MSE160 Problem Set 6

James Li

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1 Question 1: [4 pts.]

In non-steady state diffusion, the concentration of a diffusing species is a function of time and position, where C = C(x, t). To find the concentration at position x and time t, the formula involving the error function can be used.

$$\frac{C(x,t)-C_o}{C_s-C_o} = 1-\text{erf}(\frac{x}{2\sqrt{Dt}}) = 1-\text{erf}(z) \tag{1}$$

Given the values of z and the error function, erf(z):

z	erf(z)
0.5	0.5205
0.527	y
0.55	0.5633

Given the surface concentration, C_s, is 0.2% wt N, time is 10h, position is 1 mm, and the diffusion coefficient, D, is 2.5 x 10^{-11} $\frac{m^2}{s}$.

$$1-\text{erf}(\frac{x}{2\sqrt{Dt}}) = 1-\text{erf}(\frac{0.001\text{m}}{2\sqrt{2.5\times10^{-11}\frac{m^2}{s}\cdot 36000\text{s}}}) = 1-\text{erf}(0.527) \tag{2}$$

$$\frac{0.527-0.5}{0.55-0.5} = \frac{y-0.5205}{0.5633-0.5205} \tag{3}$$

$$y = 0.5437 \tag{4}$$

z	erf(z)
0.5	0.5205
0.527	0.5437
0.55	0.5633

For C_0 = 0, x = 0.001 m, t = 36000 s, the concentration is calculated as follows:

$$\frac{C(0.001,36000)-0}{0.2-0} = 1-0.5437 \tag{5}$$

$$\frac{C(0.001,36000)}{0.2} = 0.4563 \rightarrow C(0.001,36000) = 0.09126 \tag{6}$$

At a depth of 1 mm and a time of 10 hours, the concentration of the nitrogen will be **0.0913 wt% N**.

2 Question 2: [2 pts.]

To calculate the composition of lead (Pb) and tin (Sn) in atomic percent (at%), the number of moles must be determined for each element. wt% represents the mass of the element found in a 100 gram alloy, which means that in a 100 gram alloy, there is 4.5 grams of lead (Pb) and 95.5 grams of tin (Sn).

Given the molar mass of each element, the number of moles can be calculated as follows, where n is the number of moles, m is the mass of the element, and M is the molar mass of the element:

$$n_{Pb} \equiv \frac{m_{Pb}}{M_{Pb}} = \frac{4.5\text{g}}{207.19\frac{\text{g}}{\text{mol}}} = 0.0217 \text{ mol} \tag{7}$$

$$n_{Sn} \equiv \frac{m_{Sn}}{M_{Sn}} = \frac{95.5\text{g}}{118.71\frac{\text{g}}{\text{mol}}} = 0.8045 \text{ mol} \tag{8}$$

Then, the composition in atom percent can be calculated with the subsequent equation:

$$\text{atom\%} \equiv \frac{\text{moles of compound} \cdot N_A}{\text{total moles of mixture} \cdot N_A} \times 100\% \tag{9}$$

$$\text{atom\%}_{Pb} = \frac{n_{Pb}}{n_{Pb} + n_{Sn}} \times 100\% = \frac{0.0217}{0.0217 + 0.8045} \times 100\% = 2.63\% \tag{10}$$

$$\text{atom\%}_{Sn} = \frac{n_{Sn}}{n_{Pb} + n_{Sn}} \times 100\% = \frac{0.8045}{0.0217 + 0.8045} \times 100\% = 97.37\% \tag{11}$$

The composition of lead and tin in the alloy is **97.37% tin** and **2.63% lead**.

3 Question 3: [8 pts.]

3.a : [1 pts.]

When cooling from 1148°C, it is **a eutectic reaction** since the liquid carbon steel alloy transforms into two solid phases through **a eutectic transformation** (L $\longrightarrow \gamma$ + Fe_3C).

3.b : [2 pts.]

To calculate the mass of ferrite, α, in 1 kilogram of carbon steel at 726°C, the Lever Rule can be used at the tie line.

