In [312]:
```python
import matplotlib.pyplot as plt
import scipy.optimize as opt
import numpy as np
import pandas as pd

#% matplotlib inline

# DATA -------------------------------------------------------------------------
```

In [313]:
```python
import pandas as pd

df = pd.read_csv("MIDETROI.txt",delim_whitespace=True,header=None)   # df.to_csv('Mi
df.head()
```

Out[313]:

	0	1	2	3
0	1	1	1995	34.1
1	1	2	1995	21.2
2	1	3	1995	20.6
3	1	4	1995	12.5
4	1	5	1995	8.1

In [314]:
```python
df.columns=[['Month','day','year','Temp']]
df.head()
```

Out[314]:

	Month	day	year	Temp
0	1	1	1995	34.1
1	1	2	1995	21.2
2	1	3	1995	20.6
3	1	4	1995	12.5
4	1	5	1995	8.1

In [315]:
```python
df['k'] = np.arange(len(df))
```

In [316]:
```python
#df.insert(4,'k','')
```

In [317]:

```
df.columns=[['Month','day','year','Temp','k']]
df.head()
```

Out[317]:

	Month	day	year	Temp	k
0	1	1	1995	34.1	0
1	1	2	1995	21.2	1
2	1	3	1995	20.6	2
3	1	4	1995	12.5	3
4	1	5	1995	8.1	4

In [318]:

```
arr = np.array(df)
print(arr)
```

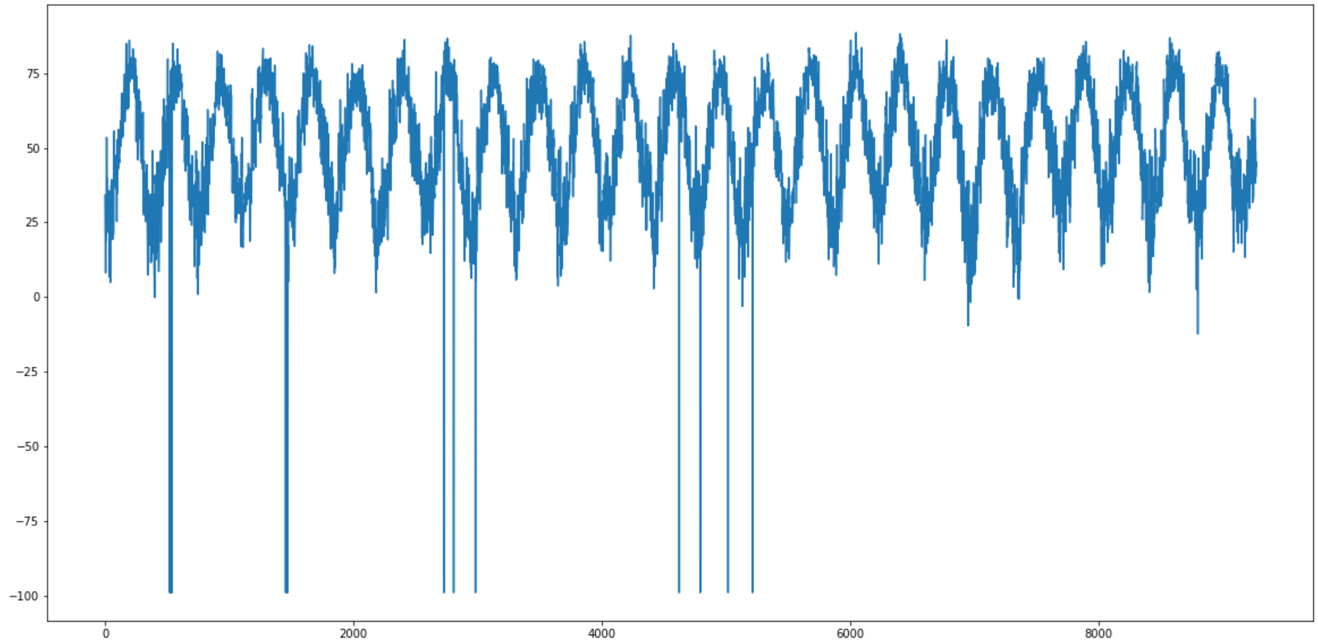
```
[[1.000e+00 1.000e+00 1.995e+03 3.410e+01 0.000e+00]
 [1.000e+00 2.000e+00 1.995e+03 2.120e+01 1.000e+00]
 [1.000e+00 3.000e+00 1.995e+03 2.060e+01 2.000e+00]
 ...
 [5.000e+00 1.100e+01 2.020e+03 4.030e+01 9.262e+03]
 [5.000e+00 1.200e+01 2.020e+03 4.540e+01 9.263e+03]
 [5.000e+00 1.300e+01 2.020e+03 4.360e+01 9.264e+03]]
```

In [319]:

```
k_values = arr[:,4]
T_values = arr[:,3]

x_samp = k_values
y_samp = T_values

import math

#x0    = numpy.array([0.0, 0.0, 0.0])
```

In [320]:
```python
plt.plot(x_samp, y_samp)
```

Out[320]: [<matplotlib.lines.Line2D at 0x1d33ce2b108>]



In [321]:
```python
# GENERAL EQUATION ---------------------------------------------------------------
def func(x, A, B, C):
    return  A+ B*np.cos(x*2*np.pi/365.25)  + C*np.sin(x*2*np.pi/365.25)

from scipy.optimize import curve_fit

#import scipy.optimize as optimization

popt, pcov = curve_fit(func, x_samp, y_samp)
popt
```

Out[321]: array([50.44610109, -22.67032479, -8.28241433])

