BIBLIOGRAPHY

General

Cohn, P.M., Algebra, Volumes 1 and 2, John Wiley and Sons, New York, 1989Dummit, D.S. and Foote, R.M., Abstract Algebra, Prentice-Hall, Upper Saddle River,NJ, 1999

Hungerford, T.M., Algebra, Springer-Verlag, New York, 1974

Isaacs, I.M., Algebra, a Graduate Course, Brooks-Cole, a division of Wadsworth, Inc., Pacific Grove, CA, 1994

Jacobson, N., Basic Algebra I and II, W.H. Freeman and Company, San Francisco, 1980 Lang, S., Algebra, Addison-Wesley, Reading, MA, 1993

Modules

Adkins, W.A., and Weintraub, S.H., Algebra, An Approach via Module Theory, Springer-Verlag, New York, 1992 Blyth, T.S., Module Theory, Oxford University Press, Oxford, 1990

Basic Group Theory

Alperin, J.L., and Bell, R.B., *Groups and Representations*, Springer-Verlag, New York, 1995

Humphreys, J.F., A Course in Group Theory, Oxford University Press, Oxford 1996 Robinson, D.S., A Course in the Theory of Groups, Springer-Verlag, New York, 1993 Rose, J.S., A Course on Group Theory, Dover, New York, 1994 Rotman, J.J., An Introduction to the Theory of Groups, Springer-Verlag, New York, 1998

Fields and Galois Theory

Adamson, I.T., Introduction to Field Theory, Cambridge University Press, Cambridge, 1982

Garling, D.J.H., A Course in Galois Theory, Cambridge University Press, Cambridge, 1986

Morandi, P., Fields and Galois Theory, Springer-Verlag, New York, 1996

Roman, S., Field Theory, Springer-Verlag, New York, 1995

Rotman, J.J., Galois Theory, Springer-Verlag, New York, 1998

Algebraic Number Theory

Borevich, Z.I., and Shafarevich, I.R., *Number Theory*, Academic Press, San Diego, 1966 Fröhlich, A., and Taylor, M.J., *Algebraic Number Theory*, Cambridge University Press, Cambridge, 1991

Gouvea, F.Q., p-adic Numbers, Springer-Verlag, New York, 1997

Janusz, G.J., Algebraic Number Fields, American Mathematical Society, Providence, 1996

Lang, S., Algebraic Number Theory, Springer-Verlag, New York, 1994

Marcus, D.A., Number Fields, Springer-Verlag, New York, 1977

Samuel, P., Algebraic Theory of Numbers, Hermann, Paris, 1970

Algebraic Geometry and Commutative Algebra

Atiyah, M.F., and Macdonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, Reading, MA, 1969

Bump, D., Algebraic Geometry, World Scientific, Singapore, 1998

- Cox, D., Little, J., and O'Shea, D., Ideals, Varieties, and Algorithms, Springer-Verlag, New York, 1992
- Eisenbud, D., Commutative Algebra with a View Toward Algebraic Geometry, Springer-Verlag, New York, 1995
- Fulton, W., Algebraic Curves, W.A. Benjamin, New York, 1969
- Hartshorne, R., Algebraic Geometry, Springer-Verlag, New York, 1977
- Kunz, E., Introduction to Commutative Algebra and Algebraic Geometry, Birkhäuser, Boston, 1985
- Matsumura, H., Commutative Ring Theory, Cambridge University Press, Cambridge, 1986
- Reid, M., *Undergraduate Algebraic Geometry*, Cambridge University Press, Cambridge, 1988
- Reid, M., *Undergraduate Commutative Algebra*, Cambridge University Press, Cambridge, 1995
- Shafarevich, I.R., Basic Algebraic Geometry, Volumes 1 and 2, Springer-Verlag, New York 1988
- Ueno, K., Algebraic Geometry 1, American Mathematical Society, Providence 1999

