MIE377: Financial Optimization Models

Project 1 (Winter 2023)

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Results from testing out multiple mean-variance optimization models in combination with a ordinary least squares method with a focus on sparse factor models were compiled throughout this project. Based on the resulting Sharpe ratios, the Ridge Regression with the Sharpe Ratio maximization model was chosen to be the most optimal and usable combination. With the provided dataset, the resulting Sharpe ratio is 0.20756 and the average turnover rate is 0.43819.

1 Introduction

The purpose of this project is to design an algorithmic trading system that maximizes the Sharpe ratio and minimizes average turnover. Leveraging concepts seen throughout the course, our team decided to edit the existing code to achieve 2 main goals: prioritizing the most relevant factors and optimizing the Sharpe ratio. Using sparse factor models and different portfolio optimization methods, we were able to compare and contrast different models to determine the model that has the best and most consistent performance under different market conditions.

2 Methodology

2.1 Changing the initial data range

In the code given, the calibration period for parameter estimation is three years long. We decided to change this to use the full five year period allowing us to have more data for parameter estimation. From course content, we learned that more data does not improve the estimate of the mean but it does improve the estimate of the variance which will improve our Q that will be used in our optimization model. With a better estimation of Q, we will be able to effectively maximize the Sharpe ratio and minimize the average turnover ratio.

2.2 Choosing OLS, LASSO, or Ridge in OLS.m

In order to create a model that would maximize Sharpe ratio and minimize average turnover for different data sets, we decided to explore other factor models. From course content, we learned that sparse factors models decide how many factors should be included to make sure the optimization model is not over fitted. Promoting sparsity means we consider all factors, but optimization selects the best-fitting subset. By finding the most significant factor coefficients, we can re-calculate our μ and Q values which can then be used in our optimization file. A sparse model uses less factors which also makes it easier to interpret.

2.2.1 Ordinary Least Squares

In the given OLS.m file, we are conducting Ordinary Least Square to estimate the coefficients of our factor model. The formula used to find the coefficients using OLS is shown below.

$$\mathbf{B} = (X^T X)^{-1} X^T Y \tag{1}$$

With OLS, we create our factor model using all 8 factors, which means the model that is being constructed may be too closely tailored to the dataset given, or being overfit. This can lead to inaccurate predictions, which cannot perform well for all new types of data.

2.2.2 Selecting Sparse Factor Models

There are 3 different types of sparse factor models to improve our factor model coefficients: Best Subset Selection (BSS), LASSO, and Ridge Regression which are shown below:

$$\min_{B_i} \ ||(r_i - XB_i)||_2^2 + \lambda||B_i||_0 \tag{2}$$

$$\min_{B_i} \ ||(r_i - XB_i)||_2^2 + \lambda||B_i||_1 \tag{3}$$

$$\min_{B_i} \ ||(r_i - XB_i)||_2^2 + \lambda||B_i||_2 \tag{4}$$

The penalized regression model of BSS utilizes the l_0 norm which is not convex therefore BSS is not convex. The l_0 norm requires multiple indicator functions in our optimization model and it is difficult to address computationally which is why we decided to not implement BSS in OLS.m.
The penalized LASSO model utilizes the l_1 norm which is continuous and convex but not smooth and the penalized Ridge Regression model utilizes the l_2 norm which is continuous, convex, and smooth. Ridge Regression does not eliminate any of the factors and instead it determines the coefficients based on the significance of a factor. LASSO does shrink the parameters and conducts factor selection automatically which is why we decided to implement LASSO and Ridge Regression and compare their results with each optimization model to determine the model with the best performance.
Note in equations (3) and (4), there is a penalty term λ which will be addressed in section 2.4 and how we will properly select this penalty term.

2.3 Choosing Optimization Model in MVO.m

Given the access to factor returns for each of the models, the need to maximize Sharpe Ratio, and minimize the turnover rate, the following optimization models were considered: Sharpe Ratio Minimization, Baseline MVO, Lambda-Based Risk-Adjusted Return MVO, and Quantile-Based MVO. We ran varied mean-variance optimization models to find the optimal weightings of the assets depending on the objective function and relevant constraints.