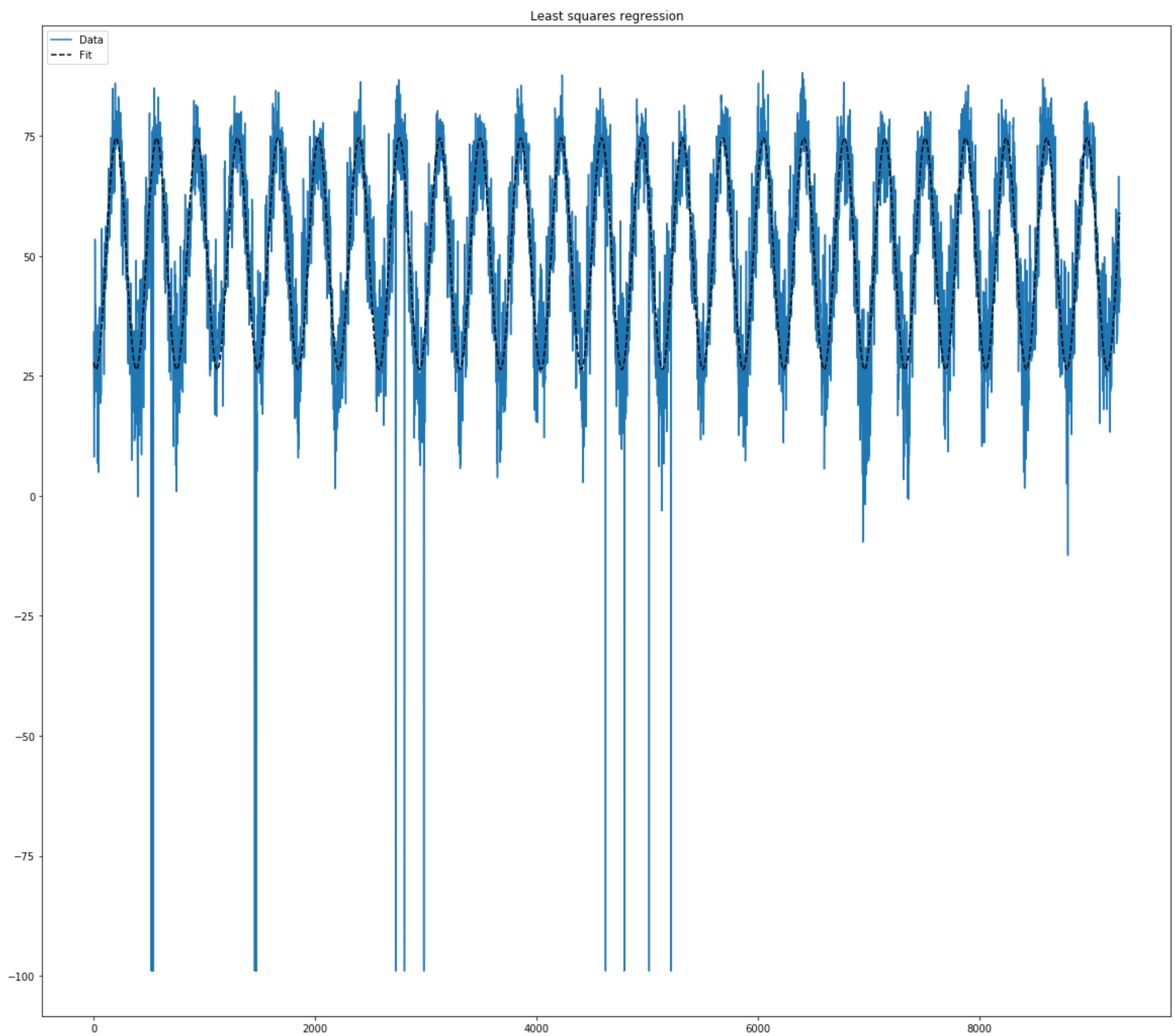
1 This is 1.a)

In [ ]:

In [322]:

```python
x_lin = np.linspace(0, 730, 730)
x_full = np.linspace(0, x_samp.max(), 1000)
y_full = func(x_full, 50.44610109, -22.67032479,  -8.28241433)
y_model = func(x_lin, 50.44610109, -22.67032479,  -8.28241433)

# PLOT ----------------------------------------------------------------------
# Visualize data and fitted curves
plt.figure(figsize=(20,18))
plt.plot(x_samp, y_samp, label="Data")
plt.plot(x_full, y_full, "k--", label="Fit")
plt.title("Least squares regression")
plt.legend(loc="upper left")
```

