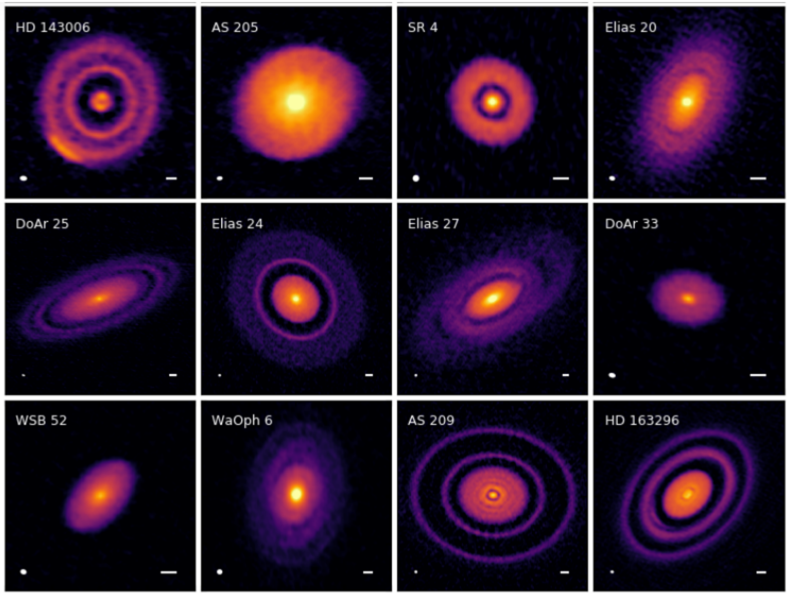
Resonant Excitation of Planetary Eccentricity due to a Dispersing Eccentric Protoplanetary Disk

Jiaru Li (Cornell, with Dong Lai)

May 11, 2023 @ 54th Annual DDA Meeting

Protoplanetary disks (PPDs):

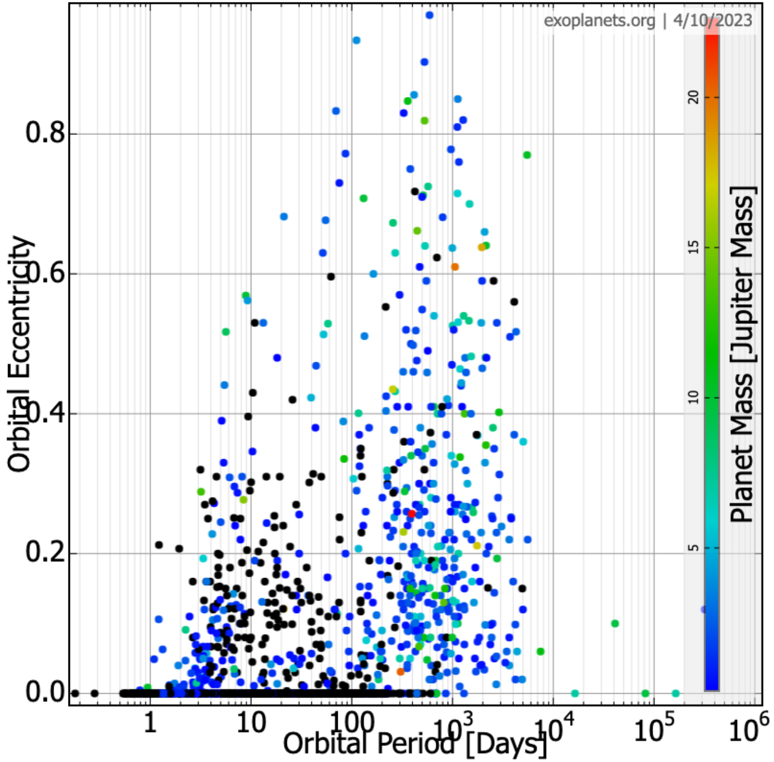
- are the **birthplaces of planets**,
- have lifetimes of around a few Myr,
- have been observed by ALMA to commonly show **signs of planet-disk interactions**.



a gallery of 1.25mm continuum image for disks in DSHARP sample
(*Andrews et al., 2018*)

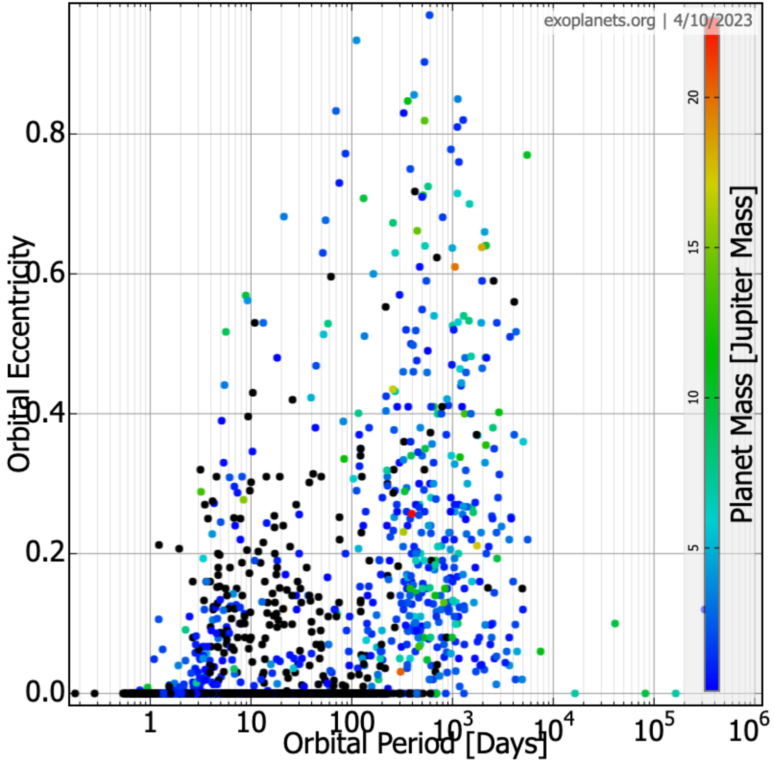
Planet eccentricity due to planet-disk interaction