Through lecture content, we were able to understand the major differences between the objective functions of each of these models to see how the minimization affects the resulting Sharpe ratio.

2.3.1 Mean Variance Optimization

$$\min_{x} \ x^T Q x$$

The baseline mean variance optimization aided in showcasing how optimizing with a objective function focused on variance minimization still provides an adequate Sharpe ratio. Additionally, this is the standard convex mean-variance optimization problem with a weight constraint and target return constraint.

2.3.2 MVO with Adjusted Target Return

Using the baseline MVO, the target return constraint is modified to ensure that the portfolio's return exceeds the mean of a defined quantile of the asset returns. Once the asset returns are sorted, the quantile parameter is defined as a percentage between 0% and 100%: Then, the mean of the sorted asset returns from the smallest to the specific percentile of asset returns is used for the return constraint.

$$\begin{aligned} \min_{x} \ & x^T Q x \\ \text{s.t. } & \mu^T x \geq \mathit{Return} \end{aligned}$$

With this in mind, this provides a more nuanced look at asset returns to see if lowering the necessary portfolio return can improve the Sharpe ratio.

2.3.3 Risk Adjusted Return MVO

$$\min_{x} \ \lambda x^T Q x - \mu^T x$$

When the lambda value is larger, the risk-adjusted return MVO focuses on minimizing the variance: with the larger λ, the objective function attempts to minimize the variance to lower the impact of the larger λ. With a small λ value, the formula shifts to maximizing returns because the most negative element of the function would be the mean return component. This risk-adjusted return MVO was considered due to its ability to balance/reconcile mean return maximization and mean variance minimization depending on which one is of more impact on the resulting Sharpe ratio.

2.3.4 Sharpe Ratio Optimization

The naive, non-convex version of Sharpe Ratio optimization goes from the following:

$$\begin{aligned} \max_{x} \ & \frac{\mu^{\mathbf{T}}\mathbf{x} - \mathbf{r_f}}{\sqrt{\mathbf{x^T Q x}}} \\ \text{s.t. } & \mathbf{1^T x = 1} \\ & \mathbf{x \geq 0} \end{aligned}$$

to a convex form in which MATLAB's `quadprog()` can solve:

$$\begin{aligned} \min_{y,\kappa} \ & \mathbf{y^T Q y} \\ \text{s.t. } & \mathbf{1^T y} = \kappa \\ & \mu^{\mathbf{T}}\mathbf{y} = \mathbf{1} \\ & \kappa, \mathbf{y} \geq \mathbf{0} \end{aligned}$$

The Sharpe ratio MVO was anticipated to be the most successful MVO due to its focus on specifically optimizing the Sharpe ratio. Since the weighting of assets would be wholly determined on how to best maximize the Sharpe ratio, the Sharpe ratio optimization focuses on turning a non-quadratic problem into a manageable convex optimization problem by redefining the variables.

In terms of Sharpe maximization, we minimize κ and y. We have also defined κ such that $y = \kappa x$.

2.4 K-Fold Cross-Validation

The penalized version of the regression models are being used in order to better adjust for the purpose of the model (see equation 4). As such, a λ penalty term needs to be properly selected to fit both the dataset and the purpose of the model. k-fold cross-validation is utilized for the λ selection by evaluating the score of the model on a specific λ; the cross-validation takes 100 λ values from $\{10^{-6}, 10^{2}\}$. By splitting the dataset into k folds, 1 fold is used as a testing dataset and $k-1$ folds are used as a training dataset. In a loop, each of the k folds will be used as a testing set with the other folds being used as a training set, and the model will be evaluated using the total error for each of the k iterations.