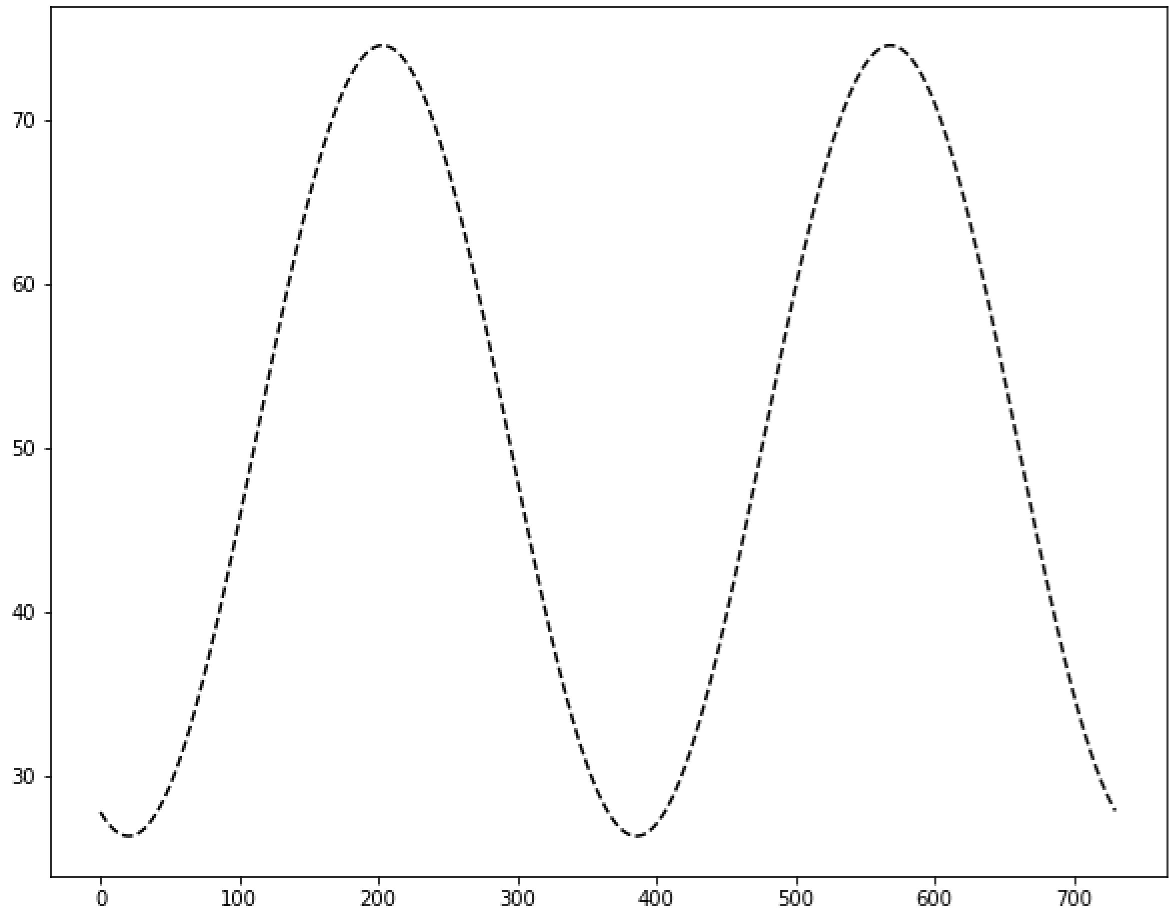
Out[322]: <matplotlib.legend.Legend at 0x1d33d2dfbc8>



In [323]:

```
plt.figure(figsize=(10,8))
plt.plot(x_lin, y_model, "k--", label="Fit")
```

Out[323]: [<matplotlib.lines.Line2D at 0x1d33c3fce88>]



In [324]:

```
def f(x): return func(x, 50.44610109, -22.67032479,  -8.28241433)
max_x = opt.fmin(lambda x: -f(x), 0)
solutionmax = opt.minimize_scalar(lambda x: -f(x), bounds=[0,365], method='bounded')
solutionmin = opt.minimize_scalar(lambda x: f(x), bounds=[0,365], method='bounded')
```

```
Optimization terminated successfully.
         Current function value: -74.582008
         Iterations: 39
         Function evaluations: 78
```

In [325]:
```
solutionmax
```

Out[325]:
```
    fun: -74.58200822476248
message: 'Solution found.'
   nfev: 9
 status: 0
success: True
      x: 202.98703242834117
```

In [326]:
```
solutionmin
```

Out[326]:
```
    fun: 26.31019395523753
message: 'Solution found.'
   nfev: 12
 status: 0
success: True
      x: 20.362033177156658
```

This finishes 1.2

In [327]:
```
def func(x, A, B, C, D):
    return  A+ B*np.cos(x*2*np.pi/365.25)  + C*np.sin(x*2*np.pi/365.25)+D*x*x
```

In [328]:
```
from scipy.optimize import curve_fit
```

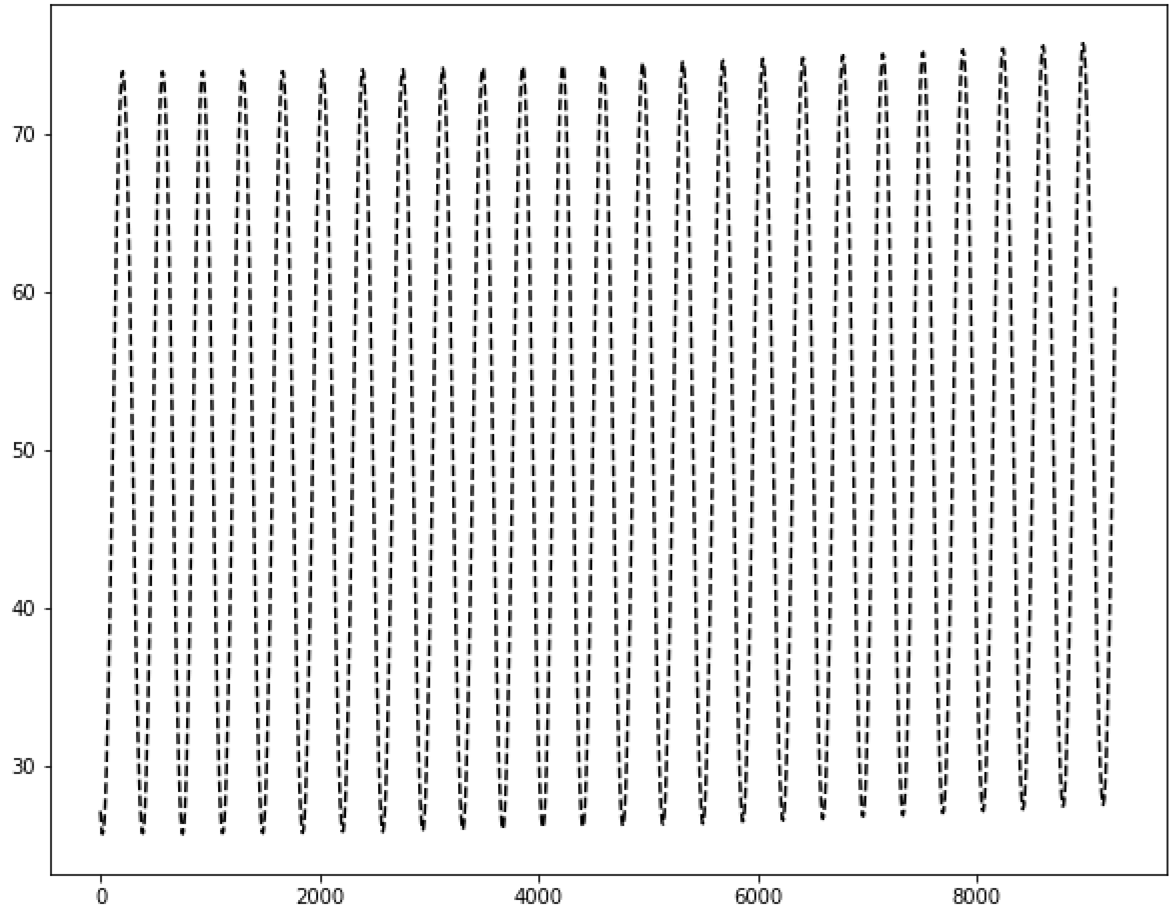
In [329]:
```
#import scipy.optimize as optimization

popt, pcov = curve_fit(func, x_samp, y_samp)
popt
```