Noncommutative Rings

- Anderson, F.W., and Fuller, K.R., Rings and Categories of Modules, Springer-Verlag, New York, 1992
- Beachy, J.A., Introductory Lectures on Rings and Modules, Cambridge University Press, Cambridge, 1999
- Farb, B., and Dennis, R.K., Noncommutative Algebra, Springer-Verlag, New York, 1993
 Herstein, I.N., Noncommutative Rings, Mathematical Association of America,
 Washington, D.C., 1968
- Lam, T.Y., A First Course in Noncommutative Rings, Springer-Verlag, New York, 1991

Group Representation Theory

- Curtis, C.W., and Reiner, I., Methods of Representation Theory, John Wiley and Sons, New York, 1981
- Curtis, C.M., and Reiner, I., Representation Theory of Finite Groups and Associative Algebras, John Wiley and Sons, New York, 1966
- Dornhoff, L., Group Representation Theory, Marcel Dekker, New York, 1971
- James, G., and Liebeck, M., Representations and Characters of Groups, Cambridge University Press, Cambridge, 1993

Homological Algebra

- Hilton, P.J., and Stammbach, U., A Course in Homological Algebra, Springer-Verlag, New York, 1970
- Hilton, P., and Wu, Y-C., A Course in Modern Algebra, John Wiley and Sons, New York, 1974
- Mac Lane, S., Categories for the Working Mathematician, Springer-Verlag, New York, 1971
- Rotman, J.J., An Introduction to Algebraic Topology, Springer-Verlag, New York, 1988
- Rotman, J.J., An Introduction to Homological Algebra, Springer-Verlag, New York, 1979
- Weibel, C.A., An Introduction to Homological Algebra, Cambridge University Press, Cambridge, 1994

List of Symbols

Throughout the text, \subseteq means subset, \subset means proper subset

	· · · · · · · · · · · · · · · · · · ·	
\mathbb{Z}_n	integers modulo n	1.1
\mathbb{Z}	integers	1.1
$\langle A \rangle$	subgroup generated by A	1.1
S_n	symmetric group	1.2
A_n	alternating group	1.2
D_{2n}	dihedral group	1.2
φ	Euler phi function	1.1, 1.3
⊴	normal subgroup	1.3
◁	proper normal subgrouad	1.3
ker	kernel	1.3, 2.2
\cong	isomorphism	1.4
Z(G)	center of a group	1.4
$H \times K$	direct product	1.5
\mathbb{Q}	rationals	2.1
$M_n(R)$	matrix ring	2.1
R[X]	polynomial ring	2.1
R[[X]]	formal power series ring	2.
End	endomorphism ring	2.1
$\langle X \rangle$	ideal generated by X	2.2
ÙFD	unique factorization domain	2.6
PID	principal ideal domain	2.6
ED	Euclidean domain	2.7
$\min(\alpha, F)$	minimal polynomial	3.1
$\bigvee_i K_i$	composite of fields	3.1
$\operatorname{Gal}(E/F)$	Galois group	3.5
0	the module $\{0\}$ and the ideal $\{0\}$	4.1
$\bigoplus_i M_i$	direct sum of modules	4.3
$\sum_{i}^{n} M_{i}$	sum of modules	4.3
$\operatorname{Hom}_R(M,N)$	set of R -module homomorphisms from M to N	4.4
$\operatorname{End}_R(M)$	endomorphism ring	4.4
$g \bullet x$	group action	5.1
G'	commutator subgroup	5.7
$G^{(i)}$	derived subgroups	5.7
$\langle S \mid K \rangle$	presentation of a group	5.8
$\mathcal{F}(H)$	fixed field	6.1
$\mathcal{G}(K)$	fixing group	6.1
$GF(p^n)$	finite field with p^n elements	6.4
$\Psi_n(X)$	n^{th} cyclotomic polynomial	6.5
Δ	product of differences of roots	6.6
D	discriminant	6.6, 7.4
N[E/F]	norm	7.3
T[E/F]	trace	7.3
char	characteristic polynomial	7.3
$n_P(I)$	exponent of P in the factorization of I	7.7
v_p	p-adic valuation	7.9
$\begin{vmatrix} p \\ p \end{vmatrix}$	p-adic absolute value	7.9
V(S)	variety in affine space	8.1
I(X)	ideal of a set of points	8.1
	polynomial ring in n variables over the field k	8.1
$k[X_1, \dots, X_n]$ \sqrt{I}	radical of an ideal	8.3
•		8.4
$k(X_1,\ldots,X_n)$	rational function field over k	0.4

