

# Testing Hypotheses about a Population Mean

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# Cartwheel Study

- 25 team members/colleagues (all adults) asked to perform a cartwheel
- **Variable:** Cartwheel Distance (in inches)



# Research Question



Is the average cartwheel distance (in inches) for adults more than 80 inches?

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Is the average cartwheel distance for adults more than 80 inches?

**Population:** All adults

**Parameter of Interest:** population mean cartwheel distance  $\mu$

Perform a one-sample test regarding the value for the mean cartwheel distance for the population of all such adults.

## Step I: Define the Null and Alternative

- Null: Population mean CW distance ( $\mu$ ) is 80 inches
- Alternative: Population mean is **greater than ( $>$ )** 80 inches

More compact notation:

- $H_0: \mu = 80$
- $H_a: \mu > 80$

where  $\mu$  represents the population mean cartwheel distance (inches) for all adults

**Significance  
Level = 5%**

## Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

```
df.describe()["CWDistance"]
```

```
count      25.000000  
mean       82.480000  
std        15.058552  
min        63.000000  
25%        70.000000  
50%        81.000000  
75%        92.000000  
max        115.000000  
Name: CWDistance, dtype: float64
```

$n = 25$  observations

Minimum = 63 inches

Maximum = 115 inches

Mean = 82.48 inches

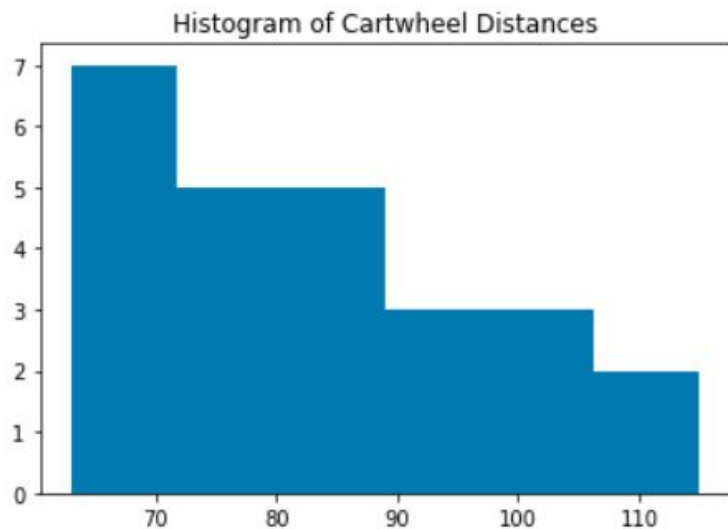
Standard Deviation = 15.06 inches

## Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

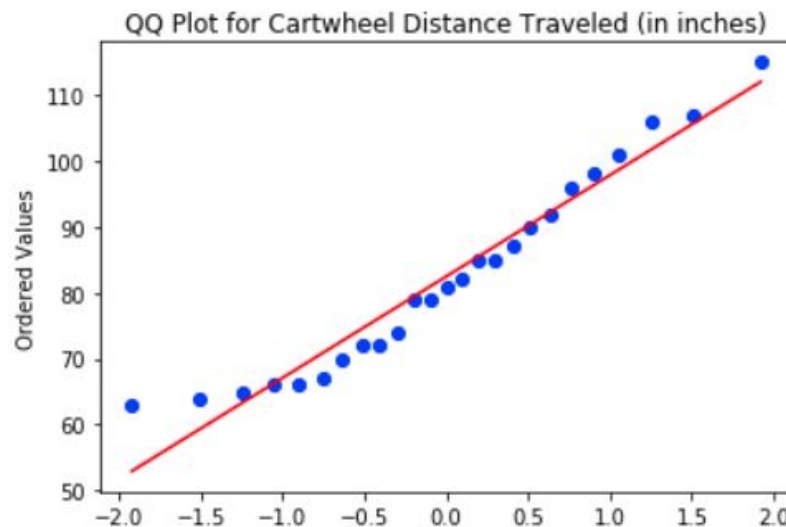
### Assumptions:

- Sample of **CW Distance** measurements considered a simple random sample
- Normal distribution for CW Distances in population  
(not as critical given large sample size, but still graph our data!)

## Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic



Histogram and Normal Q-Q Plot suggest modest deviations from **normality**



Note: reasonable sample size + CLT ... normality assumption not so crucial



## Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

$$H_0: \mu = 80$$

$$H_a: \mu > 80$$

- Is sample mean of 82.48 inches *significantly greater than* hypothesized mean of 80 inches?

$$\text{standard error of the sample mean} = \frac{\sigma}{\sqrt{n}}$$

$$\text{estimated standard error of the sample mean} = \frac{s}{\sqrt{n}}$$

## Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

**Test Statistic:** Assuming sampling distribution of sample mean is normal,

$$t = \frac{\text{best estimate} - \text{null value}}{\text{estimated std error}} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}}$$
$$= \frac{82.48 - 80}{\frac{15.06}{\sqrt{25}}} = \frac{2.48}{3.012} = \mathbf{0.82}$$

# Test Statistic Interpretation

- $$t = \frac{\text{best estimate} - \text{null value}}{\text{estimated std error}} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}} = \frac{82.48 - 80}{\frac{15.06}{\sqrt{25}}} = 0.82$$