Out[329]:
```
array([ 4.98129313e+01, -2.26820190e+01, -8.28513930e+00,  2.21349546e-08])
```

In [330]:
```
x_li = np.linspace(0, x_samp.max(), 1000)
y_lin = np.linspace(0, y_samp.max(), 100)
y_modelc = func(x_li, 4.98129313e+01, -2.26820190e+01, -8.28513930e+00,  2.21349546e
```

In [331]:
```
plt.figure(figsize=(10,8))
plt.plot(x_li, y_modelc, "k--", label="Fit")
```

Out[331]: [<matplotlib.lines.Line2D at 0x1d33c46d8c8>]



This finishes 1.3

In [ ]:

In [ ]:

In [ ]:

```python

```

In [332]:

```python
import pandas as pd

bolt = pd.read_csv('bolt.csv', sep=',')
```

In [333]:

```python
bolt.shape
```

Out[333]: (20, 3)

In [334]:

```python
bolt.columns=[['time','bolt','Thompson']]
bolt.head()
```

Out[334]:

	time	bolt	Thompson
0	0.00	0.0	0.0
1	0.01	0.0	0.0
2	1.10	5.0	4.9
3	3.00	22.5	22.6
4	4.00	34.0	34.0

In [335]:

```python
table=np.array(bolt)
table
```

Out[335]:

```
array([[0.00e+00, 0.00e+00, 0.00e+00],
       [1.00e-02, 0.00e+00, 0.00e+00],
       [1.10e+00, 5.00e+00, 4.90e+00],
       [3.00e+00, 2.25e+01, 2.26e+01],
       [4.00e+00, 3.40e+01, 3.40e+01],
       [4.50e+00, 4.13e+01, 4.11e+01],
       [5.40e+00, 5.21e+01, 5.13e+01],
       [5.80e+00, 5.59e+01, 5.53e+01],
       [6.20e+00, 6.15e+01, 6.08e+01],
       [6.50e+00, 6.48e+01, 6.39e+01],
       [6.90e+00, 6.96e+01, 6.85e+01],
       [7.30e+00, 7.33e+01, 7.21e+01],
       [7.70e+00, 7.85e+01, 7.71e+01],
       [8.00e+00, 8.17e+01, 7.99e+01],
       [8.30e+00, 8.56e+01, 8.38e+01],
       [8.60e+00, 8.92e+01, 8.75e+01],
       [8.80e+00, 9.13e+01, 8.94e+01],
       [9.40e+00, 9.86e+01, 9.64e+01],
       [9.69e+00, 1.00e+02,      nan],
       [9.89e+00,      nan, 1.00e+02]])
```

In [336]:

```
table[18,2]= (table[17,2]+table[19,2])/2;
table
```

Out[336]:

```
array([[0.00e+00, 0.00e+00, 0.00e+00],
       [1.00e-02, 0.00e+00, 0.00e+00],
       [1.10e+00, 5.00e+00, 4.90e+00],
       [3.00e+00, 2.25e+01, 2.26e+01],
       [4.00e+00, 3.40e+01, 3.40e+01],
       [4.50e+00, 4.13e+01, 4.11e+01],
       [5.40e+00, 5.21e+01, 5.13e+01],
       [5.80e+00, 5.59e+01, 5.53e+01],
       [6.20e+00, 6.15e+01, 6.08e+01],
       [6.50e+00, 6.48e+01, 6.39e+01],
       [6.90e+00, 6.96e+01, 6.85e+01],
       [7.30e+00, 7.33e+01, 7.21e+01],
       [7.70e+00, 7.85e+01, 7.71e+01],
       [8.00e+00, 8.17e+01, 7.99e+01],
       [8.30e+00, 8.56e+01, 8.38e+01],
       [8.60e+00, 8.92e+01, 8.75e+01],
       [8.80e+00, 9.13e+01, 8.94e+01],
       [9.40e+00, 9.86e+01, 9.64e+01],
       [9.69e+00, 1.00e+02, 9.82e+01],
       [9.89e+00,      nan, 1.00e+02]])
```

