Dark energy model with multiple scalar fields

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Abstract

In addition to Einstein's famous cosmological constants, a large number of dark energy theoretical models have been proposed to try to explain the expansion of the cosmos. In these dark energy models, only a single scalar field cannot make the state equation parameters $\omega = -1$. So we must consider the dark energy model with multiple scalar fields.

The No-go theorem shows that the following conditions make it impossible for w cross phantom potential well w=-1:(1)Classical physics; (2)General relativity is reasonable; (3)Single real scalar field; (4)Arbitrary Lagrangian density $L(\Phi, X)$. Among them, the $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi$ is the kinetic energy term.(5) $L(\Phi, X)$ is a continuous and sufficiently differentiable function. Therefore, to achieve the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$), at least one of the conditions in the No-go theorem above must be abandoned. Obviously, the simplest method is to consider the model containing multiple scalar fields to destroy the condition (3).

This paper studies the evolution of the dark energy model with multiple scalar fields. In addition to several common evolutionary endings, this model can give the result of the accelerated expansion of the universe and achieve the goal of the state equation crossing $\omega = -1$.

Keyword: scalar fields、dark energy、No-go theorem、quintom model

I Introduction

Since it was discovered in 1998 that the universe is accelerating expansion, dark energy, as a mysterious cause of accelerated expansion, has become an extremely important issue in the field of astrophysics and cosmology research. Because of its unique negative pressure that is different from ordinary matter and radiation, dark energy can produce a repulsive force that resists gravitational contraction, which in turn drives the universe to expand faster. The study of its essence has never ceased. In addition to Einstein's famous cosmological constants, a large number of dark energy theoretical models have been proposed to try to explain this mystery. In these dark energy models, only a single scalar field cannot make the state equation parameters $\omega = -1$. So we must consider the dark energy model with multiple scalar fields.

II Dark energy model with a single scalar field

A Standard cosmology

The distribution of matter in cosmic is uniform and isotropic on a large scale, so we have Robertson-Walker metrics.[1,2,3]

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{2.1}$$

We know that our cosmic space is approximately a flat universe, and we usually take the curvature of the three-dimensional space as 0. Substitute equation (2.1) into Einstein equation [1]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \tag{2.2}$$

In the above formula, $R_{\mu\upsilon}$ and R are curvature tensor and curvature scalar, and they are both functions related to the metric. And $T_{\mu\upsilon}$ is the energy momentum tensor of the material part. For an ideal fluid:

$$T_\nu^\mu = Diag(-\rho, p, p, p) \tag{2.3}$$

So we get:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{2.4}$$

and

$$\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2} \tag{2.5}$$

The equation (2.4) is called the Friedmann equation. Through (2.4) and (2.5), we can get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2.6}$$

Observed facts tell us that $\rho + 3p < 0$. In other words, the universe will expand faster. We have the equation of motion of the part of the ideal fluid substance：

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{2.7}$$

In cosmology, an ideal fluid has the property of the constant state equation $\omega = p/\rho = const$. For example, the state equation of the radiant matter dominating at the early point of the universe is $\omega_r = 1/3$, But the equation of state for baryon matter and cold dark matter is $\omega_m = 0$. If the universe is to accelerate expansion, there must be dark energy matter with $\omega < 1/3$.

From equation (2.7), the relationship between the energy density ρ_ω and $a(t)$ of this ideal fluid can be obtained:

$$\rho_\omega \propto a^{-3(1+\omega)} \tag{2.8}$$

It can be seen from this that the smaller the ω of an ideal fluid substance, the slower its energy density decreases with the expansion of the universe.

So the early universe was dominated by radiant matter, then it became dominated by dark matter, and finally it will be dominated by dark energy.

