AI Robotics

Week 3

Learning Objectives

- Understand the different sensor types used in robotics and where they should be applied
- Understand the Kalman filter process
- Understand the purpose of SLAM
- Understand how the Extended Kalman Filter is used to solve the problem of SLAM

Outline

1. Sensing Modalities
   - Ranging Based
   - Vision Based
   - Contact Based
   - Inertial Sensors

2. Simultaneous Localisation and Mapping (SLAM)
   - Kalman Filters
   - SLAM
   - SLAM EKF

- In order to navigate and manipulate its environment a robot needs to be able to sense and interpret the incoming information
- Six human senses: sight, smell, touch, taste, hearing, balance
- Robots can be equipped with sensors to accommodate all these senses and many unavailable to humans in high precision

Sensor Attributes

- Accuracy and Resolution
  - Precision how repeatable the measurement is
  - Accuracy how close the sensor reading is to the real value
  - The smallest discernible distance
- Bandwidth
  - Particularly important for measurement of things which change rapidly
  - Describes the response of the sensor as a function of input frequency
- Sensitivity
  - Describes the relationship between the input signal and the output signal
  - High sensitivity - Small change in input creates a large change in output

Sensor Attributes

- Stability
  - All electronics are sensitive to their environment
  - Sensors need to be chosen for their specific environment
- Field of View
  - Non contact based sensor specify an region in which they are designed to measure from
  - Field of view is not necessarily symmetric in the vertical and horizontal direction
  - Often measurement sensitivity decreases off-axis
- Power requirements
  - Mobile robots are constrained by the energy and power capacity of their energy storage
  - Lower power consumption is desired
- Size
  - Needs to fit the weight and size constraints of the robot and task

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Range Finding Sensors

- Accurate determinate of distance is a key area in which robots exceed humans
- Fast and accurate depth sensing is required for collision avoidance, mapping and manipulation
- Many sensor types exist for this with varying levels of accuracy, precision, speed and cost
- Sensors in this category typically utilise time-of-flight measurement to determine distance

Time-of-flight measurement

$$d = \frac{\nu t}{2}$$

- ν is the speed of the signal
- t is the time between emitting the signal and receiving the reflected signal

Ultrasonic Sensors

- Uses a piezo to produce ultrasonic pulses, which reflect off objects
- Reflected pulse is received by another piezo
- Delay between emission of the pulse and reception give the distance
- Typically used of ranges between a few cm to a few meters
- Inexpensive
- Has a typical field of view of 4-15°
- Can be used in any medium (except vacuum)

LIDAR Sensors

- Uses one or more lasers to produce pulses for distance measurement
- Due to low divergence of laser output range can be very high (>200m)
- Angular resolution 0.1°
- Distance accuracy on order 2cm
- Field of view can be full 360°

LIDAR Sensors

- Expensive
- Reduced performance in cloudy/foggy conditions
- Unreliable in water
- Large data streams
- Requires more power

Common problems

- Time of flight based sensors are susceptible to the following issues
  - Specular reflection: When an input wave is reflected away from the surface
  - Absorption: Emitted signal is absorbed by the target surface
  - Cross-talk: Inability to distinguish if a reflected signal was generated by itself or by another emitter

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Monocular RGB

- One of the most commonly found sensors
- CCD based image sensors are cheap
- Require additional optics for high quality image formation
- Does not need to be limited to visible light
  - IR (Thermal) Imaging
  - UV Imaging
  - X-Ray Imaging

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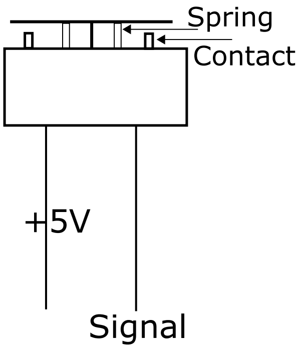
Bump Sensors

- Simple switch mechanism which is contact high



Force Sensors

- Most commonly built using strain gauges
  - The basic working principle uses the fact that Resistivity $\propto$ wire cross section
  - Applied force causes expansion and stretching of wire
- Geometry can be built to isolate different types of load
  - Compression and Tension
  - Torsion
  - Bending

Outline

Accelerometers

- Measures acceleration in one or more directions
- Most commonly now MEMS (microelectromechanical systems) based
- Bandwidth can range from 1Hz-50kHz
- Can measure large accelerations