In [337]:

```
tb = table[0:18, 0]
tt = table[0:19, 0]


b_dist = table[0:18,1]
t_dist = table[0:19,2]

b_samp = b_dist
t_samp = t_dist

tb_samp = tb
tt_samp = tt


import math

#x0    = numpy.array([0.0, 0.0, 0.0])
```

In [338]:

```
# GENERAL EQUATION ------------------------------------------------------------
def func(t, a, v0, x0):
    return  a/2*t*t+ v0*t + x0

from scipy.optimize import curve_fit

#import scipy.optimize as optimization

popt, pcov = curve_fit(func, tb_samp, b_samp)
popt
```

Out[338]:

```
array([ 0.53996374,  8.28070397, -1.71191566])
```

In [339]:

```
# GENERAL EQUATION -------------------------------------------------------------
def fun(t, a, v0, x0):
    return  a/2*t*t+ v0*t + x0

from scipy.optimize import curve_fit

#import scipy.optimize as optimization

popt, pcov = curve_fit(fun, tt_samp, t_samp)
popt
```

Out[339]:

```
array([ 0.39516373,  8.64950506, -1.91512335])
```

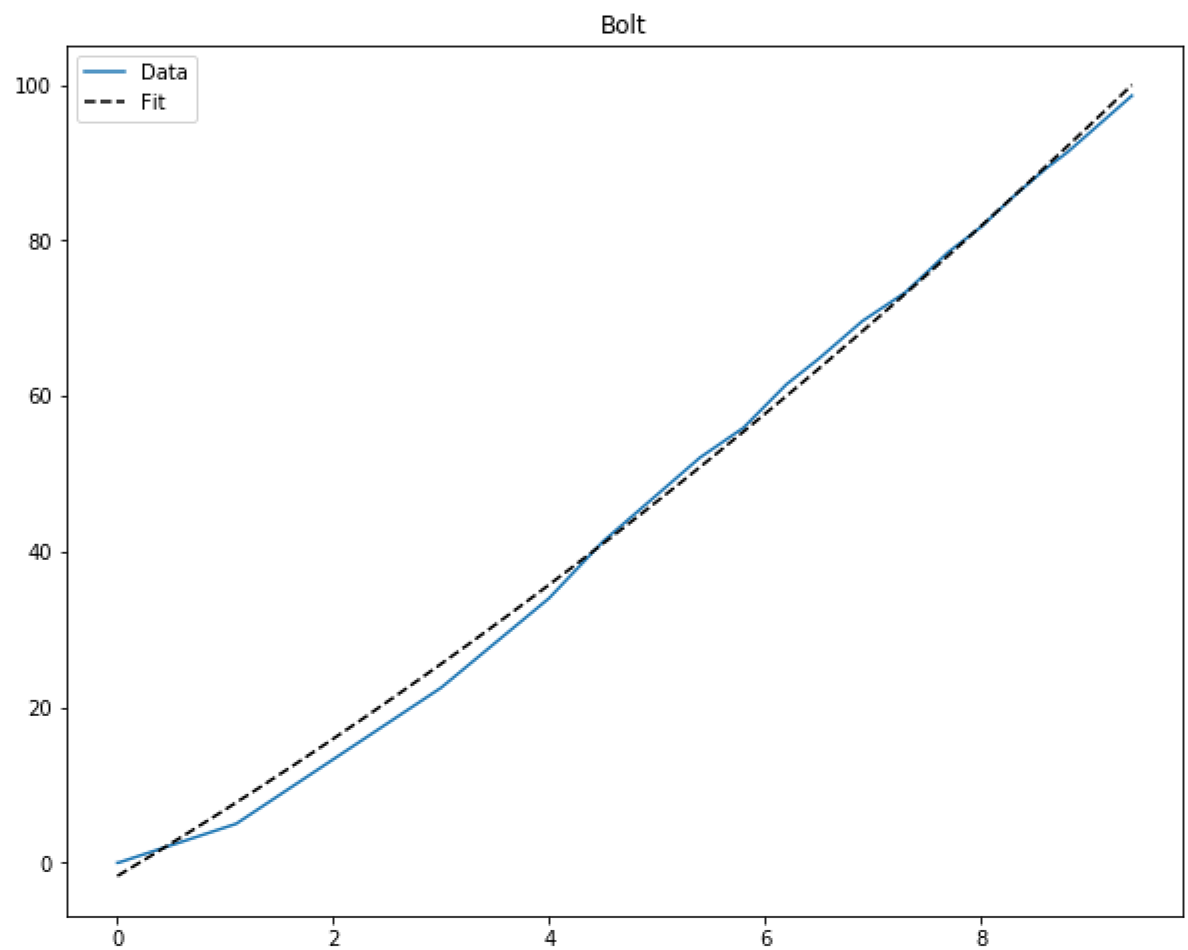
2.1 Bolt and Thompson, see the above two boxes

2.2   Not consistent, speed should be lower and x_0 should be 0.

In [340]:

```python
#b_lin = np.linspace(0, 365, 365)
tb_full = np.linspace(0, tb_samp.max(), 30)
#y_full = func(x_full, 50.44610109, -22.67032479,  -8.28241433)
b_model = func(tb_full, 0.53996374,  8.28070397, -1.71191566)

# PLOT ------------------------------------------------------------------
# Visualize data and fitted curves
plt.figure(figsize=(10,8))
plt.plot(tb_samp, b_samp, label="Data")
plt.plot(tb_full, b_model, "k--", label="Fit")
plt.title("Bolt")
plt.legend(loc="upper left")
```

