In [ ]:
```
from IPython.display import Image
```

In [ ]:
```
var('x,y,z')
# To start things off, enter the function f that you want to use.
f(x,y,z) =  1-7(0.01) -(1/2-0.07)*x-(2/7+0.01)*y-0.01*z
show(f)
```

In [ ]:
```
# Next, we need a constraint equation of the form g(x,y)= 0.  (Note, "C" is inside g.)
# Also, the constraint curve needs to be entered as a vector valued function r(t) = <u(t),v(t)>
var('t')
var('s')
g(x,y,z)= 1-x^2-9y^2-z^2 = -t



tstart=0
tend= 1/100
```

In [ ]:
```
h(x,y,z)= 3-x-9y-z = -s

sstart=0
send= 1/10
```

In [8]: `Image("Min1.png")` t=0, s=0

Out[8]:

Form the Lagrangian: $L(x,y,z,\lambda,\mu) = \left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)$.

Find all the first-order partial derivatives:

$\frac{\partial}{\partial x}\left(\left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)\right) = -2\lambda x - \mu - \frac{493}{1000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial y}\left(\left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)\right) = -2\lambda y - \mu - \frac{1}{1000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial z}\left(\left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)\right) = -18\lambda z - 9\mu - \frac{18063}{7000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \lambda}\left(\left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)\right) = -x^2 - y^2 - 9z^2 + 1$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \mu}\left(\left(-\frac{493x}{1000} - \frac{y}{1000} - \frac{18063z}{7000} + \frac{993}{1000}\right) + \lambda\left(-x^2-y^2-9z^2+1\right) + \mu\left(-x-y-9z+3\right)\right) = -x - y - 9z + 3$ (for steps, see partial derivative calculator).

Next, solve the system $\begin{cases} \frac{\partial L}{\partial x} = 0 \\ \frac{\partial L}{\partial y} = 0 \\ \frac{\partial L}{\partial z} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \\ \frac{\partial L}{\partial \mu} = 0 \end{cases}$, or $\begin{cases} -2\lambda x - \mu - \frac{493}{1000} = 0 \\ -2\lambda y - \mu - \frac{1}{1000} = 0 \\ -18\lambda z - 9\mu - \frac{18063}{7000} = 0 \\ -x^2 - y^2 - 9z^2 + 1 = 0 \\ -x - y - 9z + 3 = 0 \end{cases}$.

The system has the following real solutions: $(x,y,z) = \left(\frac{3}{11} - \frac{274\sqrt{25705}}{169653}, \frac{231345+1787\sqrt{25705}}{848265}, \frac{3}{11} - \frac{139\sqrt{25705}}{2544795}\right)$, $(x,y,z) = \left(\frac{274\sqrt{25705}+46269}{169653}, \frac{3}{11} - \frac{1787\sqrt{25705}}{848265}, \frac{139\sqrt{25705}+694035}{2544795}\right)$.

$f\left(\frac{3}{11} - \frac{274\sqrt{25705}}{169653}, \frac{231345+1787\sqrt{25705}}{848265}, \frac{3}{11} - \frac{139\sqrt{25705}}{2544795}\right) = -\frac{231345+1787\sqrt{25705}}{848265000} + \frac{5564789\sqrt{25705}}{5937855000} + \frac{11919}{77000}$

$f\left(\frac{274\sqrt{25705}+46269}{169653}, \frac{3}{11} - \frac{1787\sqrt{25705}}{848265}, \frac{139\sqrt{25705}+694035}{2544795}\right) = -\frac{2007\left(139\sqrt{25705}+694035\right)}{1979285000} - \frac{493\left(274\sqrt{25705}+46269\right)}{169653000} + \frac{1787\sqrt{25705}}{848265000} + \frac{273}{275}$

Thus, the minimum value is $-\frac{2007\left(139\sqrt{25705}+694035\right)}{1979285000} - \frac{493\left(274\sqrt{25705}+46269\right)}{169653000} + \frac{1787\sqrt{25705}}{848265000} + \frac{273}{275}$, and the maximum value is $-\frac{231345+1787\sqrt{25705}}{848265000} + \frac{5564789\sqrt{25705}}{5937855000} + \frac{11919}{77000}$.

