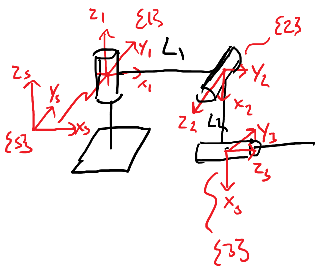
Spatial 3R Open Chain

Wednesday, October 14, 2020 12:24 PM

Example: Spatial 3R Open Chain



$\{s\} = \{s\}$

$\{b\} = \{3\}$

$M(\theta_1 = \theta_2 = \theta_3 = 0)$

$$M = \begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{sb}(\theta_1, \theta_2, \theta_3) = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} M$$

$$S_1, S_2, S_3 \Rightarrow S_1 = \begin{bmatrix} \omega_1 \\ v_1 \end{bmatrix}, S_2 = \begin{bmatrix} \omega_2 \\ v_2 \end{bmatrix}, S_3 = \begin{bmatrix} \omega_3 \\ v_3 \end{bmatrix}$$

$$S_1 : \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_2 : \omega_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad q_2 = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ -L_1 \end{bmatrix}$$

$v = -\omega_1 \times q_1$

$$S_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ -L_1 \end{bmatrix}$$

$$S_3 : \omega_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, q_3 = \begin{bmatrix} 0 \\ 0 \\ -L_2 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ -L_2 \\ 0 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -L_2 \end{bmatrix}$$

$$[S] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \qquad [\omega] = 3 \times 3$$

$$S_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -L_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$e^{[S_n]\theta_n} \qquad \theta_1 = \frac{\pi}{2}, \theta_2 = \pi, \theta_3 = \emptyset$

$e^{[S_3]\theta_3} = I_{4 \times 4}$

$$e^{[S]\theta} = \begin{bmatrix} e^{[\omega]\theta} & (I\theta + (1 - c\theta)[\omega] + (\theta - s\theta)[\omega]^2)v \\ 0 & 1 \end{bmatrix}$$

$e^{[S_2]\theta_2}$

① $e^{[\omega_2]\theta_2}$ ② $I\theta + \ldots$

$$\omega_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad e^{[\omega_2]\theta_2} = I + \sin\theta[\omega] + (1-\cos\theta)[\omega]^2$$

$$[\omega_2] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad [\omega_2]^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$e^{[\omega_2]\theta_2} = I + \overset{0}{\cancel{\sin(\pi)}}[\omega] + (1 - \overset{-1}{\cancel{\cos(\pi)}})[\omega]^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\left(I\theta + (1-\cos(\pi))[\omega] + (\pi - \sin(\pi))[\omega]^2\right) v_2 \qquad v_2 = \begin{bmatrix} 0 \\ 0 \\ -L_1 \end{bmatrix}$$

$$\left(\pi\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \pi\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right)\begin{bmatrix} 0 \\ 0 \\ -L_1 \end{bmatrix} = \begin{bmatrix} 2L_1 \\ 0 \\ 0 \end{bmatrix}$$

$$e^{[S_2]\overset{\theta_2}{\pi}} = \begin{bmatrix} -1 & 0 & 0 & 2L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$e^{[S_1]\theta_1}$ $\theta_1 = \frac{\pi}{2}$

$$[\omega_1] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [\omega_1]^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e^{[\omega]\theta_1} = I + \overset{1}{\cancel{\sin(\tfrac{\pi}{2})}}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \left(1 - \overset{0}{\cancel{\cos(\tfrac{\pi}{2})}}\right)\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left(I\theta_1 + (1-\cos\theta_1)[\omega_1] + (\theta_1 - \sin\theta_1)[\omega]^2\right)v_1 \qquad v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

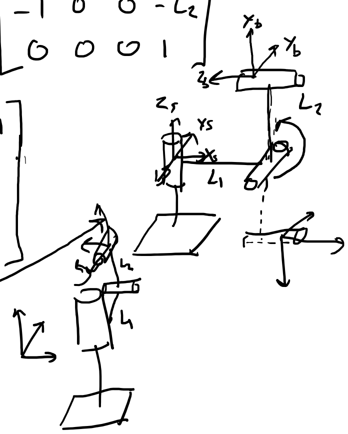
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow e^{[S_1]\frac{\pi}{2}} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{sb} = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} M$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 2L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 0 & -1 & L_1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$



$$T_{sb} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & L_1 \\ 1 & 0 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$