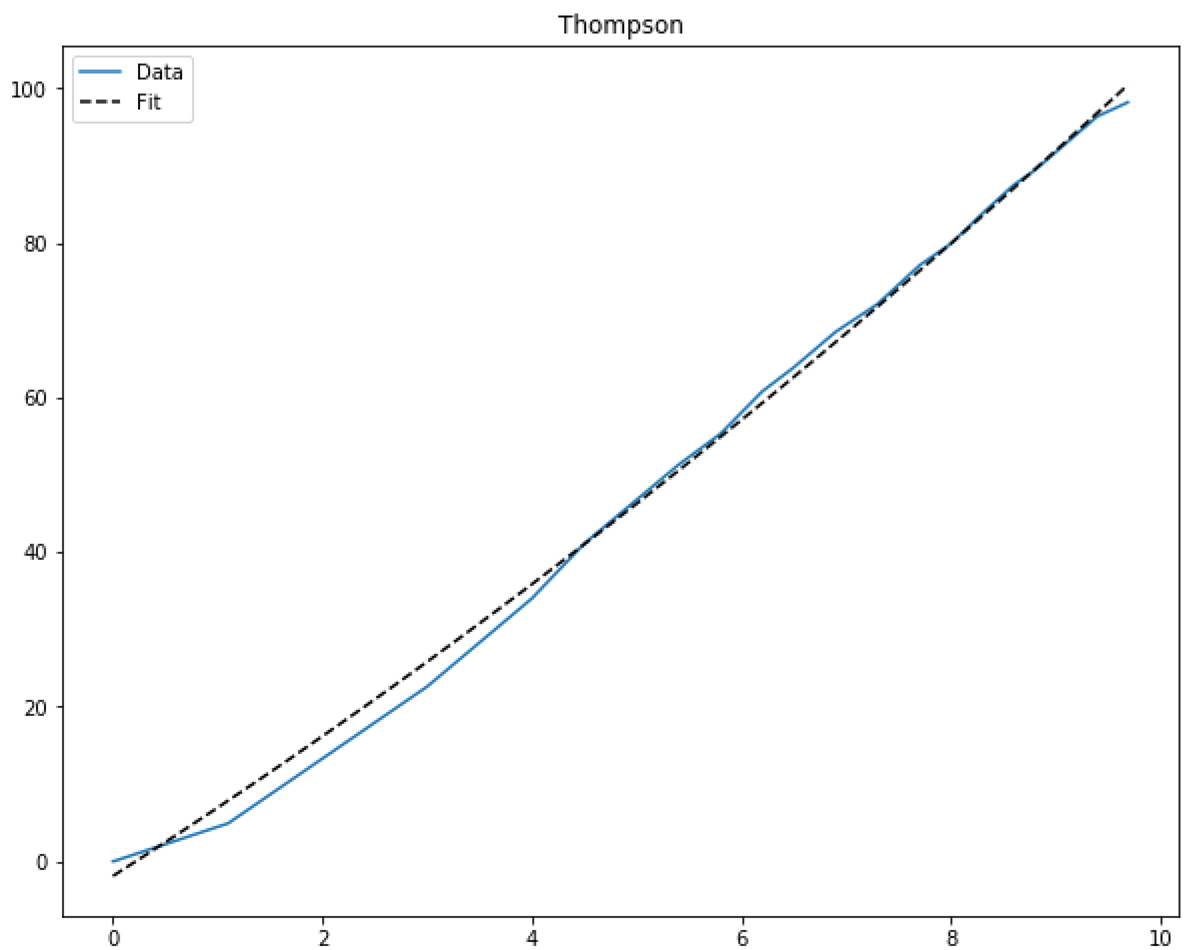
Out[340]: <matplotlib.legend.Legend at 0x1d33d327e88>



In [341]:

```python
#b_lin = np.linspace(0, 365, 365)
tt_full = np.linspace(0, tt_samp.max(), 30)
#y_full = func(x_full, 50.44610109, -22.67032479,  -8.28241433)
t_model = func(tt_full, 0.39516373,  8.64950506, -1.91512335)

# PLOT ---------------------------------------------------------------------
# Visualize data and fitted curves
plt.figure(figsize=(10,8))
plt.plot(tt_samp, t_samp, label="Data")
plt.plot(tt_full, t_model, "k--", label="Fit")
plt.title("Thompson")
plt.legend(loc="upper left")
```