$S^{-1}R$	localization of the ring R by S	8.5
$S^{-1}M$	localization of the module M by S	8.5
$\mathcal{N}(R)$	nilradical of the ring R	8.6
$M\otimes_R N$	tensor product of modules	8.7
UMP	universal mapping property	8.7
$A\otimes B$	tensor (Kronecker) product of matrices	8.7
kG	group algebra	9.5
RG	group ring	9.5
J(M), J(R)	Jacobson radical	9.7
$\operatorname{Hom}_R(M,-), \operatorname{Hom}_R(-,N)$	hom functors	10.3
$M \otimes_R -, - \otimes_R N$	tensor fuctors	10.3
\mathbb{Q}/\mathbb{Z}	additive group of rationals mod 1	10.6
	(also 1.1, P	roblem 7)
$\mathbb{Z}(p^{\infty})$	quasicyclic group	A10
G[n]	elements of G annihilated by n	A10
H_n	homology functor	S1
$f \simeq g$	chain homotopy	S1
∂	connecting homomorphism	S2, S3
$P_* o M$	projective resolution	S4
$M o E_*$	injective resolution	S4
$L_n F$	left derived functor	S5
R^nF	right derived functor	S5
Tor	derived functor of \otimes	S5
Ext	derived functor of Hom	S5

INDEX

INDEX	
abelian category 10.4	character 6.1
abelian group 1.1	characteristic of a ring or field 2.1
absolute value 7.9	characteristic polynomial 7.3
action of a group on a set 5.1	characteristic subgroup 5.7
adjoint associativity 10.7	chief series 5.6
adjoint functors 10.7	Chinese remainder theorem 2.3
affine n-space 8.1	class equation 5.2
affine variety 8.1	cokernel 10.1
AKLB setup 7.3	colorings 5.3
algebra 4.1	commutative diagram 1.4
algebraic closure 3.3, 10.9	commutative ring 2.1
algebraic curve 8.3	commutator 5.7
algebraic element 3.1	compatible morphisms 10.9
algebraic extension 3.1	complete ring of fractions 2.8
algebraic function field 6.9	composite of fields 3.1, 6.2
algebraic geometry 8.1ff	composition factors 5.6, 7.5
algebraic integers 7.1	composition length 5.6, 7.5
algebraic number 3.3, 7.1	composition of morphisms 10.1
algebraic number theory 7.1ff, 7.3	composition series 5.6, 7.5
algebraically closed field 3.3	conjugate elements 5.1, 5.2
algebraically independent set 6.9	conjugate subfields 6.2
algebraically spanning set 6.9	conjugate subgroups 5.1, 6.2
alternating group 1.2	conjugates of a field ement 3.5
annihilator 4.2, 9.2, 9.7	conjugation 5.1, 5.2-1
archimedian absolute value 7.9	connecting homomorphism S2, S3
Artin-Schreier theorem 6.7	constructible numbers and points 6.8
Artinian modules 7.5	content 2.9
Artinian rings 7.5	contravariant functor 10.3
ascending chain condition (acc) 2.6, 7.5	coproduct 10.2
associates 2.6	core 5.1
associative law 1.1, 2.1	correspondence theorem for
automorphism 1.3	groups 1.4
D 1 10 4	correspondence theorem for
Baer's criterion 10.6	modules 4.2
base change 10.8	correspondence theorem for
basis 4.3	rings 2.3
bilinear mapping 8.7	coset 1.3
binomial expansion modulo p 3.4	counting two ways 5.3
binomial theorem 2.1	covariant functor 10.3
boundary S1	cycle 1.2, S1
canonical map 1.3	cyclic extension 6.7
category 10.1	cyclic group 1.1
Cauchy's theorem 5.4	cyclic module 4.2, 9.1, 9.2, 9.7
Cayley's theorem 5.1	cyclotomic extension.5
center of a group 1.4	cyclotomic field 6.5, 7.2
center of a ring 4.1	cyclotomic polynomial 6.5
central series 5.7	decomposable module 9.6
centralizer 5.2	Dedekind domain 7.6, 7.7
chain complex S1	Dedekind's lemma 6.1, 6.7, 7.3, 7.4
chain homotopy S1	degree 2.5
chain map S1	deleted projective (or injective)
chain rule 1.3, 3.1	resolution S4
*	

