Sensitivity Analysis Tutorial

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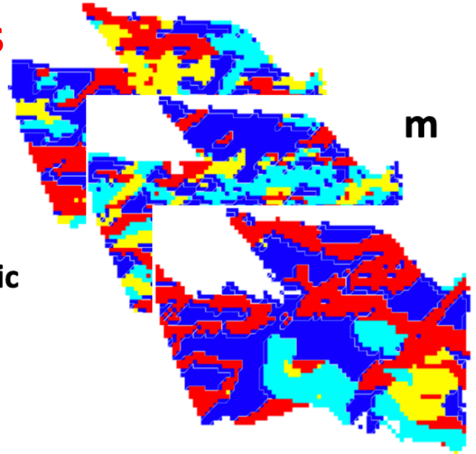
Methodology: DGSA

- Distance-Based Generalized Sensitivity Analysis, Fenwick, D., Scheidt, C. & Caers, J. Math Geosci (2014) 46: 493. https://doi.org/10.1007/s11004-014-9530-5

1. INPUT: model parameters

Sampling of input parameters:
Structure,
Rock, Fluid

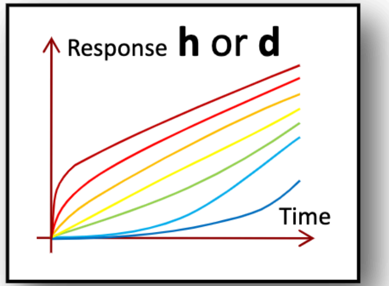
stochastic



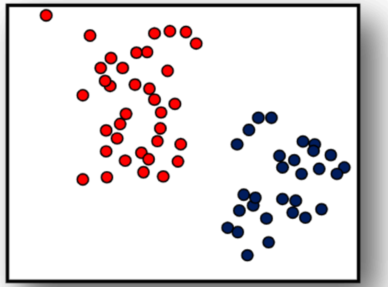
m

Flow model

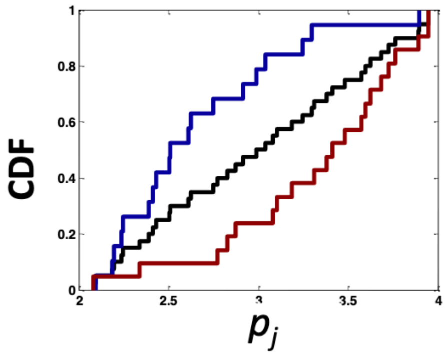
2. OUTPUT: model responses



Dim. Reduction
classification



3. Clustering



4. DGSA

Measure of sensitivity:
difference between the frequency distributions of input parameters per each class

Adapted from Prof. Jef Caers' GS 240 material

Python Code

- Tutorial & code: https://github.com/lijingwang/DGSA_Light, forked from https://github.com/sdyinzhen/DGSA_Light

- Before we apply DGSA to calculate sensitivities:
  - 1&2 Monte Carlo: Multiple input and outputs -> on your own
  - 3 Clustering: K-medoids clustering on **Euclidean** distances between outputs -> code can take care
  - 4 DGSA: python code giving you the tornado chart -> code can take care

