MSE160 Problem Set 7

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March 2022

Q1.

Length in mm (l_0) is 50.8 mm, and the area (A) of the section can be calculated as $\frac{\pi d^2}{4} \approx 153.94mm^2$. The equations to calculate stress (σ) and strain (ε) are $\frac{F}{A}$ and $\frac{\Delta l}{L_0}$, respectively.

Given the values for force and length:

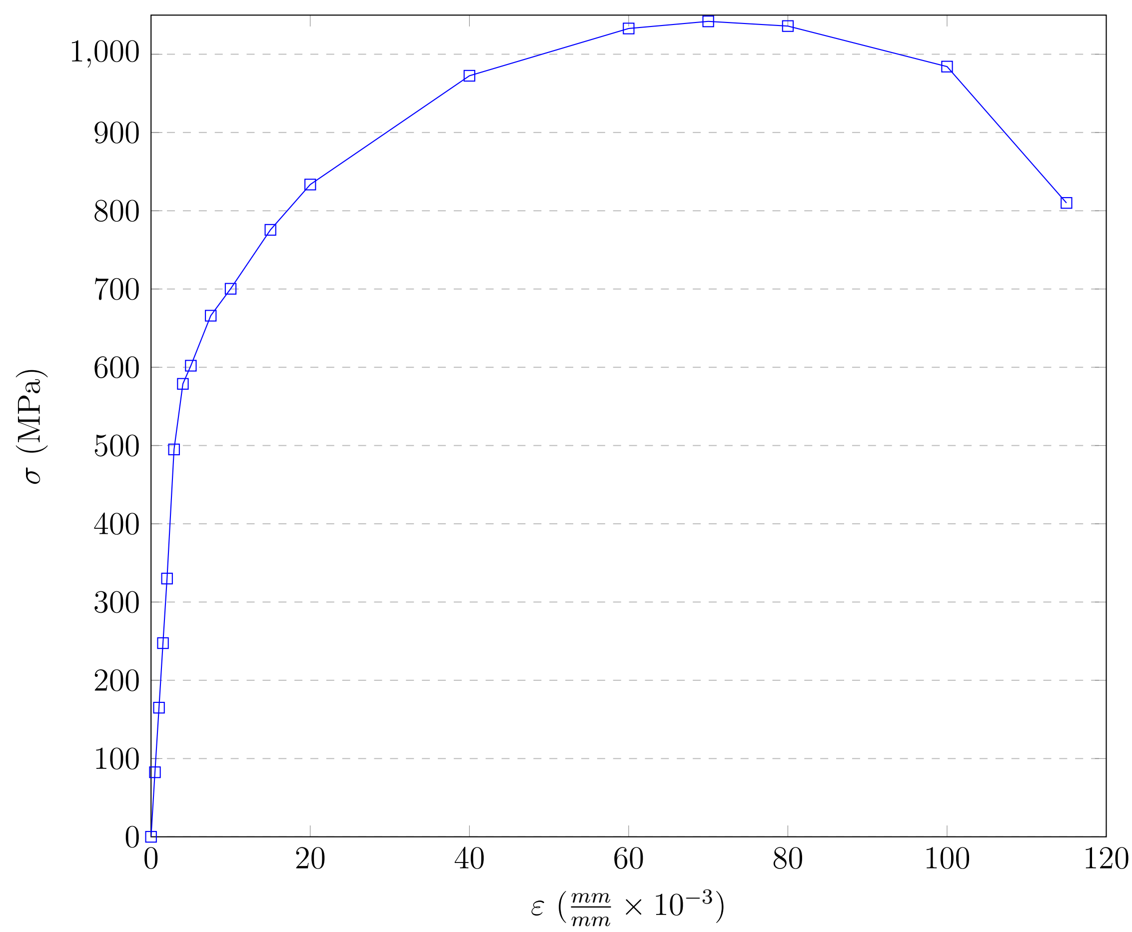
Force (N)	Length (mm)
0	50.8
12700	50.825
25400	50.851
38100	50.876
50800	50.902
$\vdots$	
124700	56.642

We can obtain the stress-strain curve by utilizing the formulas defined above:

Stress (MPa)	Strain ($\frac{mm}{mm}$)
0	0
82.5 ($\frac{12700N}{49\pi}$)	4.92e-4 ($\frac{50.825mm-50.8mm}{50.8mm}$)
165	1e-3
247.5	1.5e-3
330	2.01e-3
$\vdots$	
810	0.115