derivative of a polynomial 3.4 faithful action 5.1 derived functors S5 faithful module 7.1, 9.2, 9.4 derived length 5.7 faithful representation 9.5 derived series 5.7 Fermat primes 6.8 descending chain condition 7.5 Fermat's little theorem 1.3 diagram chasing 4.7 field 2.1 differential S1 field discriminant 7.4 dihedral group 1.2, 5.8, 5.8 finite abelian groups 4.6 (infinite dilhedral group), 9.5 finite extension 3.1 direct limit 10.9 finite fields 6.4 direct product of groups 1.5 finitely cogenerated module 7.5 direct product of modules 4.3 finitely generated algebra 10.8 direct product of rings 2.3 finitely generated module 4.4 direct sum of modules 4.3 finitely generated submodule 7.5 direct system 10.9 five lemma 4.7 directed set 10.9 fixed field 6.1 discriminant 6.6, A6, 7.4 fixing group 6.1 divides means contains 2.6, 7.7 flat modules 10.8 divisible abelian group A10 forgetful functor 10.3 divisible module 10.6 formal power series 2.1, 8.2 division ring 2.1, 9.1 four group 1.2, 1.5, A6 double centralizer 9.2 four lemma 4.7 double dual functor 10.3 fractional ideal 7.6 dual basis 7.4 Frattini argument 5.8-2 duality 10.1 free abelian gup functor 10.3 duplicating the cube 6.8 free group 5.8-1 free module 4.3,15 Eisenstein's irreducibility criterion 2.9 free product 10.2-2 elementary divisors 4.6 Frobenius automorphism 3.4, 6.4 elementary symmetric functions 6.1 full functor 10.3 embedding 3.3, 3.5 full ring of fractions 2.8 embedding in an injective module 10.7 full subcategory 10.3 endomorphism 1.3, 4.4 functor 10.3 epic 10.1 fundamental decomposition theorem epimorphism 1.3 (for finitely generated modules

embedding 3.3, 3.5
embedding in an injective module 1
endomorphism 1.3, 4.4
epic 10.1
epimorphism 1.3
equivalent absolute values 7.9
equivalent matrices 4.4
equivalent matrix
representations 9.5
Euclidean domain 2.7
Euler's identity 2.1
Euler's theorem 1.3
evaluation map 2.1
exact functor 8.5, 10.4
exact sequence 4.7
exponent of a group 1.1, 6.4
Ext S5
extension of a field 3.1
extension of scalars 8.7, 10.8

F-isomorphism, etc. 3.2 factor theorem for groups 1.4 factor theorem for modules 4.2 factor theorem for rings 2.3

exterior algebra 8.8

Galois extension 3.5, 6.1ff.
Galois group 3.5, 6.1ff
Galois group of a cubic, 6.6
Galois group of a polynomial 6.3
Galois group of a quadratic 6.3
Galois group of a quartic A6
Gauss' lemma 2.9
Gaussian integers 2.1, 2.7
general equation of degree n 6.8
general linear group 1.3
generating set 4.3
generators and relations 1.2, 4.6, 5.8
greatest common divisor 2.6, 7.7
group 1.1
group algebra 9.5