Model parameterization: The big table

Global parameters

Spatial parameters

	Symbol	Description	Type	Prior Distribution
Global/Non-gridded Parameter	μ_1^ρ	Log resistivity mean of sand and gravel ($\Omega \cdot$ m)	fixed	1.66
	s_1^ρ	Log resistivity variance/sill of sand and gravel	continuous	N(0.1311,2.3146e-5)
	n_1^ρ	Log resistivity variogram nugget of sand and gravel	continuous	U(0, 0.3)
	$\mathrm{range}_{xy_1}^{\rho}$	Log resistivity variogram xy plane range of sand and gravel (m)	continuous	U(100, 1000)
	$\mathrm{range}_{z_1}^{\rho}$	Log resistivity variogram z axis range of sand and gravel (m)	continuous	U(5, 20)
	μ_1^K	Log hydraulic conductivity mean of sand and gravel (m/s)	continuous	U(-5.5, -4.5)
	σ_1^K	Log hydraulic conductivity standard deviation of sand and gravel	continuous	U(0.5, 1.5)
	D_1	Grain size diameter of sand and gravel (μm)	continuous	U(60,200)
	μ_2^ρ	Log resistivity mean of glacial clay ($\Omega \cdot$ m)	fixed	1.45
	s_2^ρ	Log resistivity variance/sill of glacial clay	continuous	N(0.1311,2.3146e-5)
	n_2^ρ	Log resistivity variogram nugget of glacial clay	continuous	U(0, 0.3)
	$\mathrm{range}_{xy_2}^{\rho}$	Log resistivity variogram xy plane range of glacial clay (m)	continuous	U(100, 300)
	$\mathrm{range}_{z_2}^{\rho}$	Log resistivity variogram z axis range of glacial clay (m)	continuous	U(5, 20)
	μ_2^K	Log hydraulic conductivity mean of glacial clay (m/s)	continuous	U(-7.5, -6.5)
	σ_2^K	Log hydraulic conductivity standard deviation of glacial clay	continuous	U(0.5, 1.5)
	D_2	Grain size diameter of glacial clay (μm)	continuous	U(20,60)
	ρ_{solid_2}	Density of solid phase of glacial clay (Mg/m^3)	continuous	U(1.77, 2.07)
	μ_3^ρ	Log resistivity mean of hemipelagic clay ($\Omega \cdot$ m)	fixed	0.53
	s_3^ρ	Log resistivity variance/sill of hemipelagic clay	fixed	0.0915
	n_3^ρ	Log resistivity variogram nugget of hemipelagic clay	continuous	U(0, 0.3)
	$\mathrm{range}_{xy_3}^{\rho}$	Log resistivity variogram xy plane range of hemipelagic clay (m)	continuous	U(100, 300)
	$\mathrm{range}_{z_3}^{\rho}$	Log resistivity variogram z axis range of hemipelagic clay (m)	continuous	U(5, 20)
	μ_3^K	Log hydraulic conductivity mean of hemipelagic clay (m/s)	fixed	-9
	σ_3^K	Log hydraulic conductivity standard deviation of hemipelagic clay	fixed	0.1
	D_3	Grain size diameter of hemipelagic clay (μm)	continuous	U(2,20)
	ρ_{solid_3}	Density of solid phase of hemipelagic clay (Mg/m^3)	continuous	U(1.60, 2.00)
	K_{riv2}	Riverbed hydraulic conductivity for the river segment 2	continuous	log-normal(-5, 0.5)
	K_{riv3}	Riverbed hydraulic conductivity for the river segment 3	continuous	log-normal(-8, 0.5)
	D_{riv2}	Drainage Rate for the river segment 2	continuous	log-normal(-5, 1.0)
	D_{riv3}	Drainage Rate for the river segment 3	continuous	log-normal(-7, 1.0)
	rch	Recharge variable, scale from 0 to 1	continuous	trapeze(0.6, 0.75,0.85, 1.0)
	TI	Conceptual hydrostratigraphic model / Training Image	fixed	/
	σ_w	Electrical conductivity of the water (mS/m)	continuous	U(40, 70)
	μ_w	Viscosity of water (g/m $\cdot$ s)	fixed	1.4
	ρ_w	Density of water (g/cm^3)	fixed	1
	m	Cementation exponent in Archie's Law	continuous	U(1.3, 2.5)
	CEC	Cation exchange capacity in Waxman-Smits model (Meq/100g)	continuous	log-normal(-4.53, 0.77)
	g	Gravity (m/s^2)	fixed	9.816
	radius	Radius in smooth filter (m)	continuous	U(100, 300)
	depth gradient	Depth gradient in smooth filter, higher = depth increases faster with z	continuous	U(0.01, 0.3)
Spatial/Gridded Parameter	Lith	Geological model by Direct Sampling, conditioned on hard data	discrete	/
	ρ	Resistivity model	continuous	Cookie cut gaussian simulations based on lithologies
	K	Hydraulic conductivity model	continuous	Forward modeling from $\rho \to \phi \to K$ + histogram transformation

Different facies: variogram related parameters for
1. resistivity
2. hydraulic conductivity

