Estimating a Mean Difference for Paired Data

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Twin Education Levels

Twin Days in Twinsburg, Ohio annually since 1976

Variable: Education Level of Twins

Paired Values

- Want to treat the two sets of values simultaneously

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- Variable: Difference of measurements within pairs

Research Question

What is the *average* difference between the older twin's and younger twin's self-reported education?

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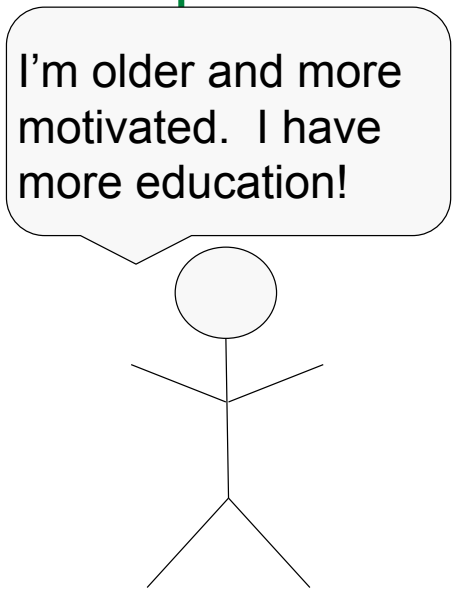

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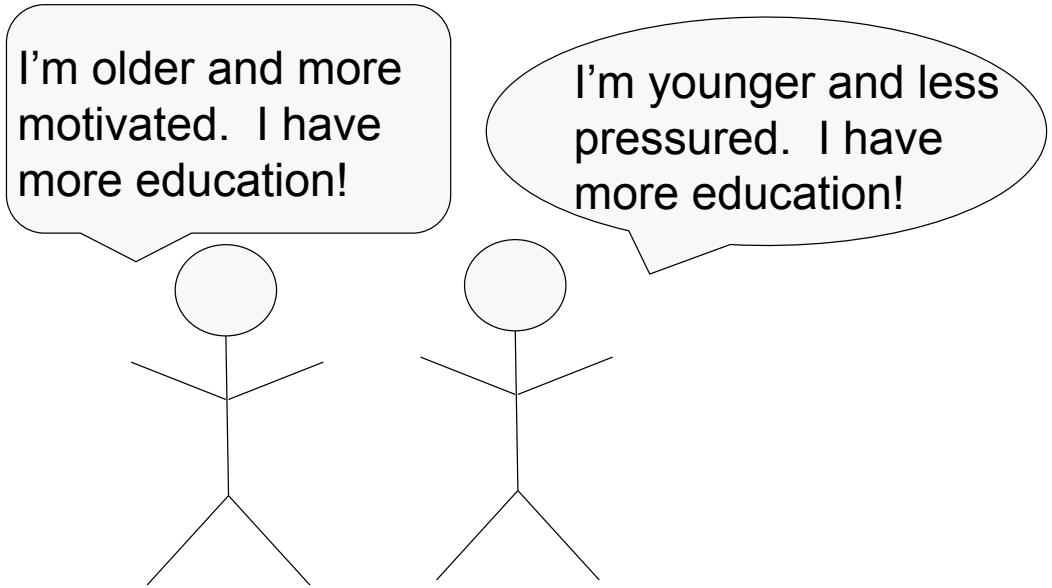

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Construct a 95% confidence interval for the mean difference of self-reported education for a set of identical twins.

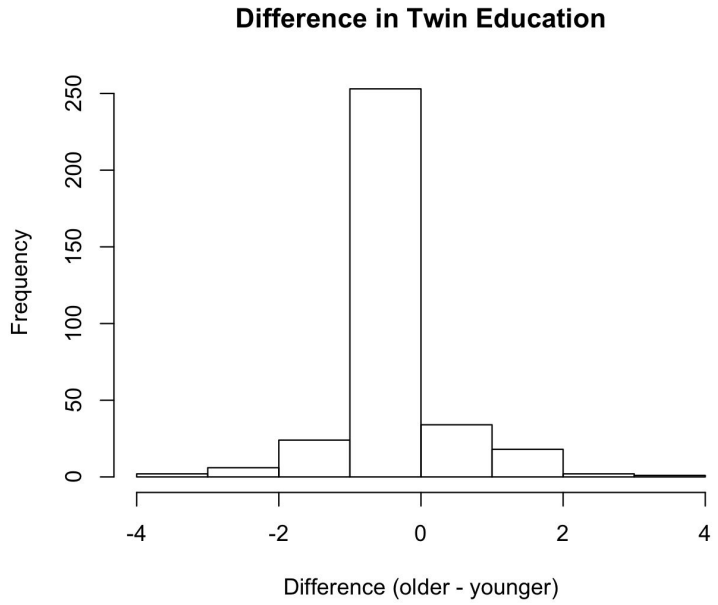
Difference Calculation

Difference = older twin - younger twin

Older twin education	Younger twin education	Difference (older - younger)
16	16	0
18	16	2
12	12	0
14	14	0
13	15	-2