$$W_\alpha = \frac{R}{R+S} \tag{12}$$

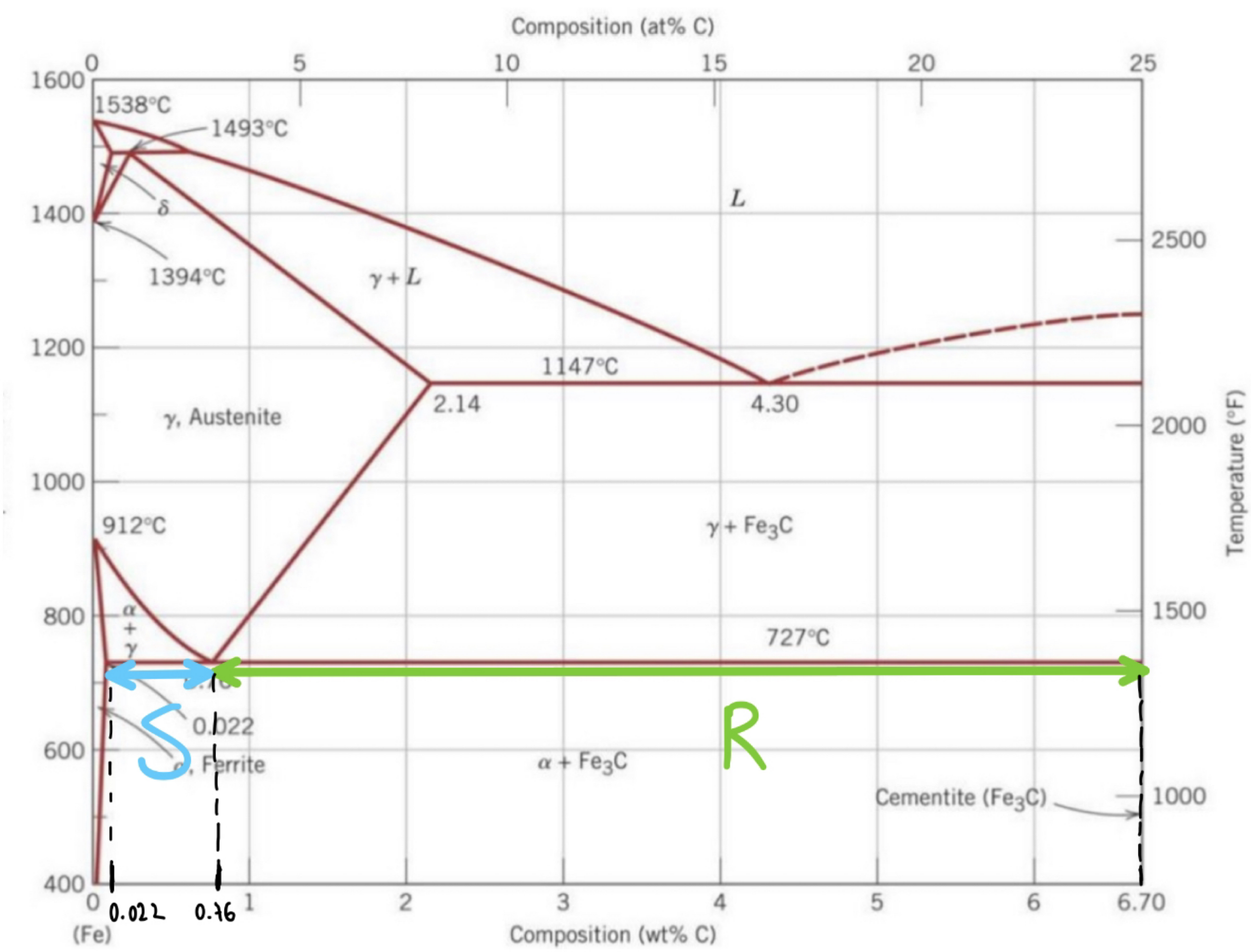
where R represents the region of the tie line lying in the cementite (Fe_3C) region (to the right of 0.76 wt% C) and S represents the region of the tie line lying in the ferrite + cementite (α + Fe_3C).

$$W_\alpha = \frac{R}{R+S} = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.7\% - 0.5\%}{6.7\% - 0.022\%} = 92.84\% \tag{13}$$

The mass in 1 kilogram of carbon steel can then be calculated by:

$$\text{Mass of Ferrite}(\alpha) = (1\text{kg}) \cdot (W_\alpha) = (1\text{kg}) \cdot (92.84\%) = 928.4 \text{ g} \tag{14}$$

There is **928.4 grams of ferrite** found in a 1 kilogram sample of 0.5 wt% carbon steel.



3.c : [2 pts.]

To calculate the maximum mass of carbon that could dissolve as a solid solution, one must determine the solubility limit at 726°C. Thus, one must determine the region where the material no longer exists at one phase, and that region is at 0.022 wt% C, where the ferrite no longer exists as pure ferrite; instead, it becomes either ferrite and austenite (α + γ) or ferrite and cementite (α + Fe_3C). Therefore, the maximum mass of carbon existing as a solid solution can be calculated as follows:

$$m_C = 0.022 \text{ wt\%} \cdot 1000 \text{ grams} = 0.22g \tag{15}$$

The maximum mass of carbon that exists at 726°C in 1 kilogram of steel is 0.022 grams.

3.d : [3 pts.]

At 726°C, the eutectoid point is at 0.76 wt% C, which means if the alloy has less than 0.76 wt% C, it is a hypoeutectoid alloy, and if the alloy has greater than 0.76 wt% C, it is a hypereutectoid alloy. As seen on the phase diagram, the weight composition of cementite (Fe_3C) is 6.7 wt% C, which means the remaining weight is 93.3 wt% Fe, whereas ferrite (α) is a form of pure iron.