Hydrological and petrophysical model parameters

Input: Monte Carlo from prior distribution

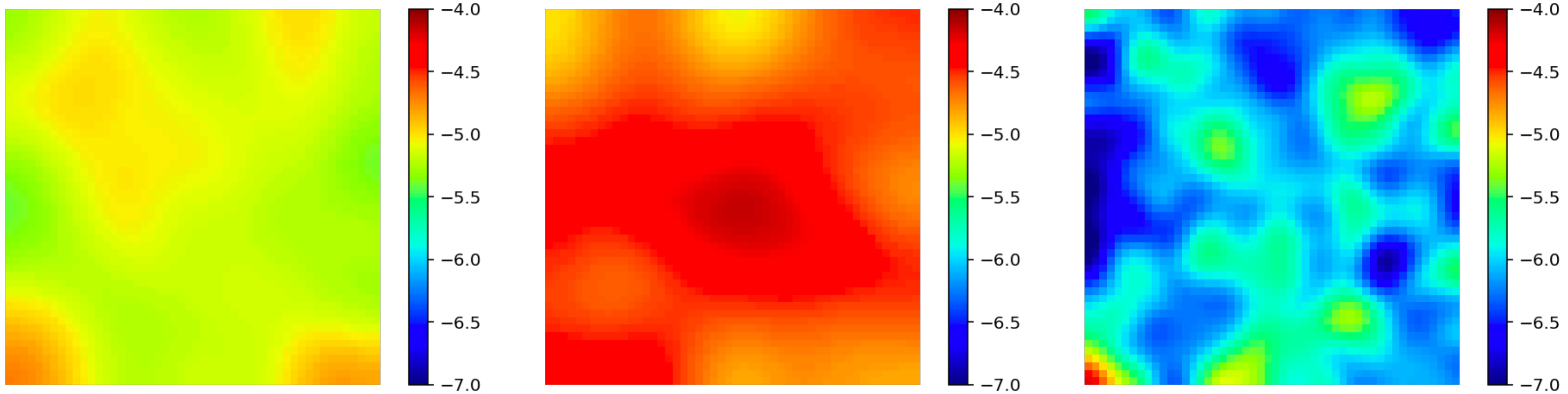
- Input, model parameters:
  - Global parameters: i.e. mean of log hydraulic conductivity
  - Sample: from a prior distribution, i.e. Gaussian or uniform: $U(-6, -4)$

log K	-5.166	-4.559	-6.000	…

  - Spatial parameters: i.e. log hydraulic conductivity field
  - Sample: spatial distributed Gaussian fields

