机器学习与数据挖掘

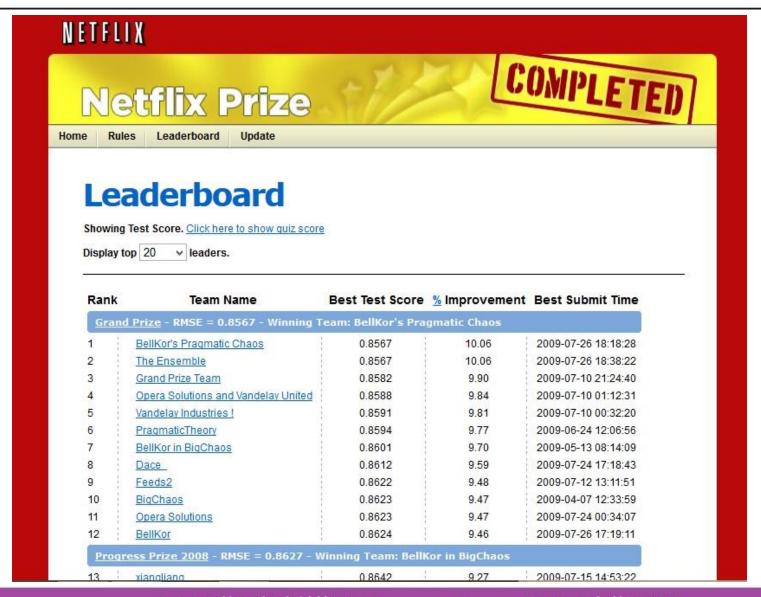
Machine Learning & Data Mining

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Preface



Lecture 19 Recommender Systems: Latent Factor Model

The Netflix Prize

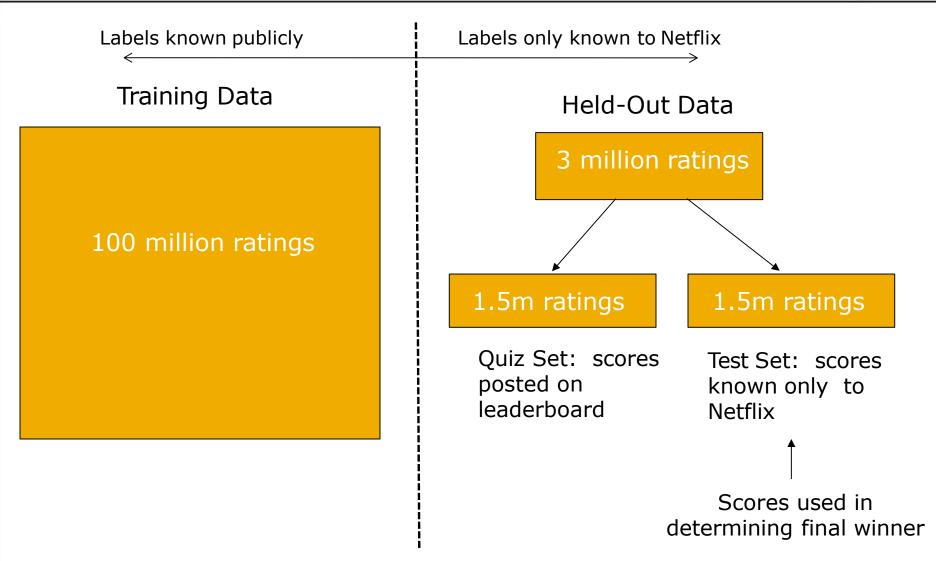
Training data

- 100 million ratings, 480,000 users, 17,770 movies
- 6 years of data: 2000-2005
- Test data
 - Last few ratings of each user (2.8 million)
 - Evaluation criterion: Root Mean Square Error (RMSE)

$$= \frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\hat{r}_{xi} - r_{xi})^2}$$

- Netflix's system RMSE: 0.9514
- Competition
 - 2,700+ teams
 - \$1 million prize for 10% improvement on Netflix