The stress-strain curve can be modelled as follows:

Stress-Strain relation for a Tensile Coupon



The Young's Modulus (E) can be calculated using the following formula in the linear section of the stress-strain curve:

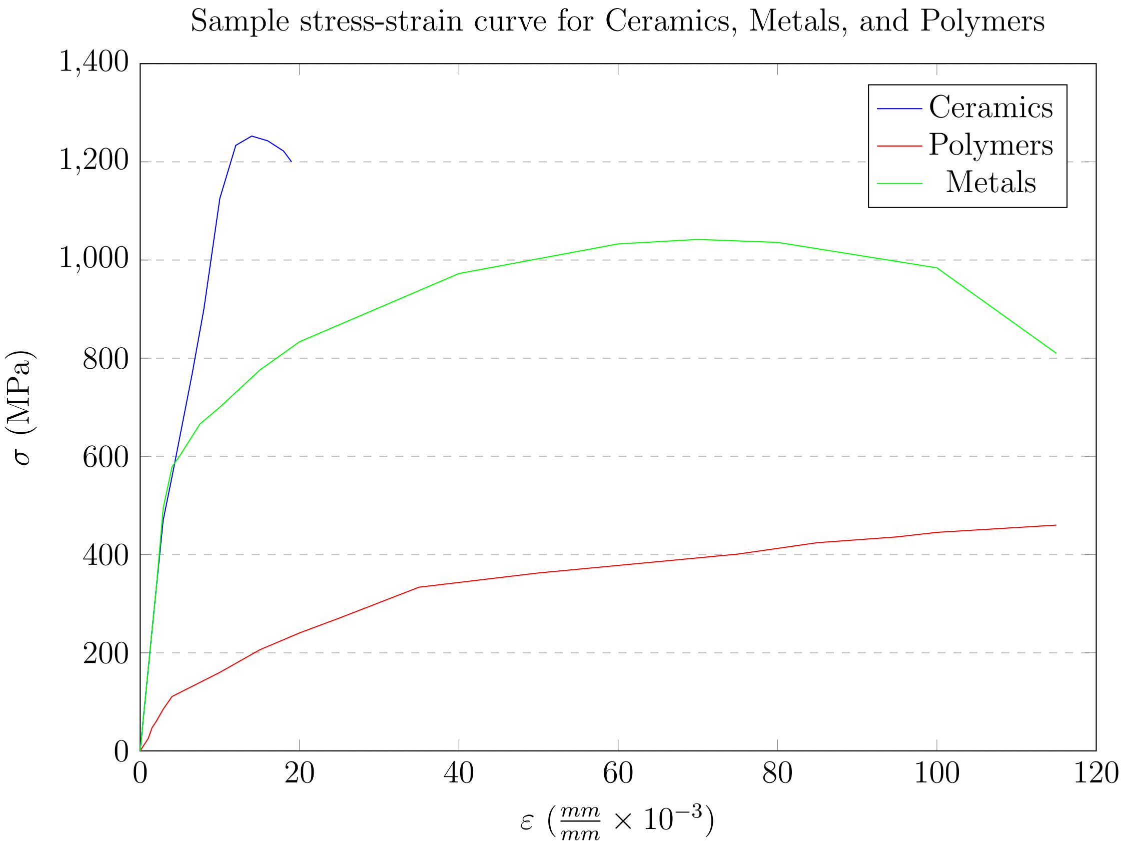
$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{165}{1 \times 10^{-3}} = 165000 \text{ MPa}$$

The yield stress is the point where the stress-strain curve is no longer linear, which is at around 578.8 MPa. Thus, $\sigma_{yield} = 578.8$ MPa.

The fracture stress is the point where the stress-strain curve ends, which is at 810 MPa. Thus, $\sigma_{fracture} = 810$ MPa.

This material is a metal since it has a mildly high yield stress, and continues to deform following the yield point until it reaches its ultimate stress and fractures. It cannot be a ceramic since ceramics have a very high yield stress but fracture past the point of the yield stress. It cannot be a polymer since they generally have very low yield stresses but can be greatly deformed prior to fracturing.



- Ceramic — High yield stress, high Young's modulus, low plastic deformation
- Polymer — Low yield stress, high plastic deformation
- Metal — High yield stress, strain hardening, average plastic deformation

Q2.

