MSE160 Problem Set 8

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Q1.

Q1. a)

We can use the strain, the force applied, and the stress provided in the second data set to calculate for the original length of the sample and the area through the strain formula and stress formula, respectively:

$$\varepsilon = \frac{\Delta l}{l_0}$$
$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{6}{0.005} = 1200 \text{ mm}$$

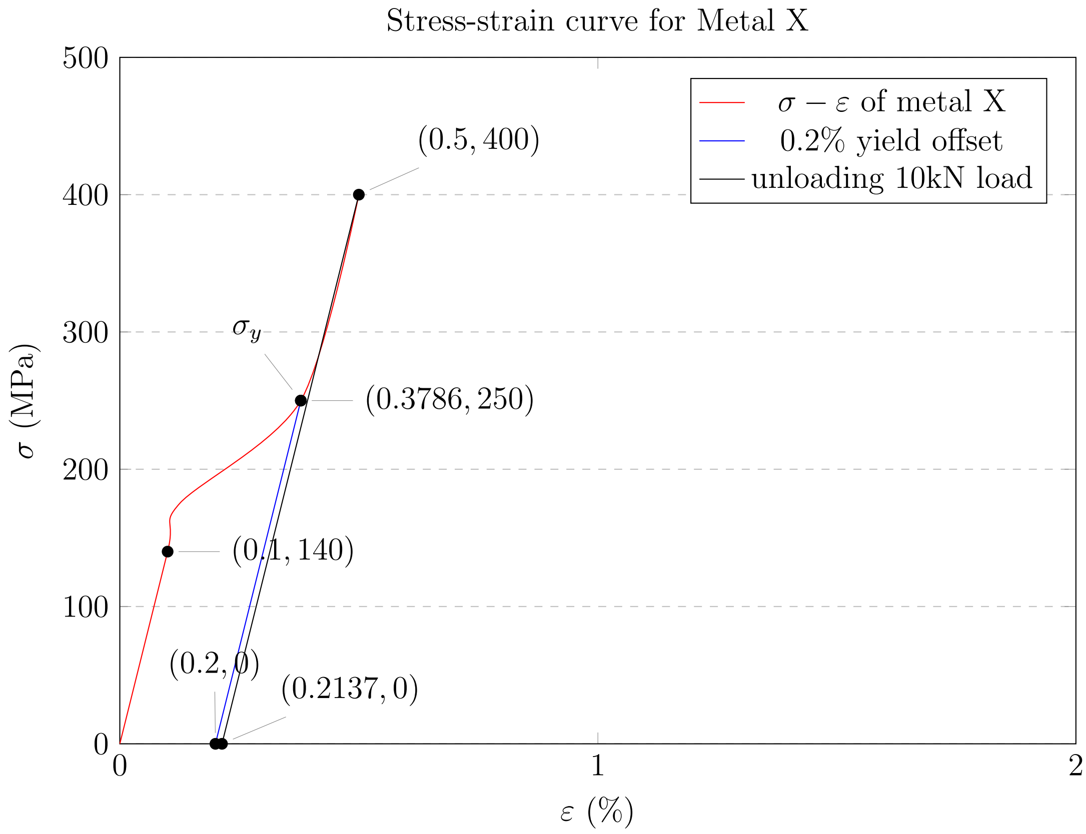
$$\sigma = \frac{F}{A}$$
$$A = \frac{F}{\sigma} = \frac{10000}{400} = 25 \text{ mm}^2$$

However, we cannot calculate the Young's Modulus using the second set of data since it has already strained past the yield strength, which means the first set of data will be used.

We can again use the strain formula and the stress formula to calculate the strain and stress, respectively, given the elongation and the original length:

$$\varepsilon = \frac{\Delta l}{l_0}$$
$$\varepsilon = \frac{1.2}{1200} = 0.001 = 0.1\%$$
$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{3500}{25} = 140 \text{ MPa}$$

At 0.1%, the stress would be 140 MPa, which means the Young's Modulus can be calculated as follows:



$$\sigma = E\varepsilon$$
$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{140}{0.001} = 140 \text{ GPa}$$

The Young's Modulus of Metal X would be 140 GPa.

Q1. b)

For the 3.5 kilonewton case, the sample metal will not deform since it is still within the elastic region of the curve, which means the length will remain 1200 mm.

However, for the 10 kilonewton case, the sample metal will deform according to the slope of the linear region ($\frac{\Delta\sigma}{\Delta\varepsilon}$, Young's Modulus). The slope is 140000, so the strain it will be unloaded to can be calculated as follows, modeled by the black line:

$$(\sigma - \sigma_0) = E(\varepsilon - \varepsilon_0)$$
$$(400 - 0) = 140000(0.005 - \varepsilon_0)$$
$$\varepsilon_0 = 0.005 - \frac{400}{140000} = 0.002143$$

The permanent deformation can be calculated using the strain formula:

$$\varepsilon = \frac{\Delta l}{l_0}$$
$$\Delta l = \varepsilon l_0 = 0.002143 \times 1200 = 2.57 \text{ mm}$$

Q1. c)

After the sample has undergone the 10 kilonewton load, the sample will experience uniform deformation and will be permanently deformed. As an effect, the sample will begin showing signs of necking and thinning, but necking will not yet occur since at 10 kilonewtons, the stress has not yet exceeded the ultimate tensile strength of metal X, which is 450 MPa. Instead, it will experience strain hardening, which basically means if it were to be reloaded then it would require more stress to cause more plastic deformation.

Q2.

Given σ_y = 560 MPa, σ_{ult} = 630 MPa, E = 200 GPa:

Q2. a)

To obtain the maximum load that can be withstood without plastic deformation, we use the yield strength and calculate the area to determine the load:

$$\text{Area} = s^2 = 10^2 \text{ mm}^2 = 100 \text{ mm}^2$$

$$\sigma = \frac{F}{A}$$
$$\sigma_y = \frac{F\text{max}}{A}$$
$$F\text{max} = \sigma_y A$$
$$F\text{max} = (560)(100)$$
$$F\text{max} = 56{,}000 \text{ N}$$

The maximum load the square cross-section can withstand is 56 kilonewtons.

Q2. b)

Given the original length, l_0, is 63 mm, and the equations for stress and strain, we can calculate the maximum elongation:

$$\sigma = E\varepsilon,\ \varepsilon = \frac{\Delta l}{l_0}$$

First, we use the equation for stress to calculate the strain at the yield strength.

$$\begin{aligned}
\sigma &= E\varepsilon \\
\varepsilon_y &= \frac{\sigma_y}{E} \\
\varepsilon_y &= \frac{560}{200000} = 2.8 \times 10^{-3}
\end{aligned}$$

Next, we use the equation for strain to calculate the elongation:

$$\begin{aligned}
\varepsilon_y &= \frac{\Delta l}{l_0} \\
\Delta l &= \varepsilon_y l_0 \\
\Delta l &= 0.1764 \text{ mm}
\end{aligned}$$

Thus, the maximum elongation prior to plastic deformation is 0.1764 millimeters.

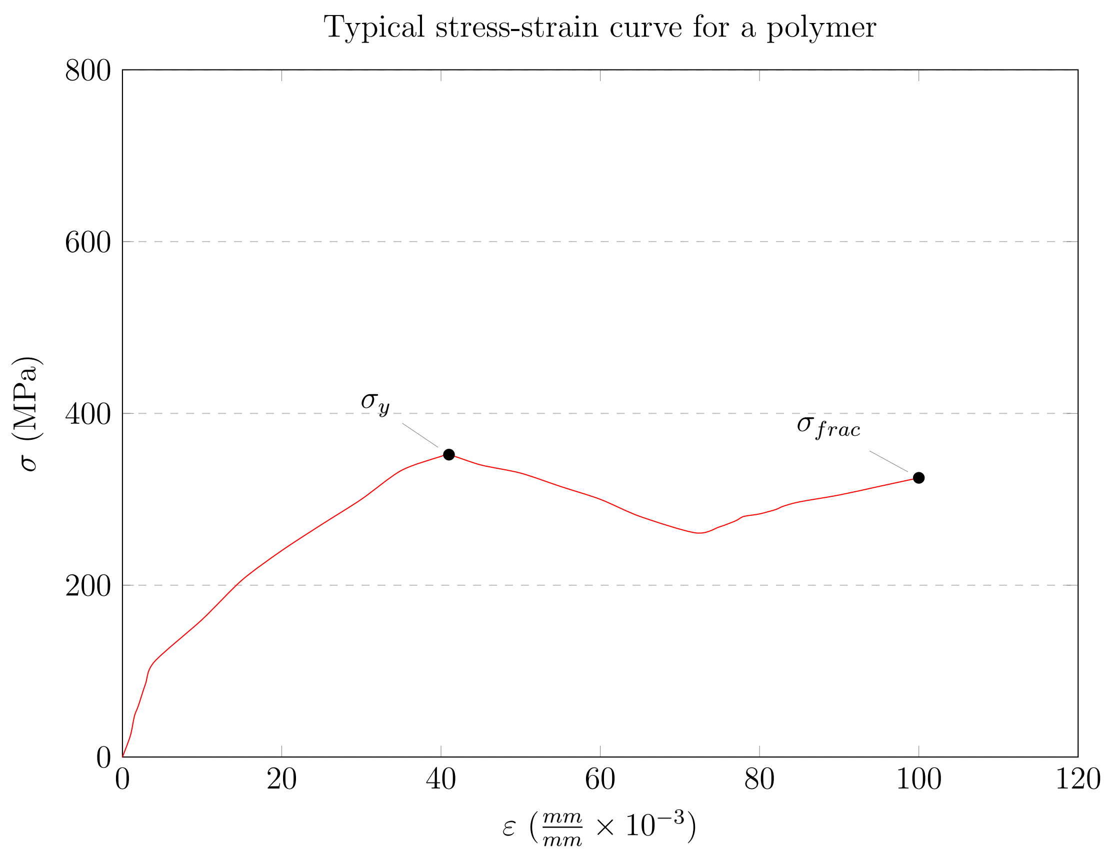
Q2. c)

