



UNIVERSITY OF  
MICHIGAN

# Estimating a Population Proportion With Confidence

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Statistics Course Developer



# Confidence Interval Basics

**Best Estimate  $\pm$  Margin of Error**

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**Best Estimate = Unbiased Point Estimate**

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**Margin of Error** = *“a few”* Estimated Standard Errors

# Confidence Interval Basics

**Best Estimate  $\pm$  Margin of Error**

**Best Estimate** = Unbiased Point Estimate

**Margin of Error** = “a few” Estimated Standard Errors

“a few” = multiplier from appropriate distribution  
based on desired confidence level and sample design

**95% Confidence Level  $\leftrightarrow$  0.05 Significance**

# C.S. Mott Children's Hospital

A hospital based in Ann Arbor MI, C.S. Mott Children's Hospital frequently conducts national polls on children's health. We will be looking at an example of children's safety precautions that parents use when driving.

# MottsPoll Background

**What proportion of parents report they use a car seat for all travel with their toddler?**

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**Population** - Parents with a toddler



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**Parameter of Interest** - A Proportion

# MottsPoll Background

**What proportion of parents report they use a car seat for all travel with their toddler?**

**Population** - Parents with a toddler

**Parameter of Interest** - A Proportion

**Construct a 95% Confidence Interval for the population proportion of parents reporting they use a car seat for all travel with their toddler**

# MottsPoll Background

A sample of 659 parents with a toddler was taken and asked if they used a car seat for all travel with their toddler.

540 parents responded 'yes' to this question.

# 95% Confidence Interval Calculations

**Best Estimate  $\pm$  Margin of Error**

# 95% Confidence Interval Calculations

**Best Estimate  $\pm$  Margin of Error**

**$\hat{p} \pm$  Margin of Error**

Best Estimate =  $\hat{p}$

**Sample Size**



$n = 659$

**Number  
Responded “yes”**



$x = 540$

$\hat{p} = x/n$

$= 540/659$

$=$ ~~0.85~~ 0.81

# 95% Confidence Interval Calculations

**Best Estimate  $\pm$  Margin of Error**

**$\hat{p} \pm$  Margin of Error**

# 95% Confidence Interval Calculations

**Best Estimate  $\pm$  Margin of Error**

**$\hat{p} \pm$  Margin of Error**

**$\hat{p} \pm \text{“a few”} \cdot \text{estimated se}(\hat{p})$**



# 95% Confidence Interval Calculations

**Best Estimate  $\pm$  Margin of Error**

**$\hat{p} \pm$  Margin of Error**

**$\hat{p} \pm$  “a few”  $\cdot$  estimated se( $\hat{p}$ )**

**$\hat{p} \pm 1.96$**

$$\cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

# 95% Confidence Interval Calculations

$$\hat{p} \pm 1.96 \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

0.81

~~$\hat{p}=0.85$~~

$n=659$

# 95% Confidence Interval Calculations

$$\hat{p} \pm 1.96 \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$0.85 \pm 1.96 \cdot \sqrt{\frac{0.85(1 - 0.85)}{659}}$$

$$0.85 \pm 0.0273$$

$$(0.8227, 0.8773)$$

$$\hat{p}=0.85$$

$$n=659$$

# Interpreting the Confidence Interval

**“range of reasonable values for our parameter”**

With 95% confidence, the population proportion of parents with a toddler who use a car seat for all travel is **estimated** to be between 82.27% - 87.73%.

# Summary

- Confidence Intervals are used to give an *interval* estimate for our parameter of interest
- Center of the Confidence Interval = our best estimate
- Margin of Error (MoE) = “a few” (estimated) standard errors  
 $z^*$