B LCDM model

The cosmological constant was first proposed by Einstein. Its original purpose was to obtain a static universe. Einstein found that adding a constant term to

Einstein's gravitational equation does not destroy the symmetry of the equation, that is, we cannot theoretically rule out the existence of this term in the equation [3]. The gravity equation after adding the cosmological constant is [4]:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=8\pi GT\mu\nu \qquad (2.9)$$

We move the cosmological constant to the right of the equation. Considering the cosmological constant as part of the energy dynamic tensor, we can get the corresponding Robertson — Walker metric equation:

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3} \qquad (2.10)$$

It is generally believed that there are two types of matter components in the universe: the radiant matter with the equation of state = 1/3 and the zero-pressure matter with u=0. At present, the uniformly distributed radiation materials on the large scale of the universe are mainly photons and neutrinos, while the zero-pressure materials include cold and dark materials and general baryon materials. So we can get:

$$\left(\frac{H}{H_0}\right)^2=\Omega_r^{(0)}a^{-4}+\Omega_m^{(0)}a^{-3}-\frac{k}{H_0^2}a^{-2}+\Omega_\Lambda^{(0)} \qquad (2.10)$$

Assuming that the universe originates from the singularity of infinite energy density (the Big Bang theory), the age of the universe can be calculated by using the equation (2.10):

$$t_0=\int_0^{t_0}dt=\int_0^1\frac{da}{a_0H_0\sqrt{\Omega_r^{(0)}a^{-4}+\Omega_m^{(0)}a^{-3}-\frac{k}{H_0^2}a^{-2}+\Omega_\Lambda^{(0)}}} \qquad (2.11)$$

Form (2.11), we get

$$H_0t_0=\int_0^1\frac{da}{\sqrt{\Omega_m^{(0)}a^{-1}+\left(1-\Omega_m^{(0)}\right)}} \qquad (2.12)$$

It can be seen from (2.12) that the existence of cosmological constants can well solve the problem of the age of the universe.

When $\Omega_m^{(0)}>1$, integral (2.12) to get[5]

$$H_0t_0=\frac{\Omega_m^{(0)}}{2\left(\Omega_m^{(0)}-1\right)^{\frac{3}{2}}}\left[\cos^{-1}\left(\frac{2}{\Omega_m^{(0)}}-1\right)-\frac{2}{\Omega_m^{(0)}}\left(\Omega_m^{(0)}-1\right)^{\frac{1}{2}}\right] \qquad (2.13)$$

When $\Omega_m^{(0)}<1$, integral (2.12) to get[5]

$$H_0t_0=\frac{\Omega_m^{(0)}}{2\left(\Omega_m^{(0)}-1\right)^{\frac{3}{2}}}\left[-\cos^{-1}\left(\frac{2}{\Omega_m^{(0)}}-1\right)+\frac{2}{\Omega_m^{(0)}}\left(1-\Omega_m^{(0)}\right)^{\frac{1}{2}}\right] \qquad (2.13)$$

If the cosmic space is flat, that is $\Omega_m^{(0)}=1$, then $H_0t_0=\frac{2}{3}$. The range of the Hubble constant measured by the Hubble telescope is: [6]

$$H_0^{-1}=9.776h^{-1}Gyr, 0.64<h<0.80 \quad (2.13)$$

And then , the age of the flat universe is $t_0=8-10Gyr$ [6].

But why does the cosmological constant take such a tiny value. What is its specific source? Can its size be derived from some basic theory? These questions are still not well answered. In field theory, there is a huge contradiction in the existence of the cosmological constant. Since the cosmological constant does not change with time, and the current observations show that the current universe is dominated by dark energy, the magnitude of the cosmological constant should be of the same order as the current energy density of the universe.

But when we fitted this model in the laboratory, we found that $\omega = -1$ [5]. This phenomenon has been difficult to explain with a dark energy model containing a scalar field. Therefore, we need to turn our attention to the dark energy model with multiple scalar fields.

III Dark energy model with double scalar fields

Let's discuss the dark energy model with two scalar fields. One of them is the quintessence field and the other is the phantom field.What we consider is the general situation where there is an interaction between them, and the interaction is expressed by the coupling of the two fields in the potential energy term. The evolution of the universe depends on the role of the entire system composed of these two scalar fields. Due to their respective properties, this model is able to achieve equations of state exceeding -1 and may avoid "big tear".