ANSWER

Maximum

$-\frac{231345+1787\sqrt{25705}}{848265000} + \frac{5564789\sqrt{25705}}{5937855000} + \frac{11919}{77000} \approx 0.304436374346745$ A **at** $\left(\frac{3}{11} - \frac{274\sqrt{25705}}{169653}, \frac{231345+1787\sqrt{25705}}{848265}, \frac{3}{11} - \frac{139\sqrt{25705}}{2544795}\right) \approx (0.01378806004285, 0.610482289564545, 0.263969961154734)$ A.

Minimum

$-\frac{2007\left(139\sqrt{25705}+694035\right)}{1979285000} - \frac{493\left(274\sqrt{25705}+46269\right)}{169653000} + \frac{1787\sqrt{25705}}{848265000} + \frac{273}{275} \approx 0.004602586692216$ A **at** $\left(\frac{274\sqrt{25705}+46269}{169653}, \frac{3}{11} - \frac{1787\sqrt{25705}}{848265}, \frac{139\sqrt{25705}+694035}{2544795}\right) \approx (0.531666485411696, -0.06502774411, 0.281484584299812)$ A.

In [9]:
```
Image("Min2.png")    t=1/100, s=1/10
```

Out[9]:

Rewrite the constraint $-x^2-y^2-9z^2+1=-\frac{1}{100}$ as $-x^2-y^2-9z^2+\frac{101}{100}=0$.

Rewrite the constraint $-x-y-9z+3=-\frac{1}{10}$ as $-x-y-9z+\frac{31}{10}=0$.

Form the Lagrangian: $L(x,y,z,\lambda,\mu)=\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z+\frac{31}{10}\right)$.

Find all the first-order partial derivatives:

$\frac{\partial}{\partial x}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z\right.\right.$
$-2\lambda x-\mu-\frac{4993}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial y}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z\right.\right.$
$-2\lambda y-\mu-\frac{1}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial z}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z\right.\right.$
$-18\lambda z-9\mu-\frac{180063}{70000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \lambda}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z\right.\right.$
$-x^2-y^2-9z^2+\frac{101}{100}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \mu}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+\frac{101}{100}\right)+\mu\left(-x-y-9z\right.\right.$
$-x-y-9z+\frac{31}{10}$ (for steps, see partial derivative calculator).

Next, solve the system $\begin{cases}\frac{\partial L}{\partial x}=0\\ \frac{\partial L}{\partial y}=0\\ \frac{\partial L}{\partial z}=0\\ \frac{\partial L}{\partial \lambda}=0\\ \frac{\partial L}{\partial \mu}=0\end{cases}$, or $\begin{cases}-2\lambda x-\mu-\frac{4993}{10000}=0\\ -2\lambda y-\mu-\frac{1}{10000}=0\\ -18\lambda z-9\mu-\frac{180063}{70000}=0\\ -x^2-y^2-9z^2+\frac{101}{100}=0\\ -x-y-9z+\frac{31}{10}=0\end{cases}$.

The system has the following real solutions: $(x,y,z)=\left(\frac{31}{110}-\frac{353\sqrt{123535}}{543554},\frac{765917+2239\sqrt{123535}}{2717770},\frac{31}{110}-\frac{79\sqrt{123535}}{4076655}\right)$, $(x,y,z)=\left(\frac{1765\sqrt{123535}+765917}{2717770},\frac{31}{110}-\frac{2239\sqrt{123535}}{2717770},\frac{158\sqrt{123535}+2297751}{8153310}\right)$.