For the original length, l_0, equal to 420 mm, and the area equal to 25 mm^2, the strain can be calculated using the formula stated in question 1:

$$\Delta l = 2.05 \text{ mm}, \ \varepsilon = \frac{\Delta l}{l_0}$$

$$\varepsilon = \frac{2.05}{420} = 4.88 \times 10^{-3}$$

The stress can then be determined by matching the strain to the stress shown on the graph, which is approximately 1000 MPa.

The force can be then calculated using the formula of stress stated in question 1:

$$\sigma = 1000 \text{ MPa}, \ \sigma = \frac{F}{A}, \ F = \sigma A$$

$$F = 1000 \cdot 25 = 25 \text{ kN}$$

A force of 25 kilonewtons will produce an elongation of 2.05 mm.

Since the material has not reached past its yielding point, which is approximately 1500 MPa of stress, the material will not experience any form of permanent deformation, which means the final sample length will still be 420 mm.

Q3.

When an elastic deformation occurs, a material's shape is physically deformed, but this change is reversible and non-permanent, which means the macroscopic sample deformations will increase in size when it is strained but return to its original size. Similarly, the atomic position will change temporarily as the material is being loaded but will return to its original position when unloaded. The elastic deformation is located at the linear portion of the stress-strain curve.

When a plastic deformation occurs, a solid material will undergo a physical deformation that is permanent and irreversible, which means the macroscopic sample deformations and the atomic position will not return to its original values after being unloaded. The transition from elastic deformation to plastic deformation occurs near the end of the linear portion of the stress-strain curve, when the curve begins to become non-linear.

Q4.

The yielding strain of these materials can be calculated using the equation found in question 1:

$$\sigma = E\varepsilon \ , \ \varepsilon = \frac{\sigma}{E}$$

Material	σ_{yield} (MPa)	E (GPa)	$\varepsilon_{yield}(\frac{mm}{mm} \times 10^{-3})$
Tungsten	550	407	1.351
Steel	180	207	0.870
Nickel	138	190	0.726
Titanium	450	107	4.206
Brass	77	97	0.794
Aluminum	35	69	0.507
Magnesium	130	45	2.889

For part a, the equation that can be used for strain is:

$$\varepsilon = \frac{\Delta l}{l_0}$$

Given $l_0 = 500$ mm and $\Delta l = 0.4$ mm, strain can be calculated:

$$\varepsilon = \frac{0.4}{500} = 8 \times 10^{-4}$$

This value closely matches the yielding strain of **brass**.

The material with the lowest strain at yield strength is **aluminum**, and the highest strain at yield strength is **titanium**.

For part d, the equation that can be used to determine the relationship between applied force and cross-sectional area is the stress formula:

$$\sigma = \frac{F}{A}$$

Given F = 22.5 kN and A = 50 mm^2:

$$\sigma = \frac{22500\text{N}}{50\ \text{mm}^2} = 450\ \text{MPa}$$

The material that requires 22.5 kilonewtons to deform a 50 squared millimeter cross-sectional area is also **titanium**, which has a yield stress of 450 MPa.

Q5.

For "beams", the equation needed to calculate the flexural stress is as follows:

$$\sigma = \frac{3FL}{2wh^2},\ w = \frac{3FL}{2\sigma h^2}$$

where F is the force exerted on the beam, L is the length of the beam, w is the width of the beam, and h is the height of the beam.

The gravitational force that is exerted by a 86 kilogram mass on the beam is:

$$F_g = mg = 86 \times 9.81 = 843.66\ \text{N}$$

Material	Flexural Strength (MPa)	L (mm)	w (mm)	h (mm)	F (N)
Moleculia	170	500	20 to 27	12	843.66
Materialia	210	300	15 to 18	10	843.66

Given the values of σ, L, and h, we can calculate the width that can withstand the gravitational force.

$$w_1 = \frac{3(843.66)(500)}{2(170)(12)^2} = 25.85\ \text{mm}$$

$$w_2 = \frac{3(843.66)(300)}{2(210)(10)^2} = 18.07\ \text{mm}$$

If flexural strength is constant, then force and width of the beam are inversely proportional. That means as width increases, force decreases and vice versa, which means that materialia cannot meet the force requirements and will, therefore, yield. However, the **moleculia** can meet the force requirements if the width of the beam is between 25.85 mm and 27 mm.