Our **sample** mean is only 0.82  
(estimated) standard errors  
above null value of 80 inches

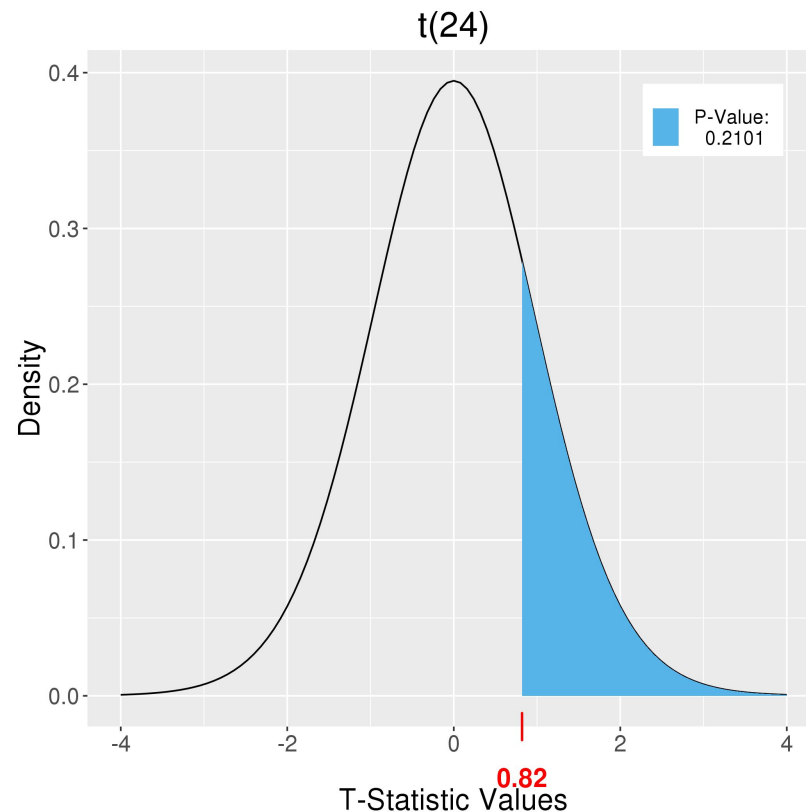
## Step 3: Determine P-Value

- If null hypothesis was true, would a test statistic value of only  $t = 0.82$  be unusual enough to reject the null?
- **P-value** = Probability of seeing test statistic of 0.82 or *more extreme* assuming the null hypothesis is true.
- If null hypothesis was true, t test statistic follows a Student t distribution with degrees of freedom  $n - 1 = 25 - 1 = 24$ .
- Since we have a one tailed test to the right *right skewed*
  - More extreme measured to the right (upper tail).

## Step 3: Determine P-Value

**P-value = 0.21**

If population mean CW distance was really was 80 inches, then observing a sample mean of 82.48 inches (i.e. a t statistic of 0.82) or larger is **quite likely**.



## Step 4: Make a Decision about the Null

Since our P-value is much bigger than 0.05 significance level,  
weak evidence against the null  
→ we **fail to reject the null!**

Based on estimated mean (82.48 inches),  
we ***cannot support***  
the population mean CW distance  
is greater than 80 inches

# 90% Confidence Interval Estimate

Mean = 82.48 inches

Standard Deviation = 15.08 inches

$n = 25$  observations  $\rightarrow t^* = 1.711$

$$\bar{x} \pm t^* \left( \frac{s}{\sqrt{n}} \right)$$

$$82.48 \pm 1.711 \left( \frac{15.06}{\sqrt{25}} \right)$$

$$82.48 \pm 1.711(3.012)$$

$$82.48 \pm 5.15$$

$$(77.33 \text{ inches}, 87.63 \text{ inches})$$

Note: 80 inches is IN confidence interval of reasonable values for population mean CW distance

## What if Normality Doesn't Hold?

- Not convinced that CW Distance follows a normal distribution in the population?
  - **non-parametric test** that does not assume normality
- Non-parametric analog of the one sample t-test
  - = **Wilcoxon Signed Rank Test**
  - ~ uses median to examine location of distribution of measurements



## What if Normality Doesn't Hold?

Wilcoxon Signed Rank Test Result:  $p\text{-value} \gg 0.05$

Fail to reject the null that population median CW distance 80 inches

Conclusion is robust to potential violations of normality!

For the population of interest (all adults)

~ regardless of assumptions made and inference approach used

☐ There is **not** sufficient evidence to support that the population mean CW distance is more than 80 inches

# Summary

- Hypothesis Tests are used to put theories about a parameter of interest to the test ~ **parameter = population mean**
- Basic Steps:
  - **State hypotheses (and select significance level)** *set up before looking at data*
  - **Examine results, check assumptions, summarize via test statistic**
  - **Convert test statistic to P-value**
  - **Compare P-value to significance level to make decision**
- Assumptions for One-sample (t) Test for Population mean
  - **Data considered a random sample**
  - **Population of responses is normal (else n large helps)**
- Know how to interpret the **p-value, decision, and conclusion**