- Disk vs Planet Eccentricity:
  - Disks are typically believed to damp planetary eccentricities (e.g., *Tanaka & Ward 2004*)
  - In some situations, disks can boost planetary eccentricities (e.g., *Goldreich & Tremaine 1980, 1981; Eklund & Masset 2017*). **However, $e_p < 0.1$ in most cases.**



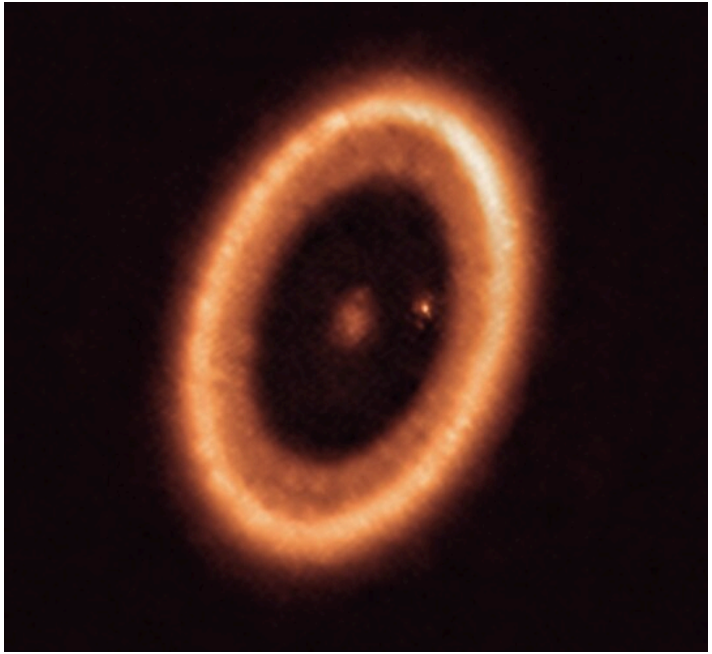
Planet eccentricity due to planet-disk interaction (*Li and Lai, submitted*)

- New mechanism
  - inner planet + outer disk
  - use disk eccentric mode
  - produce **e_p up to 0.6**



Planet eccentricity due to planet-disk interaction (*Li and Lai, submitted*)

- New mechanism
  - inner planet + outer disk
  - use disk eccentric mode
  - produce **e_p up to 0.6**
- General Picture
  1. The outer disk is massive and slightly eccentric.
  2. The disk loses mass slowly.
  3. Eccentricity transfer from the disk to the inner planet.



ALMA image of PDS 70
(*ALMA (ESO/NAOJ/NRAO)/Benisty et al.*)

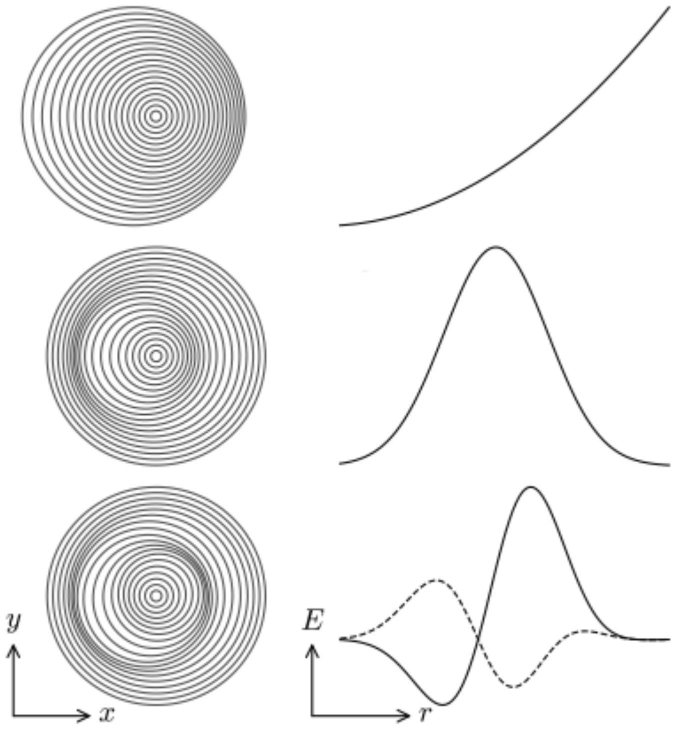
Disk eccentricity

Definition:

- *E(r)*: complex eccentricity of a disk at each radius:
  - |*E(r)*|: "orbital" eccentricity
  - arg[*E(r)*]: longitude of pericenter

