



UNIVERSITY OF  
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# Understanding Confidence Intervals

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# Understanding Confidence Intervals

- How to **interpret** confidence **intervals**?
- What does that **confidence level** really mean?
- What if we want to be **99% confident** instead?

## Car Seats for Toddlers Example

In a sample of 659 parents with a toddler, 540 (or **85%**) stated they **use a car seat** for all travel with their toddler.



95% confidence interval:  
**(0.8227, 0.8773) or about 82.3% to 87.7%**

Confidence Interval for \_\_\_\_ ??? \_\_\_\_

We make a confidence interval for a parameter.

parameter

OR

statistic

## Car Seats for Toddlers Example



**(0.8227, 0.8773)** is a confidence interval for the **POPULATION PROPORTION** of all parents with toddlers who report they use a car seat for all travel with their toddler

Just reporting interval with **good context**  
**Improve?** more of interpretation that conveys  
is an estimate based on data, with confidence  
level

# Interpreting the Confidence Interval

We estimate, **with 95% confidence**, the population proportion of parents with toddlers who report they use a car seat for all travel with their toddler is somewhere between 0.8227 and 0.8773.

**OR**

Based on our sample of 659 parents with toddlers, **with 95% confidence**, we estimate between 82.3% and 87.7% of all such parents report they use a car seat for all travel with their toddler

## Think About It ...

Does our confidence interval of  $(0.8227, 0.8773)$  contain the ***sample proportion*** of parents with toddlers who report they use a car seat for all travel with their toddler?

**Yes**, it most certainly does ... our interval is centered at that sample proportion of 0.85 or 85%.

## Think About It ...

Does our confidence interval of  $(0.8227, 0.8773)$  contain the ***population proportion*** of parents with toddlers who report they use a car seat for all travel with their toddler?

**We Don't Know...**



# **Wrong** Understanding of Confidence Level

95% chance or probability  
that the population proportion is in  
this already computed interval of (0.8227, 0.8773)

*the population proportion is fixed*

# Correct Understanding of Confidence Level

95% confidence level refers to our confidence in the (statistical procedure) that was used to make this interval

# Understanding Confidence Level

**True Proportion**

0.01 0.5 0.99

0.01 0.11 0.21 0.31 0.41 0.51 0.61 0.71 0.81 0.91 0.99

Note: We generally do not know the True Proportion but we get to set the value here to see how well the confidence interval estimation process works

**Enter Your Sample Size ( $\geq 10$ )**

25

**What Confidence Level do you want to use?**

0.9 0.95 0.99

0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99

**How Many Confidence Interval Should We Make?**

10 100 200

10 29 48 67 86 105 124 143 162 181 200

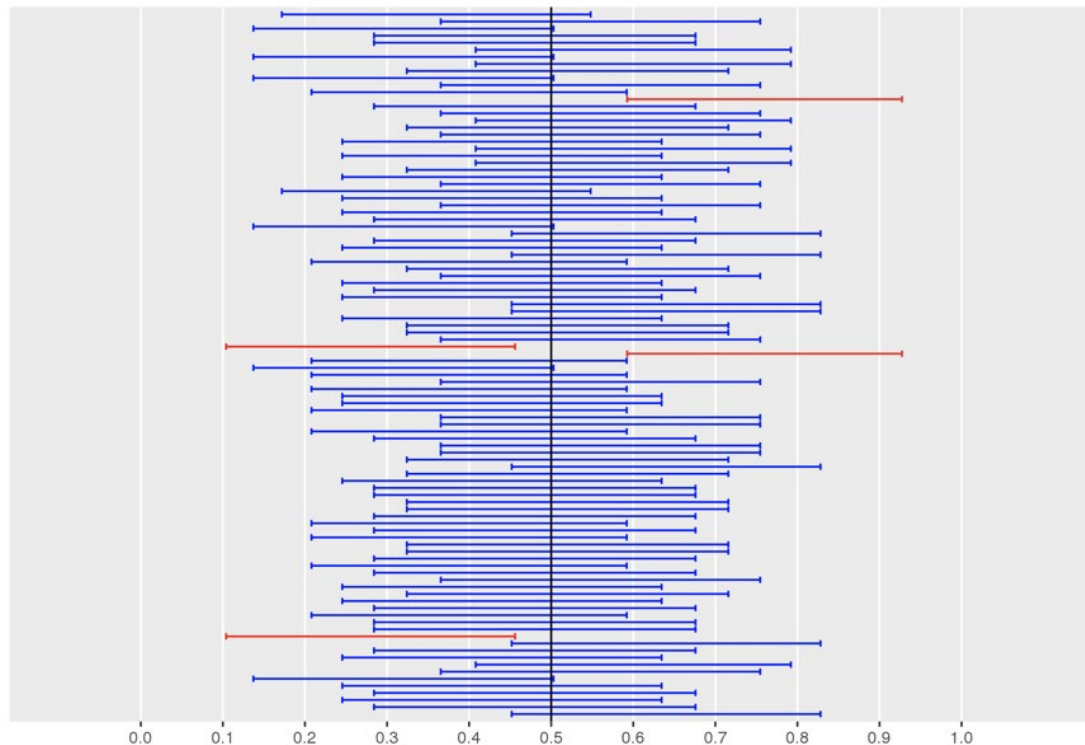
**Create Confidence Intervals**

Population Proportion = 0.50

Take 100 samples each of size 25

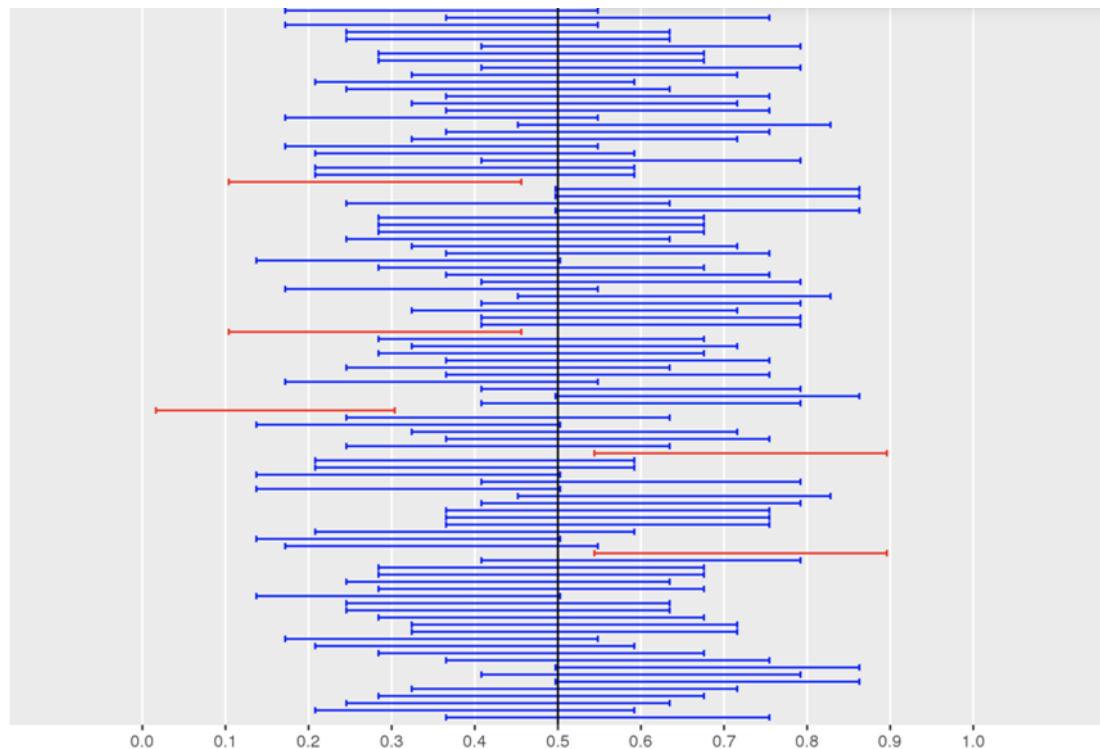
For each sample,  
create a 95% confidence  
interval for the population  
proportion

# Understanding Confidence Level



**96** of these **100**  
generated  
intervals  
**did** contain the true  
proportion of **0.5**  
while **4** did not.

# Understanding Confidence Level




**95** of these **100**  
generated  
intervals  
**did** contain the true  
proportion of **0.5**  
while **5** did not.

# Understanding Confidence Level



With a **95%** confidence level, we would expect (in the long run) about **95%** of the intervals to contain the true proportion.