The loss function is used to evaluate the quality of the model by comparing the model using the training dataset versus the testing dataset. Initially, the loss was evaluated by determining the total estimation error of the test return for all k folds across all assets and time periods:

$$\mathcal{L} = \sum_{j=1}^{k}\sum_{t=1}^{T}\sum_{i=1}^{n} \mathbf{r}_{tk}^{(i)} - \mathbf{X}_{tk}\beta^{(i)}$$

whereas the final model implemented a loss function that determined the Sharpe Ratio loss, which would ultimately outrank the previous loss function for Sharpe Ratio maximization.

$$\mathcal{L} = \frac{\prod_{i=1}^{n}(\mu_i + 1) - 1}{\sigma_i}$$

3 Analysis from Training, Validation, and Testing

3.1 Analysis Criteria

The given project handout emphasized two testing criteria: Sharpe Ratio and Average Turnover. However, it was also emphasized that 80% of the "score" for each trial will be placed upon the Sharpe Ratio maximization. Thus, this is one of the main aspects that the model aimed to maximize. Although there is a heavy emphasis on maximizing the Sharpe Ratio, there is a concern of over fitting the model to the provided dataset, which is why a cross-validation step is needed for the penalty term selection. This step allows the model to appropriately adapt to each dataset and dynamically select the parameter based on the losses. In the following section, we will analyze and test every combination of our factor models (OLS, LASSO, and Ridge) with our optimization models (baseline MVO, MVO with Adjusted Target Return, Risk Adjusted Return MVO, and Sharpe Ratio Optimization) to determine the models that have that improve the Sharpe ratio and the average turnover ratio. The best models will then be compared to choose which one has the best and most consistent performance.

3.2 Ridge Regression Training Set

Since factor returns were given as part of the dataset, it was apparent that optimizing the factor coefficients was necessary in order to optimize the model. This led to choosing between two sparse factor models, LASSO Regression ($\ell 1$) and Ridge Regression ($\ell 2$), since the $\ell 0$ model is not convex and cannot be solved using MATLAB. Ultimately, it was found that Ridge Regression is easier to implement through MATLAB, using the `ridge()` function. Additionally, utilizing the unconstrained version of Ridge Regression allows to adjust the penalty term (λ) for different re-balancing periods:

$$\min_{\beta^{(\mathbf{i})}} ||\mathbf{r}^{(i)} + \mathbf{X}\beta^{(i)}|| - \lambda||\beta^{(i)}||$$

where the adjusting of the penalty term will be implemented using k-fold cross-validation.

3.2.1 K-Fold Cross-Validation

In the k-fold cross-validation, there were many parameters that were varied during the testing process. These include the number of folds, $k \in \{5, 6, 10\}$, the number of months of data used for parameter estimation, $\{36, 48, 60\}$, and using different loss functions to evaluate the cross-validation. Comparing the two loss functions:

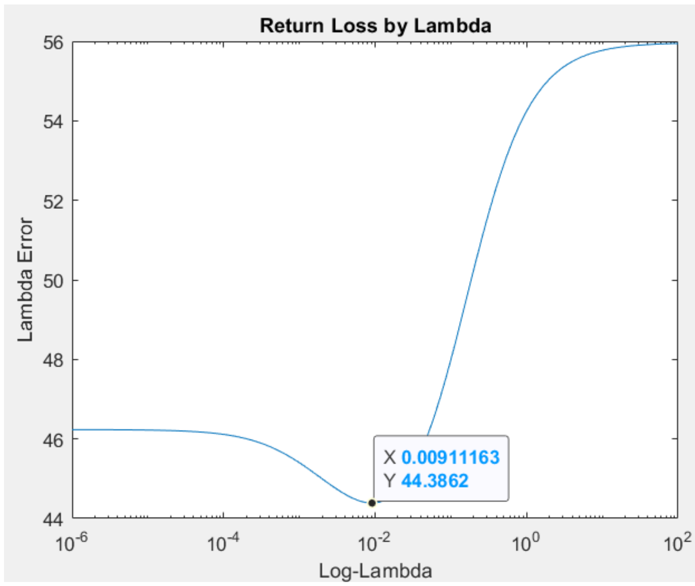


Figure 1: Semilog Plot of Lambdas - Sharpe ratio = 0.2023, average turnover = 0.48557