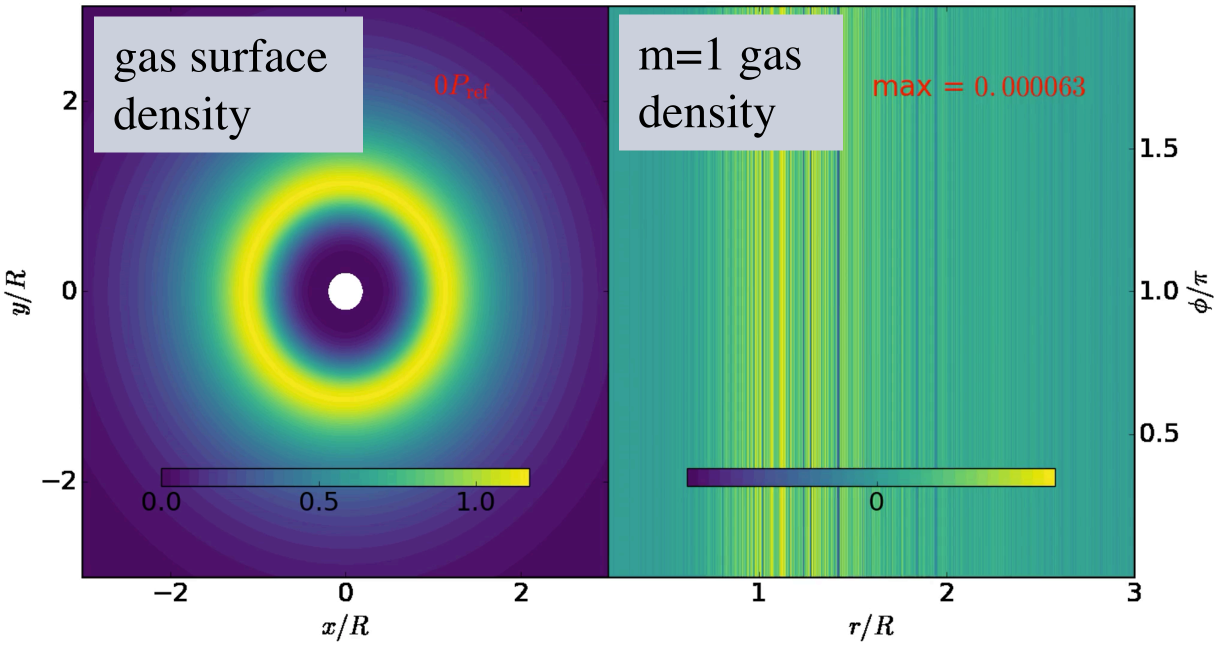
Meanings:

- describes the m=1 asymmetry of a disk
- reflects the angular momentum profile of a disk



an illustration of disk eccentricity (*Lee+ 2019*)

Why massive and eccentric disk?

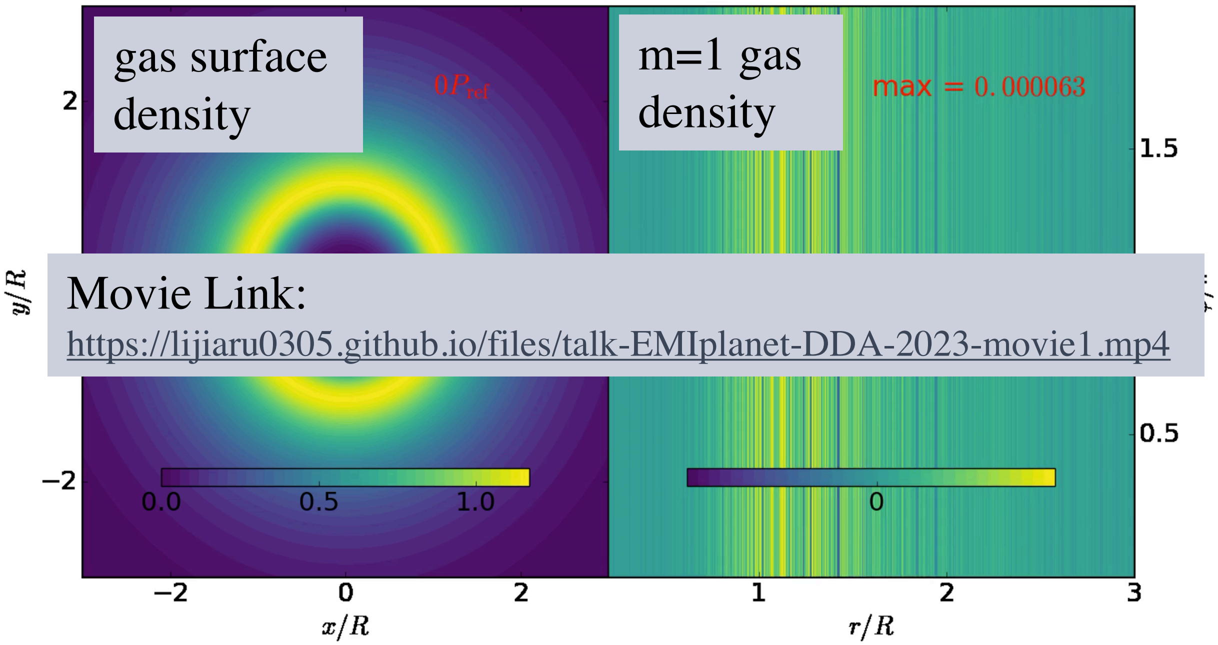


Spontaneous growth of disk eccentricity. (*Li+ 2021*)

High-res 2D hydrosimulation

- LANL code
- (nearly) locally isothermal
- disk self-gravity (but gravitationally stable)
- **no perturber (e.g., no planet for now)**

Why massive and eccentric disk?