Dark energy initially showed the evolutionary behavior of the Phantom field. In the end, this situation needs to select a special potential energy to make the Phantom field finally withdraw from the evolution. The Quintessence field oscillates because it obtains a large mass, so that the dark energy finally evolves according to the behavior of matter, which avoids the failure caused by the Phantom field. Stability and the emergence of "big tear" in the future.

If early Quintessence dominates, then $\omega > -1$. Whether it is the Quintessence field or the Phantom field, as long as they evolve independently, according to the law of conservation of energy, they all satisfy:

$d\rho / dt = 3H\rho$,which is $\rho \propto a^{-3(\omega+1)}$.

Therefore, as the energy density of the phantom field increases with time, the quintessence energy density decreases at any time, and the phantom field always plays a leading role at the end. At this time, $\omega < -1$. That is to say, regardless of the initial training and the energy density of the two scalar fields, the energy density of the quintessence field is always smaller than the energy density of the phantom field to a certain stage of evolution. If the initial conditions are set to make the density of the quintessence field larger. That is, in the early stage of evolution, $\omega > -1$, the dual-field model can make w exceed -1. The ultimate fate of the universe is deSitter space-time or "big tear".

In this case, we need to choose the appropriate interaction items. Of course, this type of interacting dual-field dark energy model, the final result also includes "large tear" because the state equation W has always been less than -1, or the universe evolved into DeSitter because the state equation ω eventually tended to -1 . If the scalar fields of these two momentum terms have opposite signs, and the potential energy satisfies a certain condition, the kinetic energy terms of the Phantom field will rapidly decay in evolution, with the following form: $\dot{\psi} \propto a^{-3}$. In this way, the evolution of the Phantom field is "frozen".

To achieve this goal, a negative kinetic energy field with only nonindependent potential energy terms can be selected. This kind of field is coupled with a scalar field with positive kinetic energy terms. This situation will be discussed in detail below.

Here is a special case. Let the Lagrangian of the dual-field model be[7]:

$$L = -\frac{1}{2}\partial_{\mu}\psi\partial^{\mu}\psi + \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - V_{eff}(\psi,\phi) \tag{3.1}$$

At the beginning, we can choose that the derivative of the potential energy to the field is close to 0, which means that the potential energy is almost constant in this stage, and the two fields are close to decoupling. We call this phase the "weak coupling" period, at which time they can be regarded as mutually independent. On the contrary, when the derivative of the potential energy with respect to the field is not 0, the two scalar fields are in the "strong coupling" period. At this time, they have a clear interaction.

In the weakly coupled period, the phantom field appears as a massless scalar field, and its energy density is attenuated rapidly according to the rules of a. The mouth here is the scale factor of the universe. The corresponding quintessence field has a qualitative term, and its evolutionary behavior is determined by the ratio m/H. If m/H is much smaller than 1, it will show a "slow roll" behavior, similar to a cosmological constant. However, when m/H is far greater than 1, its kinetic energy and potential energy terms will oscillate together, and will attenuate according to the law of n, showing the material behavior in the oscillation.

During the strong coupling phase, the two scalar fields will quickly evolve towards their equilibrium point, as shown in Figure 1[7].

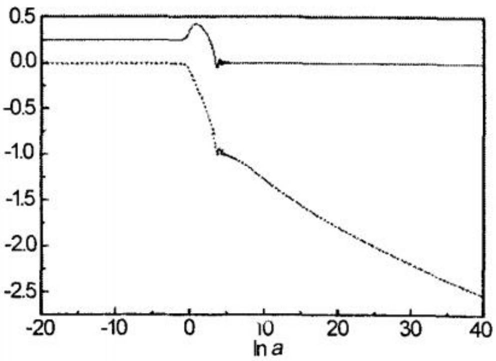

Fig.1 The scalar field as a function of $\ln a$

Because the equilibrium point determines the evolution trend of cosmos at a certain degree, it is necessary to analyze the equilibrium point of these two scalar fields in detail. The partial derivative of the effective potential for the two scalar fields can be obtained by simple calculation, and the equilibrium point is reached when the partial derivative is equal to zero. We find that the equilibrium point of ψ is infinity, while the equilibrium point of ϕ is 0.