Competition Structure



The Netflix Utility Matrix

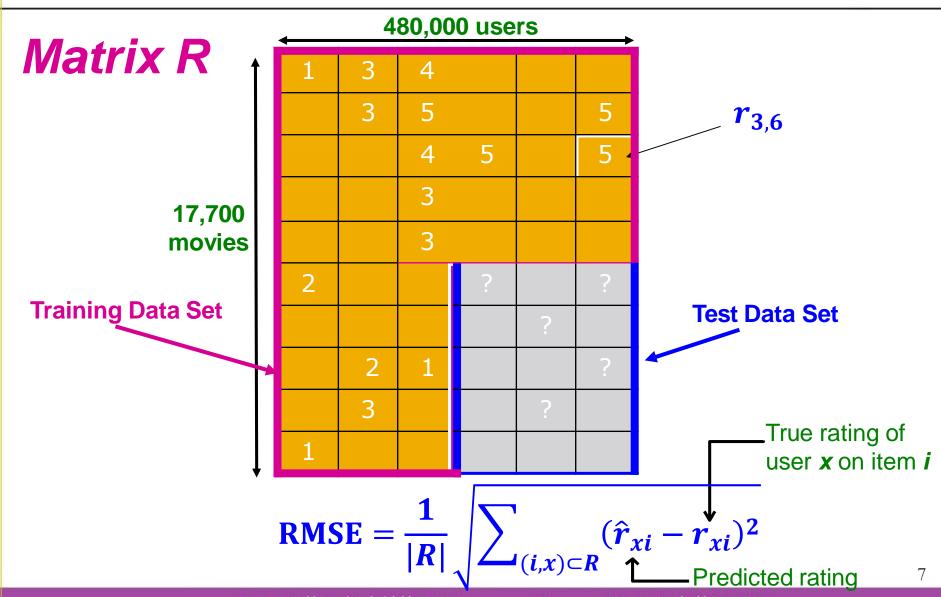
Matrix R

17,700 movies

+00,000 d3c13											
1	3	4									
	3	5			5 5						
		4	5		5						
		3									
		3									
2			2		2						
				5							
	2	1			1						
	3			3							
1											

480.000 users

Utility Matrix R: Evaluation



BellKor Recommender System

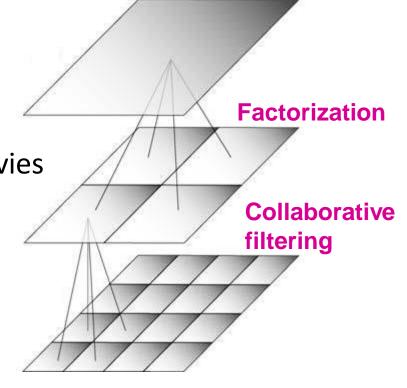
- The winner of the Netflix Challenge
- Multi-scale modeling of the data:

Combine top level, "regional" modeling of the data, with a refined, local view:

Global:

Overall deviations of users/movies

- Factorization:
 - Addressing "regional" effects
- Collaborative filtering:
 - Extract local patterns



Global effects

Model Local & Global Effect

Global:

- Mean movie rating: 3.7 stars
- The Sixth Sense is 0.5 stars above avg.
- Joe rates 0.2 stars below avg.
 - **⇒** Baseline estimation:

Joe will rate The Sixth Sense 4 stars

- Local neighborhood (CF/NN):
 - Joe didn't like related movie Signs
 - ⇒ Final estimate: Joe will rate The Sixth Sense 3.8 stars







Recap: Collaborative Filtering

- Earliest and most popular collaborative filtering method
- Derive unknown ratings from those of "similar" movies (item-item variant)
- Define similarity measure s_{ii} of items i and j
- Select k-nearest neighbors, compute the rating
 - N(i; x): items most similar to i that were rated by x

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}... similarity of items *i* and *j*r_{xj}...rating of user *x* on item *j*N(i;x)... set of items similar to item *i* that were rated by *x*

Model Local & Global Effect

In practice we get better estimates if we model deviations:

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$\boldsymbol{b}_{xi} = \boldsymbol{\mu} + \boldsymbol{b}_x + \boldsymbol{b}_i$$

 μ = overall mean rating b_x = rating deviation of user x = (avg. rating of user x) – μ b_i = (avg. rating of movie i) – μ

Problems/Issues:

- 1) Similarity measures are "arbitrary"
- 2)Pairwise similarities neglect interdependencies among users
- 3)Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weight Wij