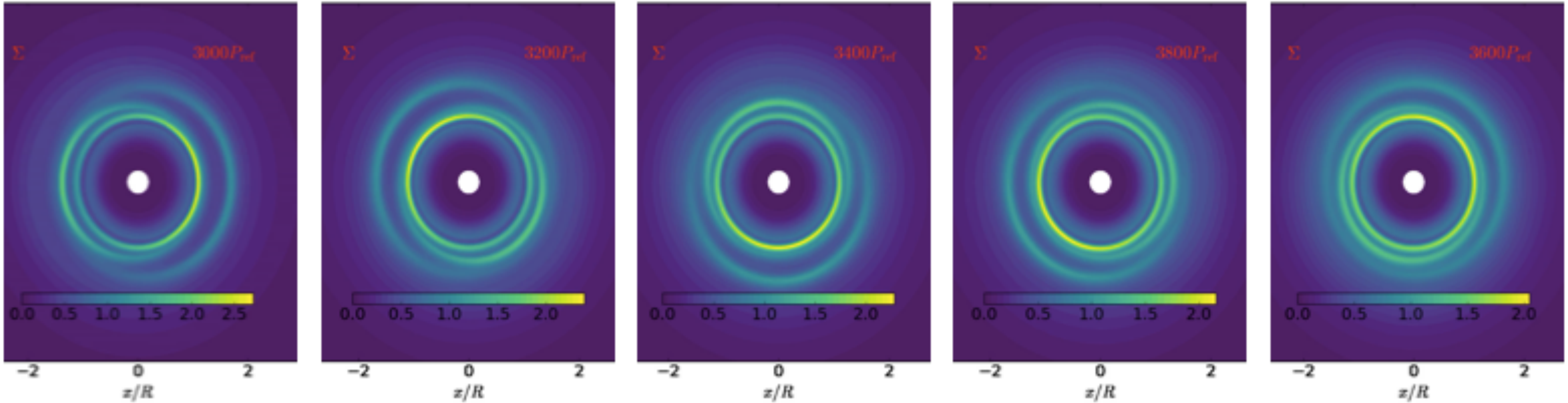


Spontaneous growth of disk eccentricity. (*Li+ 2021*)

High-res 2D hydrosimulation

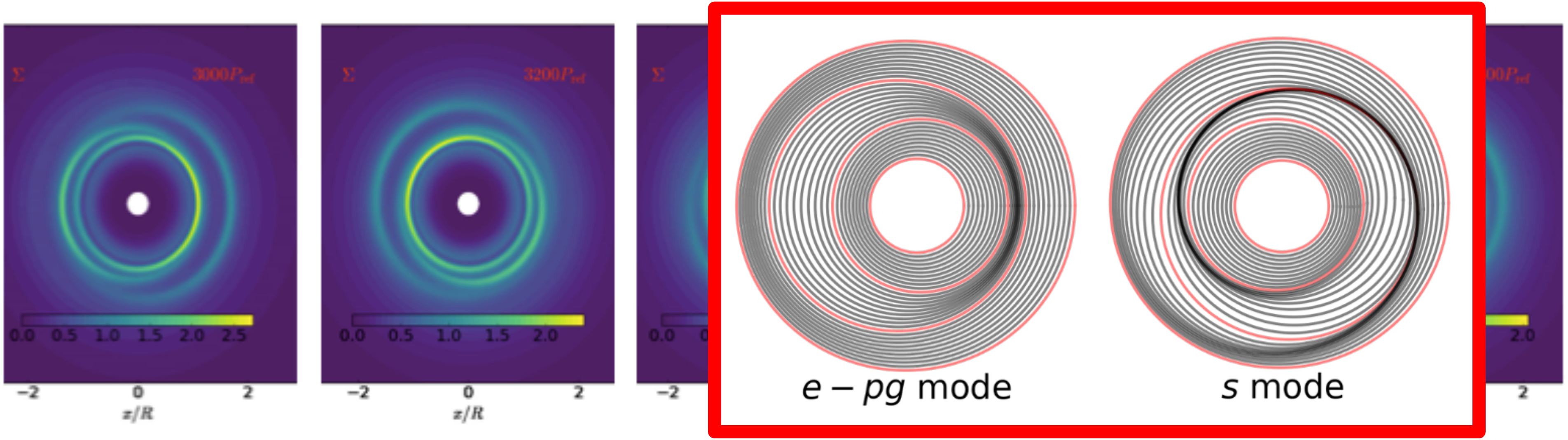
- LANL code
- (nearly) locally isothermal
- disk self-gravity (but gravitationally stable)
- **no perturber (e.g., no planet for now)**

Spontaneous emergence of disk eccentricity



Coherent disk precession like a rigid body; the pattern and precession rate can be calculated by a linear mode theory. (*Lee+ 2019*, ***Li****+ 2021*)

Spontaneous emergence of disk eccentricity



Coherent disk precession like a rigid body; the pattern and precession rate can be calculated by a linear mode theory. (*Lee+ 2019*, ***Li****+ 2021*)

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (*Teyssandier & Lai 2019*)

$$\begin{aligned}\frac{dE_{\rm d}}{dt} &= i(\omega_{\rm d,m}+\omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}\\ \frac{dE_{\rm p}}{dt} &= -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}\end{aligned}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (*Teyssandier & Lai 2019*)

disk precession frequency

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (*Teyssandier & Lai 2019*)

mutually induced precession rate

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r, t; M_d)$ has the same "shape" as the e mode of the disk

$$E(r, t; M_{\rm d}) = E_{\rm m}(r; M_{\rm d}) E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (*Teyssandier & Lai 2019*)

eccentricity coupling rate

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p}) E_{\rm d} - i\nu_{\rm d,p} E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d} E_{\rm d} + i\omega_{\rm p,d} E_{\rm p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (*Teyssandier & Lai 2019*)

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

$$\boxed{\begin{aligned}
\omega_{\rm d,p} &= \frac{1}{J_{\rm d}}\int GM_{\rm p}\Sigma K_1(r,a_{\rm p})|E_{\rm m}|^2 2\pi r dr \\
\omega_{\rm p,d} &= \frac{1}{J_{\rm p}}\int GM_{\rm p}\Sigma K_1(r,a_{\rm p}) 2\pi r dr \\
\nu_{\rm d,p} &= \frac{1}{J_{\rm d}}\int GM_{\rm p}\Sigma K_2(r,a_{\rm p})E_{\rm m} 2\pi r dr \\
\nu_{\rm p,d} &= \frac{1}{J_{\rm p}}\int GM_{\rm p}\Sigma K_2(r,a_{\rm p})E_{\rm m} 2\pi r dr
\end{aligned}}$$

Fiducial numerical example

Result of the long-term time evolution in the fiducial system:

- Planet: inside the cavity

$$a_{\mathrm{p}} = 0.2 r_0, \ M_{\mathrm{p}} = 3 \times 10^{-4} M_\odot$$

- Disk: gradually losing mass

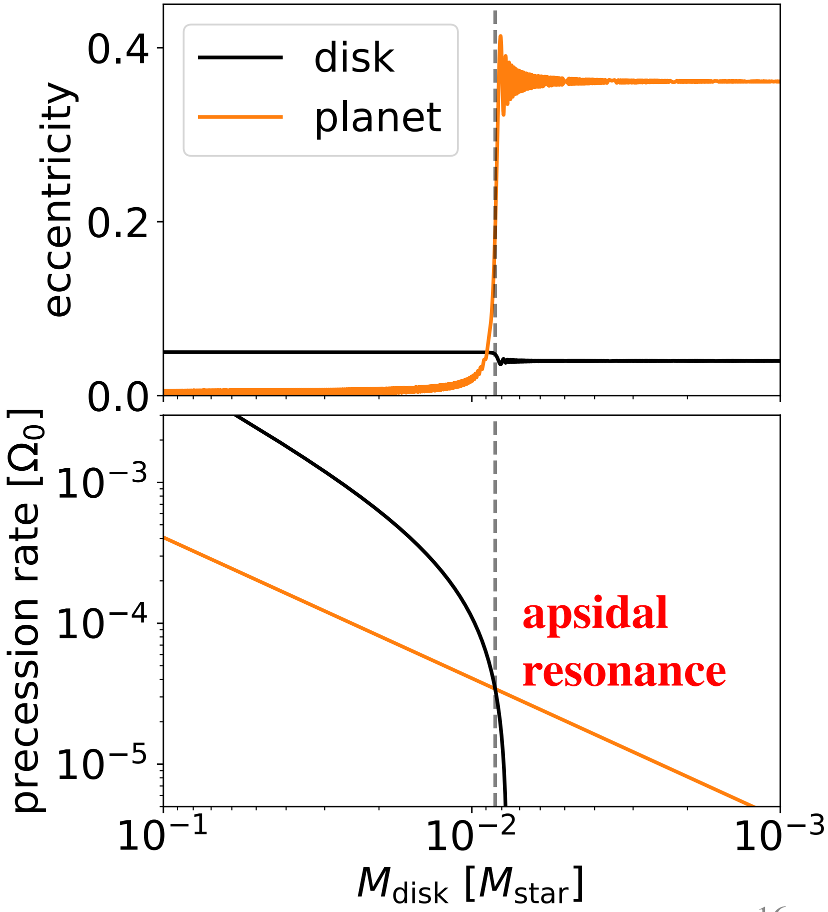
$$M_{\mathrm{d}}(t) = \frac{M_{\mathrm{d},0}}{1 + t/\tau_{\mathrm{d}}}, \quad \tau_{\mathrm{d}} = 10^5 P_0$$

$$M_{\mathrm{d},0} = 0.1 M_\star = 0.1 M_\odot$$

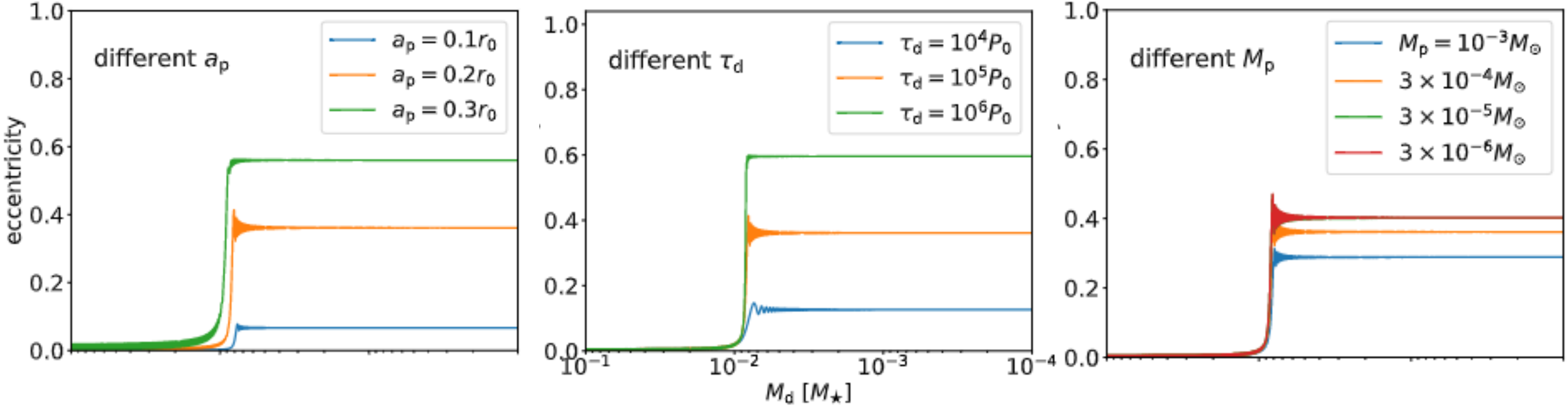
- With equations of motion

$$\frac{dE_{\mathrm{d}}}{dt} = i(\omega_{\mathrm{d,m}} + \omega_{\mathrm{d,p}}) E_{\mathrm{d}} - i\nu_{\mathrm{d,p}} E_{\mathrm{p}}$$

$$\frac{dE_{\mathrm{p}}}{dt} = -i\nu_{\mathrm{p,d}} E_{\mathrm{d}} + i\omega_{\mathrm{p,d}} E_{\mathrm{p}}$$