fundamental theorem of Galois theory

over a PID) 4.6

Jacobson radical 9.7 group representations 9.5 group ring 9.5 Jacobson's theorem 9.2 Jordan-Holder theorem 5.6, 7.5 Hermite normal form 4.5 Hilbert basis theorem 8.2 kernel 1.3, 2.2, 10.1 Hilbert's Nullstellensatz 8.3, 8.4 kernel of an action 5.1 Hilbert's Theorem 90 7.3 Kronecker product of matrices 8.7 hom functors 10.3-1Krull-Schmidt theorem 9.6 homology functors S1 Kummer extension 6.7 homology group S1 Lagrange interpolation formula 2.5 homology module S1 Lagrange's theorem 1.3 homomorphism from R to M deter-Laurent series 7.9 mined by what it does to the leading coefficient 2.5 identity, 9.4, S6 least common multiple 2.6, 7.7 homomorphism of algebras 4.1 left adjoint 10.7 homomorphism of groups 1.3 left cancellable 10.1 homomorphism of modules 4.1 left derived functors S5 homomorphism of rings 2.2 left exact functor 10.4 Hopkins-Levitzki theorem 9.8 left ideal 2.2 hypersurface 8.2 left resolution S4 ideal 2.2, 8.1 left-Noetherian ring 9.8 ideal class group 7.8 left-quasiregular element 9.7 idempotent linear transformation 9.5 left-semisimple ring 9.6 image 2.3, 4.1 length of a module 7.5 indecomposable module 9.6 lifting of a map 4.3, 10.2 index 1.3 linearly indepdent set 4.3 inductive limit 10.9 local ring 2.4, 7.9, 8.5 initial object 10.1 localization 2.8, 8.5 injection (inclusion) 4.7 long division 6.4 injective hull 10.7 long exact homology sequence S3 injective modules 10.6 injective resolution S4 Maschke's theorem 9.6 inner automorphism 1.4, 5.7 matrices 2.1, 4.4 integral basis 7.2, 7.4 maximal ideal 2.4, 8.3 integral closure 7.1 maximal submodule 9.7 integral domain 2.1 metric 7.9 integral extensions 7.1 minimal generating set 9.8 integral ideal 7.6 minimal left ideal 9.3 integrally closed 7.1 minimal polynomial 3.1 invariant factors 4.5 minimal prime ideal 8.4 inverse limit 10.9 modding out 5.7 inverse system 10.9 modular law 4.1 inversions 1.2 module 4.1 irreducible element 2.6 modules over a principal ideal domain irreducible ideal 8.6 4.6, 10.5 irreducible polynomial 2.9 monic 10.1 irreducible variety 8.1 monoid 1.1 isomorphic groups 1.1 monomorphism 1.3 isomorphism 1.3 morphism 10.1 isomorphism extension theorem 3.2 isomorphism theorems for groups 1.4 Nakayama's lemma 9.8 isomorphism theorems for modules 4.2 natural action 5.3, 6.3

natural map 1.3

isomorphism theorems for rings 2.3

natural projection 4.7, 7.5, 9.2, 9.4, 9.5, 10.5 natural transformation 10.3 naturality S3 Newton's identities A6 nil ideal 9.7 nilpotent element 8.6, 9.7 nilpotent group 5.7 nilpotent ideal 9.7 nilradical 8.6 Noetherian modules 4.6, 7.5 Noetherian rings 4.6, 7.5 nonarchimedian absolute value 7.9 noncommuting indeterminates 9.8 nontrivial ideal 2.2	principal ideal domain 2.6 product 10.2 product of an ideal and a module 9.3 product of ideals 2.3, 8.5, 7.6 projection 4.7, 9.2, 9.4, 9.5, 10.5 projection functor 10.3 projective basis lemma 10.5 projective limit 10.9 projective modules 9.8, 10.5 projective resolution S4 proper ideal 2.2 Prufer group A10 pullback 10.6 purely inseparable 3.4 pushout 10.6,3
norm, 7.1, 7.3 normal closure, 3.5	quadratic extensions 6.3, 7.2
normal closure, 3.5 normal extension 3.5	quasicyclic group A10
normal series 5.6	quasi-regular element 9.7
normal Sylow p-subgroup 5.5	quaternion group 2.1
normalizer 5.2	quaternions 2.1
Nullstellensatz 8.3, 8.4	quotient field 2.8
number field 7.1	quotient group 1.3
objects 10.1	quotient ring 2.2
objects 10.1 opposite category 10.1	R-homomorphism on R 9.4
opposite ring 4.4	Rabinowitsch trick 8.4
orbit 5.2	radical extension 6.8
orbit-counting theorem 5.3	radical of an ideal 8.3
orbit-stabilizer theorem 5.2	rank of a free module 4.4
order 1.1	rational integer 7.2
order ideal 4.2	rational root test 2.9
orthogonal idempotents 9.6	rationals mod 1 1.1 (Problem 7), 10.4, 10.6, A10
Ostrowski's theorem 7.9	refinement 5.6
p-adic absolute value 7.9	regular action 5.1, 5.2
p-adic integers 7.9	regular n-gon 6.8
p-adic numbers 7.9	regular representation 9.5
p-adic valuation 7.9	relatively prime ideals 2.3
p-group 5.4	remainder theorem 2.5
perfect field 3.4	representation 9.5
permutation 1.2	residue field 9.8
permutation group 1.2	resolvent cubic A6
permutation module 9.5	restriction of scalars 8.7, 10.8
polynomial rings 2.1, 2.5	right adjoint 10.7
polynomials over a field 3.1	right cancellable 10.1
power sums, A6	right derived functors S5
preordered set 10.2 primary component A10	right exact functor 10.4 right ideal 2.2
primary decomposition 8.6	right resolution S4
primary ideal 8.6	right-Noetherian ring 9.8
prime element 2.6hbn	right-quasiregular element 9.7
prime ideal 2.4	right-semisimple ring 9.6
primitive element, theorem of 3.5, 6.6	ring 2.1
primitive polynomial 2.9	ring of fractions 2.8, 8.5

