Descriptive Inference Examples for Single Variables Using Hypothesis Testing

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Example 1: One-Sample Tests for Proportions

Research Question:

Did 33% (one-third) of non-Hispanic African-Americans age 18+ in U.S. in 2015-2016 have systolic blood pressure greater than 130 mmHg, *or* was population proportion different than one-third?

Inference Approach:

Perform a one-sample test (two-tailed) to either **reject** or **fail to reject** null hypothesis: population proportion = 0.33

Step 1: Define the Null and Alternative

- Null: Population proportion (p) is 0.33
- Alternative: Population proportion is not equal to 0.33

Alternative allows proportion to be *either* greater or less than 0.33 → **two -tailed test** need more evidence against null hypothesis to reject it!

Significance Level = 5%

Step 2: Compute the Test Statistic

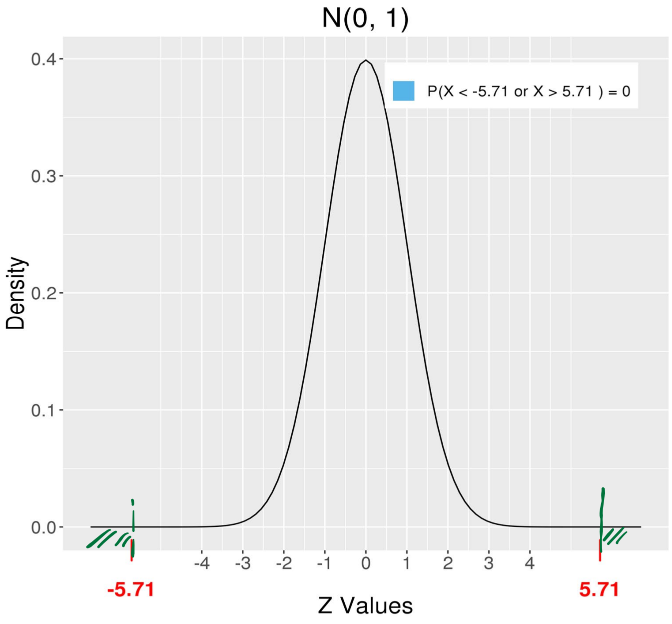
- **Best Point Estimate:** Assuming simple random sample of black adults, sample proportion is $\frac{465}{1135} = 0.4097$
- **Test Statistic:** Assuming sampling distribution of estimated proportion is normal,

$$z = \frac{\text{best estimate} - \text{null value}}{\text{null standard error}} = \frac{0.4097 - 0.33}{\sqrt{\frac{0.33(1-0.33)}{1135}}} = \mathbf{5.71}$$

Estimate is *more than* 5 standard errors from 0.33 null value

Step 3: Determine P-Value

- If null hypothesis was true, would a test statistic value of z = 5.71 be considered unusual enough to reject the null?
- **P-value** = Probability of seeing test statistic of 5.71 or *more extreme* assuming the null hypothesis is true.
- If null hypothesis true, Z follows "standard" normal distribution, and two-tailed test → more extreme measured in both tails.



P-value ≅0

Step 4: Make a Decision about the Null

- If population proportion really was 0.33, then observing a sample proportion of 0.4097 or more extreme is not likely.
- Since our P-value is much less than 0.05 significance level, strong evidence against the null → we **reject the null!**

Based on estimated proportion (0.4097),
we **support** the population proportion is not 0.33
(and likely larger)

Example 2: One-Sample Tests for

Research Question:

Was the **mean** systolic blood pressure for non-Hispanic African-Americans age 18+ in U.S. in 2015-2016 equal to 128 mmHg or was the population mean different from 128?

Inference Approach:

Perform a **one-sample test** (two-tailed!) to either **reject** or **fail to reject** null hypothesis: population mean = 128 mmHg

Step 1: Define the Null and Alternative

- Null: Population mean (μ) is 128 mmHg
- Alternative: Population mean is **not equal** to 128 mmHg

Alternative allows mean to be *either* greater or less than 128 mmHg → **two-tailed test** need more evidence against null hypothesis to reject it!

Significance Level = 5%

Step 2: Compute the Test Statistic

- **Best Point Estimate:** Assuming simple random sample of black adults, sample mean is 128.252 mmHg
- **Test Statistic:** Assuming sampling distribution of estimated mean is normal,

$$t = \frac{\textit{best estimate} - \textit{null value}}{\textit{estimated std error}} = \frac{128.252 - 128}{\frac{19.958}{\sqrt{1135}}} = \mathbf{0.425}$$

Estimate is ***less than*** ½ standard error **from 128 null value**

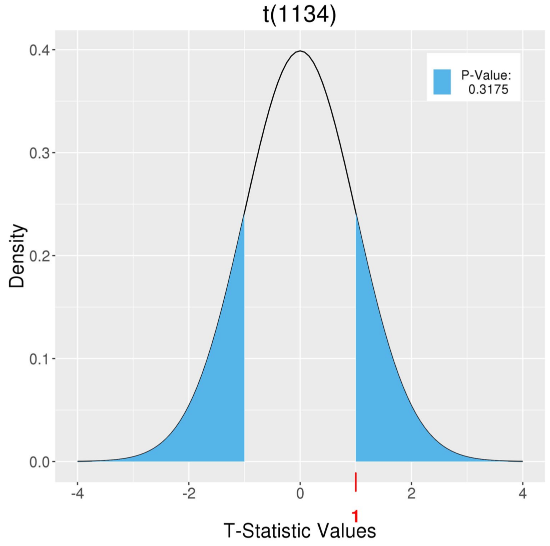
$sd = \frac{s}{\sqrt{n}}$

Step 3: Determine P-Value

- If null hypothesis was true, would a test statistic value of only t = 0.425 be unusual enough to reject the null?
- **P-value** = Probability of seeing test statistic of 0.425 or *more extreme* assuming the null hypothesis is true.
- If null hypothesis was true, t follows a Student t distribution with degrees of freedom n – 1 = 1134, and two tailed test

→ More extreme measured in both tails.

P-value = 0.3175



Step 4: Make a Decision about the Null

- If population mean really was 128 mmHg then observing a sample mean of 128.252 or more extreme is quite likely.
- Since our P-value is much bigger than 0.05 significance level, weak evidence against the null → we **fail to reject the null!**

Based on estimated mean (128.252 mmHg),
we ***cannot support***
the population mean differs from 128 mmHg

What's Next?

How to make inferences about
differences between subgroups

- **Confidence intervals** for differences in means and proportions
- **Hypothesis testing** for comparing means and proportions