Logistic Regression Inference

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Statistics with Python Course Developer

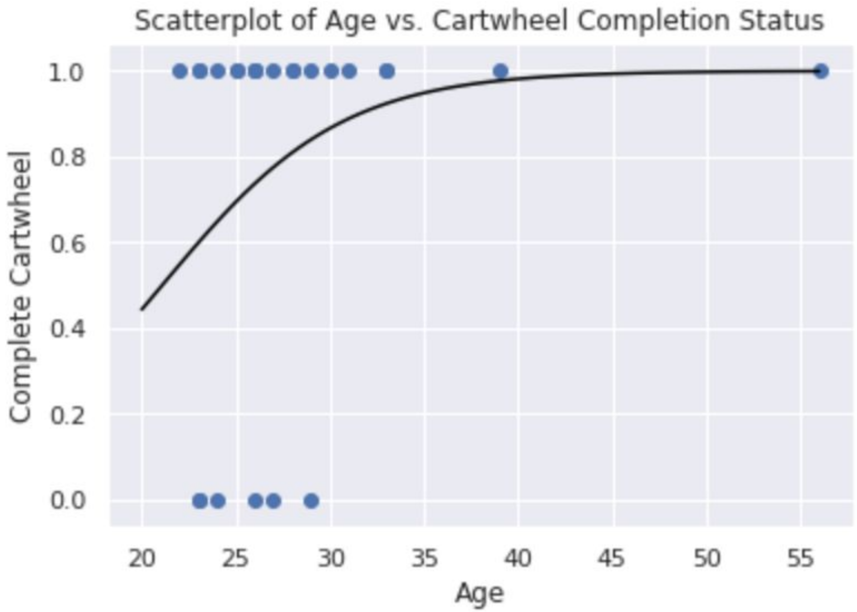
Cartwheel Data

Random sample of 25 adults attempted cartwheels

Primary Variable of interest: Cartwheel completion

Based on age, can we predict whether a cartwheel is completed?

Logistic Regression Line



Generalized Linear Model Regression Results

Dep. Variable:	CompleteGroup	**No. Observations:**	25
Model:	GLM	**Df Residuals:**	23
Model Family:	Binomial	**Df Model:**	1
Link Function:	logit	**Scale:**	1.0
Method:	IRLS	**Log-Likelihood:**	-12.534
Date:	Tue, 27 Nov 2018	**Deviance:**	25.068
Time:	16:11:20	**Pearson chi2:**	22.4
No. Iterations:	6		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-4.4213	4.429	-0.998	0.318	-13.101	4.259
Age	0.2096	0.171	1.225	0.221	-0.126	0.545

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Best Estimate ± Margin of Error

Sample slope ± "a few" · estimated standard error

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0 is included

IVQ

Does it seem reasonable that there is a significant slope?

No

We'll continue working through the hypothesis testing framework before coming back to the answer of the IVQ.

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need significance level

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With our p-value of about 0.221, we would fail to reject the null hypothesis and cannot conclude that we have a significantly linear relationship between age and the log odds of the probability of successfully completing a cartwheel.

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Coming up, we'll look at an example from NHANES using blood pressure and smoking