During the evolution of ϕ, the zero point is the smallest value. In the initial state, if ϕ is not nearly equal to 0, the evolutionary behavior is a Phantom-like scalar field, and ψ will drive the potential energy to increase. This direction of evolution tends to be negative infinity along ψ. In this process, the evolutionary line is a scalar field similar to quintessence, and f gradually evolves toward the zero point. In summary, the entire system will evolve towards the weak coupling stage, and ϕ will gain a large quality.

In this example, the initial condition selected is the dark energy state equation $\omega > -1$. Because in the early stages of evolution, radiation first dominates, and then matter dominates. The evolution of both fields is frozen, and the equation of state is always -1. After dark energy begins to evolve, the energy density of the phantom field increases with time, and the energy density of the quintessence field decreases with time. The state equation of the system will evolve from greater than -1 to less than -1. As shown in Figure 1, the quintessence field is drawn with solid lines, and the dashed line represents the phantom field. When the potential energy increases to a certain extent, the contribution of the phantom field to the evolution of potential energy becomes negligible, and both fields have entered the stage of independent evolution. At this time, phantom exits the evolution, and the universe re-enters the dominant phase of the quintessence field, and the quintessence field begins to oscillate due to the acquisition of a mass much larger than H, which causes the state equation w to oscillate around zero. In this way, the phenomenon of "big tear" is avoided. The evolution of the equation of state w is shown in Figure 2[7].

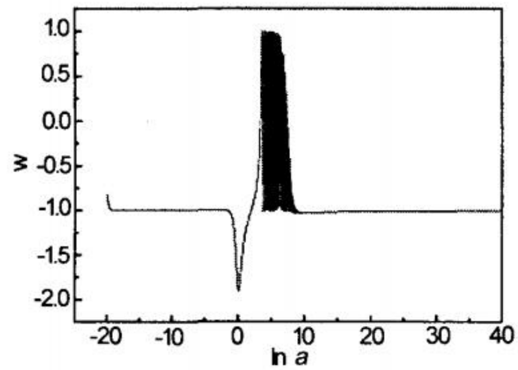

Fig.2 The equation of state ω as a function of $\ln a$

The special feature of this model is that it is first directed by quintessence to be directed by phantom, but it ends with quintessence. The last evolutionary behavior is similar to the evolution of matter, so that the universe stops accelerating expansion.

IV Dark energy model with double scalar fields

A Quintom model

Recently, cosmologists have made a lot of efforts in constructing models that enable $\omega = -1$ in the phantom field. Obviously, only a single-field phantom model ($\omega < -1$) or quintessence model ($\omega > -1$) can not make the state equation parameter of dark energy w cross phantom potential well $\omega = -1$. The No-go theorem shows that the following conditions make it impossible for w cross phantom potential well w=-1:(1)Classical physics; (2)General relativity is reasonable; (3)Single real scalar field; (4)Arbitrary Lagrangian density $L(\Phi, X)$. Among them, the $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi$ is the kinetic energy term.(5) $L(\Phi, X)$ is a continuous and sufficiently differentiable function. Therefore, to achieve the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$), at least one of the conditions in the No-go theorem above must be abandoned. Obviously, the simplest method is to consider the model containing multiple scalar fields to destroy the condition (3).

The paper [8] considers a double real scalar field model and discusses the possibility of w cross phantom potential well $\omega = -1$. Feng Wang and Zhang proposed a quintom model in [7] (quintom is named after combining the word prefix of quintessence and the word suffix of phantom). Naturally, its Lagrangian density has the following form[7]:

$$L = \frac{1}{2} \dot{\Phi}_1^2 - \frac{1}{2} \dot{\Phi}_2^2 - V(\Phi_1, \Phi_2) \qquad (4.1)$$

Among them, Φ_1 and Φ_2 play the roles of quintessence field and phantom field respectively. Consider the RIV metric for flat spacetime and assume that both Φ_1 and Φ_2 are uniformly isotropressure and energy density of the quintom field are:

$$P=\frac{1}{2}\dot{\Phi}_1^2-\frac{1}{2}\dot{\Phi}_2^2-V(\Phi_1,\Phi_2) \qquad (4.2)$$

$$\rho=\frac{1}{2}\dot{\Phi}_1^2-\frac{1}{2}\dot{\Phi}_2^2+V(\Phi_1,\Phi_2) \qquad (4.3)$$