Use a weighted sum rather than weighted avg.:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj})$$

- A few notes:
 - N(i;x)... set of movies rated by user x that are similar to movie i
 - \mathbf{w}_{ij} is the interpolation weight (some real number)
 - We allow: $\sum_{j \in N(i,x)} w_{ij} \neq 1$
 - \mathbf{w}_{ij} models interaction between pairs of movies (it does not depend on user \mathbf{x})

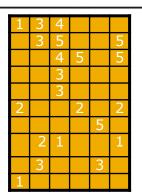
Idea: Interpolation Weight Wij

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij} (r_{xj} - b_{xj})$$

- How to set w_{ij} ?
 - Remember, error metric is: $\frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\hat{r}_{xi} r_{xi})^2}$ or equivalently SSE: $\sum_{(i,x) \in R} (\hat{r}_{xi} r_{xi})^2$
 - Find w_{ii} that minimize SSE on training data!
 - Models relationships between item i and its neighbors j
 - w_{ij} can be learned/estimated based on x and all other users that rated i

Recommendations via Optimization

- Goal: Make good recommendations
 - Quantify goodness using RMSE:
 Lower RMSE ⇒ better recommendations



- Want to make good recommendations on items that user has not yet seen. Can't really do this!
- Let's build a system such that it works well on known (user, item) ratings
 And hope the system will also predict well the unknown ratings

Recommendations via Optimization

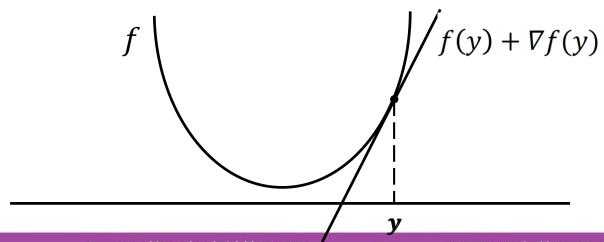
- Idea: Let's set values w such that they work well on known (user, item) ratings
- How to find such values w?
- Idea: Define an objective function and solve the optimization problem
- Find w_{ii} that minimize SSE on training data!

$$J(w) = \sum_{x,i} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$
Predicted rating

Think of w as a vector of numbers

Detour: Minimizing a function

- A simple way to minimize a function f(x):
 - Compute and take a derivative Vf
 - Start at some point y and evaluate $\nabla f(y)$
 - Make a step in the reverse direction of the gradient: $y = y \nabla f(y)$
 - Repeat until converged



Interpolation Weights

• We have the optimization problem, now what?

$$J(w) = \sum_{x} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$

- Gradient decent:
 - Iterate until convergence: $w \leftarrow w \eta \nabla_w J$ η ... learning rate
 - where $\nabla_w J$ is the gradient (derivative evaluated on data):

$$\nabla_{w}J = \left[\frac{\partial J(w)}{\partial w_{ij}}\right] = 2\sum_{x,i} \left(\left[b_{xi} + \sum_{k \in N(i;x)} w_{ik}(r_{xk} - b_{xk})\right] - r_{xi}\right) (r_{xj} - b_{xj})$$

$$\text{for } \boldsymbol{j} \in \{\boldsymbol{N}(\boldsymbol{i};\boldsymbol{x}), \forall \boldsymbol{i}, \forall \boldsymbol{x}\}$$

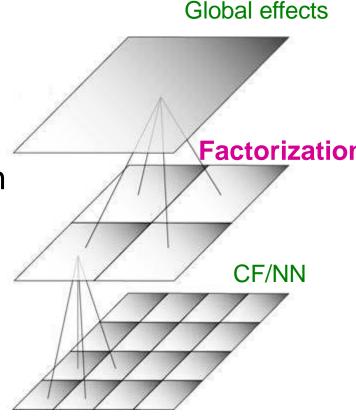
$$\text{else } \frac{\partial J(w)}{\partial w_{ij}} = \boldsymbol{0}$$

■ Note: We fix movie i, go over all r_{xi} , for every movie $j \in N(i; x)$, we compute $\frac{\partial J(w)}{\partial w_{ij}}$ while $|w_{now}|$ -

while
$$|w_{new} - w_{old}| > \varepsilon$$
:
 $w_{old} = w_{new}$
 $w_{new} = w_{old} - \eta \cdot \nabla w_{old}$

