Conservative Approach & Sample Size Consideration

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95% Confidence Intervals for p

Best Estimate ± Margin of Error

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Best Estimate ± Margin of Error

Best Estimate ± “a few” (estimated) standard errors

z^*

2 Line Segment Title

95% Confidence Intervals for p

Best Estimate ± Margin of Error

Best Estimate ± "a few" (estimated) standard errors

$$\hat{p} \pm 1.96 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Motts Car Seat Example

In a sample of 659 parents with a toddler, 540 (or **85%**) stated they **use a car seat** for all travel with their toddler.

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p̂=0.85

n=659

2 Line Segment Title

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$$\hat{p} \pm 1.96 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

p̂=0.85

n=659

(0.823, 0.877)

Closer Look at estimated SE

$$\text{estimated standard error} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Closer Look at estimated SE

estimated standard error = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

What if **p̂ is not accurate?**

Closer Look at estimated SE

estimated standard error = $\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

What if **p̂ is not accurate?**

which is maximized when **p̂=0.5**

conservative standard error = $\dfrac{1}{2\sqrt{n}}$

Back to Motts Car Seat Example

$$\hat{p} \pm \overset{1.96}{2} \cdot \frac{1}{2\sqrt{n}}$$

$\hat{p}=0.85$

$n=659$

Back to Motts Car Seat Example

$$\hat{p} \pm \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{n}}$$

$$\hat{p} \pm \frac{1}{\sqrt{n}}$$

$\hat{p}=0.85$

$n=659$

Back to Motts Car Seat Example

$\hat{p} = 0.85$

$n = 659$

$$\hat{p} \pm 2 \cdot \frac{1}{2\sqrt{n}}$$

$$\hat{p} \pm \frac{1}{\sqrt{n}}$$

95% Margin of Error is only dependent on sample size

Back to Motts Car Seat Example

p̂=0.85

n=659

$$\hat{p} \pm 2 \cdot \frac{1}{2\sqrt{n}}$$

95% Margin of Error is only dependent on sample size

$$\hat{p} \pm \frac{1}{\sqrt{n}}$$

(0.81, 0.89)

Conservative 95% Confidence Interval, 4% Margin of Error

Sample Size Determination

Margin of Error (MoE) is only dependent on:

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1) our confidence level (typically 95%) and

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1) our confidence level (typically 95%) and
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What sample size would we need to have a 95% *(conservative)* confidence interval with a Margin of Error of only 3% (0.03)?

Sample Size Determination

$$\text{MoE} = \frac{1}{\sqrt{n}}$$

For 95% Confidence

Sample Size Determination

For 95% Confidence

$$\text{MoE} = \frac{1}{\sqrt{n}}$$

MoE = 0.03

$$n = (1/\text{MoE})^2$$

Sample Size Determination

For 95% Confidence

$$\text{MoE} = \frac{1}{\sqrt{n}}$$

MoE = 0.03

$$n = (1/\text{MoE})^2$$

$$n = (1/0.03)^2$$

$$n = 1{,}111.11$$

Sample Size Determination

For 95% Confidence

$$\text{MoE} = \frac{1}{\sqrt{n}}$$

MoE = 0.03

$$n = (1/\text{MoE})^2$$

$$n = (1/0.03)^2$$

$$n = 1{,}111.11$$

$$n \geq 1{,}112$$

What if 3% MoE @ 99% Confidence?

$$\hat{p} \pm Z^* \cdot \frac{1}{2\sqrt{n}}$$

$p = 0.5$

What if 3% MoE @ 99% Confidence?

$$\hat{p} \pm \boxed{Z^* \cdot \frac{1}{2\sqrt{n}}}$$

What if 3% MoE @ 99% Confidence?

$$\hat{p} \pm \boxed{Z^* \cdot \frac{1}{2\sqrt{n}}}$$

$$\text{MoE} = Z^* \cdot \frac{1}{2\sqrt{n}}$$

2 Line Segment Title

What if 3% MoE @ 99% Confidence?

$$\hat{p} \pm Z^* \cdot \frac{1}{2\sqrt{n}}$$

$$\mathbf{MoE} = Z^* \cdot \frac{1}{2\sqrt{n}}$$

$$\boxed{n = \left(\frac{Z^*}{2 \cdot MoE}\right)^2}$$

2 Line Segment Title
Segment Title Here!
2 Line Segment Title

IVQ HERE

What if 3% MoE @ 99% Confidence?

$$n = \left(\frac{Z^*}{2 \cdot MoE}\right)^2$$

What if 3% MoE @ 99% Confidence?

$$n = \left(\frac{Z^*}{2 \cdot MoE}\right)^2$$

Z* = 2.576 (99%)
MoE = 0.03

What if 3% MoE @ 99% Confidence?

$$n = \left(\frac{Z^*}{2 \cdot MoE}\right)^2$$

Z* = 2.576 (99%)
MoE = 0.03

n = 1843.27

What if 3% MoE @ 99% Confidence?

$$n = \left(\frac{Z^*}{2 \cdot MoE}\right)^2$$

Z* = 2.576 (99%)
MoE = 0.03

at most 3%. So round up

n = 1843.27

n ≥ 1844

Summary

- Estimated standard error may be too small, or inaccurate based off sample so can employ conservative approach.

maximize se by set $p=0.5$

Summary

- Estimated standard error may be too small, or inaccurate based off sample so can employ conservative approach.
- Conservative approach → determine sample size needed based on a confidence level and desired margin of error.