Thus, the state equation parameter is:

$$\omega=\frac{P}{\rho}=\frac{\frac{1}{2}\dot{\Phi}_1^2-\frac{1}{2}\dot{\Phi}_2^2-V(\Phi_1,\Phi_2)}{\frac{1}{2}\dot{\Phi}_1^2-\frac{1}{2}\dot{\Phi}_2^2+V(\Phi_1,\Phi_2)} \qquad (4.4)$$

Obviously, when $\dot{\Phi}_1^2 \geq \dot{\Phi}_2^2$, $\omega \geq -1$ and when $\dot{\Phi}_1^2 \leq \dot{\Phi}_2^2$, $\omega \leq -1$. So the quintom model achieve the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$).

B Model with three real scalar fields

We consider the following model with three real scalar fields to break the third condition of No-go theorem. Its Lagrangian density has the following form:

$$L=\frac{1}{2}\dot{\Phi}_1^2-\frac{1}{2}\dot{\Phi}_2^2-\frac{1}{2}\dot{\Phi}_3^2-V\left(\Phi_1^2,\Phi_2^2-\Phi_3^2\right) \qquad (4.5)$$

Consider the RW metric in flat space:

$$ds^2=dt^2-a^2(t)dX^2 \qquad (4.6)$$

It is obtained by the Euler-Lagrange equation:

$$\begin{aligned}&3H\sqrt{-g}\dot{\Phi}_1+\sqrt{-g}\ddot{\Phi}_1+\sqrt{-g}V_1'=0\\&\Rightarrow \ddot{\Phi}_1+3H\dot{\Phi}_1+V_1'=0\end{aligned} \qquad (4.7)$$

$$\begin{aligned}&3H\sqrt{-g}\dot{\Phi}_2+\sqrt{-g}\ddot{\Phi}_2-\sqrt{-g}V_2'=0\\&\Rightarrow \ddot{\Phi}_2+3H\dot{\Phi}_2-V_2'=0\end{aligned} \qquad (4.8)$$

$$\begin{aligned}&3H\sqrt{-g}\dot{\Phi}_3+\sqrt{-g}\ddot{\Phi}_3-\sqrt{-g}V_3'=0\\&\Rightarrow \ddot{\Phi}_3+3H\dot{\Phi}_3-V_3'=0\end{aligned} \qquad (4.9)$$

Let

$$\begin{aligned}\Phi_1&=\Phi\cosh\alpha\cosh\beta\\ \Phi_2&=\Phi\cosh\alpha\sinh\beta\\ \Phi_3&=\Phi\sinh\alpha\end{aligned} \qquad (4.10)$$

We can calculate that

$$\Phi_1^2-\Phi_2^2-\Phi_3^2=\Phi^2 \qquad (4.11)$$

So

$$\begin{aligned}\dot{\Phi}_1&=\dot{\Phi}\cosh\alpha\cosh\beta+\Phi\,\dot{\alpha}\sinh\alpha\cosh\beta+\Phi\,\dot{\beta}\cosh\alpha\sinh\beta\\ \dot{\Phi}_2&=\dot{\Phi}\cosh\alpha\sinh\beta+\Phi\,\dot{\alpha}\sinh\alpha\sinh\beta+\Phi\,\dot{\beta}\cosh\alpha\cosh\beta\\ \dot{\Phi}_3&=\dot{\Phi}\sinh\alpha+\dot{\Phi}\,\dot{\alpha}\cosh\alpha\end{aligned} \qquad (4.12)$$

Combining (4.11) and (4.12), we can get the following formula￥:

$$\begin{aligned}&\ddot{\Phi}+3H\dot{\Phi}+V'+\Phi\,\dot{\alpha}^2+\cosh^2\left(\alpha\left(\Phi\,\dot{\beta}\right)^2\right)=0\\&\ddot{\alpha}+3H\alpha+2\frac{\dot{\Phi}}{\Phi}\left(\dot{\alpha}-\dot{\beta}\right)^2\cosh\alpha\sinh\alpha=0\\&\ddot{\beta}+3H\beta+2\frac{\dot{\Phi}}{\Phi}\dot{\beta}+2\dot{\beta}\tanh\alpha=0\end{aligned} \qquad (4.13)$$