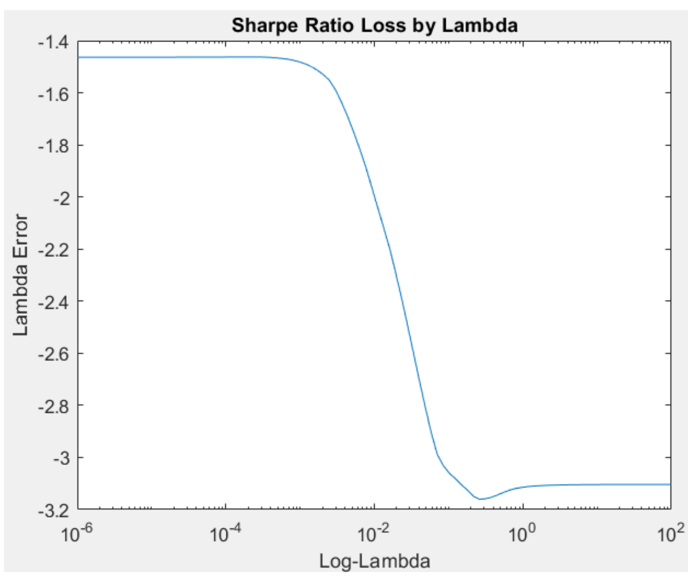


Figure 2: Semilog Plot of Lambdas - Sharpe ratio = 0.20756, average turnover: 0.43819

Next, it was noted that more months of data used in parameter estimation led to an improved Sharpe Ratio. This trend is easily explainable by the fact that despite returning an ex post Sharpe Ratio, there is no sufficient historical data for portfolio returns and variances. Thus, estimations for these parameters must be made; the more accurate the parameter estimation, the better the ex post Sharpe Ratio. The following data will present results of using 5-fold, 6-fold and 10-fold cross-validation against 36, 48 and 60 months of parameter estimation in an ordered pair designating {Sharpe Ratio, Average Turnover}.

	$t = 36$	$t = 48$	$t = 60$
$k = 5$	{0.16998, 0.6221}	{0.13146, 0.51599}	{0.20588, 0.64156}
$k = 6$	{0.16483, 0.62683}	{0.1419, 0.58575}	{0.18689, 0.48391}
$k = 10$	{0.15182, 0.67025}	{0.15553, 0.62967}	$\{0.20756, 0.43819\}$

Table 1: Outcomes when varying number of months used for parameter estimation (t) and number of folds for cross-validation (k); output given as {SR, AVG. TO}.

One parameter that was changed but saw little difference in results was the number of λ's tested, varying from 50 up to 1000. It was noted that the cross-validation would lead to very similar λ's being chosen despite the higher density of points. In addition, the number of folds to split the data into was chosen to ideally be multiples of the number of observations since it would yield an equal number of observations within each subset of data.

3.3 Comparing Different Optimization Models w/ OLS, Ridge, and LASSO

3.3.1 Baseline MVO:

In terms of the Baseline MVO, we used the provided code to come to an understanding of what to beat for Sharpe ratio and average turnover rate. Table 2 has results for each OLS Method for the Baseline MVO, and the baseline MVO is the following minimization: $x^T * Q * x$

OLS Method	MVO Method	Sharpe Ratio	Average Turnover
OLS	Baseline	0.17171	0.48735
Lasso	Baseline	0.13947	0.56553
Ridge	Baseline	0.17217	0.30487

Table 2: Baseline MVO

Our results show that using the LASSO model, the Sharpe ratio has significantly decreased and average turnover rate has clearly increased relative to the other results. However, with the OLS model, the Sharpe ratio decreased while the average turnover has significantly increased. Using the Ridge Regression model, the dharpe ratio has been at its highest. The results of using Ridge Regression with this MVO yield the best results for the Sharpe ratio and average turnover compared to all of the other combinations that have been tested with the baseline MVO.

3.3.2 Adjusting target return for baseline MVO:

We chose a range of quantiles from 5% to 90% to see how the Sharpe ratio and average turnover changed when the quantile of the expected value of μ changed. Table 3 includes the results of adjusting target return for baseline MVO using OLS, LASSO, and Ridge Regression.

Our results show that using the LASSO model, the Sharpe ratio and average turnover improve with a quantile of 5% to 75%. With OLS, the Sharpe Ratio has decreased and the average turnover has increased with the adjusted target return which shows how baseline MVO with OLS is preferable to adjusted target return. With Ridge Regression, the Sharpe ratio and average turnover have slightly decreased showing an improvement with the average turnover ratio for Adjusted Target return and Ridge Regression.