Difference Summary

Difference = older twin - younger twin



n = 340 observations

Minimum = -3.5 years
Maximum = 4 years
72.1% had a difference of 0 years

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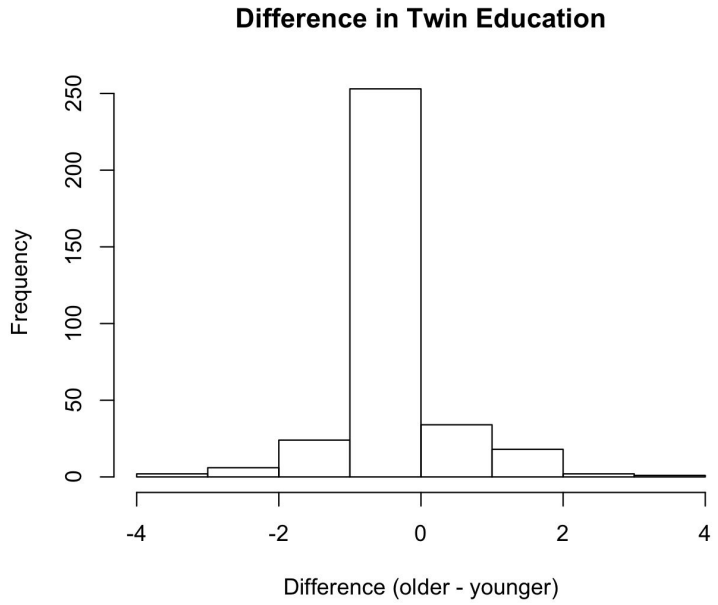

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Maximum = 4 years
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Mean = 0.0838 years
Standard Deviation = 0.7627 years

Confidence Interval Basics

Best Estimate ± Margin of Error

95% Confidence Interval Calculations

Best Estimate ± Margin of Error

Sample mean difference ± "a few" · estimated standard error

$$\bar{x}_d \pm t^* \left(\frac{s_d}{\sqrt{n}}\right)$$

small sample size

t^* multiplier comes from a t-distribution with $n - 1$ degrees of freedom

95% confidence

$n = 25 \rightarrow t^* = 2.064$

$n = 1000 \rightarrow t^* = 1.962$

Mean Difference Confidence Interval

Mean = 0.084 years
Standard Deviation = 0.76 years

n = 340 observations ⟶ t* = 1.967

$$\bar{x}_d \pm t^* \left(\frac{s_d}{\sqrt{n}}\right)$$

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$$0.084 \pm 1.967\ (0.76/\sqrt{340})$$

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$0.084 \pm 1.967\ (0.04)$

0.084 ± 0.0814

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n = 340 observations ⟶ t* = 1.967

$$\bar{x}_d \pm t^* \left(\frac{s_d}{\sqrt{n}}\right)$$

$0.084 \pm 1.967\ (0.76/\sqrt{340})$

$0.084 \pm 1.967\ (0.04)$

0.084 ± 0.0814

(0.0025, 0.1652) years

Interpreting the Confidence Interval

“range of reasonable values for our parameter”

Interpreting the Confidence Interval

"range of reasonable values for our parameter"

With 95% confidence, the population mean difference of the older twin's and younger twin's self-reported education is estimated to be between 0.0025 years and 0.1652 years.

IVQ

Is there a difference between education levels of the older and younger twin?

Intervals for Differences

Is there a mean difference between the education level of twins?

If education levels are generally equal → mean difference is **0**

If education levels are unequal → mean difference is not **0**

Look for **0** in the range of reasonable values

Assumptions

We need to assume that we have a **random sample of identical twin sets**.

2. **Population of differences is normal** (or a **large enough sample size** can help to bypass this assumption).

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Extension of the one mean confidence interval

~use difference variable now

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Data need to be paired to calculate a difference variable

~two measurements on same individual

~two measurements on similar, matched individuals

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Data need to be paired to calculate a difference variable

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0 in the confidence interval

~implies the mean difference is 0 → no true difference