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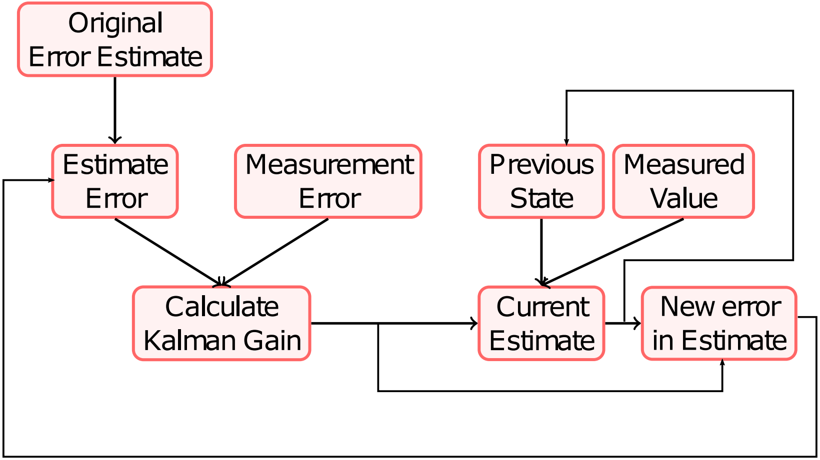
Kalman Filters

- A mathematical process which is used to estimate the true value of a measured variable
- Both the measurement model and the process model can include uncertainty
- No measurement can be made without some non-zero amount of uncertainty
- Uncertainty can come from systematic sources. e.g. Finite measuring resolution, calibration factors or random sources e.g. electrical noise

Example problem

- A car is traveling in one dimension (x) we want to know its position x and its velocity $\dot{x}$

Kalman Filter Overview



Process Model

- The process model describes the time evolution of the state vector
$$x_k = Ax_{k-1} + Bu_k + w_k$$
- A is the transition matrix
- B is the control-input matrix
- w is the noise vector

Process Model

- In our example we have motion in the x dimension and we are tracking the position of the car
- Position is given by $x = x + vt + \frac{1}{2}at^2$
- What are our vectors x and u?
- What are our transition matrix A and control-input B?

Process Model

$$x_k = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, u_k = \begin{bmatrix} a_x \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{2} a \Delta t^2 \\ \Delta t \end{bmatrix}$$

$$X_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} a \Delta t^2 \\ \Delta t \end{bmatrix} \begin{bmatrix} a_x \end{bmatrix}$$

- How does this extend in 2D and 3D?

Measurement Model

- Measurement model describes the relationship between the state and measurement at a given timestep
$$z_k = Hx_k + \nu_k$$
- H is the measurement matrix
  - The structure of H depends on the sensor used as sensors do not typically output values directly in the units of the state description
- ν_k is the measurement noise vector

Covariance Matrices

- Covariance describes the strength of correlations between two random variables
- Three covariance matrices are used in the basic Kalman Filter
  - P - Predicted error covariance
  - Q - Process noise covariance
  - R - Measurement noise covariance

Noise Vectors

- The process noise w and measurement noise ν are typically assumed to be Gaussian with covariance Q and R

$$w \sim \mathcal{N}(0, Q)$$
$$\nu \sim \mathcal{N}(0, R)$$

Error covariance

- The predicted error covariance describes the models uncertainty in the estimate

$$P_k = AP_{k-1}A^T + Q$$

Kalman Gain

- Describes the belief which we attach to the either the estimate and provides an output over the interval (0,1]

$$K_k = \frac{P_{k-1}H^T}{HPH^T + R}$$

- Recall that R is the measurement uncertainty
- If the sensor is exactly accurate $K_t = 1$
- As the sensor uncertainty increases K approaches 0

Kalman Filter Update

- First step is to compute the measurement residual
- Second step is the calculation of Kalman Gain
- Update the State estimate
- Update the Error Covariance

Outline

What is SLAM

- When navigating an environment a robot needs to be able to estimate its current position (localisation) a robot also needs to build a map of the positions of objects and obstacles in the environment (mapping)
- SLAM is the process of simultaneously mapping the environment and determining its position
- Extended Kalman Filter estimates the position of the robot through the combination of odometry data and observed landmarks

When is SLAM Used

- SLAM is needed when the robots odometry cannot be fully trusted
- Odometery data provides an estimated position based on the robots knowledge of its own movement