The elastic strain is the strain at the yield strength, where elastic deformation occurs and no permanent deformation occurs (sample size will return to original shape). Whereas, the total strength includes the plastic strain, where plastic deformation occurs and permanent deformation will occur (sample size will not return to original shape). Since the elastic strain is previously calculated, we only have to calculate for the total strain:

$$\begin{aligned}
\sum \varepsilon &= \varepsilon_{\text{elastic}} + \varepsilon_{\text{plastic}} \\
\varepsilon_{\text{plastic}} &= \varepsilon_{\text{total}} - \varepsilon_{\text{elastic}} \\
\varepsilon_{\text{total}} &= \frac{\sigma_{ult}}{E} \\
\varepsilon_{\text{total}} &= \frac{630}{200000} = 3.15 \times 10^{-3} \\
\varepsilon_{\text{plastic}} &= 3.15 \times 10^{-3} - 2.8 \times 10^{-3} = 3.5 \times 10^{-4}
\end{aligned}$$

The difference in the total strain and the elastic strain can be represented as the plastic strain, therefore the difference in strain is 3.5 $\times 10^{-4}$.

Q3.



Polymers have a property that allow it to continue to take load after the peak stress, unlike metals, which will break very quickly following the peak stress since the metal cannot strengthen fast enough to overcome the rapidly decreasing cross-sectional area during necking. Polymers have a microstructural mechanism that allows them to strengthen greatly when the sudden decrease in cross-sectional area occurs.

Q4.

The equations to calculate number average and weight average molecular weights are as follows:

$$\overline{M}_{number} = \sum_{n=1}^{i} M_n x_n$$
$$\overline{M}_{weight} = \sum_{n=1}^{i} M_n w_n$$

where M_n is the molecular weight, x_n is the number of molecules in the group divided by the total number of molecules, and w_n is the mass of the molecules in the group divided by the total mass of molecules.

	No. of Molecules	Molecular Weight (M_n)	Number Fraction (x_n)	Weight Fraction (w_n)
	12	11,000 g/mol	$\frac{12}{97} = 0.1237$	0.052
	29	23,000 g/mol	$\frac{29}{97} = 0.299$	0.263
	32	28,000 g/mol	$\frac{32}{97} = 0.330$	0.353
	24	35,000 g/mol	$\frac{24}{97} = 0.247$	0.331
Total	97			

Calculations for the weight fractions:

$$w_1 = \frac{12 \times 11000}{(12 \times 11000) + (29 \times 23000) + (32 \times 28000) + (24 \times 35000)} = 0.052$$
$$w_2 = \frac{29 \times 23000}{(12 \times 11000) + (29 \times 23000) + (32 \times 28000) + (24 \times 35000)} = 0.263$$
$$w_3 = \frac{32 \times 28000}{(12 \times 11000) + (29 \times 23000) + (32 \times 28000) + (24 \times 35000)} = 0.353$$
$$w_4 = \frac{24 \times 35000}{(12 \times 11000) + (29 \times 23000) + (32 \times 28000) + (24 \times 35000)} = 0.331$$

Calculation for the number average molecular weight ($\overline{M}_{number}$):

$$\begin{aligned}\overline{M}_{number} &= \sum_{n=1}^{i} M_n x_n \\ &= (11000 \cdot 0.1237) + (23000 \cdot 0.299) + (28000 \cdot 0.33) + (35000 \cdot 0.247) \\ &= 26122.7 \frac{\text{g}}{\text{mol}}\end{aligned}$$

Calculation for the weight average molecular weight ($\overline{M}_{weight}$):

$$\begin{aligned}\overline{M}_{weight} &= \sum_{n=1}^{i} M_n w_n \\ &= (11000 \cdot 0.052) + (23000 \cdot 0.263) + (28000 \cdot 0.353) + (35000 \cdot 0.331) \\ &= 28090 \frac{\text{g}}{\text{mol}}\end{aligned}$$