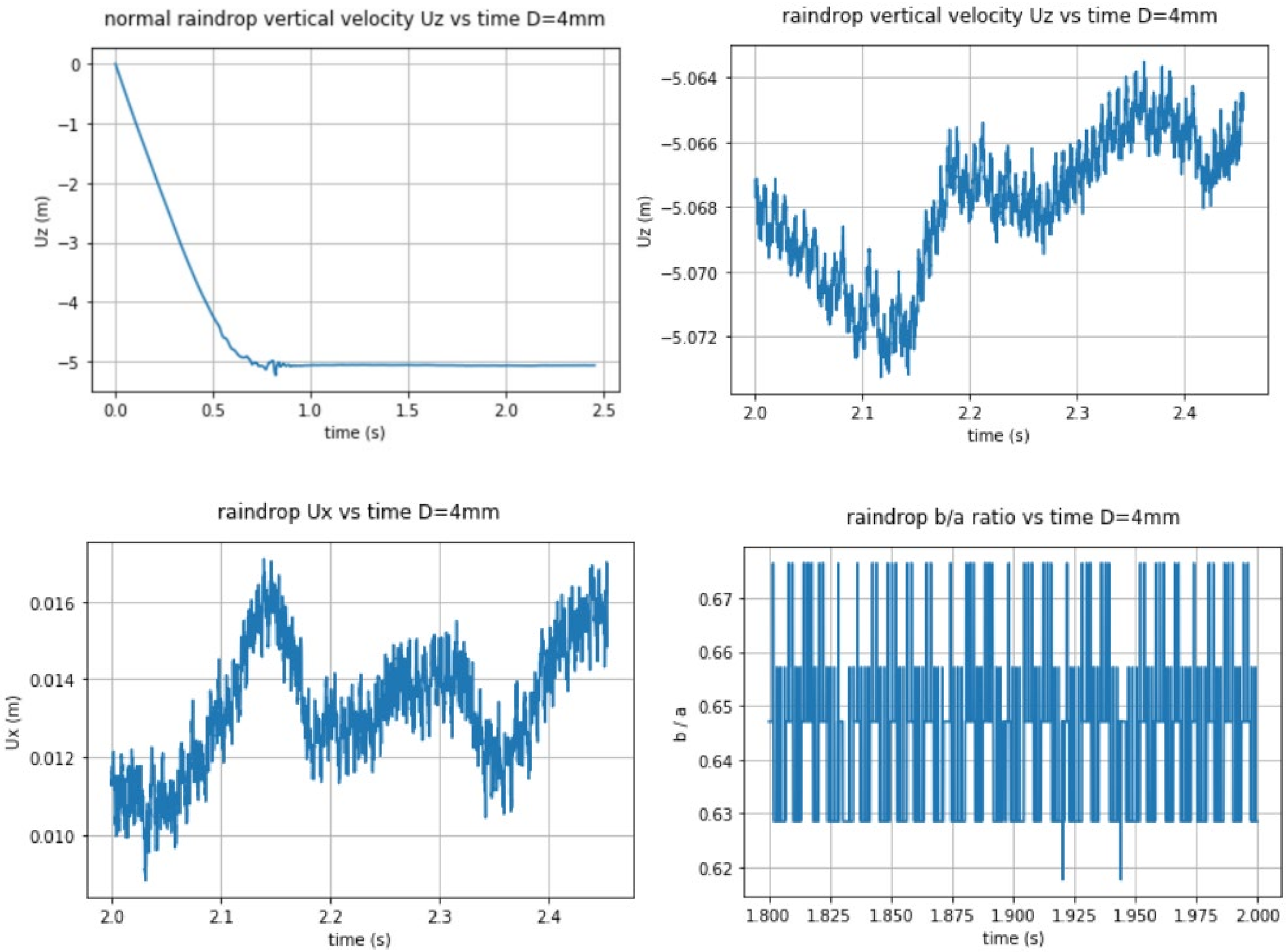
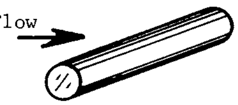
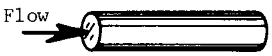
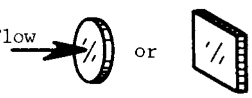
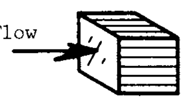
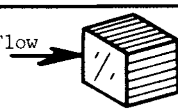
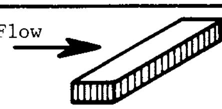
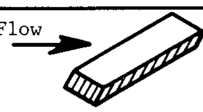
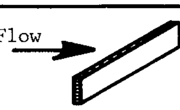

normal raindrop vertical velocity Uz vs time D=4mm
Uz (m)
time (s)
raindrop vertical velocity Uz vs time D=4mm
Uz (m)
time (s)
raindrop Ux vs time D=4mm
Ux (m)
time (s)
raindrop b/a ratio vs time D=4mm
b / a
time (s)

Table 3-2. Drag Coefficients, C_D, of Various Shapes
[Source: Hoerner (1958)]

SHAPE	SKETCH	C_D
Right Circular Cylinder (long rod), side-on		1.20
Sphere		0.47
Rod, end-on		0.82
Disc, face-on		1.17
Cube, face-on		1.05
Cube, edge-on		0.80
Long Rectangular Member, face-on		2.05
Long Rectangular Member, edge-on		1.55
Narrow Strip, face-on		1.98