Interpolation Weights

- So far: $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} b_{xj})$
 - Weights \mathbf{w}_{ij} derived based on their role; no use of an arbitrary similarity measure $(\mathbf{w}_{ii} \neq \mathbf{s}_{ii})$
 - Explicitly account for interrelation the neighboring movies
- Next: Latent factor model
 - Extract "regional" correlations



Performance of Various Methods

Global average: 1.1296

<u>User a</u>verage: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

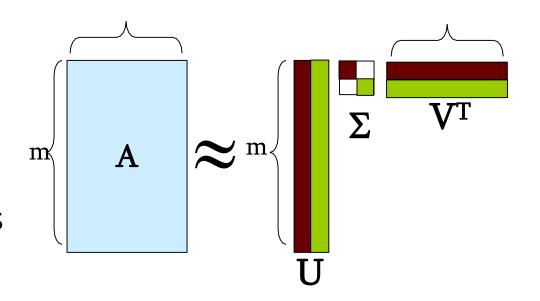
CF+Biases+learned weights: 0.91

Grand Prize: 0.8563

Recap: SVD

Remember SVD:

- A: Input data matrix
- U: Left singular vecs
- V: Right singular vecs
- Σ: Singular values



So in our case:

"SVD" on Netflix data: $R \approx Q \cdot P^T$

$$A = R$$
, $Q = U$, $P^{T} = \sum V^{T}$

$$\hat{\boldsymbol{r}}_{xi} = \boldsymbol{q}_i \cdot \boldsymbol{p}_x$$

SVD: More good stuff

 We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):

$$\min_{U,V,\Sigma} \sum_{ij\in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij} \right)^{2}$$

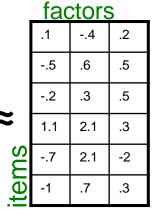
- Note two things:
 - SSE and RMSE are monotonically related:
 - $RMSE = \frac{1}{c}\sqrt{SSE}$ Great news: SVD is minimizing RMSE • Complication: The sum in SVD error term is over
 - Complication: The sum in SVD error term is over all entries (no-rating in interpreted as zero-rating). But our R has missing entries!

Latent Factor Model

SVD: $A = U \Sigma V^T$

"SVD" on Netflix data: R ≈ Q · P^T

_	users													
	1		3			5			5		4			
S			5	4			4			2	1	3		
items	2	4		1	2		3		4	3	5			
ite		2	4		5			4			2		•	
			4	3	4	2					2	5		
	1		3		3			2			4			
	R													



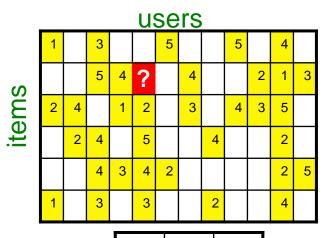
	users												
1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4			
8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1			
2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6			

PT

- For now let's assume we can approximate the rating matrix R as a product of "thin" $Q \cdot P^T$
 - R has missing entries but let's ignore that for now!
 - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i? $\hat{x} = a$





$r_{xi} = q$	i Px
$=\sum_{\boldsymbol{\epsilon}}\boldsymbol{q}$	$_{if}\cdot p_{xf}$

$$q_i$$
= row i of Q
 p_x = column x of P^T

	.1	4	.2
()	5	.6	.5
items	2	.3	.5
ite	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3

_	users											
S	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9
cto	8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
fa	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1
•												

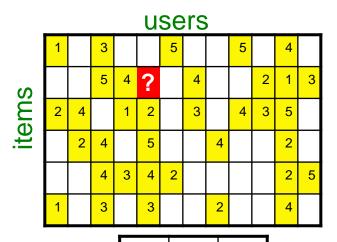
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factors

Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i? $\hat{x} = a$





$\hat{r}_{xi} =$	q_i .	$\boldsymbol{p}_{\boldsymbol{x}}$
$=\sum_{\epsilon}$	q_{if}	$\cdot p_{xf}$

$$q_i$$
= row i of Q
 p_x = column x of P^T

	.1	4	.2					
()	5	.6	.5					
items	2	.3	.5					
ite	1.1	2.1	.3					
	7	2.1	-2					
	-1	.7	.3					
factors								