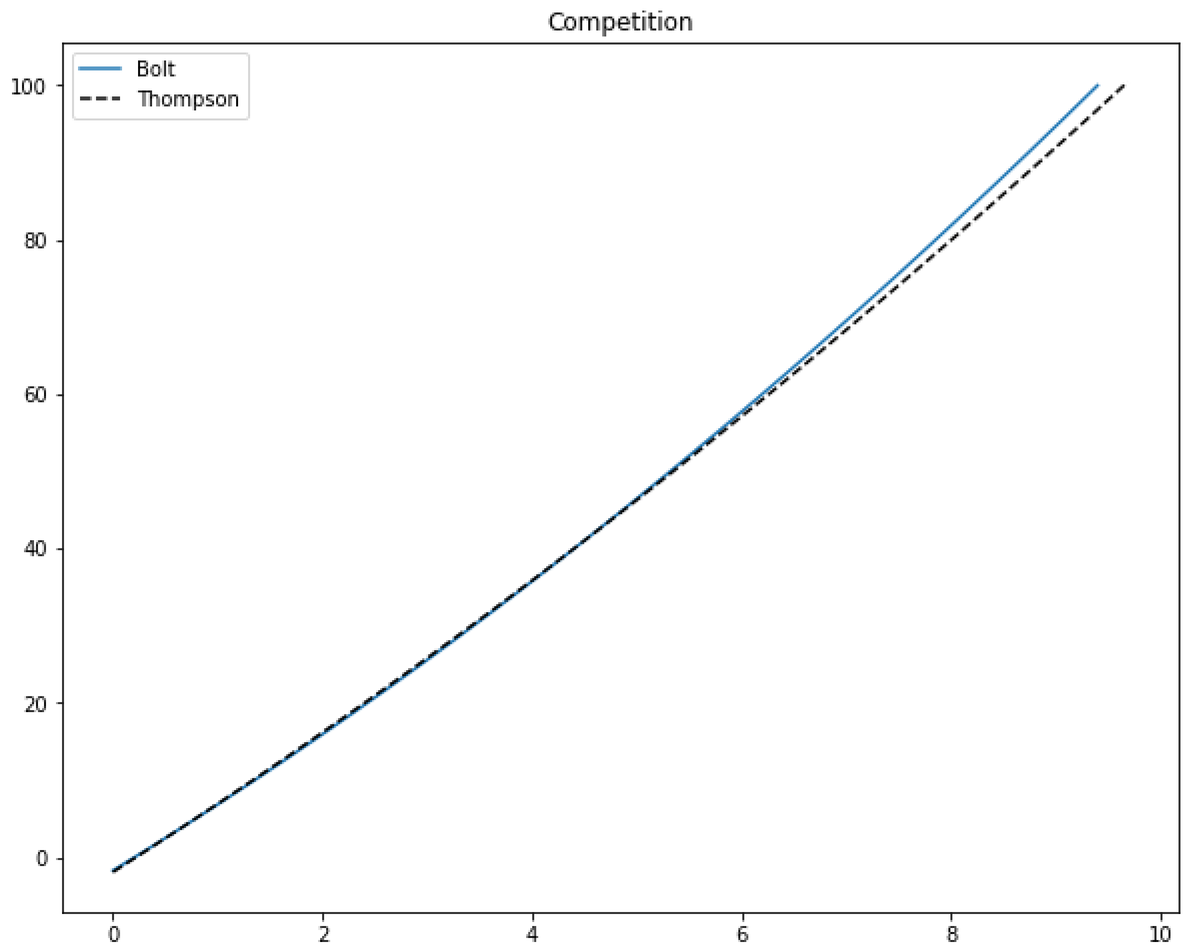
Out[341]: <matplotlib.legend.Legend at 0x1d33d393cc8>



In [342]:

```python
plt.figure(figsize=(10,8))
plt.plot(tb_full, b_model, label="Bolt")
plt.plot(tt_full, t_model, "k--", label="Thompson")
plt.title("Competition")
plt.legend(loc="upper left")
```

Out[342]: <matplotlib.legend.Legend at 0x1d33d84ad08>



In [343]:

```python
t5 = table[0:5, 0]
#tt5 = table[0:5, 0]


b5_dist = table[0:5,1]
t5_dist = table[0:5,2]

b5_samp = b5_dist
th5_samp = t5_dist

t5_samp = t5
#tt_samp = tt
```

In [344]:

```
popt, pcov = curve_fit(func, t5, b5_dist)
popt
```

Out[344]:

```
array([ 2.42389158,  3.74319987, -0.12451425])
```

In [ ]:

In [345]:

```
popt, pcov = curve_fit(fun, t5_samp, th5_samp)
popt
```

Out[345]:

```
array([ 2.43373304,  3.73897625, -0.14345813])
```

2.3 The above data are more consistent.

In [ ]:

In [ ]:

