Firstly, thank you for bringing up another possible approach to achieve the stability of $Symp$. To our best understanding, this approach is trying to do the following: take a minimal ruled surface M as an example in your paper [8], let u, u' be two symplectic classes such that they are in the same integral interval. $\mathcal{A}_u$ and $\mathcal{J}_u$ are the spaces of almost complex structures which are compatible and tamed by some form in the class u, respectively (here cohomologous is equivalent to isotopic since it's a minimal ruled surface). Let G_u be $Symp(M,\omega) \cap \mathrm{Diff}_0(M)$ for some ω in class u. Then we are trying to make the following diagram homotopy commute:

$$
\begin{tikzcd}
G_u \arrow[r] \arrow[d] & \mathrm{Diff}_0(M) \arrow[r] \arrow[d, "="'] & \mathcal{A}_u \arrow[d, "h"] & \mathcal{A}_u \arrow[l, "="', "f"] \arrow[d, hook, "i"] \\
G_{u'} \arrow[r] & \mathrm{Diff}_0(M) \arrow[r] & \mathcal{A}_{u'} & \mathcal{J}_{u'}. \arrow[l, "\sim"', "g"]
\end{tikzcd}
\tag{1}
$$

Here i is the inclusion $\mathcal{A}_u \hookrightarrow \mathcal{J}_{u'}$ obtained by the inflation for a compatible pair (ω, J), and g is a homotopy equivalence. The upshot is to claim is that h is a (weak) homotopy equivalence. It seems that to establish this claim, one needs to show that i is a homotopy equivalence by constructing some homotopy inverse map. This may be achievable. But it does not seem very obvious to us: the classes u and u' are different and there's no obvious map $\mathcal{J}_{u'} \to \mathcal{A}_u$ since we don't have the J-tame inflation.

If we try to apply this idea to our case, especially at the level stability Theorem 1.3 in [4], one needs a map $\mathcal{A}_u \hookrightarrow \mathcal{J}_{u'}$ which is expected to have a homotopy inverse for every stratum. This could be very complicated.

Secondly, we agree that our approach does not look very straightforward at first glance since it does need a deep result of $K_J^c = K_J^t$. But it seems that unless the local tame/compatible conjecture along a curve can be proved, our approach is by far the only approach to consolidate the proofs of stability of $Symp$ in the literature [5, 2, 3, 1, 7], etc. Also, there are other results (cone theorem in [9] for example) that need the equality $\mathcal{A}_u = \mathcal{A}_{u'}$. Those proofs that originally rely on the J-tamed inflation, now can be strengthened simply by using our section 4.4.

For your following comments on [4]:

Condition 1 is now removed since it's not explicitly used. See the next 2 paragraphs.

The hypothesis $\chi(M) \le 12$ is indeed necessary. We wrote a Remark 5.5. Here Condition 1 implicitly shows up: to establish the almost Kähler Nakai-Moishezon criterion, one needs enough embedded curve with negative intersection to do inflation while taming any given J. For $\chi(M) > 12$, Condition 1 fails and we may only have very singular components in the stable curve of an exceptional class, which we cannot inflate along.

For the argument that $U_{u,\mathcal{C}}$ are submanifolds, this is now stated as Prop 4.4 and the proof is rewritten. The old argument does use Condition 1, but not the new argument.

The 3 other comments:

1. Thanks for pointing out the oversight that the connectedness of space of cohomologous forms! The revised fibration (2) in [4] replaces Diff_0 by Diff_u, which preserves the symplectic class, to take the connected components into account. We believe this fibration could be useful in other circumstances. Section 2 of the current file mainly justifies this new approach.

2. ArXiv1611.07436 was split into 2 parts [6, 7] for publication, and [6] is the first part. It has the correct Lemma 2.16 we want to apply to prove Lemma 2.6 in the old version. We apologize for not keeping arXiv updated on time. Now the arXiv version for both files are up-to-date.

3. The discussion of inflation has been changed following your suggestion. This is section 4.4 now.

Besides those changes, we also improved the exposition and included examples to illustrate the symplectic cone and its chamber structure.

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Center for Mathematical Analysis, Geometry and Dynamical Systems, Department of Mathematics, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail address: `sanjos@math.tecnico.ulisboa.pt`

Department of Mathematics, University of Michigan, Ann Arbor, MI 48109
E-mail address: `lijungeo@umich.edu`

School of Mathematics, University of Minnesota, Minneapolis, MN 55455
E-mail address: `lixxx248@umn.edu`

Department of Mathematics, University of Western Ontario, Canada
E-mail address: `mpinson@uwo.ca`