Numerical examples: parameter study



Analysis

- We may transfer the eccentricity equation

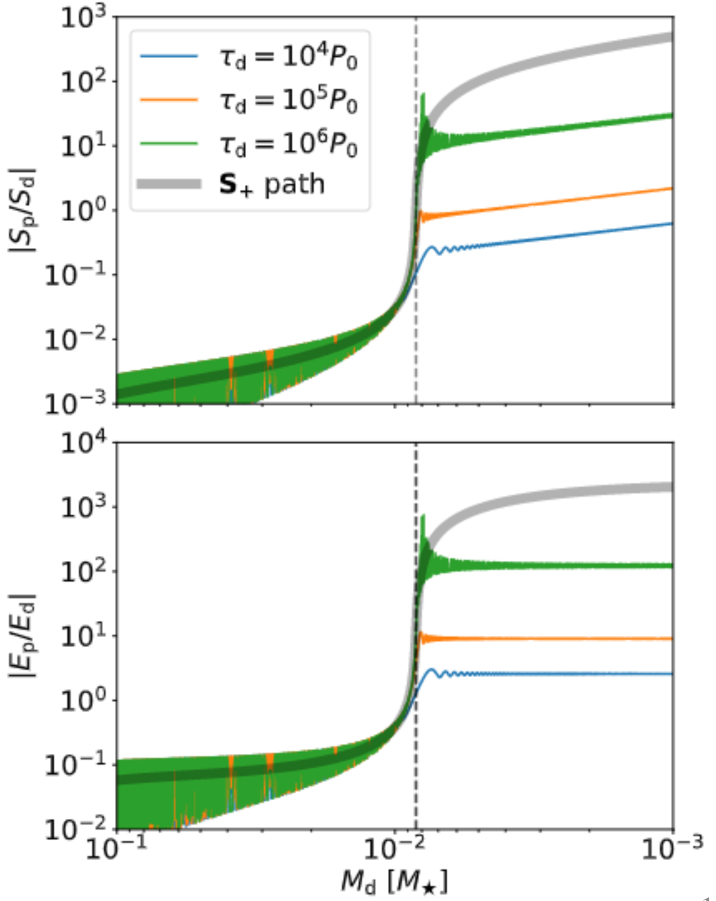
$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$
$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

- in to an equation for angular momentum deficit (AMD) evolution:

$$\frac{d}{dt}\begin{pmatrix} S_{\rm d} \\ S_{\rm p} \end{pmatrix} = i\begin{pmatrix} \omega + \Delta\omega + i\frac{1}{2\tau_{\rm J}} & -\nu \\ -\nu & \omega - \Delta\omega \end{pmatrix}\begin{pmatrix} S_{\rm d} \\ S_{\rm p} \end{pmatrix}$$

- We can find their eigenstates.

resonance



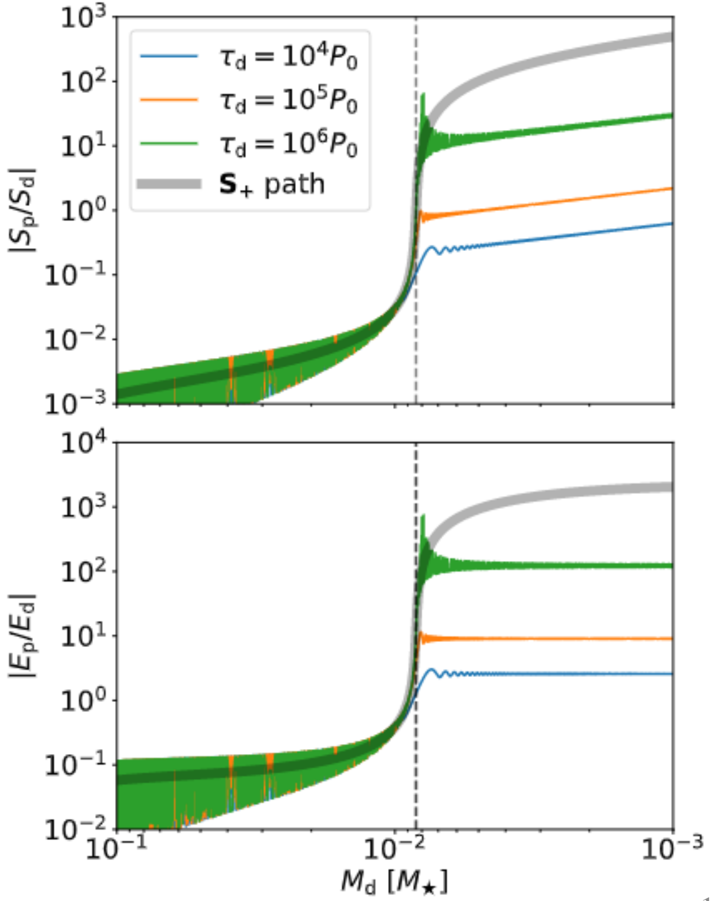
Analysis

- If the disk loses mass with time slowly, the eigenstate also evolves slowly (from $E_d > E_p$ to $E_p >> E_d$).

- By adiabatic theorem, the eccentricities of the planet and the disk evolves accordingly to remain in the eigenstate.

