MSE160 Problem Set 2

James Li

January 2022

1 Question 1: [3 pts]

To calculate the energy within the material, we can use the Schrodinger's Equation for in an electron in a box where:

$$E_n = \frac{h^2}{8mL^2} \cdot n^2 \tag{1}$$

h is Planck's constant, L is the position, and m is the mass of the electron.

However, since it requires the lowest possible energy, we calculate the energy at the first energy level, n = 1:

$$E = \frac{h^2}{8mL^2} \cdot n^2 = \frac{(6.626 \times 10^{-34})^2 \cdot 1}{(8)(2.36 \times 10^{-31})(10^{-6})} = 2.33 \times 10^{-25}\mathrm{J} \tag{2}$$

If we assume that the energy is all kinetic, we can calculate for the velocity:

$$E = \mathrm{KE} + \mathrm{PE} = \frac{1}{2}mv^2 + 0 \tag{3}$$

$$v = \sqrt{\frac{2.33 \times 10^{-25}\mathrm{J}}{\frac{1}{2}(2.36 \times 10^{-31})}} \approx 1404\frac{m}{s} \tag{4}$$

The velocity of the electron is approximately 1404 meters per second.

2 Question 2: [5 pts]

2.a :

The normalized wavefunction of a particle inside a box can be represented with the equation:

$$\psi_n = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}) \tag{5}$$

Substituting L = q and n = 1 (because ground state), we obtain a graph that is shown below:

$$\psi = \sqrt{\frac{2}{q}} \sin(\frac{\pi x}{q}) \tag{6}$$

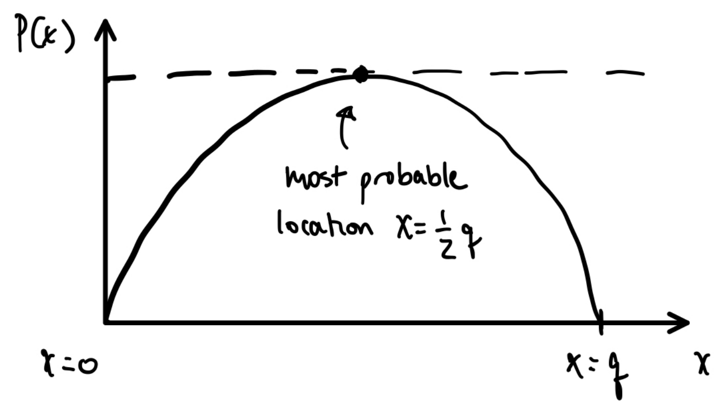


Fig 1: Graph of the probability of the electron's occurrence versus the length of the box, represented by the equation $\psi = \sqrt{\frac{2}{q}} \sin(\frac{\pi x}{q})$.

This electron is most likely to be found at x = $\frac{q}{2}$.

2.b :

The probability of finding a particle between x and x+Δx can be represented by:

$$P_n(x) = \frac{2}{L} \sin^2(\frac{n\pi x}{L})\Delta x \tag{7}$$

The probability of finding an electron between 0.41q and 0.42q (0.41q + 0.01q) is approximately 1.844%:

$$P \approx \frac{2}{q} \sin^2(\frac{(1)(\pi)(0.415q)}{(q)})(0.01q) \approx 0.0186 = 1.86\% \tag{8}$$

3 Question 3: [4 pts]

The restrictions on the wave function based on the Born interpretation are:

- The wave, ψ(x), has to be continuous at all of x.
- The wave, ψ(x), must have a single value at any point in space.
- The wave, ψ(x), must be finite everywhere.
- The wave, ψ(x), cannot be zero everywhere.
- The wave, ψ(x), must be normalisable.