Lagrangian density has the following form:

$$L=\frac{1}{2}\dot{\Phi}^2-\frac{1}{2}\Phi^2\dot{\alpha}^2-\frac{1}{2}\Phi^2\dot{\beta}^2\cosh^2\alpha-V\left(\Phi^2\right) \qquad (4.13)$$

Then the energy density and pressure are:

$$\rho=\frac{1}{2}\dot{\Phi}^2-\frac{1}{2}\Phi^2\dot{\alpha}^2-\frac{1}{2}\Phi^2\dot{\beta}^2\cosh^2\alpha+V\left(\Phi^2\right) \qquad (4.13)$$

$$P=\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\phi^2\dot{\alpha}^2-\frac{1}{2}\phi^2\dot{\beta}^2\cosh^2\alpha-V(\phi^2)$$
(4.14)

State equation parameters:

$$\omega=\frac{P}{\rho}=\frac{\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\phi^2\dot{\alpha}^2-\frac{1}{2}\phi^2\dot{\beta}^2\cosh^2\alpha-V(\phi^2)}{\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\phi^2\dot{\alpha}^2-\frac{1}{2}\phi^2\dot{\beta}^2\cosh^2\alpha+V(\phi^2)}$$
(4.15)

Obviously, We can draw that when $\dot{\phi}^2 \geq \phi^2\left(\dot{\alpha}^2+\dot{\beta}^2\cosh^2\alpha\right)$, $\omega \geq -1$ and when $\dot{\phi}^2 \leq \dot{\phi}^2\left(\dot{\alpha}^2+\dot{\beta}^2\cosh^2\alpha\right)$, $\omega \leq -1$ .So the dark energy mode with three scalar fields achieves the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$).

V Conclusion

This paper first analyzes the development status of various scalar field dark energy models and dark energy models, and focuses on three types of models: dark energy models with a single scalar field, dark energy models with dual scalar fields, and three scalar fields Dark energy model. We also introduced the characteristics and limitations of these three types of models.

Through our discussion, we can know that with the development of astronomical observation technology in recent years, when domestic and foreign cosmologists use experimental data to fit the dark energy model, they find that the state equation of dark energy has a tendency to cross -1 . So since then, many scholars have begun to try to establish a new dark energy model to solve this problem.

This paper reviews the standard cosmological model and the LCDM dark energy model. This scalar field model can easily obtain the oscillating H and w<1 equations of state. Obviously, only a single-field phantom model ($\omega < -1$) or quintessence model ($\omega > -1$) can not make the state equation parameter of dark energy w cross phantom potential well $\omega = -1$.

We know that the following conditions make it impossible for w cross phantom potential well w=-1:(1)Classical physics; (2)General relativity is reasonable; (3)Single real scalar field; (4)Arbitrary Lagrangian density $L(\phi, X)$. Among them, the $X=\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$ is the kinetic energy term.(5) $L(\phi, X)$ is a continuous and sufficiently differentiable function.

In order to destroy the condition (3), we introduced a dark energy model with two scalar fields. In this part of the discussion,the quintom model can achieve the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$). For the dark energy model with three scalar fields, it can also achieve the transition from $\omega > -1$ to $\omega < -1$ (or from $\omega < -1$ to $\omega > -1$).

According to [3], we know that the dark energy model with multiple scalar fields can obtain different fate of the universe's evolution: DeSitter space-time (the universe will continue to expand, and the later evolutionary behavior is similar to the cosmological constant); 2) "Great tear" (In a limited time, the scale factor of the universe, the energy density of dark energy, and the pressure all reach infinity); 3) Dark energy evolves in a behavior similar to matter, and the universe stops accelerating its expansion. The construction of dark matter models with different number of scalar fields is much helpful for humans to understand the universe and predict the fate of the universe.

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