- The eigenstate path naturally guide the system to a high-planet-eccentricity configuration.

resonance

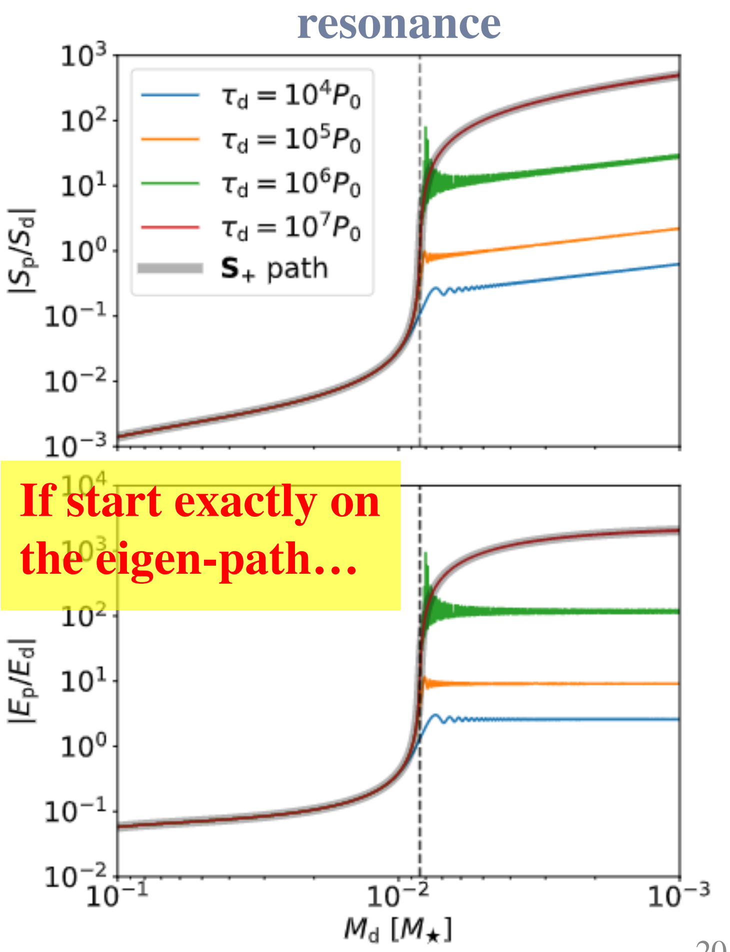


Analysis

- If the disk loses mass with time slowly, the eigenstate also evolves slowly (from $E_d > E_p$ to $E_p >> E_d$).

- By adiabatic theorem, the eccentricities of the planet and the disk evolves accordingly to remain in the eigenstate.

- The eigenstate path naturally guide the system to a high-planet-eccentricity configuration.



Summary

- Our previous work shows that eccentric mode instability allows massive disks to become eccentric.

- As the disk gravity becomes weaker, the disk and its companion inner planet may encounter an apsidal resonance, which can significant excite the planetary eccentricity.

- The resonant excitation of planetary eccentricity can be explained by an simple eigenstate analysis.

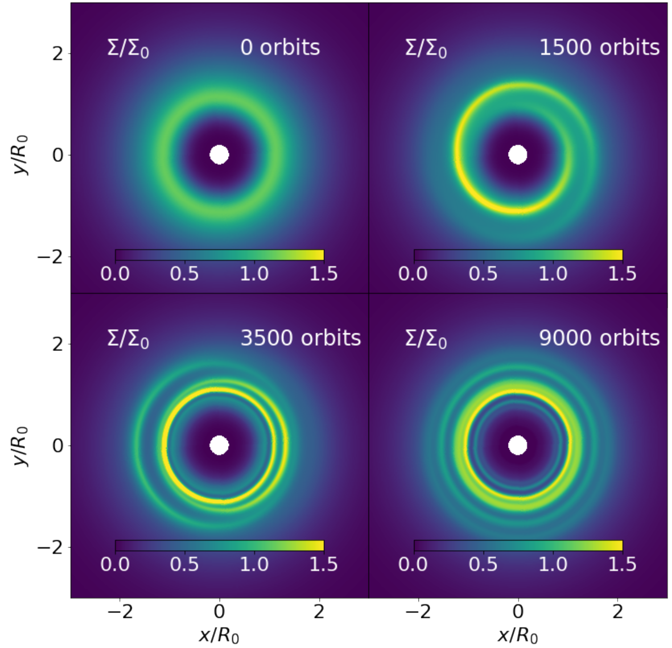
Related papers

- **[Ring Formation in Protoplanetary Disks Driven by an Eccentric Instability]**
  Jiaru Li, Adam Dempsey, Hui Li, and Shengtai Li, *ApJ* 910, 79, 2021

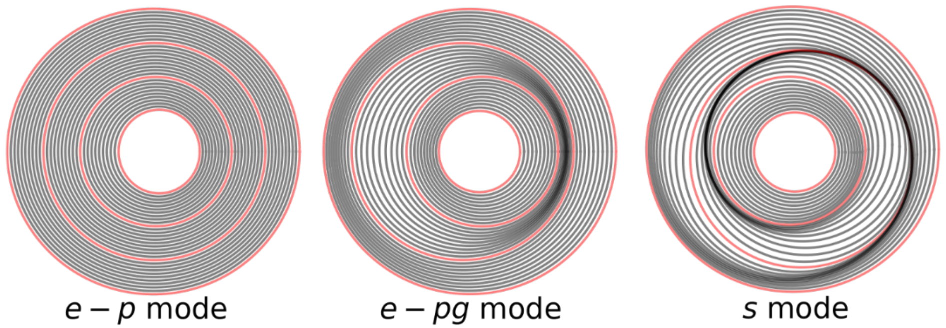
- **[Resonant Excitation of Planetary Eccentricity due to a Dispersing Eccentric Protoplanetary Disk: a New Mechanism of Generating Large Planetary Eccentricities]**
  Jiaru Li and Dong Lai, *submitted (arXiv:2211.07305)*

- Backup slides..

