

continuous line segment: has more points than discrete sequence

Argument: Assume they are 1-1 corresponding to each other.

Step 1: find the 1st pair of numbers in $\{r_i\}_{i=0}^{\infty}$.
s.t. the pair is in $[a, b]$.

Step 2: name the image of pair in step 1
 $[a_1, b_1]$

$(a_1, b_1) \subseteq [a, b]$
Repeat step 1: find a pair in $\{r_i\}_{i=0}^{\infty}$
that's in $[a_1, b_1]$

Step n: $[a_{n-1}, b_{n-1}]$ repeat,

one get $[a_n, b_n] \subseteq [a_{n-1}, b_{n-1}]$

$\bigcap_{k=1}^{\infty} [a_k, b_k] \neq \emptyset \Rightarrow$ Cantor Thm

Is \mathbb{R} or $[a, b]$ complete
↓
"hole?"

7. Completeness Axioms

① (least upper bound property)

Every non-empty subset of \mathbb{R} , if it has an upper bound, then it has a unique least upper bound.



However,

greatest lower bound.

$$\sup(S = \{x \in \mathbb{Q} \mid x^2 < 2\}) = \sqrt{2}$$

(2) Dedekind's cut. Idea: complete \mathbb{Q}

We can cut \mathbb{R} at a point x .

(1) $x \in \mathbb{Q}$.

(2) $x \notin \mathbb{Q}$, then it can be named using a sequence of rational numbers.

E.g. $\{x \mid x^2 > 2 \text{ and } x \in \mathbb{Q}\}$ $\{x \mid x^2 < 2 \text{ or. } x \in \mathbb{Q}\}$
This cut defines irrational number. $\sqrt{2}$.

(3) \mathbb{R} is Cauchy complete.

Def: (Cauchy sequence)

$$\{a_i\}_{i=1}^{\infty}$$

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H $\{ e <$

$\exists N \in \mathbb{N}$, s.t. $\forall m, n > N$

Cauchy sequence in \mathbb{Q} may

Take all limit of Cauchy sequences

Name them a number,

completion

\mathbb{R} is Cauchy complete.

④

Nested interval Theorem

$$I_n = [a_n, b_n]$$

s.t.

$$I_1 \supset I_2 \supset I_3 \supset \dots \supset T$$

$$\forall \epsilon \quad |a_m - a_n| < \epsilon$$

not converge in \mathbb{Q} .

a is \mathbb{Q}

call the

\mathbb{R}

\dots

$$I_1 \supset I_2 \supset I_3 \supset \dots \supset I_n$$

Moreover, $(b_n - a_n) \rightarrow 0$

$\bigcap_{n=1}^{\infty} I_n$ contains exactly one point.

$$\sqrt{2} = \sqrt{1.4142358\ldots}$$

$$[1, 2] \supset [1.4, 1.5]$$

⑤ monotone convergence theorem

$$\{x_n\} \rightarrow x$$

non-in.

Y . . ~ ~

$\alpha \mapsto \infty$

try one point.

$$\geq \overline{[(.4), 1.42]} > \dots$$

hm.

$\{ b_i \}_{i=1}^{\infty}$

non-d

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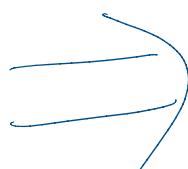
Bolzano Weierstrass

Every, bounded seq'

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Intermediate Value Th

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decreasing, bounded sequence.

it has a finite number as its

number has a converging subsequence.

Theorem.



limit

enh