Possibly require dimension reduction

Spatial log K

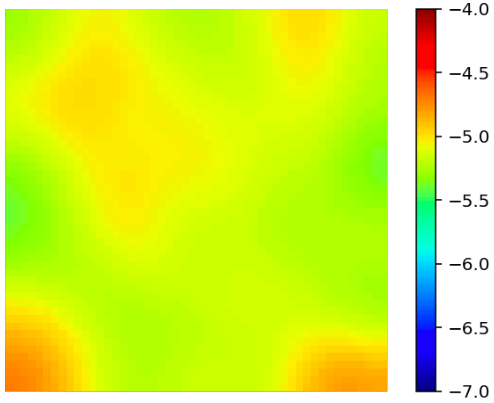


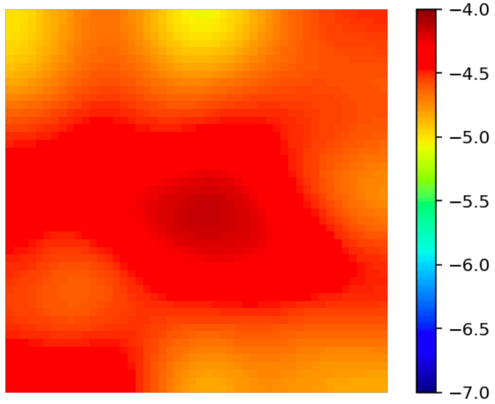
Forward function: from input to output

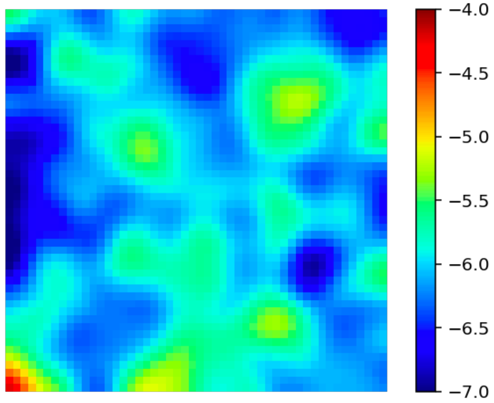
- MODFLOW, CrunchTope …

log K   -5.166   -4.559   -6.000   …

Spatial log K





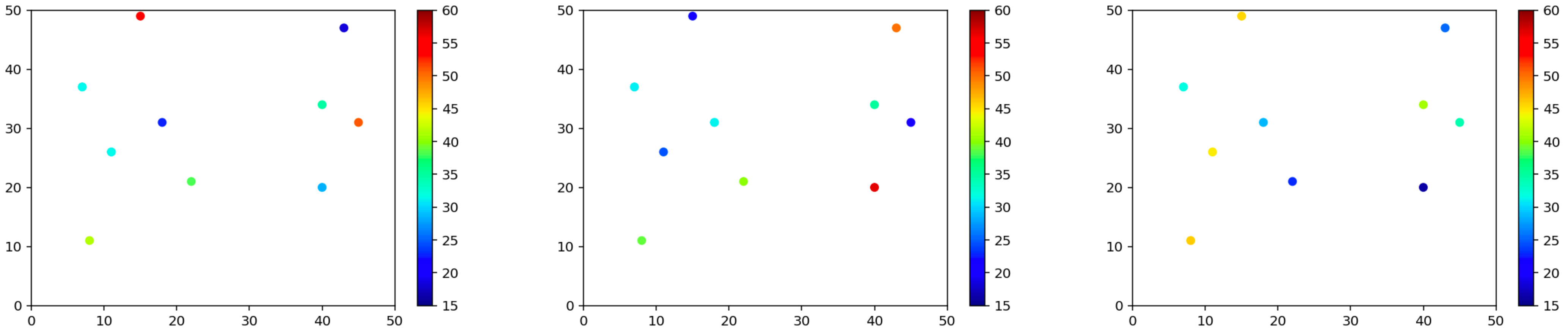


…

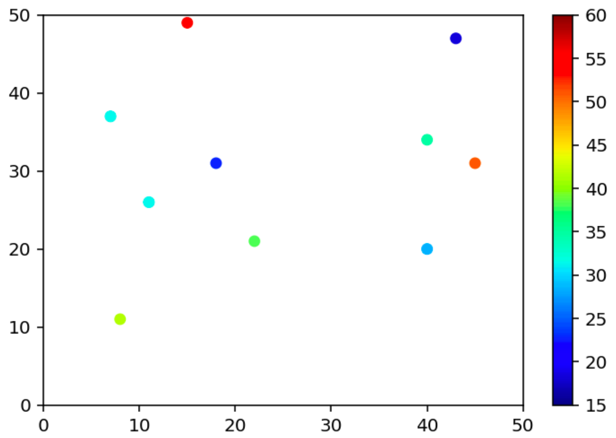
Forward functions

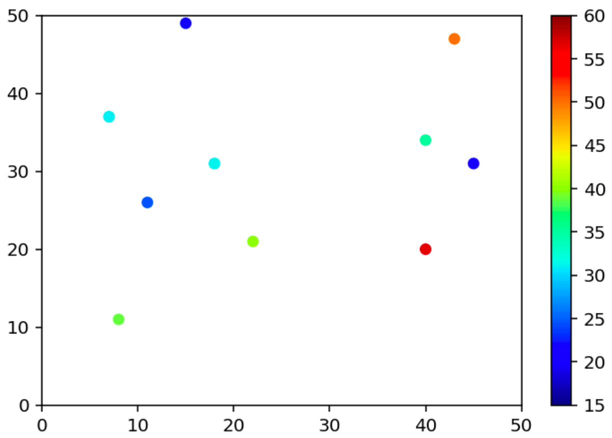
Output

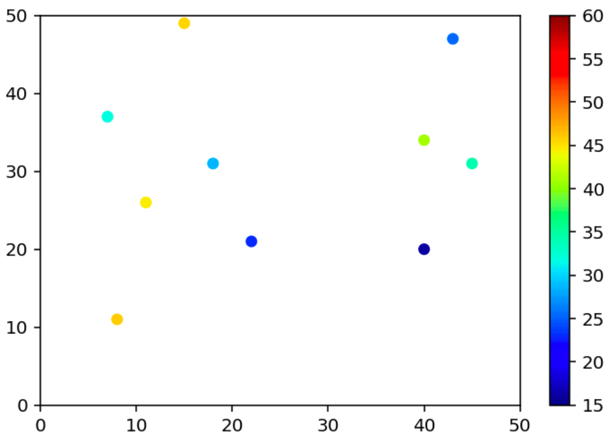
- Output, model responses, i.e. head maps



