Assignment2

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Assignment +2_Updated

MIE376 Assignment 2 Winter 2024

1. Solve the following problem in two ways, first **by hand**, and second by writing **Python code using revised simplex** method.

$$\text{Min } -3x_1 + 8x_2$$
$$s.t.\ \ 4x_1 + 2x_2 \le 12$$
$$2x_1 + 3x_2 \le 6$$
$$all\ x_i \ge 0$$

2. Solve the following problem in two ways, first by writing **Python code using revised simplex** method, and second by **writing python code using Gurobi**.

Maximize:

$$3x_1 + 2x_2 - x_3 - 2x_4 + x_5 + 2x_6 - x_7 + 3x_8 + 4x_9 - 3x_{10}$$

Constraints:

$$2x_1 + x_2 + 3x_3 + x_4 + 2x_5 + x_6 + 4x_7 + x_8 - 2x_9 + 3x_{10} \le 80$$
$$x_1 - 4x_2 + x_3 + 2x_4 + 3x_5 + x_6 - x_7 + 4x_8 + x_9 + 2x_{10} \le 50$$
$$3x_1 + 2x_2 - 2x_3 - x_4 + x_5 + 3x_6 + 2x_7 + x_8 + x_9 + x_{10} \le 40$$
$$2x_1 + 3x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + 3x_9 + 2x_{10} \le 90$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \le 50$$

$b = \begin{bmatrix} 80 \\ 50 \\ 40 \\ 90 \\ 50 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 1 & 3 & 1 & 2 & 1 & 4 & 1 & -2 & 3 & 1 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 2 & 3 & 1 & -1 & 4 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 3 & 2 & -2 & -1 & 1 & 3 & 2 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$C = [3\ 2\ -1\ -2\ 1\ 2\ -1\ 3\ 4\ -3\ 0\ 0\ 0\ 0\ 0]$

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1) $\text{Min } -3x_1 + 8x_2$ — Basic

s.t. $4x_1 + 2x_2 + x_3 = 12 \rightarrow x_3 = 12 - 4x_1 - 2x_2$

$2x_1 + 3x_2 + x_4 = 6 \rightarrow x_4 = 6 - 3x_2 - 2x_1$

$x_i \ge 0\ \forall i$

$B = \{x_3, x_4\}$ $N = \{x_1, x_2\}$

→ set $x_1, x_2 = 0$: $B = \{x_3 = 12, x_4 = 6\}$

→ pick variable to enter basic set based off objective z

$z = 0 - 3x_1 + 8x_2$ → x_1 enters

→ pick variable to depart basic set

$x_3 = 12 - 4x_1 \rightarrow x_1 = 3$ → x_3 departs (smaller index)

$x_4 = 6 - 2x_1 \rightarrow x_1 = 3$

$B = \{x_1, x_4\}$ $N = \{x_3, x_2\}$

→ determining pivot: $x_3 = 12 - 4x_1 - 2x_2$

$x_4 = 6 - 2x_1 - 3x_2$

→ div by 2:

$\frac{x_3}{4} = 3 - x_1 - \frac{x_2}{2}$

$x_1 = 3 - \frac{x_2}{2} - \frac{x_3}{4}$

→ plug x_1 into x_4: $x_4 = 6 - 2(3 - \frac{x_2}{2} - \frac{x_3}{4}) - 3x_2 = -2x_2 + \frac{x_3}{2}$

$z = -3(3 - \frac{x_2}{2} - \frac{x_3}{4}) + 8x_2$ → set $x_2, x_3 = 0$

$= -9 + \frac{19}{2}x_2 + \frac{3}{4}x_3$

coefficients ⊕, cannot min. more

→ ∴ $z = -9$. The minimum objective value is -9.

$x_3\ x_4$ $x_1\ x_2$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $N = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$, $a = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Iter #1:

$C_B^T B^{-1} N - C_N^T = [0\ 0] B^{-1} N - [-3\ 8] = [3\ -8]$

$B = \begin{bmatrix} 4 & 0 \\ 2 & 1 \end{bmatrix}$ $N = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 8 \\ -3 \\ 0 \end{bmatrix}$ → $c_B = [-3\ 0]$, $c_N = [0\ 8]$

$x_B = B^{-1} b = \begin{bmatrix} 0.25 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

Iter #2:

$C_B^T B^{-1} N - C_N^T = [-3\ 0] \begin{bmatrix} 0.25 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - [0\ 8]$

$= [-\frac{3}{4}\ \ 0] \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - [0\ \ 8]$

$= [-\frac{3}{4}\ \ -\frac{3}{2}] - [0\ \ 8]$

$= [-\frac{3}{4}\ \ -\frac{19}{2}]$