## Different Z Multipliers



| 90%   | 95%  | 98%   | 99%   |
|-------|------|-------|-------|
| 1.645 | 1.96 | 2.326 | 2.576 |

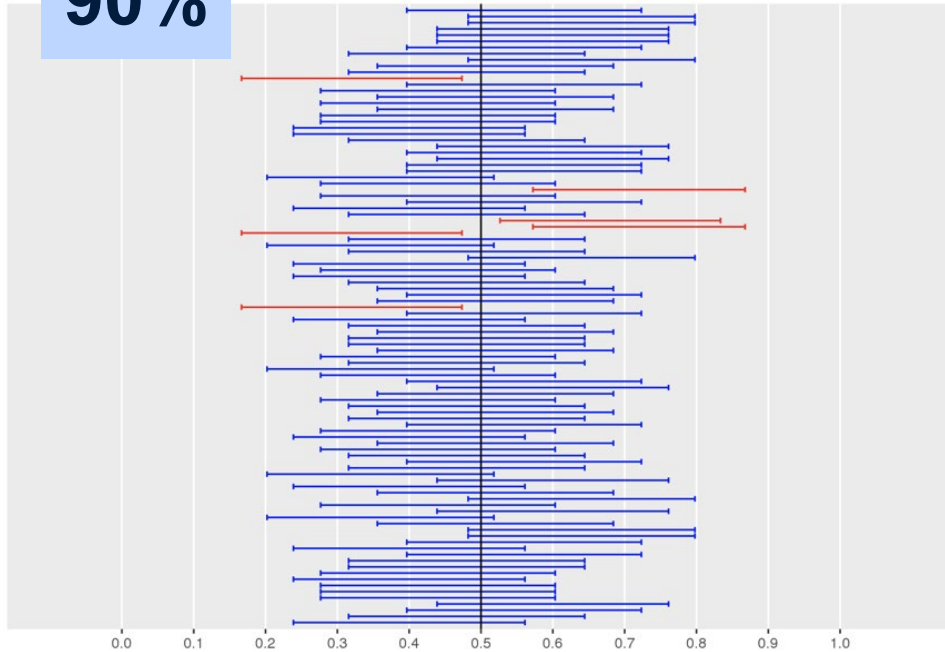
**Best Estimate  $\pm$  Margin of Error**

**Best Estimate  $\pm$  “a few” (estimated) standard errors**

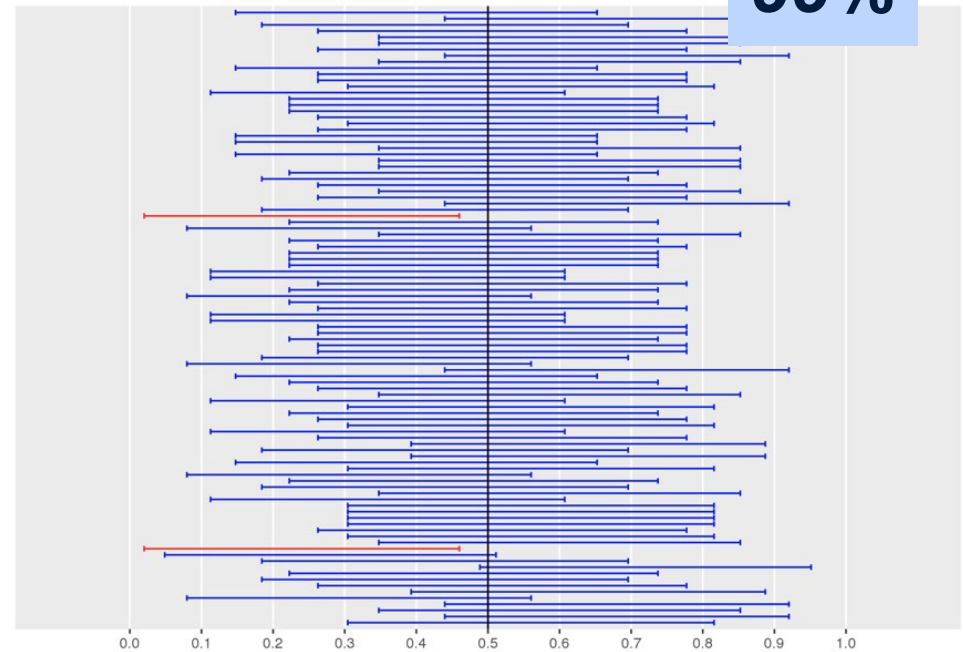
More confident  $\rightarrow$  Larger Multiplier  $\rightarrow$  Wider Interval

# Changing Confidence Level

90%



99%





# Car Seats for Toddlers Example

In a sample of 659 parents with a toddler, 540 (or **85%**) stated they **use a car seat** for all travel with their toddler.



**90% CI:**

**$0.85 \pm 0.0229$**

**82.7% to 87.3%**

**95% CI:**

**$0.85 \pm 0.0273$**

**82.3% to 87.7%**

**99% CI:**

**$0.85 \pm 0.0358$**

**81.4% to 88.6%**

# Understanding Confidence Intervals

- We know how to **interpret** confidence **intervals**
- We understand what that **confidence level** really means
- We have options for the desired **confidence level**