Spontaneous emergence of disk eccentricity



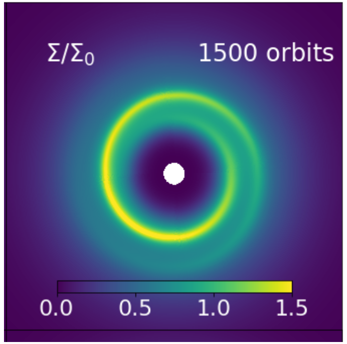
Ring and gap formations driven by the eccentric mode instability. (***Li****+ 2021*)

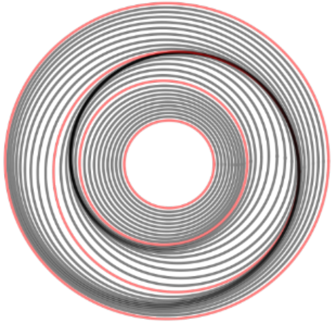


Eccentric mode instability (EMI):

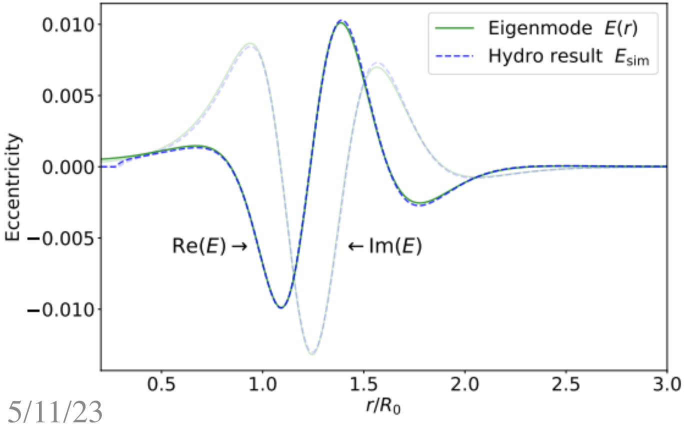
- the s mode grows exponentially by itself (*Lin 2015, Lee+ 2019*);
- the disk transform into other e modes after saturation (***Li****+ 2021*).

EMI analysis: hydro simulation vs linear theory

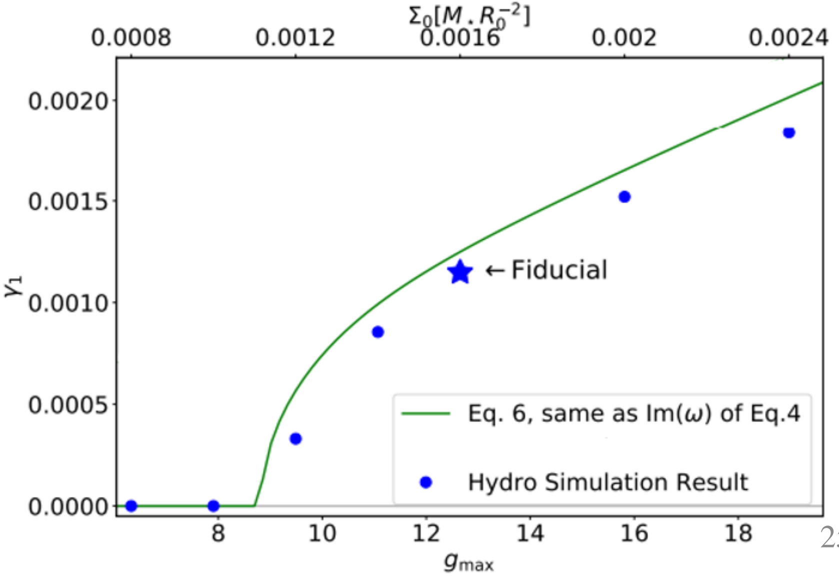




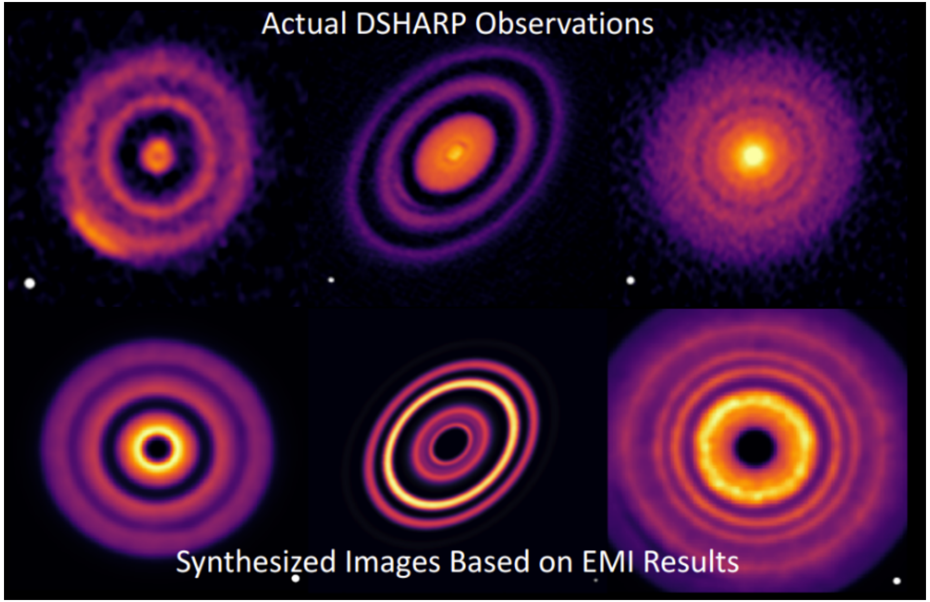
s mode



*The linear mode (theory, Lee+2019; **Li**+ 2021) matches the hydro simulation result (numerical, **Li**+ 2021) precisely.*



Formation of ALMA rings (*Li+ 2021, Li+ in prep*)



A new mechanism to form ALMA rings

1. Start with a ringless disk and **no planet.**
2. As EMI saturates, large E drives **radial mass transfer** (non-linear coupling).
3. Rings may be circularized by viscosity (or boundary effects).

Synthesized images:

1. Run hydro simulations with gas and **dust**.
2. Use **radiative transfer code (RADMC-3D)** to get synthetic images.