To determine the amount of carbon given the mass fractions of ferrite and cementite, the Lever Rule can be applied for both ferrite and cementite:

Given the mass fraction of cementite (Fe_3C) as 0.12:

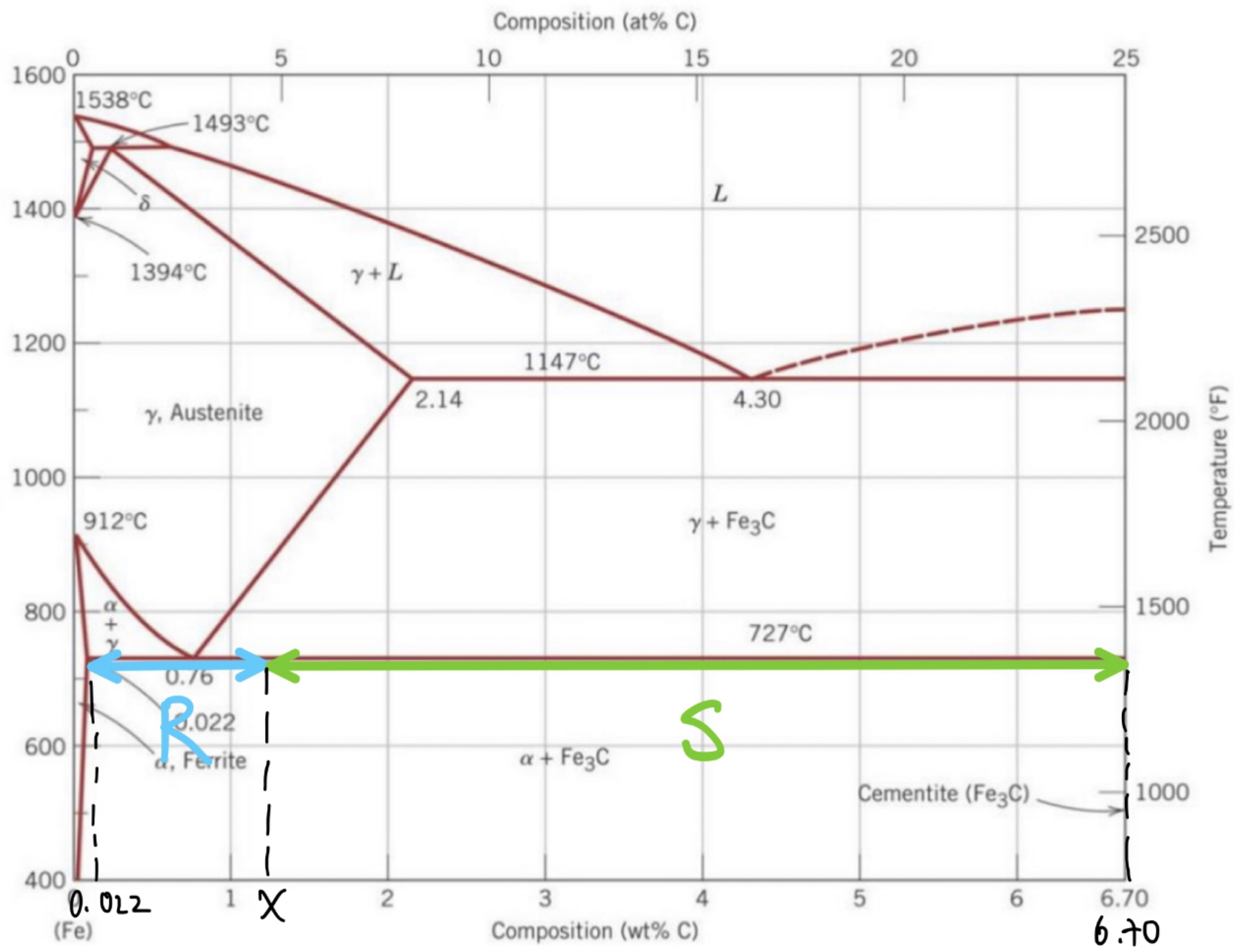
$$0.12 = \frac{R}{R+S} \tag{16}$$

where R is the region on the Tie Line from ferrite (0.022 wt% C) to the weight percent of carbon that satisfy the mass fractions of ferrite and cementite (x wt% C), and S is the region of the Tie Line from the satisfactory carbon wt% (x wt% C) to cementite (6.7 wt% C).

$$0.12 = \frac{x - 0.022}{6.7 - 0.022} \tag{17}$$

$$x - 0.022 = 0.80136 \rightarrow x = 0.823 \text{ wt\% C} \tag{18}$$

Since the carbon percent in weight percent of the iron-carbon alloy is greater than the carbon percent at the eutectoid point (0.823 wt% C is greater than 0.76 wt% C), the alloy is a **hypereutectoid alloy**.


Composition (at% C)
0
5
10
15
20
25
1600
1538°C
1493°C
1400
1394°C
δ
L
γ + L
1200
1147°C
2.14
4.30
γ, Austenite
1000
912°C
γ + Fe$_3$C
800
α
+
γ
727°C
0.76
0.022
R
S
600
α, Ferrite
α + Fe$_3$C
Cementite (Fe$_3$C)
400
0
0.022
(Fe)
1
X
2
3
4
5
6
6.70
6.70
Composition (wt% C)
2500
2000
1500
1000
Temperature (°F)