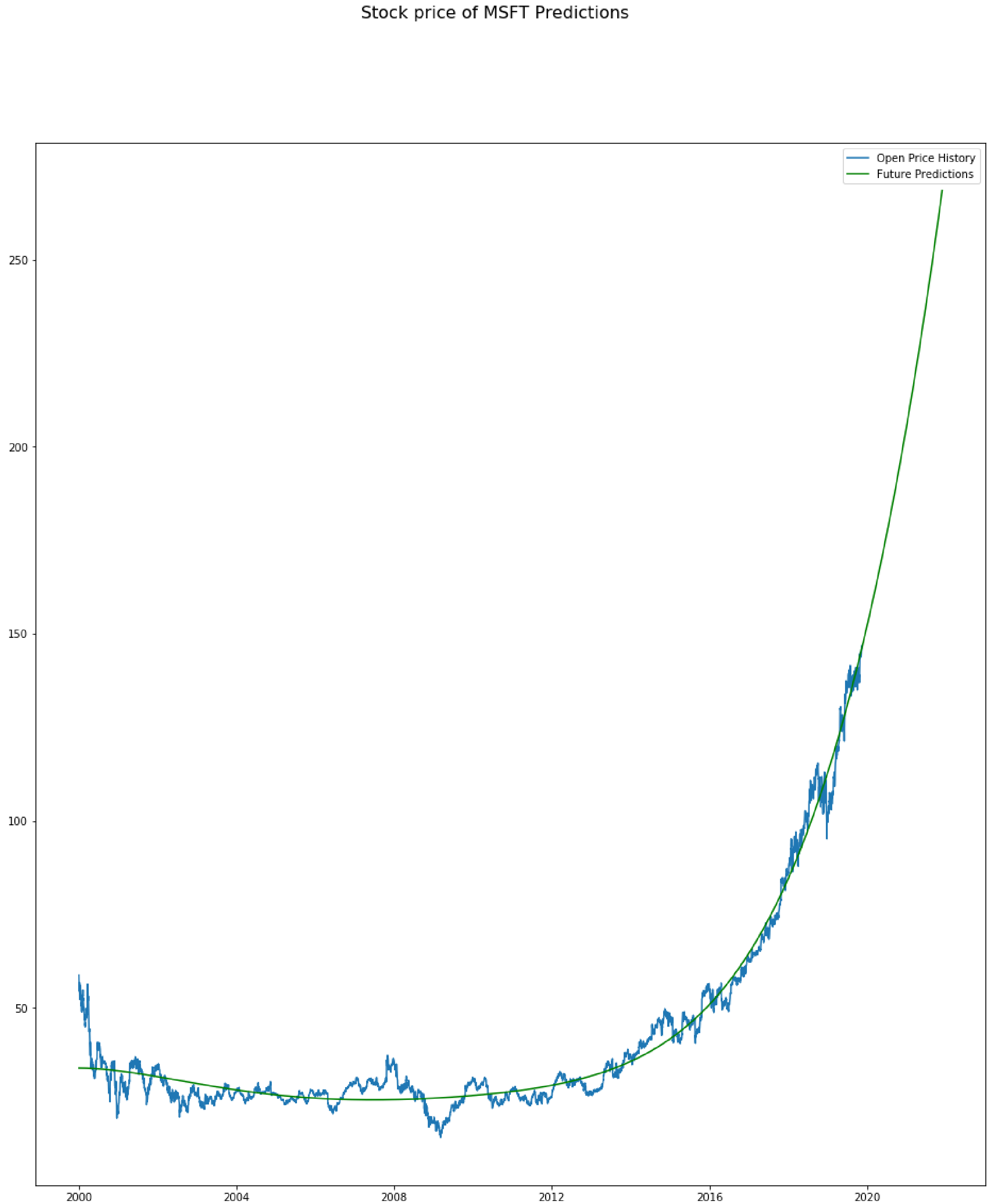
Below is an example of stock market

In [350]:

```python
#import packages
import pandas as pd
import numpy as np

df = pd.read_csv('MSFTHistoricalData.csv')
df.head()

from sklearn.linear_model import LinearRegression

#for polynomial regression
from sklearn.preprocessing import PolynomialFeatures

#to plot within notebook
import matplotlib.pyplot as plt

#function to calculate compound annual growth rate
def CAGR(first, last, periods):
    return ((last/first)**(1/periods)-1) * 100

#Read the data file
df = pd.read_csv('MSFTHistoricalData.csv')

#Setting index as date
df['Date'] = pd.to_datetime(df.Date)
df.index = df['Date']

#Converting dates into number of days as dates cannot be passed directly
#to any regression model
df.index = (df.index - pd.to_datetime('2000-01-01')).days

#Convert the pandas series into numpy array, we need to further
#massage it before sending it to regression model
a = np.asarray(df['Open'])
b = np.asarray(df.index.values)

y = a.astype(np.float)
x = b.astype(np.float)

df.shape

#Model initialization
#by default the degree of the equation is 1.
#Hence the mathematical model equation is y = mx + c,
#which is an equation of a line.
regression_model = LinearRegression()

#Choose the order of your polynomial. Here the degree is set to 5.
#hence the mathematical model equation is
#y = c0 + c1.x**1 + c2.x**2+....+ c5.x**5
poly = PolynomialFeatures(5)



#Convert dimension x in the higher degree polynomial expression
X_transform = poly.fit_transform(x.reshape(-1, 1))

#Fit the data(train the model)
regression_model.fit(X_transform, y.reshape(-1, 1))
```

```
# Prediction for historical dates. Let's call it learned values.
y_learned = regression_model.predict(X_transform)

#Now, add future dates to the date index and pass that index to
#the regression model for future prediction.
#As we have converted date index into a range index, hence, here we
#just need to add 3650 days ( roughly 10 yrs)
#to the previous index. x[-1] gives the last value of the series.
newindex = np.asarray(pd.RangeIndex(start=x[-1], stop=x[-1] + 8000))

#Convert the extended dimension x in the higher degree polynomial expression
X_extended_transform = poly.fit_transform(newindex.reshape(-1, 1))

#Prediction for future dates. Let's call it predicted values.
y_predict = regression_model.predict(X_extended_transform)

#Print the last predicted value

print (" Disclaimer:  Not an investing advice, take your own risk!    Data from inve
print ("openinging price of MSFT at 2023-0101 would be around ", y_predict[-1])




#Convert the days index back to dates index for plotting the graph
x = pd.to_datetime(df.index, origin='2000-01-01', unit='D')
future_x = pd.to_datetime(newindex, origin='2000-01-01', unit='D')

#Print CAGR for next ten years.
print ('Your investments will have a CAGR of ',(CAGR(y[-1], y_predict[-1], 10)), '%'

#Setting figure size
from matplotlib.pylab import rcParams
rcParams['figure.figsize'] = 20,10

#Plot the actual data
plt.figure(figsize=(16,18))
plt.plot(x,df['Open'], label='Open Price History')

#Plot the regression model
#plt.plot(x,y_learned, color='r', label='Mathematical Model')

#Plot the future predictions
plt.plot(future_x,y_predict, color='g', label='Future Predictions')

#Set the title of the graph
plt.suptitle('Stock price of MSFT Predictions', fontsize=16)

#Set the title of the graph window
fig = plt.gcf()
fig.canvas.set_window_title('Stock price of MSFT Predictions')

#display the legends
plt.legend()
#display the graph
plt.show()
```

```
 Disclaimer:  Not an investing advice, take your own risk!    Data from investin
g.com
openinging price of MSFT at 2023-0101 would be around  [268.54248639]
Your investments will have a CAGR of  [16.42463622] %
```



3.2   Data from investing.com

3.3 Disclaimer: Not an investing advice, take your own risk!   It doesn't really reflect common sense.