Programming theory - problems sheet - 1

1. Decide whether the following statements are true or false. Explanation is required.

   (a) $5 > 3$

   (b) $\exists x \in \mathbb{N}\colon 2|x$

   (c) $\forall x \in [32..40]\colon \neg prime(x)$

   (d) $\exists x \in \emptyset\colon 2|x$

   (e) $\forall x \in \emptyset\colon x^2 + 5 < 0$

   (f) $\forall x \in \mathbb{N}\colon (\exists y \in \mathbb{N}\colon y|x)$

   (g) $\forall x \in \mathbb{N}\colon (\exists y \in \mathbb{N}\colon (x = y^2 + 1 \wedge 5 > 8))$

2. Let A denote the set $\mathbb{N} \times \mathbb{N}$. Given relation $R \subseteq A \times A$ as follows:
   $R = \{((x, y), (x + y, y))|x, y \in \mathbb{N}\}$.
   Let $H = \{(a, b)|a, b \in \mathbb{N} \wedge a + b < 5\}$ be also given.

   - Find all the elements of A, to which the value $(8, 6)$ is assigned by relation R.
   - Find all the elements of A, to which R assigns the value $(6, 8)$.
   - Find all elements of A, that are in the domain of R and each value that are assigned to them is an element of H.

3. Draw a picture to illustrate the following problems:

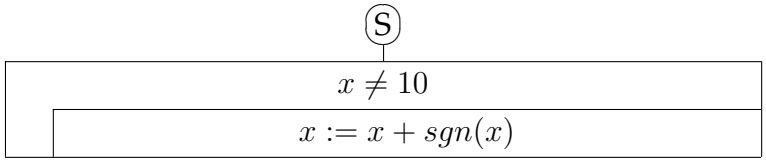
   (a) Decide whether natural number d is a divisor of a given natural number n.

   (b) Find a non-trivial divisor of a given positive integer.

   (c) Find a prime number that is closer to the endpoint of a given interval than any prime number of the interval is.

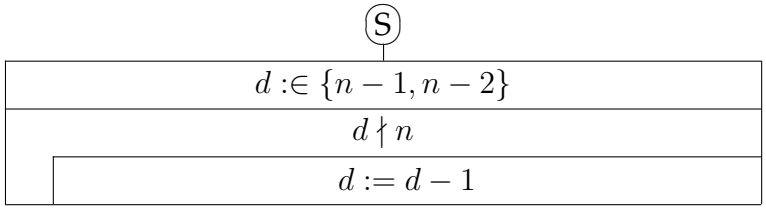
4. Let $A = (x{:}\mathbb{N}, y{:}\mathbb{N}, d{:}\mathbb{N})$ be the statespace of problems F_1 and F_2. What is the relationship between the two problems?
   $F_1 = \{(a, b) \in A \times A|d(b)|x(a) \wedge d(b)|y(a) \wedge \forall k \in \mathbb{N}\colon (k > d(b) \rightarrow \neg(k|x(a) \wedge k|y(a)))\}$
   $F_2 = \{(a, b) \in A \times A|x(a) = x(b) \wedge y(a) = y(b) \wedge d(b)|x(a) \wedge d(b)|y(a)\}$

5. Let H denote the set $\{a \in \mathbb{Z} \mid a \geqslant -5\}$.
   $A = (x{:}H)$



   - What does S assign to the following states: $4, 13, -2, 0$ and 10?
   - Is it true that S is a function?
   - Find the states to which only finite sequences are assigned by S.
   - Determine the elements of A, to which only sequences ending in $fail$ belong.

6. $A = (n{:}\mathbb{N}, d{:}\mathbb{N})$



   Let problem $F \subseteq A \times A$ be given in the following way:
   $F = \left\{ \left(\{n : x, d : y\}, \{n : u, d : v\} \right) \in A \times A \mid x = u \wedge v | x) \right\}$

   - What does S assign to the states $(6, 11), (8, 11), (2, 11)$ and $(1, 11)$?
   - Is it true that program S solves problem F?