Calculus revision

Numbers:

natural numbers: $\mathbb{N} = \{1, 2, 3, ...\}$

integers: $\mathbb{Z} = \{0, 1, 2, ..., -1, -2, ...\}$

rationals: $\mathbb{Q} = \{\frac{p}{q},\ \text{where } p, q \in \mathbb{Z},\ q \neq 0\}$

reals: $\mathbb{R}$

- power: a^b, $\left(a^b\right)^c = a^{bc} \neq a^{(b^c)}$
- inverse: $a^{1/b}$: for x^2: $x^{1/2} = \sqrt{x}$

complex: $\mathbb{C} = \{z = a + bi :\ \text{where } a, b \in \mathbb{R}\}$

- imaginary unit: i (or j): $i = \sqrt{-1}$
- complex conjugate: $\bar{z} = z^* = a - bi$
- $z\bar{z} = (a + bi)(a - bi) = a^2 - b^2 i^2 = a^2 + b^2 = r^2 = |z|^2$
- so absolute value: $|z| = r = \sqrt{z\bar{z}}$

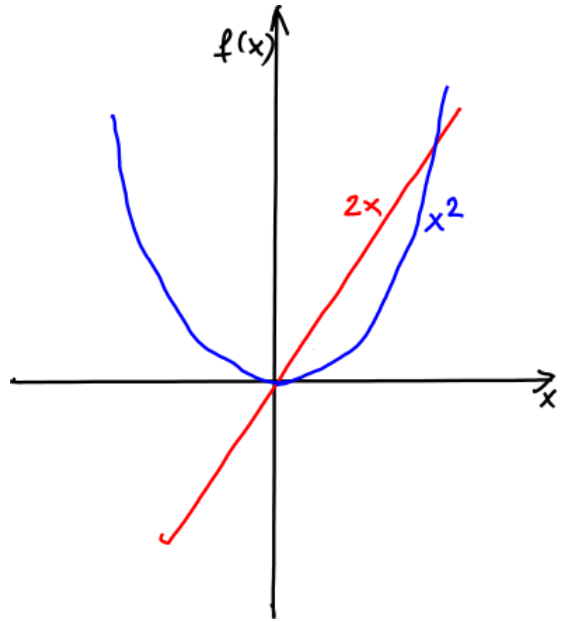
in this course: no complex analysis (poles, residuals, contour integrals etc.)

Calculus revision

Functions:

$f\colon \ \mathbb{R} \to \mathbb{R}, \quad x \mapsto 2x, \quad$ or $f(x) = 2x$

$\qquad\qquad\quad x \mapsto x^2, \quad$ or $f(x) = x^2$



polynomials:

$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ $\quad$ n-degree polynomial

exponential:

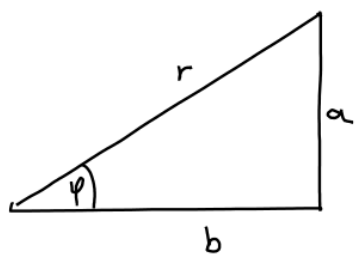
a^x $\quad$ $\exp(x) = e^x$, where $e = 2.7172\ldots$

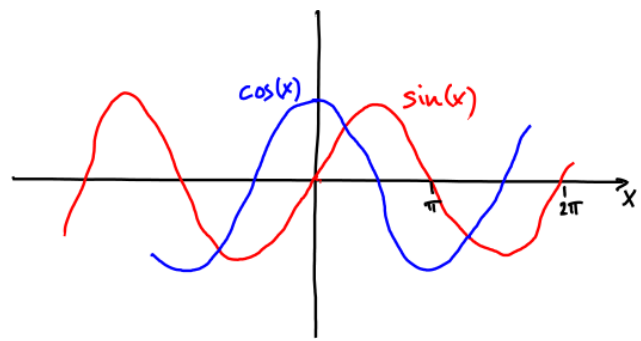
trigonometric:

$\sin(\phi) = \frac{a}{r}$

$\cos(\phi) = \frac{b}{r}$

$\tan(\phi) = \frac{a}{b} = \frac{\sin\phi}{\cos\phi}$





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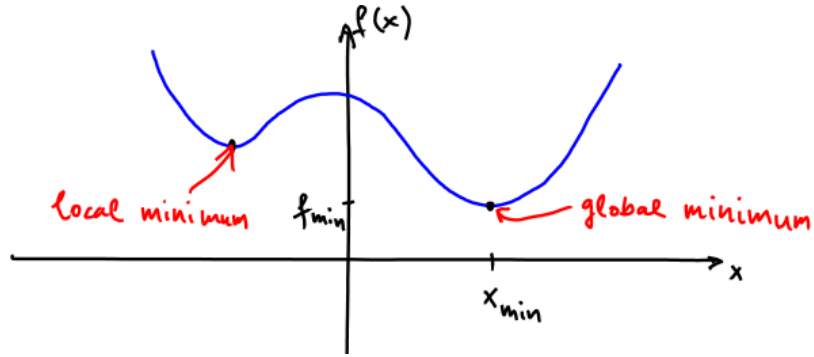


Function properties:

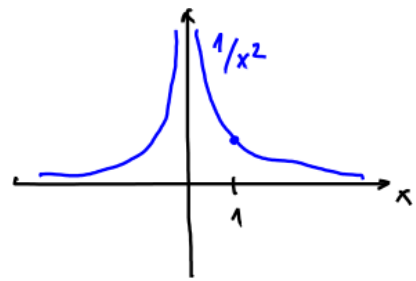
extremum: minimum or maximum

eg. min: $f_{\text{min}} = f(x_{\text{min}})$ if $\forall x: \; f(x) \geq f(x_{\text{min}})$

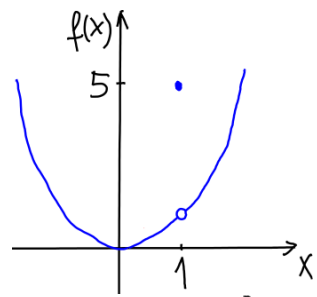
limit: "what it approaches to",

proper definition based on tolerances (ε, δ)

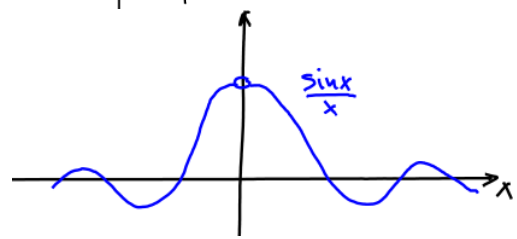
$\lim_{x \to 1} \frac{1}{x^2} = 1, \qquad \lim_{x \to \infty} \frac{1}{x^2} = 0, \qquad \lim_{x \to 0} \frac{1}{x^2} = \infty$



$$f(x) = \begin{cases} x^2 \text{ if } x \neq 1 \\ 5 \text{ if } x = 1 \end{cases} \quad : \lim_{x \to 1} = 1, \text{ not } 5.$$



$$f(x) = \frac{\sin(x)}{x} \to 1 \text{ as } x \to 0$$



continuity: "no jump" at a: $\lim_{x \to a} f(x) = f(a)$