$f\left(\frac{31}{110}-\frac{353\sqrt{123535}}{543554},\frac{765917+2239\sqrt{123535}}{2717770},\frac{31}{110}-\frac{79\sqrt{123535}}{4076655}\right)=-\frac{765917+2239\sqrt{123535}}{27177700000}+\frac{71171833\sqrt{123535}}{190243900000}+\frac{128647}{962500}$

$f\left(\frac{1765\sqrt{123535}+765917}{2717770},\frac{31}{110}-\frac{2239\sqrt{123535}}{2717770},\frac{158\sqrt{123535}+2297751}{8153310}\right)=-\frac{60021\left(158\sqrt{123535}+2297751\right)}{190243900000}-\frac{4993\left(1765\sqrt{123535}+765917\right)}{27177700000}+\frac{2239\sqrt{123535}}{27177700000}+\frac{1099199}{1100000}$

Thus, the minimum value is $-\frac{60021\left(158\sqrt{123535}+2297751\right)}{190243900000}-\frac{4993\left(1765\sqrt{123535}+765917\right)}{27177700000}+\frac{2239\sqrt{123535}}{27177700000}+\frac{1099199}{1100000}$, and the maximum value is $-\frac{765917+2239\sqrt{123535}}{27177700000}+\frac{71171833\sqrt{123535}}{190243900000}+\frac{128647}{962500}$.

ANSWER

Maximum

$-\frac{765917+2239\sqrt{123535}}{27177700000}+\frac{71171833\sqrt{123535}}{190243900000}+\frac{128647}{962500}\approx 0.265091990775311$ A **at** $\left(\frac{31}{110}-\frac{353\sqrt{123535}}{543554},\frac{765917+2239\sqrt{123535}}{2717770},\frac{31}{110}-\frac{79\sqrt{123535}}{4076655}\right)\approx(0.05355965020752,0.571376738348647,0.275007067938204)$ A .

Minimum

$-\frac{60021\left(158\sqrt{123535}+2297751\right)}{190243900000}-\frac{4993\left(1765\sqrt{123535}+765917\right)}{27177700000}+\frac{2239\sqrt{123535}}{27177700000}+\frac{1099199}{1100000}\approx$

0.002170087146767 A **at**

$$\left(\frac{1765\sqrt{123535}+765917}{2717770}, \frac{31}{110} - \frac{2239\sqrt{123535}}{2717770}, \frac{158\sqrt{123535}+2297751}{8153310}\right) \approx$$

$(0.510076713428843, -0.007740374712284, 0.28862929569816)$ A .

In [10]:
```
Image("Min3.png")     t=1/100, s=0
```

Out[10]:

Form the Lagrangian: $L(x,y,z,\lambda,\mu) = \left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z + 3\right).$

Find all the first-order partial derivatives:

$\frac{\partial}{\partial x}\left(\left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z\right.\right.$
$-2\lambda x - \mu - \frac{4993}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial y}\left(\left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z\right.\right.$
$-2\lambda y - \mu - \frac{1}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial z}\left(\left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z\right.\right.$
$-18\lambda z - 9\mu - \frac{180063}{70000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \lambda}\left(\left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z\right.\right.$
$-x^2 - y^2 - 9z^2 + \frac{101}{100}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \mu}\left(\left(-\frac{4993x}{10000} - \frac{y}{10000} - \frac{180063z}{70000} + \frac{9993}{10000}\right) + \lambda\left(-x^2 - y^2 - 9z^2 + \frac{101}{100}\right) + \mu\left(-x - y - 9z\right.\right.$
$-x - y - 9z + 3$ (for steps, see partial derivative calculator).

Next, solve the system $\begin{cases} \frac{\partial L}{\partial x} = 0 \\ \frac{\partial L}{\partial y} = 0 \\ \frac{\partial L}{\partial z} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \\ \frac{\partial L}{\partial \mu} = 0 \end{cases}$, or $\begin{cases} -2\lambda x - \mu - \frac{4993}{10000} = 0 \\ -2\lambda y - \mu - \frac{1}{10000} = 0 \\ -18\lambda z - 9\mu - \frac{180063}{70000} = 0 \\ -x^2 - y^2 - 9z^2 + \frac{101}{100} = 0 \\ -x - y - 9z + 3 = 0 \end{cases}$.