OLS Method	MVO Method	Sharpe Ratio	Average Turnover
Lasso	Quantile 5%	0.18459	0.42364
Lasso	Quantile 10%	0.18459	0.42364
Lasso	Quantile 20%	0.18459	0.42364
Lasso	Quantile 50%	0.18459	0.42363
Lasso	Quantile 75%	0.18064	0.42185
Lasso	Quantile 90%	0.16505	0.46323
OLS	Quantile 5%	0.15815	0.70606
OLS	Quantile 10%	0.15815	0.70606
OLS	Quantile 20%	0.15816	0.70606
OLS	Quantile 50%	0.15815	0.70606
OLS	Quantile 75%	0.15815	0.70606
OLS	Quantile 90%	0.15815	0.70606
Ridge	Quantile 5%	0.16347	0.43876
Ridge	Quantile 10%	0.16347	0.43876
Ridge	Quantile 20%	0.16347	0.43876
Ridge	Quantile 50%	0.16347	0.43876
Ridge	Quantile 75%	0.16347	0.43876
Ridge	Quantile 90%	0.16347	0.43876

Table 3: Adjusted Target Return MVO data

3.3.3 Risk-Adjusted Return MVO:

We had used a range of lambdas for risk-adjusted return from 10^{-3} to 10^{3} for the risk-adjusted return MVO. Then, based on the resulting Sharpe ratios and turnover rates from changing lambda values, we formulated Table 3 to showcase the results. The objective function for this is the minimization of the following mean-variance optimization function:

$$z = \lambda x^T Q x - \mu^T x$$

Based on Table 4, the trend for the original least squares method with a risk-adjusted return is that there is a positive correlation between lambda and the Sharpe ratio while also having a negative correlation between lambda and turnover rate. In terms of the LASSO method and the risk-adjusted return for that combination, for turnover rate, it increases from 0 until 1 but drops to a lower value when lambda is 10 or larger. With the Sharpe ratio and LASSO method, the ratio is larger when closer to 0 or closer to 10^{3} and significantly when in between. For the Ridge, the turnover rate is larger when lambda is closer to 0, and the Sharpe ratio is greatest when 10^{-2} to 1.

OLS Method	MVO Method	Sharpe Ratio	Average Turnover
OLS	Risk-Adjusted Lambda = 10^{-3}	0.029822	0.73684
OLS	Risk-Adjusted Lambda = 10^{-2}	0.026513	0.73442
OLS	Risk-Adjusted Lambda = 10^{-1}	0.096704	0.71141
OLS	Risk-Adjusted Lambda = 1	0.096704	0.71141
OLS	Risk-Adjusted Lambda = 10	0.14708	0.7055
OLS	Risk-Adjusted Lambda = 10^{2}	0.16536	0.48961
OLS	Risk-Adjusted Lambda = 10^{3}	0.17164	0.48728
Lasso	Risk-Adjusted Lambda = 10^{-3}	0.12133	0.94691
Lasso	Risk-Adjusted Lambda = 10^{-2}	0.10547	0.95833
Lasso	Risk-Adjusted Lambda = 10^{-1}	0.080505	1.0371
Lasso	Risk-Adjusted Lambda = 1	0.080505	1.0371
Lasso	Risk-Adjusted Lambda = 10	0.13803	0.5601
Lasso	Risk-Adjusted Lambda = 10^{2}	0.13935	0.56492
Lasso	Risk-Adjusted Lambda = 10^{3}	0.13947	0.5655
Ridge	Risk-Adjusted Lambda = 10^{-3}	0.19314	0.52632
Ridge	Risk-Adjusted Lambda = 10^{-2}	0.19314	0.52632
Ridge	Risk-Adjusted Lambda = 10^{-1}	0.21442	0.38308
Ridge	Risk-Adjusted Lambda = 1	0.21047	0.3539
Ridge	Risk-Adjusted Lambda = 10	0.14217	0.51525
Ridge	Risk-Adjusted Lambda = 10^{2}	0.1799	0.29192
Ridge	Risk-Adjusted Lambda = 10^{3}	0.1719	0.3026

Table 4: Risk-Adjusted MVO

3.3.4 Optimizing Sharpe Ratio MVO:

Table 5 includes the results of altering MVO.m to optimize the Sharpe ratio using OLS, LASSO, and Ridge Regression. This is based on the following optimization model with minimization:

$$
\begin{aligned}
\min_{y,\kappa} \quad & \mathbf{y^T Q y} \\
\text{s.t.} \quad & \mathbf{1^T y} = \kappa \\
& \mu^{\mathbf{T}}\mathbf{y} = \mathbf{1} \\
& \kappa, \mathbf{y} \geq \mathbf{0} \\
& y = \kappa x
\end{aligned}
$$

OLS Method	MVO Method	Sharpe Ratio	Average Turnover
Lasso	Sharpe	0.043491	0.9149
OLS	Sharpe	0.16245	0.75314
Ridge	Sharpe	0.20756	0.43819

Table 5: Optimizing Sharpe Ratio MVO data

Our results show that using the LASSO model, the Sharpe ratio has significantly decreased and average turnover has significantly increased. Using the OLS model, the Sharpe ratio has slightly decreased and average turnover has significantly increased. Using the Ridge Regression model, the Sharpe ratio has increased and average turnover has decreased. Using Ridge Regression with this MVO yields the best results for the Sharpe ratio and average turnover compared to OLS and LASSO.

4 Discussion and Conclusion

4.1 Choosing our Final Model

OLS Method	MVO Method	Sharpe Ratio	Average Turnover
Ridge	Sharpe	0.20756	0.43819
Ridge	Risk-Adjusted Lambda = 10^{-1}	0.21442	0.38308
Ridge	Risk-Adjusted Lambda = 1	0.21047	0.3539

Table 6: Sharpe Ratio & Turnover Rate

Based on the results, the top three best combinations based on average turnover rate and mean-variance optimization include the following: ridge regression with risk-adjusted return MVO with a lambda of 10^{-1}, ridge regression with risk-adjusted return MVO with a lambda of 1, and Sharpe ratio minimization MVO with the ridge regression.

Our results show that there are better results for risk-adjusted return MVO, but this MVO is dependent on a parameter whereas the Sharpe ratio optimization is independent of parameters. Since we want our model to be generalized and adaptable for other datasets, we have chosen the Sharpe Ratio optimization MVO method with Ridge Regression. With the use of k-fold cross validation, we will be choosing the best lambda to create a factor model for each dataset that reduces over fitting. The use of the Sharpe ratio optimization is not dependent on any parameters which is why it is the most favourable optimization model.

4.2 Comparison of Original Model with Final Model

The portfolio wealth evolution for the original model and the penalized ridge regression with 10-fold cross-validation using the Sharpe Ratio optimizer are as follows:

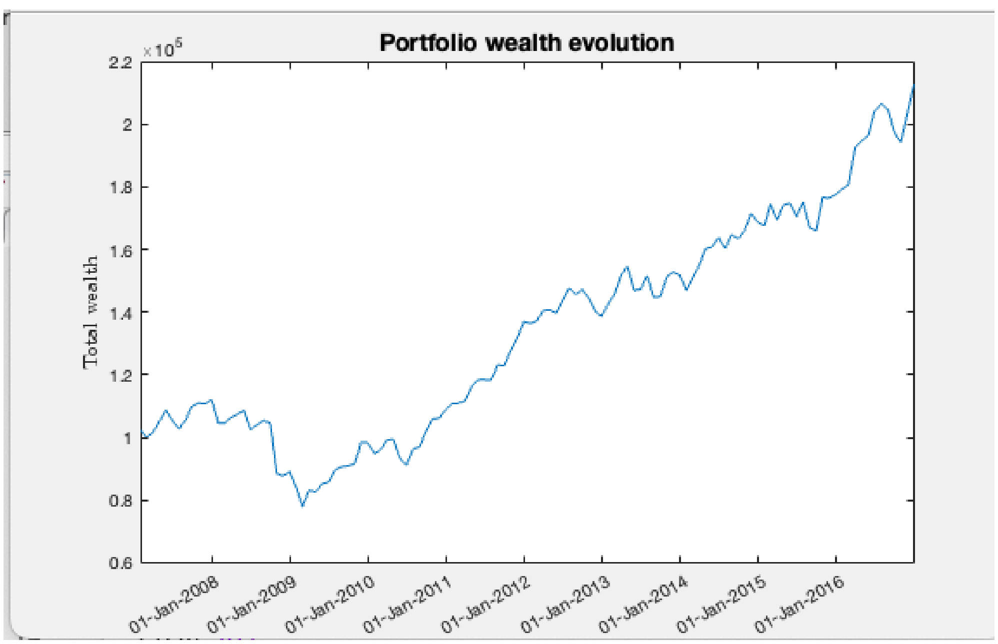


Figure 3: Portfolio Wealth Evolution for Original Model - Sharpe ratio = 0.17171, average turnover = 0.48735

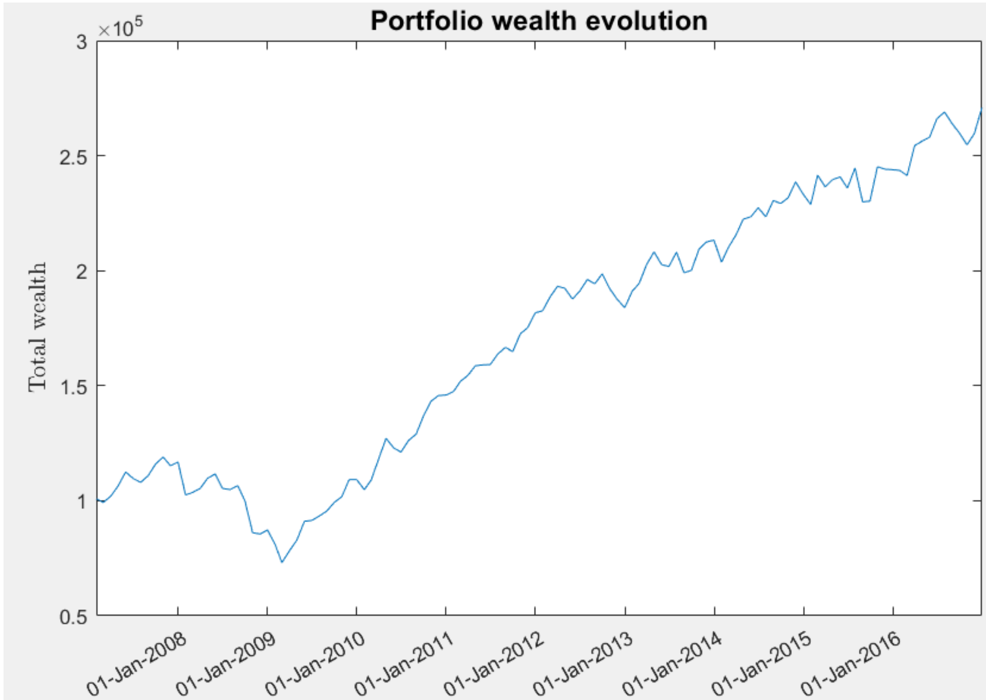
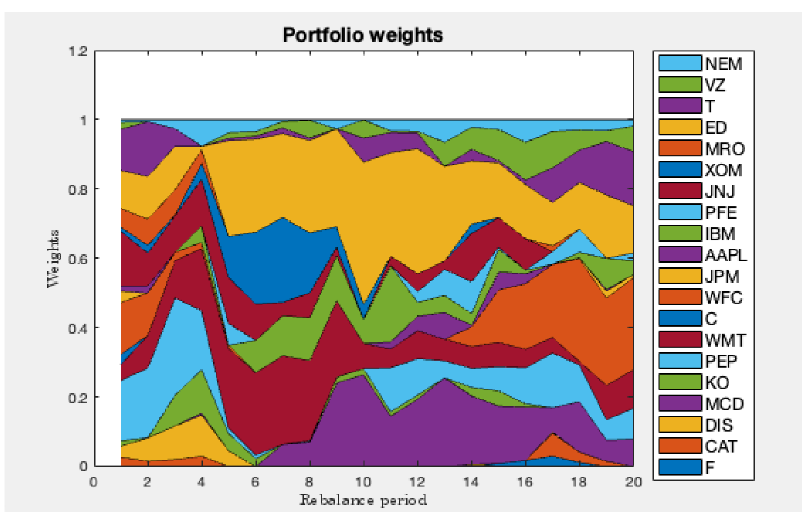


