

20.

解:  $\because$  母体是常態分配, 兩組樣本是獨立小樣本,

$\therefore (\bar{x} - \bar{y})$  會服從  $t$  分配.

$$(1) \because \sigma_1^2 \neq \sigma_2^2, \text{ 則自由度 } V = \frac{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{\left( \frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left( \frac{s_2^2}{n_2} \right)^2}{n_2 - 1}}$$

$\therefore \mu_1 - \mu_2 \pm 100(1-\alpha)\% \text{ 信賴區間為}$

$$(\bar{x} - \bar{y}) \pm t_{\frac{\alpha}{2}}(V) \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(2) \sigma_1^2 \text{ 之 } 100(1-\alpha)\% \text{ 信賴區間 } \left( \sqrt{\frac{(n_1-1)s^2}{\chi_{\frac{\alpha}{2}}^2(n_1-1)}}, \sqrt{\frac{(n_1-1)s^2}{\chi_{1-\frac{\alpha}{2}}^2(n_1-1)}} \right)$$

(3).  $\frac{\sigma_1^2}{\sigma_2^2} \pm 100(1-\alpha)\% \text{ 信賴區間為}$

$$\left( \frac{s_1^2}{s_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{s_1^2}{s_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)} \right)$$

$$\text{其中, } F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = \frac{1}{F_{\frac{\alpha}{2}}(n_2-1, n_1-1)}$$