The system has the following real solutions: $(x,y,z) = \left(\frac{3}{11} - \frac{353\sqrt{156395310}}{16306620}, \frac{22236300+2239\sqrt{156395310}}{81533100}, \frac{3}{11} - \frac{79\sqrt{156395310}}{122299650}\right)$, $(x,y,z) = \left(\frac{353\sqrt{156395310}+4447260}{16306620}, \frac{3}{11} - \frac{2239\sqrt{156395310}}{81533100}, \frac{79\sqrt{156395310}+33354450}{122299650}\right)$.

$f\left(\frac{3}{11} - \frac{353\sqrt{156395310}}{16306620}, \frac{22236300+2239\sqrt{156395310}}{81533100}, \frac{3}{11} - \frac{79\sqrt{156395310}}{122299650}\right) = -\frac{22236300+2239\sqrt{156395310}}{815331000000} + \frac{71171833\sqrt{156395310}}{5707317000000} + \frac{124419}{770000}$

$f\left(\frac{353\sqrt{156395310}+4447260}{16306620}, \frac{3}{11} - \frac{2239\sqrt{156395310}}{81533100}, \frac{79\sqrt{156395310}+33354450}{122299650}\right) = -\frac{20007\left(79\sqrt{156395310}+33354450\right)}{951219500000} - \frac{4993\left(353\sqrt{156395310}+4447260\right)}{163066200000} + \frac{2239\sqrt{156395310}}{815331000000} + \frac{1374}{1375}$

Thus, the minimum value is $-\frac{20007\left(79\sqrt{156395310}+33354450\right)}{951219500000} - \frac{4993\left(353\sqrt{156395310}+4447260\right)}{163066200000} + \frac{2239\sqrt{156395310}}{815331000000} + \frac{1374}{1375}$, and the maximum value is $-\frac{22236300+2239\sqrt{156395310}}{815331000000} + \frac{71171833\sqrt{156395310}}{5707317000000} + \frac{124419}{770000}$.

ANSWER

Maximum

$-\frac{22236300+2239\sqrt{156395310}}{815331000000} + \frac{71171833\sqrt{156395310}}{5707317000000} + \frac{124419}{770000} \approx 0.317472449445422$ **at** $\left(\frac{3}{11} - \frac{353\sqrt{156395310}}{16306620}, \frac{22236300+2239\sqrt{156395310}}{81533100}, \frac{3}{11} - \frac{79\sqrt{156395310}}{122299650}\right) \approx (0.002005854040157, 0.616152347197784, 0.26464908875134)$.

Minimum

$-\frac{20007\left(79\sqrt{156395310}+33354450\right)}{951219500000} - \frac{4993\left(353\sqrt{156395310}+4447260\right)}{163066200000} + \frac{2239\sqrt{156395310}}{815331000000} + \frac{1374}{1375} \approx 0.005639238866266$ **at** $\left(\frac{353\sqrt{156395310}+4447260}{16306620}, \frac{3}{11} - \frac{2239\sqrt{156395310}}{81533100}, \frac{79\sqrt{156395310}+33354450}{122299650}\right) \approx (0.543448691414388, -0.070697801743238, 0.280805456703206)$.

In [11]: `Image("Min4.png")    t=0, s=1/10`

Out[11]:

Rewrite the constraint $-x-y-9z+3=-\frac{1}{10}$ as $-x-y-9z+\frac{31}{10}=0$.

Form the Lagrangian: $L(x,y,z,\lambda,\mu)=\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\frac{31}{10}\right)$.

Find all the first-order partial derivatives:

$\frac{\partial}{\partial x}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\right.\right.$
$-2\lambda x-\mu-\frac{4993}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial y}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\right.\right.$
$-2\lambda y-\mu-\frac{1}{10000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial z}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\right.\right.$
$-18\lambda z-9\mu-\frac{180063}{70000}$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \lambda}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\right.\right.$
$-x^2-y^2-9z^2+1$ (for steps, see partial derivative calculator).

$\frac{\partial}{\partial \mu}\left(\left(-\frac{4993x}{10000}-\frac{y}{10000}-\frac{180063z}{70000}+\frac{9993}{10000}\right)+\lambda\left(-x^2-y^2-9z^2+1\right)+\mu\left(-x-y-9z+\right.\right.$
$-x-y-9z+\frac{31}{10}$ (for steps, see partial derivative calculator).

