

Comparing Proportions for Two Independent Samples: An Example

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Example: Comparing Proportions in Two Groups

Research Question:

Considering elderly Hispanic adults (age 80+) living in the U.S. in 2015-2016, did the **proportions of males and females who smoked vary significantly?**

Inference Approaches:

- Form a confidence interval for the difference in the two proportions
- Perform a chi-square test to test the significance of the difference in the two population proportions
- Be sure to check assumptions!

Approach 1: Form a Confidence Interval

- **Males:** Proportion = 0.565, $n = 16$ (small sample!)
- **Females:** Proportion = 0.250, $n = 32$

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Males: Proportion = 0.565, $n = 16$ (small sample!)

Females: Proportion = 0.250, $n = 32$

- **Best Point Estimate:** Difference in sample proportions is
$$0.565 - 0.250 = 0.315$$
- **Interpretation:** In 2015-2016, we estimate that the percentage of all **male** elderly Hispanics who smoked was 31.5 percentage points *higher* than for all **female** elderly Hispanics

Approach 1: Form a Confidence Interval

Males: Proportion = 0.565, $n = 16$ (small sample!)

Females: Proportion = 0.250, $n = 32$

Assumptions:

1. Is sampling distribution of difference in sample proportions normal? **Unlikely...** *small sample size*
2. Do we have at least 10 outcomes (smoker/non-smoker) in each group? **No...**
3. Are the two samples independent? **Yes**
4. Are the observations independent? **Yes, assumed for now...**

Approach 1: Form a Confidence Interval

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

- Calculate **estimated standard error of the difference**:

$$\text{Estimated SE} = \sqrt{\frac{0.565(1-0.565)}{16} + \frac{0.250(1-0.250)}{32}} = \mathbf{0.146}$$

- Determine the “few”: a critical value for a 95% CI: **z = 1.96**
- Add and subtract margin of error from best estimate of the difference:

$$\mathbf{0.315 \pm 1.96 \times 0.146}$$

- 95% CI for difference in population proportion is **(0.027, 0.598)**
- Interval doesn't include 0 → **Significant difference!**

Approach 1: Check Robustness

Given concerns about some assumptions due to small sample sizes,
can compute an **exact 95% confidence interval**
for the difference in population proportions
(more computationally intensive)

Resulting exact 95% confidence interval: **(0.015, 0.574)**

Similar result, but evidence is not as overwhelming!

Approach 2: Chi-square Test

- **Null:** Equal population proportions of elderly male and female Hispanics who smoke
- **Alternative:** Males and Females have different population proportions who smoke

Alternative allows male proportion to be *either* greater or less than female proportion
→ **two-tailed test!** Need more evidence against null hypothesis to reject it!

**Significance
Level = 5%**

Approach 2: Chi-square Test

Assumptions:

- Are all **expected counts** for each cell of the 2 x 2 table *under the null hypothesis* **greater than 5**? Δ *χ^2 test requires at 5 counts in each cell*
- **Yes:** if overall sample rate of smokers, $\frac{17}{48} = 0.354$, is applied to each sample size of 16 (males) and 32 (females)
 - expect about 6 males and about 11 females to be smokers!
- **Note: we assume independent observations for now!**

Approach 2: Chi-square Test

Resulting test statistic and p-value:

$$\chi^2 = 4.554, df = 1, p\text{-value} = 0.033$$

Reject null hypothesis →

support the population proportions of smokers are **different!**

Note: if had initially selected 1% significance level

(Type I error rate = 0.01),

we would have **failed to reject** the null hypothesis!

Evidence is not overwhelming.

Chi-square Test: Check Robustness

Consider **Fisher's Exact Z Test** given small sample sizes...

p-value is 0.054 →

do not have overwhelming evidence against the null hypothesis

Overall Conclusion

Conclusion: We have weak evidence of a significant difference in the population proportions of smokers for elderly male and female Hispanics living in the U.S. in 2015-2016
(small sample sizes and limited statistical power)

Notes:

1. If same difference in proportions (0.315) were to emerge with larger sample sizes in each group, would likely find it significant, regardless of significance level → big difference in reality!
2. Need to allow for larger degree of uncertainty in our analysis due to small samples sizes.