Programming theory - problems sheet - 5

1. Let $F \subseteq A \times A$ denote the problem given by the following specification:
   $A = (x{:}\mathbb{N}, d{:}\mathbb{N})$
   $B = (x'{:}\mathbb{N})$
   $Q = (x = x' \wedge x > 0)$
   $R = \left(Q \wedge 10^{d-1} \leqslant x < 10^d\right)$

   (a) Determine the truth-set of the logical function $Q_{\{x':6854\}} : A \to \mathbb{L}$.
   (b) Determine the truth-set of the logical function $R_{\{x':6854\}} : A \to \mathbb{L}$.
   (c) What does F assign to the states $\{x{:}6854, d{:}2\}$ and $\{x{:}7267363, d{:}123\}$?
   (d) Find all the states, such that their image by F equals to the image of state $\{x{:}6854, d{:}2\}$.
   (e) Describe in your own words, what problem F is about.
   (f) Write down problem F in the form of a set.

2. Let $F \subseteq A \times A$ denote the problem given by the following specification:
   $A = (x{:}\mathbb{Z}, y{:}\mathbb{Z}, z{:}\mathbb{Z})$
   $B = (x'{:}\mathbb{Z}, y'{:}\mathbb{Z})$
   $Q = (x = x' \wedge y = y')$
   $R = ((z = x' \vee z = y') \wedge z \geqslant x' \wedge z \geqslant y')$

   (a) Determine the truth-set of the logical function $Q_{\{x':6,y':5\}} : A \to \mathbb{L}$.
   (b) Determine the truth-set of the logical function $R_{\{x':6,y':5\}} : A \to \mathbb{L}$.
   (c) What does F assign to the state $\{x{:}6, y{:}5, z{:}3\}$?
   (d) Describe in your own words, what problem F is about.

3. Let $F \subseteq A \times A$ denote the problem given by the following specification:
   $A = (x{:}\mathbb{Z}, y{:}\mathbb{Z}, z{:}\mathbb{Z})$
   $B = (x'{:}\mathbb{Z}, y'{:}\mathbb{Z})$
   $Q = (x = x' \wedge y = y' \wedge x' > 5)$
   $R = (Q \wedge x > y \to z = x)$

   (a) Determine the truth-set of the logical function $Q_{\{x':4,y':2\}} : A \to \mathbb{L}$.
   (b) What does F assign to the states $\{x{:}4, y{:}2, z{:}1\}$, $\{x{:}8, y{:}5, z{:}7\}$, $\{x{:}9, y{:}3, z{:}10\}$ and $\{x{:}6, y{:}9, z{:}4\}$?

4. Let $F \subseteq A \times A$ denote the problem given by the following specification:
   $A = (x{:}\mathbb{N}, y{:}\mathbb{N}, z{:}\mathbb{N})$
   $B = (x'{:}\mathbb{N}, y'{:}\mathbb{N})$
   $Q = (x = x' \wedge y = y' \wedge x \leqslant y + 1)$
   $R = (Q \wedge prime(z) \wedge min(|x - z|, |y - z|) < 3)$

(a) What does F assign to the states $\{x{:}10, y{:}19, z{:}13\}$ and $\{x{:}26, y{:}34, z{:}31\}$?

(b) Describe in your own words, what problem F is about.

5. Let $F \subseteq A \times A$ denote the problem given by the following specification:

$A = (n{:}\mathbb{N}, p{:}\mathbb{N})$
$B = (n'{:}\mathbb{N})$
$Q = (n = n')$
$R = (Q \wedge prime(p) \wedge \forall i \in \mathbb{N}^+ : prime(i) \to |n - i| \geqslant |n - p|)$

(a) What does F assign to the states $\{n{:}9, p{:}5\}$ and $\{n{:}10, p{:}1\}$?

(b) Describe in your own words, what problem F is about.

6. Let $F \subseteq A \times A$ denote the problem given by the following specification:

$A = (x{:}\mathbb{N}, y{:}\mathbb{N}, p{:}\mathbb{N})$
$B = (x'{:}\mathbb{N}, y'{:}\mathbb{N})$
$Q = (x = x' \wedge y = y' \wedge x \leqslant y + 1)$
$R = (Q \wedge prime(p) \wedge \forall i \in [x..y] : prime(i) \to |y - p| \leqslant |y - i|)$

(a) What does F assign to the states $\{x{:}20, y{:}28, p{:}12\}$, $\{x{:}20, y{:}35, p{:}23\}$ and $\{x{:}24, y{:}28, p{:}22\}$?

(b) Describe in your own words, what problem F is about.