Schreier refinement theorem 5.6 Schur's lemma 9.2 semidirect product 5.8 semigroup 1.1 semisimple module 9.1 semisimple ring 9.3 separable element 3.4 separable extension 3.4 separable polynomial 3.4 series for a module 7.5 simple group 5.1, 5.5 simple left ideal 9.3 simple module 7.5, 9.1, 9.2 simple ring 9.3, 9.5 simplicity of the alternating group 5.6 simultaneous basis theorem 4.6 skew field 2.1 Smith normal form 4.5 snake diagram S2 snake lemma S2 solvability by radicals 6.8 solvable group 5.7 spanning set 4.3, 6.9 special linear group 1.3 split exact sequence 4.7, 5.8 splitting field 3.2 squaring the circle 6.8 standard representation of a p-adic integer 7.9 Steinitz exchange 6.9 Stickelberger's theorem 7.4 subcategory 10.3 subgroup 1.1 submodule 4.1 subnormal series 5.6 subring 2.1 sum of ideals 2.2 sum of modules 4.3 Sylow p-subgroup 5.4 Sylow theorems 5.4 symmetric group 1.2, 6.6 symmetric polynomial 6.1 tensor functors 10.3 tensor product of matrices 8.7 tensor product of module homomorphisms 8.7 tensor product of modules 8.7 terminal object 10.1 Tor S5 torsion abelian group 8.7 (Problem 3), torsion element 4.6

torsion module 4.6

torsion subgroup A10 torsion submodule 4.6 torsion-free module 4.6 trace 7.3 transcendence basis 6.9 transcendence degree 6.9 transcendental element 3.1 transcendental extension 6.9 transcendental number 3.3 transitive action 5.2 transitive subgroup of S_n 6.3 transitivity of algebraic extensions 3.3 transitivity of separable extensions 3.4 transitivity of trace and norm 7.3 transposition 1.2 trisecting the angle 6.8 trivial absolute value 7.9 trivial action 5.1, 5.2 twisted cubic 8.3 two-sided ideal 2.2

ultrametric inequality 7.9
underlying functor 10.3
unimodular matrix 7.4
unique factorization domain 2.6
unique factorization of ideals 7.7
unit 2.1
universal mapping property (UMP)
8.7, 10.2
upper central series 5.7

valuation 7.9 valuation ideal 7.9 valuation ring 7.9 Vandermonde determinant A6, 7.4 vector space as an F[X]-module 4.1 Von Dyck's theorem 5.8

weak Nullstellensatz 8.3 Wedderburn structure theorem 9.5 Wedderburn-Artin theorem 9.4

Zariski topology 8.1 Zassenhaus lemma 5.6 zero object 10.1