Next, solve the system $\begin{cases}\frac{\partial L}{\partial x}=0\\ \frac{\partial L}{\partial y}=0\\ \frac{\partial L}{\partial z}=0\\ \frac{\partial L}{\partial \lambda}=0\\ \frac{\partial L}{\partial \mu}=0\end{cases}$, or $\begin{cases}-2\lambda x-\mu-\frac{4993}{10000}=0\\ -2\lambda y-\mu-\frac{1}{10000}=0\\ -18\lambda z-9\mu-\frac{180063}{70000}=0\\ -x^2-y^2-9z^2+1=0\\ -x-y-9z+\frac{31}{10}=0\end{cases}$.

The system has the following real solutions: $(x,y,z)=\left(\frac{31}{110}-\frac{353\sqrt{103028190}}{16306620},\frac{2239\sqrt{103028190}+22977510}{81533100},\frac{31}{110}-\frac{79\sqrt{103028190}}{122299650}\right)$, $(x,y,z)=\left(\frac{353\sqrt{103028190}+4595502}{16306620},\frac{31}{110}-\frac{2239\sqrt{103028190}}{81533100},\frac{79\sqrt{103028190}+34466265}{122299650}\right)$.

$f\left(\frac{31}{110}-\frac{353\sqrt{103028190}}{16306620},\frac{2239\sqrt{103028190}+22977510}{81533100},\frac{31}{110}-\frac{79\sqrt{103028190}}{122299650}\right)=-\frac{2239\sqrt{103028190}+22977510}{815331000000}+\frac{71171833\sqrt{103028190}}{5707317000000}+\frac{128647}{962500}$

$f\left(\frac{353\sqrt{103028190}+4595502}{16306620},\frac{31}{110}-\frac{2239\sqrt{103028190}}{81533100},\frac{79\sqrt{103028190}+34466265}{122299650}\right)=-\frac{20007\left(79\sqrt{103028190}+34466265\right)}{951219500000}-\frac{4993\left(353\sqrt{103028190}+4595502\right)}{163066200000}+\frac{2239\sqrt{103028190}}{815331000000}+\frac{1099199}{1100000}$

Thus, the minimum value is $-\frac{20007\left(79\sqrt{103028190}+34466265\right)}{951219500000}-\frac{4993\left(353\sqrt{103028190}+4595502\right)}{163066200000}+\frac{2239\sqrt{103028190}}{815331000000}+\frac{1099199}{1100000}$, and the maximum value is $-\frac{2239\sqrt{103028190}+22977510}{815331000000}+\frac{71171833\sqrt{103028190}}{5707317000000}+\frac{128647}{962500}$.

ANSWER

Maximum

$-\frac{2239\sqrt{103028190}+22977510}{815331000000}+\frac{71171833\sqrt{103028190}}{5707317000000}+\frac{128647}{962500}\approx 0.260179988052319$ A **at** $\left(\frac{31}{110}-\frac{353\sqrt{103028190}}{16306620},\frac{2239\sqrt{103028190}+22977510}{81533100},\frac{31}{110}-\frac{79\sqrt{103028190}}{122299650}\right)\approx(0.06208846818232,0.560557461608944,0.275261563356526)$ A.

Minimum

$-\frac{20007\left(79\sqrt{103028190}+34466265\right)}{951219500000}-\frac{4993\left(353\sqrt{103028190}+4595502\right)}{163066200000}+\frac{2239\sqrt{103028190}}{815331000000}+\frac{1099199}{1100000}\approx 0.007082089869759$ A **at**
$\left(\frac{353\sqrt{103028190}+4595502}{16306620},\frac{31}{110}-\frac{2239\sqrt{103028190}}{81533100},\frac{79\sqrt{103028190}+34466265}{122299650}\right)\approx$

$\left(\frac{\quad}{16306620}, \frac{\quad}{110} \frac{\quad}{81533100}, \frac{\quad}{122299650} \right) \approx$
$(0.501547895454044, 0.00307890202742, 0.288374800279837)$ A .

In [ ]: