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Assignment 6

Problem-1

Solution

To prove that Integer Programming is in NPC, we can firstly prove that Integer Programming is NP; then we know that 3SAT is NPC, if we can prove that 3SAT $\leq_p$ Integer Programming, then Integer Programming is in NPC. Firstly, we can verify the constraint of $Ax \geq b$ in polynomial time, so Integer Programming is NP. Secondly, we can prove that 3SAT is polynomially reducible to Integer Programming by the following steps. For each input and output of 3SAT, we can construct a vector that satisfy $Ax \geq b$. If the clauses are satisfiable, we regard it as one truth assignment, we can treat every row of Ax to one element of b as one constrain, and the factors of this row can be constructed as a clause of 3SAT, e.g.: if there is 4 variables marked by x_1, x_2, x_3, x_4, the corresponding variables of y is y_i. If there is one clause $y_1 \vee y_3 \vee y_4$, then the corresponding constraints of x is $x_1 + x_3 + (1 - x_4) \geq 1$. And if y_i is true then x_i is 1, if y_i is false then x_i is 0. So we can construct the matrix of A row by row according to each clause. Finally, we can move the constant value from the left to right to get the vector b. Then, if one assignment is true, both the clause and the constraints of matrix will be satisfied. Therefore, 3SAT is polynomially reducible to Integer Programming, so Integer Programming is in NPC.

Problem-3

Solution

To prove that Half-3SAT is in NPC, we can firstly prove that Half-3SAT is NP; then we know that 3SAT is NPC, if we can prove that 3SAT $\leq_p$ Half-3SAT, then Half-3SAT is in NPC. Firstly, for one regular Half-3SAT problem, if there are n variables and m clauses, we can set their value to be true or false, according to the formulation of clauses, we can get its boolean value. If half of the values are true, then it will be one true assignment. It is obvious that we only need to set the value for every variable once, we can finish the step in polynomial time. Now we only need to prove that 3SAT is polynomially reducible to HALF-3SAT problem. It means that for one input of instance in 3SAT, with one transformation, there will be one instance of HALF-3SAT problem. And the instance of 3SAT will be satisfied if and only if the instance of HALF-3SAT problem is satisfied. For one 3SAT problem with n variables and m clauses, the value of $C_1 \wedge C_2 \wedge C_3 \wedge ... \wedge C_m$ is defined to be ϕ. The corresponding HALF-3SAT problem is 4m clauses. The first part of these clauses are the same as ϕ, then there will be m clauses which formulation is $s_1 \vee s_2 \vee \neg s_2$. It is obvious that these clauses are always true no matter what kind of assignment it is. Finally, the left 2 m clauses are constructed to be false or all true. E.g., $s_1 \vee s_2 \vee s_3$ is the formulation of all of clauses. We know that if 3SAT is satisfied, ϕ is true, half of the clauses in HALF-3SAT is true, then the last 2m clauses are set to be false, the requirement of HALF-3SAT is satisfied, the true assignment of ϕ is transformed to a ideal HALF-3SAT assignment. And if HALF-3SAT is satisfied, because the last 2m clauses must to be false, or 3/4 of clauses are true and the requirement cannot be satisfied, thus, the first m clauses are true. We can take the assignment of first m clauses as 3SAT assignment. It is obvious that the corresponding requirement is satisfied.