_													
<u>rs</u>	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9	
ctc	8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3	
fa	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1	
-													

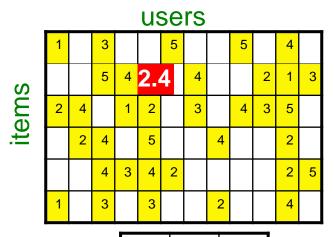
users

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Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i? $\hat{x} = a$





\hat{r}_{x}	_i =	q_i .	$\boldsymbol{p}_{\boldsymbol{x}}$
=	\sum_{ϵ}	q_{if}	$\cdot \boldsymbol{p}_{xf}$
	f		

$$q_i$$
= row i of Q
 p_x = column x of P^T

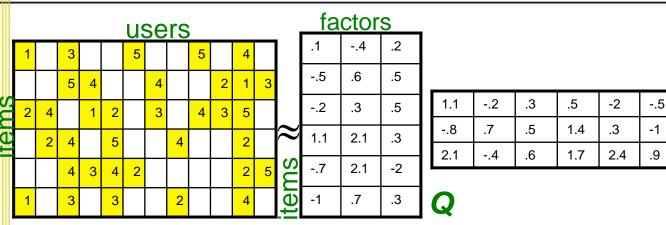
	.1	4	.2
	5	.6	.5
	2	.3	.5
	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3
,	fa	ctors	3

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)rs	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9
		.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
fa	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1

users

PT

Latent Factor Models



users						
5	.8	4	.3	1.4	2.4	9
-1	14	29	- 7	12	- 1	13

PT

- SVD isn't defined when entries are missing!
- Use specialized methods to find P, Q

$$\min_{P,Q} \sum_{(i,x)\in \mathbb{R}} (r_{xi} - q_i \cdot p_x)^2$$

$$\hat{r}_{xi} = q_i \cdot p_x$$

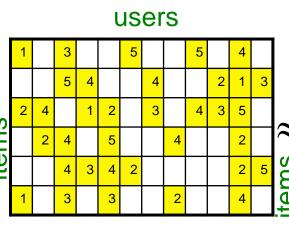
- Note:
 - We don't require cols of P, Q to be orthogonal/unit length
 - P, Q map users/movies to a latent space
 - The most popular model among Netflix contestants

Finding the Latent Factors

Latent Factor Model

Our goal is to find P and Q such tat:

$$\min_{P,Q} \sum_{(i,x)\in R} (r_{xi} - q_i \cdot p_x)^2$$



<u> </u>							
	.1	4	.2				
	5	.6	.5				
	2	.3	.5				
>	1.1	2.1	.3				
2	7	2.1	-2				
פֿ	-1	.7	.3				

factors

П	S	6	rs
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1.1	2	.3	.5	-2	5 -1	.8	4	.3	1.4	2.4	9
8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1

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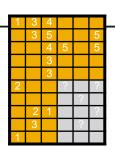
Back to Our Problem

- Want to minimize SSE for unseen test data
- Idea: Minimize SSE on training data
 - Want large k (# of factors) to capture all the signals
 - But, SSE on test data begins to rise for k > 2
- This is a classical example of overfitting:
 - With too much freedom (too many free parameters) the model starts fitting noise
 - That is it fits too well the training data and thus not generalizing well to unseen test data



Dealing with Missing Entries

To solve overfitting we introduce regularization:



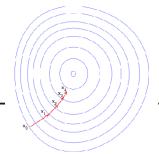
- Allow rich model where there are sufficient data
- Shrink aggressively where data are scarce

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} \|p_x\|^2 + \lambda_2 \sum_{i} \|q_i\|^2 \right]$$
"error"
"length"

 $\lambda_1, \lambda_2 \dots$ user set regularization parameters

Note: We do not care about the "raw" value of the objective function, but we care in P, Q that achieve the minimum of the objective

Gradient Decent



Want to find matrices P and Q:

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} \|p_x\|^2 + \lambda_2 \sum_{i} \|q_i\|^2 \right]$$

- Gradient decent:
 - Initialize P and Q (using SVD, pretend missing ratings are 0)
 - Do gradient descent:

■
$$P \leftarrow P - \eta \cdot \nabla P$$

•
$$Q \leftarrow Q - \eta \cdot \nabla Q$$

How to compute gradient of a matrix?