Figure 4: Portfolio Wealth Evolution for the Final Model

When comparing the two portfolio wealth evolution graphs from the original model with the our improved model, our total wealth has increased over the time period from 2008 to 2016. By January 2016, our improved model has exceeded 2.5×10^5 in total wealth compared to the original model which reached approximately 1.7×10^5 in total wealth by January 2016. It is also evident that the slope of a trendline of the portfolio wealth evolution graph has increased in our improved model: this shows that our total wealth is increasing at a faster rate compared to the original model.

The portfolio weight for the original model and the penalized ridge regression with 10-fold cross-validation using the Sharpe Ratio optimizer are as follows:



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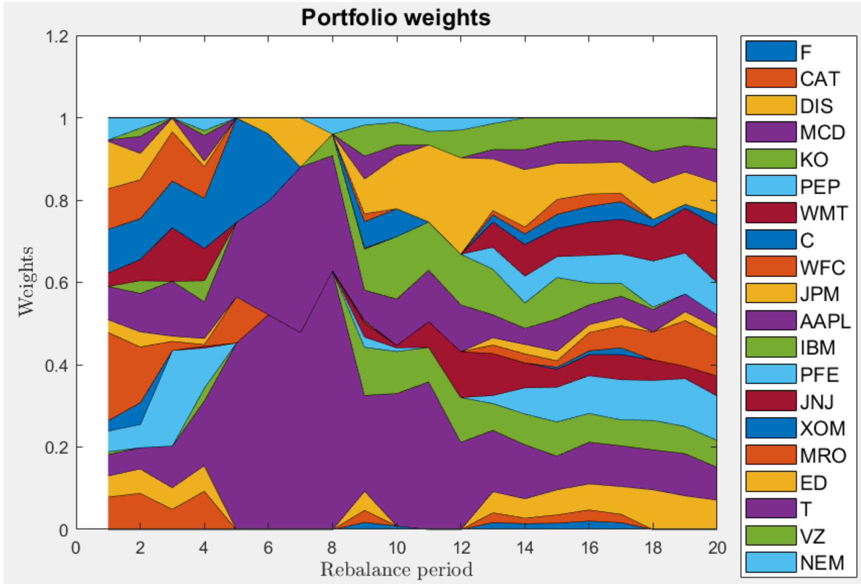

Figure 6: Portfolio Weights for the final model.

When directly comparing Figure 5 and Figure 6, the portfolio weights rebalancing is visually less extreme with fewer spikes in reweighting for the 12 to 20 rebalance period in Figure 6 compared to Figure 5. The original model has consistently many spikes which are indicative of major immediate re-weightings between periods, especially in periods 1 to 6. Other than T and AAPL in Figure 6, which have major spikes in the rebalance periods from 4 to 9, the overarching majority of Figure 6 is clearly more smooth in direct contract to Figure 5. This is a testament to our efforts to lower the average turnover rate.

From the original model to our improved model: the penalized ridge regression with 10-fold cross-validation using the Sharpe Ratio optimizer, we can see that the Sharpe ratio has improved by 0.03585 and the average turnover ratio has improved by 0.04916.

4.3 Conclusion

After testing multiple combinations using different factor models and optimization models, we believe that the model that will perform the best and most consistent under different market conditions is the model that uses ridge regression and Sharpe ratio optimization. Our implementation of ridge regression using k-fold cross validation allows the best penalty term λ to be chosen with each dataset. This will allow us to calculate μ and Q that will be used in our optimization formulation to consistently maximize the Sharpe ratio and reduce average turnover.

5 Appendix

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Table 7: Sharpe Ratio Turnover Rate