Programming theory - problems sheet - 2

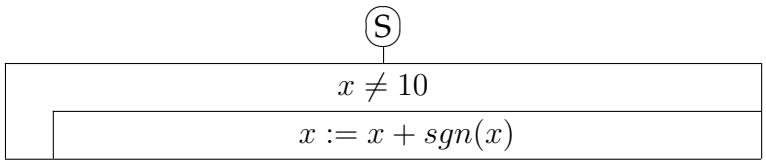
1. Let $A = [1..5]$ be a statespace, and let $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ denote the following relation over A:

$$S = \left\{ \begin{array}{lll} 1 \rightarrow < 1,2,5,1 > & 1 \rightarrow < 1,4,3,5,2 > & 1 \rightarrow < 1,3,2,3,\ldots > \\ 2 \rightarrow < 2,1 > & 2 \rightarrow < 2,4 > & 3 \rightarrow < 3,3,3,\ldots > \\ 4 \rightarrow < 4,1,5,4,2 > & 4 \rightarrow < 4,3,1,2,5,1 > & 5 \rightarrow < 5,2,3,4 > \\ 5 \rightarrow < 5,2,fail > & 5 \rightarrow < 5,3,4 > & \end{array} \right\}$$

   Let $F \subseteq A \times A$ be a problem: $F = \{ (2,1), (2,4), (4,1), (4,2), (4,5) \}$

   - Is S a program?
   - Determine the following sets: $S(2)$, $D_{p(S)}$, $p(S)(4)$, $p(S)(3)$, and $p(S)$.
   - Determine $\tilde{p}(S)$, the weak programfunction of S.
   - Does S solve F?

2. Let $A = (x\colon H)$ be a statespace, where $H = \{a \in \mathbb{Z} \mid a \geqslant -5\}$



   Determine the relation $p(S)$.

3. $A = \{1,2,3\}$. Let $F \subseteq A \times A$ be the following problem: $F = \{ (1,1), (1,2), (2,3) \}$

   - Find a program S that solves F.
   - Find a program over A, such that its programfunction equals to $p(S)$.
   - Does the latter program solve problem F?

4. Let A be any arbitrary statespace. Let $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be a program and let $F \subseteq A \times A$ be a problem, such that S solves F. Decide whether the following statements are true:

   - In case $S \subseteq S_2$, then program S_2 also solves F.
   - In case $F_2 \subseteq F$, then program S solves F_2 as well.
   - In case $S_2 \subseteq S$, then program S_2 also solves F.

5. Let A be any arbitrary statespace. Let $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be a program and let $F \subseteq A \times A$ be a problem, such that S solves F. Decide whether the following statements are true:

   - In case F is non-deterministic, then S is also non-deterministic.
   - In case F is deterministic, then $p(S)$ is also deterministic.
   - In case S is non-deterministic, then $p(S)$ is also non-deterministic.