Compute gradient of every element independently!

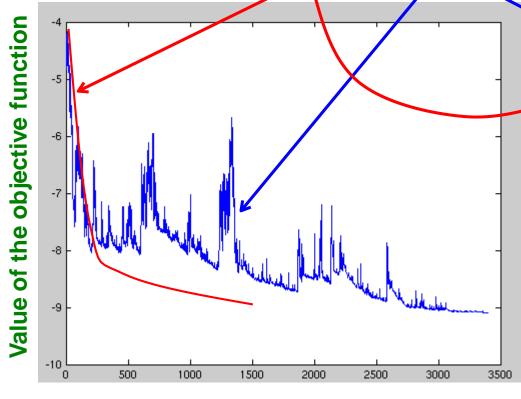
• where ∇Q is gradient/derivative of matrix Q:

$$abla Q = [
abla q_{if}]$$
 and $abla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x)p_{xf} + 2\lambda_2 q_{if}$

- Here q_{if} is entry f of row q_i of matrix Q
- Observation: Computing gradients is slow!

SGD vs. GD

Convergence of GD vs. SGD



Iteration/step

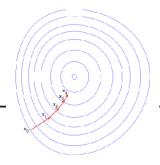
GD improves the value of the objective function at every step.

SGD improves the value but in a "noisy" way.

GD takes fewer steps to converge but each step takes much longer to compute.

In practice, **SGD** is much faster!

Stochastic Gradient Decent



Stochastic gradient decent:

- Initialize P and Q (using SVD, pretend missing ratings are 0)
- Then iterate over the ratings (multiple times if necessary) and update factors.

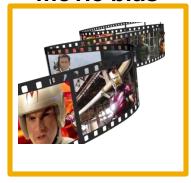
Extending Latent Factor Model to Include Biases

Model Biases and Interaction

user bias



movie bias



user-movie interaction



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition
 - $\mu = \mu$ = overall mean rating
 - $\mathbf{b}_{\mathbf{x}} = \text{bias of user } \mathbf{x}$
 - \mathbf{b}_i = bias of movie \mathbf{i}

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

Baseline Predictor

We have expectations on the rating by user x of movie i, even without estimating x's attitude towards movies like i







- Rating scale of user x
- Values of other ratings user gave recently (day-specific mood, anchoring, multi-user accounts)

- (Recent) popularity of movie i
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting it All Together

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

Overall Bias for Bias for Movie interaction interaction

Example:

- Mean rating: $\mu = 3.7$
- You are a critical reviewer: your ratings are 1 star lower than the mean: $b_x = -1$
- Star Wars gets a mean rating of 0.5 higher than average movie: $b_i = +0.5$
- Predicted rating for you on Star Wars:

$$= 3.7 - 1 + 0.5 = 3.2$$

Fitting the New Model

Solve:

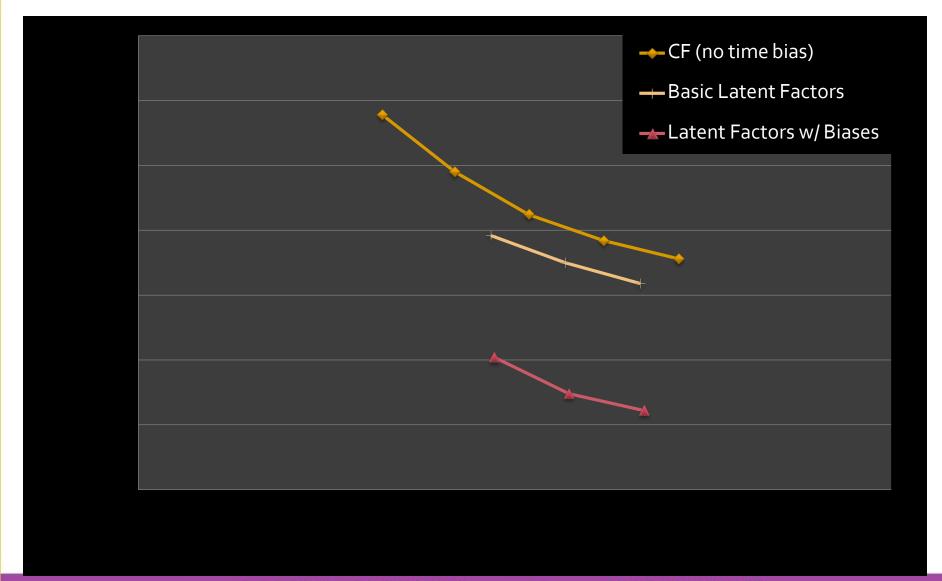
$$\min_{Q,P} \sum_{(x,i)\in R} (r_{xi} - (\mu + b_x + b_i + q_i p_x))^2$$
goodness of fit