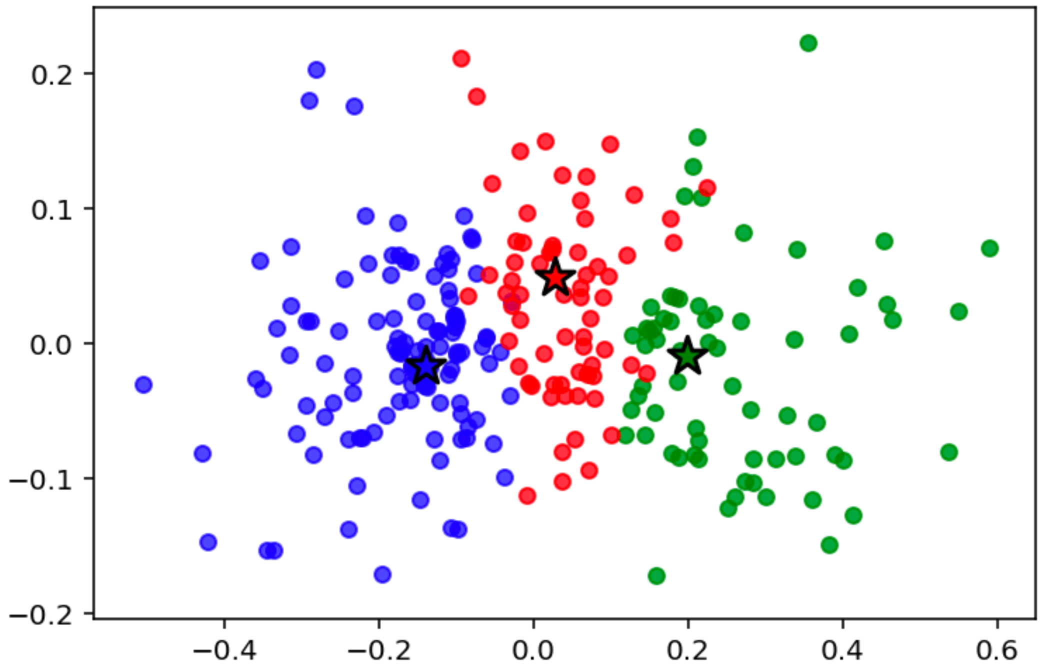
Clustering: K-medoid







- Euclidean distances: K-medoid.



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Trusted | Python 3

Run Code

In [1]:
```python
import numpy as np
from DGSA_light import DGSA_light
from gsa_pareto_plt import gsa_pareto_plt
import pandas as pd
%matplotlib inline
%config InlineBackend.figure_format = 'retina'
```

Step 1, input

In [2]:
```python
# 1 input, model_parameters:  L x N_input  dimension
model_parameters = pd.read_csv('./tutorial_data/m_parameters.csv')
```

Step 2, output

In [3]:
```python
# 2 output, model_responses: L x N_output dimension
model_responses = pd.read_csv('./tutorial_data/m_responses.csv')
```

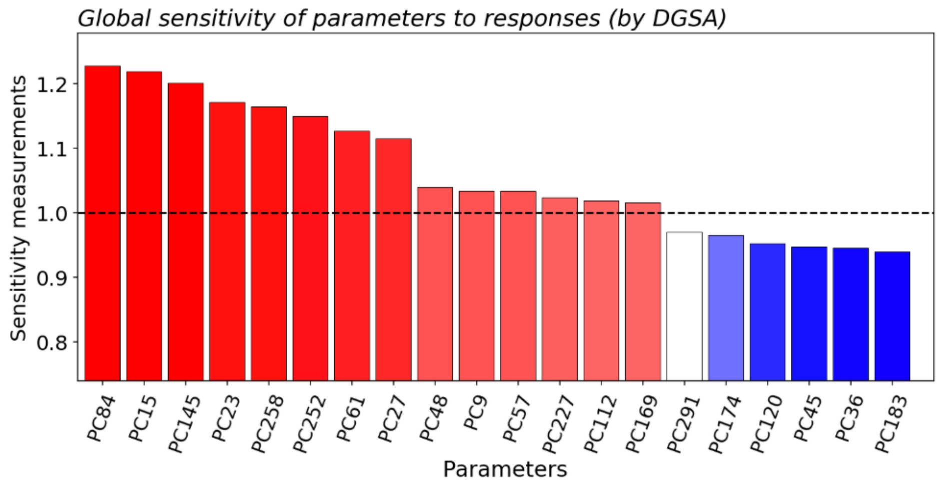
Step 3 & 4, Clustering + DGSA

In [4]:
```python
# Call DGSA
# 3 clusutering by K-medoids, Euclidean distance
# 4 DGSA: calculating sensitivities
dgsa_measures = DGSA_light(model_parameters.values, model_responses.values,model_parameters.columns)
```

