Testing a One Population Proportion

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Hypotheses

$H_0 : p = 0.52$

$H_a : p > 0.52$

Best Estimate of p is $\hat{p} = 0.56$

Where p is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

$\alpha = 0.05$

Test Statistic

Best estimate - Hypothesized estimate

Standard error of estimate

$$\frac{\hat{p} - p_o}{s.e.}$$

$$s.e.(\hat{p}) = \sqrt{\frac{p \cdot (1-p)}{n}} \quad \rightarrow \quad s.e.(\hat{p}) = \sqrt{\frac{p_0 \cdot (1-p_o)}{n}}$$

Test Statistic

$$\frac{\hat{p} - p_o}{s.e.} \qquad \text{Null } s.e.(\hat{p}) = \sqrt{\frac{p_0 \cdot (1 - p_o)}{n}}$$

$$Z = \frac{0.56 - 0.52}{0.0157}$$

Z = 2.555

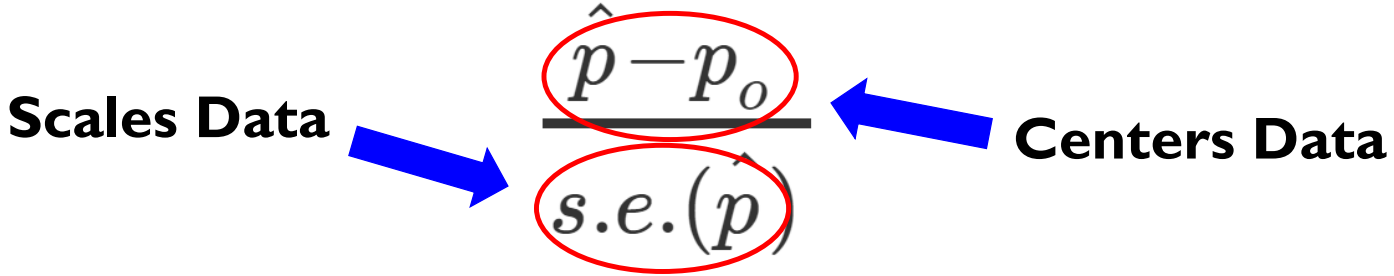
Test Statistic Interpretation

$$Z = 2.555$$

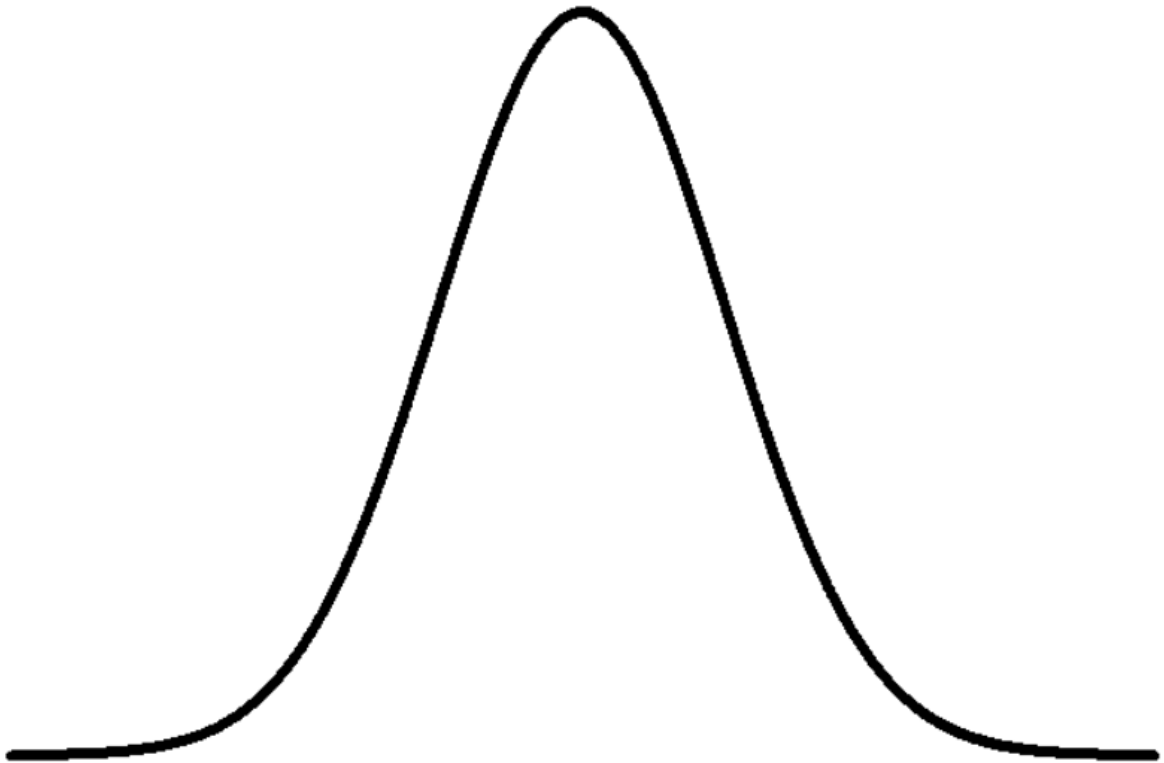
That means that our observed sample proportion is 2.555 null standard errors above our hypothesized population proportion

Test Statistic Distribution

- A Z test statistic is another random variable! It has a distribution.
- The Z test statistic will always follow a N(0,1)
- This is due to us centering and scaling our original data



The P-Value



Conclusions

p-value = 0.0053 < α = 0.05

Reject the null hypothesis (H_0: p = 0.52)

There is sufficient evidence to conclude that the population proportion of parents with a teenager who believe that electronics and social media is the cause for lack of sleep is greater than 52%.

Summary

- 4 main steps to a hypothesis test
  - Stating hypothesis & select significance level (α)
  - Checking assumptions
  - Calculating a test statistic and getting a p-value from the test statistic
  - Drawing a conclusions from the p-value
- The Z test statistic distribution is N(0,1)