$$+ \left(\frac{\lambda_{1}}{1} \sum_{i} \|q_{i}\|^{2} + \lambda_{2} \sum_{x} \|p_{x}\|^{2} + \lambda_{3} \sum_{x} \|b_{x}\|^{2} + \lambda_{4} \sum_{i} \|b_{i}\|^{2} \right)$$
regularization

 λ is selected via gridsearch on a validation set

- Stochastic gradient decent to find parameters
 - Note: Both biases b_x , b_i as well as interactions q_i , p_x are treated as parameters (we estimate them)

Performance of Various Models



Performance of Various Methods

Global average: 1.1296

<u>User a</u>verage: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

Latent factors+Biases: 0.89

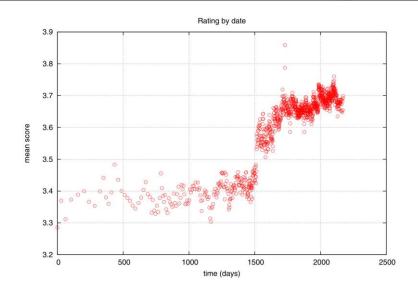
Grand Prize: 0.8563

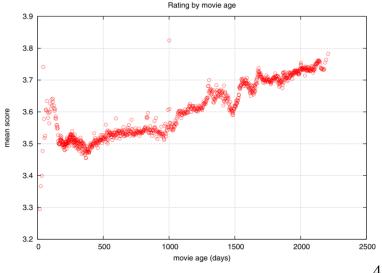
The Netflix Challenge 2006-09

Temporal Biases of Users

- Sudden rise in the average movie rating (early 2004)
 - Improvements in Netflix
 - GUI improvements
 - Meaning of rating changed
- Movie age
 - Users prefer old movies without any reasons
 - Older movies are just inherently better than newer ones

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09





Temporal Biases & Factors

Original model:

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

Add time dependence to biases:

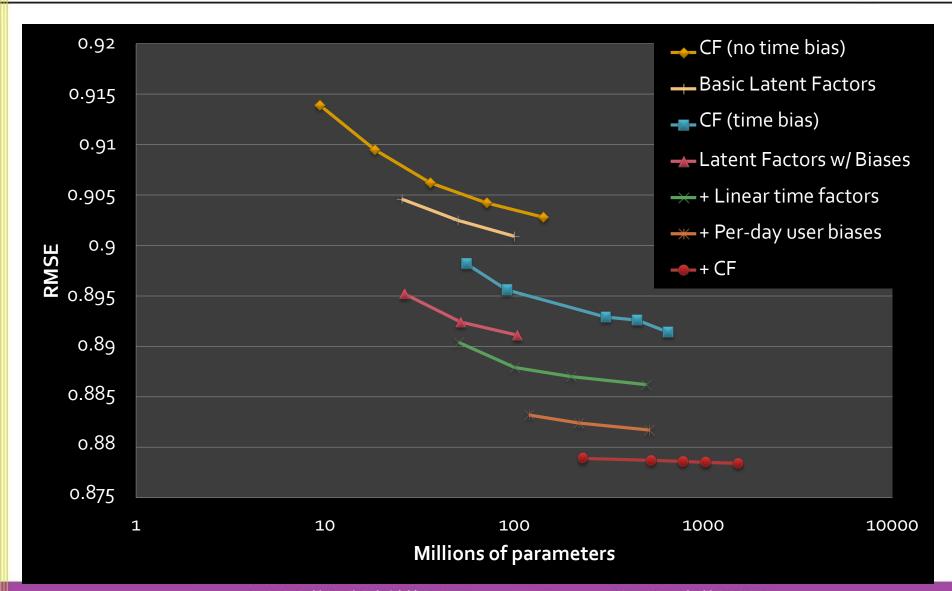
$$r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$$