Extended Kalman Filter

- Regular Kalman filter is limited to linear dynamic systems, in the case of nonlinear dynamics the extended kalman filter can be used.
- No longer view the process and control models as matrices but non-linear functions which act on our state and control vectors

State transition and measurement

- Replace the State Transition Matrices F and B and the measurement matrix H with the following

$$x_k = g(x_{k-1}, u_{k-1}) + w_{k-1}$$
$$z_k = h(x_k) + \nu_k$$

Jacobians

- Given some vector valued function $g(x)$ and some input vector x the Jacobian is given as

$$G_x = \begin{bmatrix} \frac{\delta g_1}{\delta x_1} & \frac{\delta g_1}{\delta x_2} & \frac{\delta g_1}{\delta x_2} & \cdots & \frac{\delta g_1}{\delta x_n} \\ \frac{\delta g_2}{\delta x_1} & \frac{\delta g_2}{\delta x_2} & \frac{\delta g_2}{\delta x_3} & \cdots & \frac{\delta g_2}{\delta x_n} \\ \frac{\delta g_3}{\delta x_1} & \frac{\delta g_3}{\delta x_2} & \frac{\delta g_3}{\delta x_3} & \cdots & \frac{\delta g_3}{\delta x_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\delta g_m}{\delta x_1} & \frac{\delta g_1}{\delta x_2} & \frac{\delta g_m}{\delta x_1} & \cdots & \frac{\delta g_m}{\delta x_n} \end{bmatrix}$$

EKF in 1 Slide

Input: x_{k-1}, u_k, P_{k-1}, z_k
$x_k = g(x_{k-1}, u_{k-1})$
$P_t = G_k P_{k-1} G_k^T + R_k$
$y_k = z_k - h(x_k)$
$K = P_k H_k^T (H_k P_k H_k^T + Q_k)^{-1}$
$x_k = x_k + K_k y_k$
$P_t = (I - K_k H_k) P_k$
Result: x_k, P_k

Outline

System state vector

- As previosuly mentioned the position of the robot is encoded as (x, y, θ). The x,y coordinates on the given world frame and the orientation.
- Landmarks can be designates simply with their x and y coordinates (x_i, y_i).

$$X = \begin{bmatrix} x_r & y_r & \theta_r & x_1 & y_1 & \dots & x_n & y_n \end{bmatrix}^T$$

Covariance Matrix

- The covariance matrix represent how strongly correlated elements of the state vector are

$$P_k = \begin{bmatrix} P_R & P_{RL_1} & P_{RL_2} & \cdots & P_{RL_n} \\ P_{L_1 R} & P_{L_1} & P_{L_1 L_2} & \dots & P_{L_1 L_n} \\ P_{L_2 R} & P_{L_2 L_1} & P_{L_2} & \dots & P_{L_2 L_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{L_n R} & P_{L_n L_1} & P_{L_n L_2} & \dots & P_{L_n} \end{bmatrix}$$

SLAM Measurement Model

- Measurement is based on the distance and angle to landmarks

$$h = \begin{bmatrix} \sqrt{(\lambda_x - x)^2 + (\lambda_y - y)^2} + \nu_r \\ \tan^{-1}(\frac{\lambda_y - y}{\lambda_x - x}) - \theta + \nu_\theta \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{x-\lambda_x}{r} & \frac{y-\lambda_y}{r} & 0 \\ \frac{\lambda_y - y}{r^2} & \frac{\lambda_x - x}{r^2} & -1 \end{bmatrix}$$

Step 1

- First step in SLAM is to use the current odometry data to update the state
- In ideal case the robot tracks its change in x, y and θ

$$x_k = \begin{bmatrix} x + \delta x \\ y + \delta y \\ \theta + \delta\theta \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & -\delta y \\ 0 & 1 & \delta x \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2

- The second step in SLAM is to refine the predicted state based on the observation of landmarks

Step 3

- Steps 1 and 2 cover the normal EKF Prediction and Update cycle. Step 3 is unique to SLAM in that the state vector x and covariance matrix P are updated with new landmarks
- Any newly detected landmarks are appended to the state vector $x = \begin{bmatrix} x & x_N & y_N \end{bmatrix}^T$

Summary

- Introduced key characteristics of sensors
- Introduced the Kalman Filter and Extended Kalman Filter and how they relate to SLAM

- Next Lecture
  - Spatial Descriptions and Configuration Space of rigid bodies