The wave function itself determines the shape of the wave, whereas the probability of locating a particle, P(x), is proportional to the absolute square of the wave function, ψ(x), in space at any given point.

$$P(x) \propto |\psi(x)|^2 dx \tag{9}$$

4 Question 4: [8 pts]

4.a :

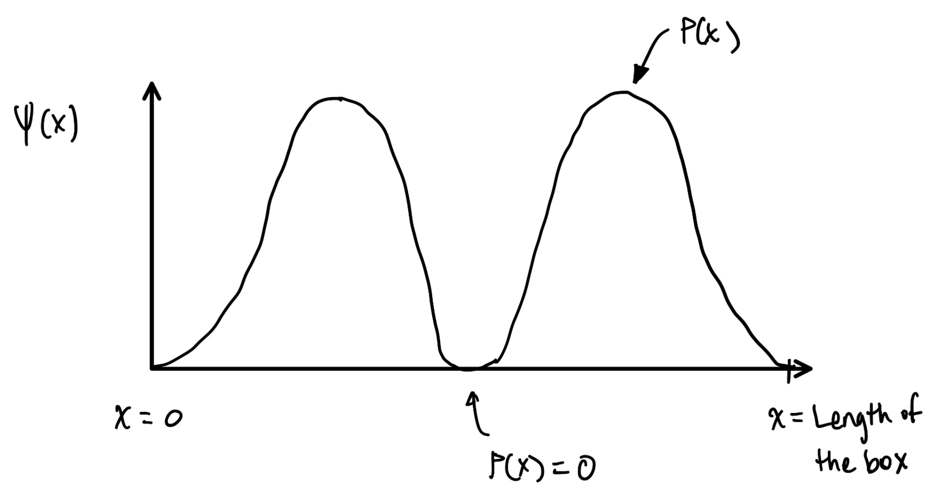
Given the initial energy to be 1.00 microJoules, Planck's constant to be 6.626 $\times 10^{-34}$ J·s, the mass to be 0.001 kg, and L to be 0.01 m:

$$E = \frac{h^2}{8mL^2} \cdot n^2 \tag{10}$$

$$n = \sqrt{\frac{8EmL^2}{h^2}} = \sqrt{\frac{8(1 \cdot 10^{-3})(0.001)(0.01)^2}{(6.626 \cdot 10^{-34})^2}} = 4.27 \times 10^{28} \tag{11}$$

4.b :

At the excited state (n = 2), the graph for $\psi(x)^2$ looks like:



As seen from the graph, there is a node at the centre of the length, which means the probability density is 0 at the centre.

Another way to mathematically determine the probability is by using the relationship:

$$P(x) = |\psi(x)|^2 dx \tag{12}$$

and:

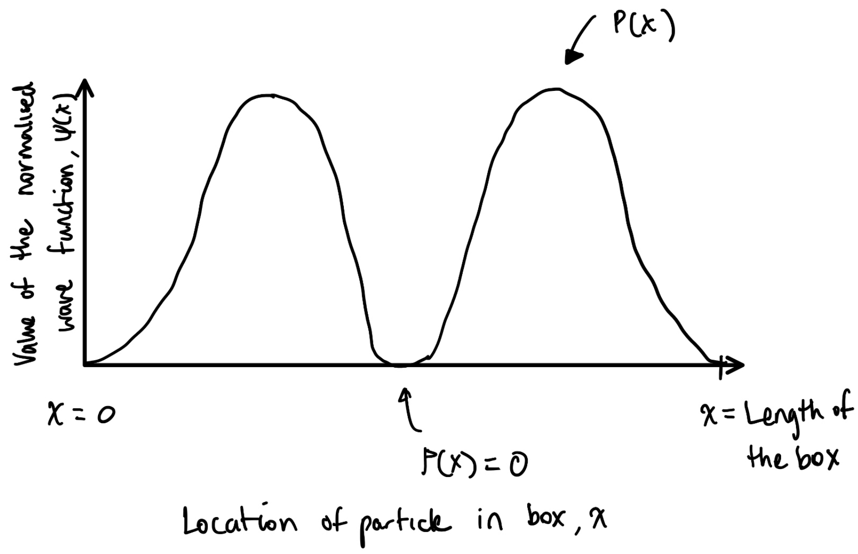
$$\psi(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}) \tag{13}$$

In this case, the value of x is the centre of the box, which means x = $\frac{L}{2}$.

$$\psi(\frac{L}{2}) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi \frac{L}{2}}{L}) = 0 \tag{14}$$

Since $\psi(\frac{L}{2})$ is 0, then $\psi(\frac{L}{2})^2$ is also 0, which means the probability is 0.

4.c :


P(x)
Value of the normalised wave function, ψ(x)
x = 0
P(x) = 0
x = Length of the box
Location of particle in box, x