AI Robotics

Learning Objectives

- Apply the formalism of product of exponential
- Understand how to calculate the forward kinematics of a robot

Outline

Kinematics

- Kinematics is the description of the motion of particles and systems of particles without regarding the forces which produce the movement.
- Forward kinematics is the determination of the position and orientation of the end-effector based on the configuration of its joint variables.

- Two common approaches to solving the problem of kinematics are the Denavit-Hartenberg method and the Product of Exponential (PoE) method
- In the Denavit-Hartenberg method each link is assigned a frame and the final end-effector position is determined through the sequence of frames.
- The atlternative method is the Product of Exponential method: In this method the end-effector frame is directly determined relative to a fixed frame with the application of rotation about a screw axis.

Forward Kinematics

- Given a planar robot with n joints and n links, where each joint is described by a single degree of freedom which is paramaterised by θ.
- Reference frames are assigned as following:
  - A fixed frame {s} is assigned sometimes referred to as {0}
  - Each joint is assigned a reference frame {i}
  - The end-effector is assigned the reference frame {b}

Example: Planar 3R

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Product of Exponential

- Using the Product of exponential formalism only the fixed frame {s} and the end-effector frame {b} need to be defined.
- In addition a home or zero position needs to be defined. This is a definition which sets the joint variable θ for each joint to a convenient value. $M = T_{sb}(\theta_0^0, \ldots, \theta_n^0)$
- A set of screw axis $S_1, \cdots, S_n$ needs to be defined in the {s} frame.
- To determine the effect of a screw motion on joint n the displacement becomes

$$T = e^{[S_n]\theta_n} M$$

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$S_n = (\omega_n, v_n)$ is the screw axis of joint n in the fixed frame.

- In the case of the revolute joint:
  - $\omega_n \in \mathbb{R}^3$ describes the unit vector in the positive direction of the joint axis
  - $v_v = -\omega_n \times q_n$
  - θ_n is the angle of rotation about the joint axis
- For a prismatic joint:
  - $\omega_n = 0$
  - $v_n \in \mathbb{R}^3$ is a unit vector in the direction of positive translation
  - θ_n is the amount if extension or retraction of the joint

- If now joint $n-1$ is varied the end-effector frame displacement becomes

$$T = e^{[S_{n-1}]\theta_{n-1}} \quad e^{[S_n]\theta_n} M$$

- This can be extended to variation of all joints as

$$T = e^{[S_1]\theta_1} \cdot \cdot \; e^{[S_{n-1}]\theta_{n-1}} e^{[S_n]\theta_n} M$$

Summary of PoE

1. Define a configuration M which specifies the robot in its zero/home position
2. Determine the screw axes $S_1, \ldots, S_n$ in the fixed frame
3. Determine the joint variables $\theta_1, \ldots, \theta_n$
4. Apply the PoE formula

Example: Spatial 3R Open Chain

PoE in the End-Effector frame

- If the screw axis is defined in the body frame the PoE formula can be re-written as

$$T = Me^{[B_1]\theta_1} \cdots e^{[B_{n-1}]\theta_{n-1}} e^{[B_n]\theta_n}$$

Here B is the screw axes in the body-frame

Summary

- Forward kinematics is the process of determining the configuration of the end-effector based on the configuration of the joints
- Two approaches are used Denavit-Hartenberg and the product of exponentiasl.
- In the DH approach a reference frame needs to be assigned to each frame and the transformation matrices of each joint frame is determined relative to the previous and the final transformation is the product of each of these
- In the PoE method, two frames are assigned, one to the fixed frame, the other to the end-effector. The final position is determined by the product of exponential mappings of screw motions applied to a given start configuration
- Next Lecture
  - Velocity kinematics and statics