- Make parameters b_x and b_i to depend on time
- (1) Parameterize time-dependence by linear trends
 - (2) Each bin corresponds to 10 consecutive weeks

$$b_i(t) = b_i + b_{i,\operatorname{Bin}(t)}$$

- Add temporal dependence to factors
 - $p_x(t)$... user preference vector on day t

Adding Temporal Effects



Performance of Various Methods

Global average: 1.1296

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Movie average: 1.0533

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Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

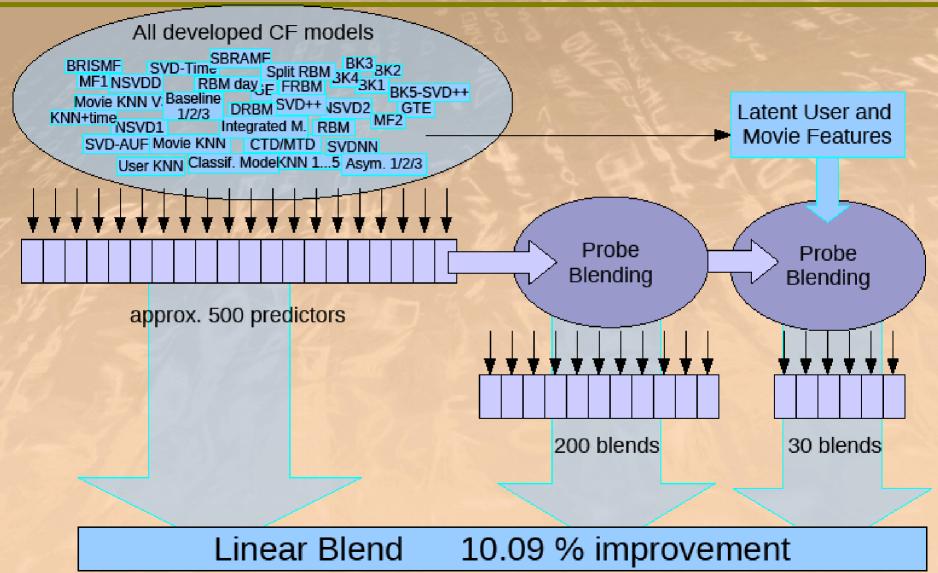
Latent factors+Biases: 0.89

Latent factors+Biases+Time: 0.876

Still no prize! (2)
Getting desperate.
Try a "kitchen sink" approach!

Grand Prize: 0.8563

The big picture solution of BellKor's Pragmatic Chaos



The Last 30 Days

Ensemble team formed

- Group of other teams on leaderboard forms a new team
- Relies on combining their models
- Quickly also get a qualifying score over 10%

BellKor

- Continue to get small improvements in their scores
- Realize they are in direct competition with team Ensemble

Strategy

- Both teams carefully monitoring the leaderboard
- Only sure way to check for improvement is to submit a set of predictions
 - This alerts the other team of your latest score

24 Hours from the Deadline

Submissions limited to 1 a day

- Only 1 final submission could be made in the last 24h
- 24 hours before deadline...
 - BellKor team member in Austria notices (by chance) that
 Ensemble posts a score that is slightly better than BellKor's
- Frantic last 24 hours for both teams
 - Much computer time on final optimization
 - Carefully calibrated to end about an hour before deadline
- Final submissions
 - BellKor submits a little early (on purpose), 40 mins before deadline
 - Ensemble submits their final entry 20 mins later
 -and everyone waits....

Netflix Prize



Home

Rules

Leaderboard

Update

Progress Prize 2007 - RMSE = 0.8723 - Winning Team: KorBell

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Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 ‡ leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand	Prize - RMSE = 0.8567 - Winning To	sam: BellKor's Prace	natic Chane	
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.0002	5.50	24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries!	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progr	ess Prize 2008 - RMSE = 0.8627 - W	inning Team: BellKo	r in BigChaos	
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50

Million \$ Awarded Sept 21st 2009



Acknowledgement

CS246: Mining Massive Datasets, Stanford University

Thank you!

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