

張軍力 企管四甲

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課本例題 6.4.

$$\text{依 } E(x_i) = \mu, \quad V(x) = \sigma^2 = E(x_i^2) - \mu^2$$

$$\text{則 } E(\bar{x}) = \mu, \quad V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n} \sigma^2$$

$$E(\hat{\theta}_2) = E\left(\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

因此,  $\theta_2 = \sum_{i=1}^n (x_i - \bar{x})^2 / (n-1)$  為母體變異數  $\sigma^2$  之 unbiased 估計量.

而  $\theta_1 = \sum_{i=1}^n (x_i - \bar{x})^2 / n$  為母體變異數  $\sigma^2$  之 biased 估計量.