What are Multilevel Models, and Why Do We Fit Them?

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Models need to reflect the **correlations**!

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Longitudinal Study Example

Predictor Time → **Outcome of Interest**

intercept and slope allowed to randomly vary across randomly sampled subjects

→ each subject have own unique intercept and slope!

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→ Expand types of inferences:

- What are the relationships between predictors and outcomes?
- How variable are coefficients in larger population from which clusters (schools, clinics, etc.) were randomly sampled? **NEW!**
- Can we explain that variance with cluster-level variables? **NEW!**

What are Multilevel Models?

Q: What changes allow coefficients to randomly vary?

A: **Random effects** of higher-level, randomly sampled clusters!

Level 1: $y_{ij} = \beta_{0j} + \beta_{1j} x_{1ij} + e_{ij}$

Random coefficients (**not parameters!**)

Level 2:

$$\beta_{0j} = \beta_0 + u_{0j}$$
$$\beta_{1j} = \beta_1 + u_{1j}$$

random variables

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Random effects (u) allow each cluster denoted by j to have unique coefficients!

Random effects are **random variables:** values for different clusters assumed to be random (depending on which clusters randomly sampled!) from normal distribution with mean 0 and some variance.

Interested in **estimating that variance**!

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- Without random effects, assuming observations from same cluster are **independent**
- Accounting for correlations often **substantially improves model fit** when working with dependent data!

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Key research question!

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- **How much** of unexplained variance due to **between-cluster variance** in intercepts or slopes for given model?

Key research question!

- **Need explicit research interest in estimating variances of these random coefficients**; otherwise, should consider other models for dependent data.

Why do we Fit Multilevel Models?

All need to be true to warrant multilevel modeling…

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- Have **explicit research interest** in estimating between-cluster variance in selected regression coefficients in our model.

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Examples:

How much of unexplained variance among hospitals in mean patient satisfaction is due to the size of the hospital?

How much variance is there in long-term trends of substance use for a sample of drug users?

Why do we Fit Multilevel Models?

Advantages over other approaches for dependent data:

- Estimate **one parameter** representing variance of given random coefficient across clusters, rather than **unique regression coefficient for every possible cluster.** More Efficient!

- Clusters with smaller sample sizes do not have as pronounced of an effect on variance estimate as larger clusters; their effects shrink toward overall mean of outcome when using random effects.

What are Multilevel Models?

Estimating variance in given random coefficient across higher-level clusters → can add cluster-level predictors to those Level-2 equations for random coefficients, and explain variance in random effects!

Example: y = outcome, x = age, t = time point, i = subject

Level 1: $$y_{ti} = \beta_{0i} + \beta_{1i}x_{1ti} + e_{ti}$$

Level 2: $$\beta_{0i} = \beta_{00} + \beta_{01}T_i + u_{0i}$$
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Add regression parameters for subject-level covariate T (treatment), to **explain variance** in random intercepts and random slopes!

View Level 2 equations for random coefficients like mini-regression models; adding cluster-level predictors like T → explain variance in random effects denoted by *u*

Test hypotheses about regression parameters for T → **"45% of between-subject variance in the age – Y relationship is due to T!"**

What's Next?

- **Visualize** ideas multilevel modeling **online**!
- **Details** about fitting multilevel models to *different kinds of dependent variables*
- **Examples**

Remember: *need explicit research interest in estimating between-cluster variance in regression coefficients.* Other modeling approaches for dependent (correlated) data don't need random effects!