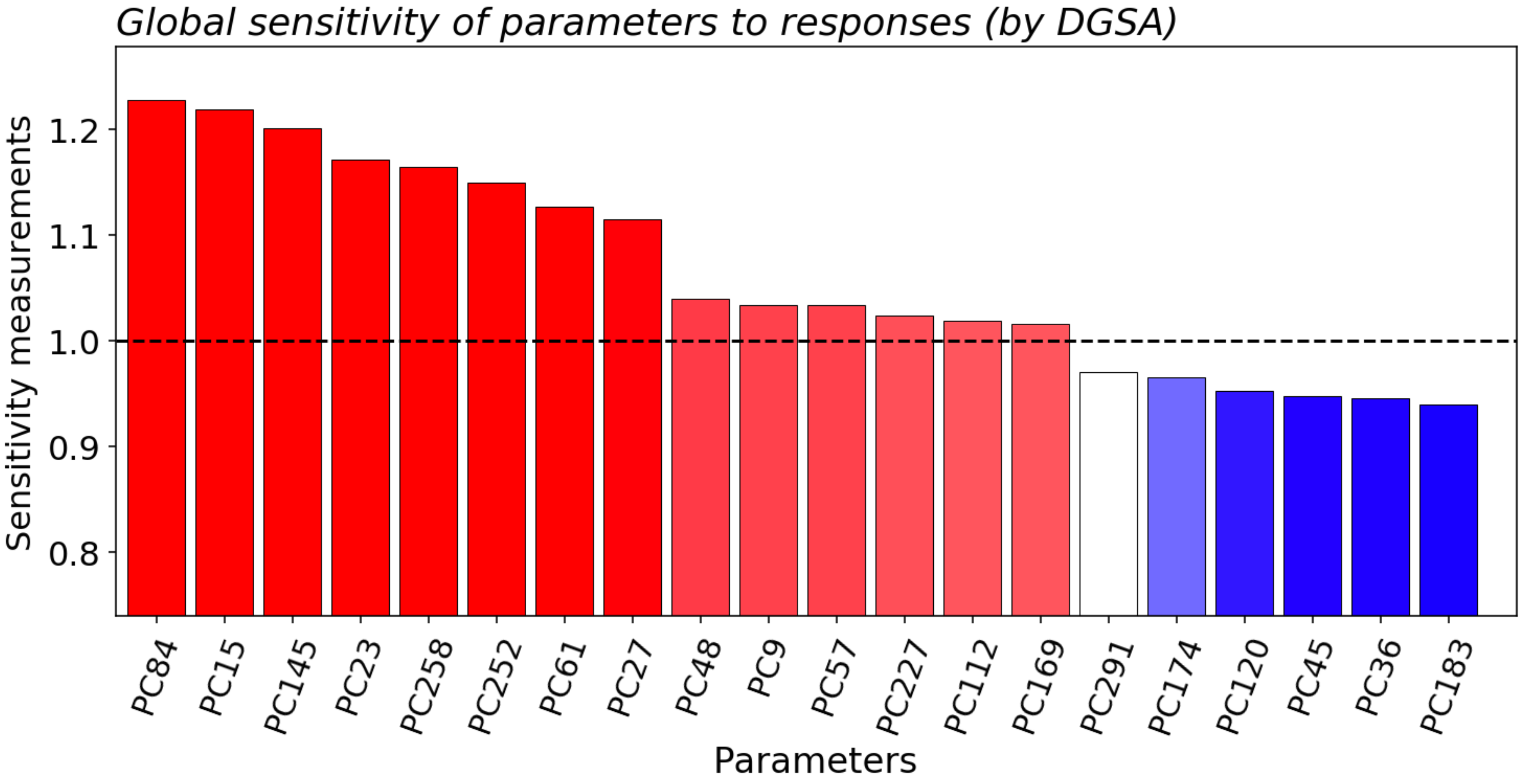
```
100%|██████████| 3000/3000 [00:32<00:00, 93.41it/s]
```

Visualization: the Pareto plot

In [5]:
```python
gsa_pareto_plt(dgsa_measures)
```



DGSA result


Global sensitivity of parameters to responses (by DGSA)
Sensitivity measurements
1.2
1.1
1.0
0.9
0.8
PC84
PC15
PC145
PC23
PC258
PC252
PC61
PC27
PC48
PC9
PC57
PC227
PC112
PC169
PC291
PC174
PC120
PC45
PC36
PC183
Parameters