Tarski fixed point theorem

Thursday, March 2, 2023

11:08 AM

5.3. RULE INDUCTION

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Exercise 5.8 The set S is defined to be the least subset of natural numbers \mathbb{N} such that:

 $1 \in S$:

if $n \in S$, then $3n \in S$;

if $n \in S$ and n > 2, then $(n - 2) \in S$.

Show that $S = \{m \in \mathbb{N} \mid \exists r, s \in \mathbb{N} \cup \{0\}, m = 3^r - 2s\}$. Deduce that S is the set of odd numbers.

for all (X/8) \in R.

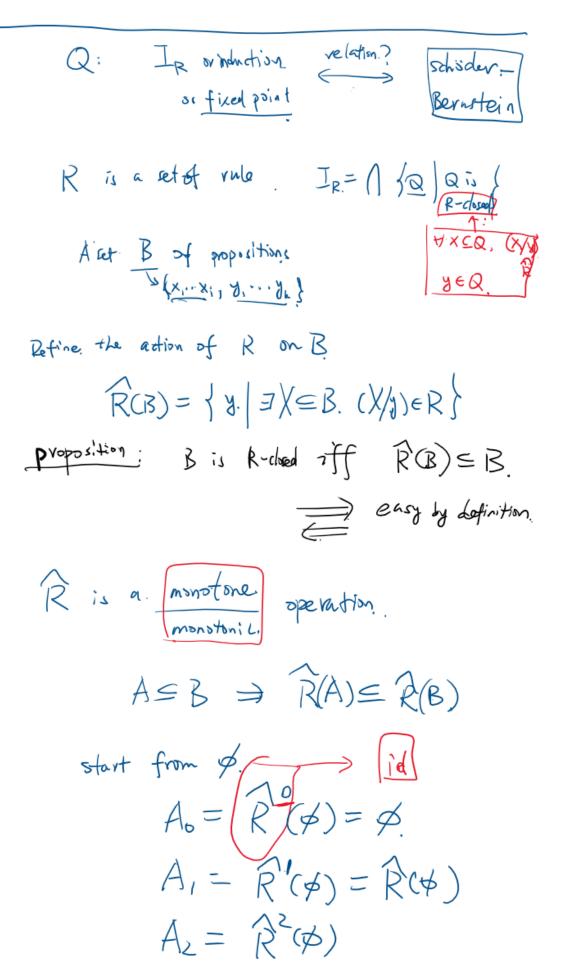
(VXEX. XGIR & POX)) => (YGIR & POD)

Became IR IS R-closed,

VXEX. XGIR & POX)

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A, consilts all the axioms
Anti is the aclusions imediately follows from An
$A_0 \subseteq A_1 \subseteq A_2 \subseteq \cdots A_n \subseteq A_{n+1} \subseteq \cdots = A_n \subseteq $
then D Ais R-chied.
\bigcirc $\widehat{R}(A) = A$.
(3) A) is the least. R-closed set> 4=
Proof: Take B R-closed and ACB. S) = B
Now we show the Mo, AnEB.
base Ao S. B. forse. Q.E. angest
inductive: assume $A_n \subseteq B$. $A_{n+1} = R(A_n) \subseteq R(B) = B$.

Taxski's Fixed point Theorem

Thm (minimal version) U is quest, P(U) power set,

Let $\varphi: P(V) \rightarrow P(V)$ monotoniz. (total) function

Define m = \ { S=U. | \ \phi(s) = s \}.

Then. In is a fixed point of Q.

movement In is a least five-fixed point of q.

S s.t. 8(s)=15

Proof: X= { SSU \ 9 (5) E S}

Take SEX, then MSS

 $P(m) \subseteq \varphi(S)$ by monotonicity.

 $\varphi(s) \subseteq \varsigma$

then. Am) E.S for any SEX

 $\varphi(m) \subseteq \bigwedge X = m$ $\Rightarrow \lim_{n \to \infty} \sum_{n \to \infty} \sum_$

P(m) = M = m = m is the minimal greation. by def.

Now: P(m) = m or me. P(m)

Then, P(m) = X=1 Sey P(s) = Structure of the print of the minimal greation.

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