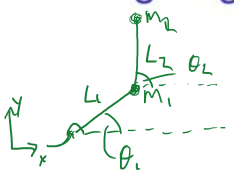
Lagrangian Example Planar 2R

M - mass

m - moment



$$\mathcal{L}(q,\dot{q}) = K(q,\dot{q}) - P(q), \quad f = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q}$$

Position + Velocity of m_1

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 \\ L_1 \cos\theta_1 \end{bmatrix} \dot{\theta}_1$$

Position + Velocity of m_2

2×2

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -L_1\sin\theta_1 - L_2\sin(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2) \\ L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Generalised coordinates $- \ \theta = (\theta_1, \theta_2)$

Generalised forces $- \ \tau = (\tau_1, \tau_2)$

$$\mathcal{L}(\theta,\dot{\theta}) = \sum_{i=1}^{2} (K_i - P_i)$$

Kinetic Energy:

$$K_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) = \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2, \quad K_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2} m_2 \Big[(L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2)\dot{\theta}_1^2 + 2(L_2^2 + L_1L_2\cos\theta_2)\dot{\theta}_1\dot{\theta}_2 + L_2^2\dot{\theta}_2^2 \Big]$$

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Potential energy

$$P_1 = m_1 g y_1 = m_1 g L_1 \sin\theta_1$$

$$P_2 = m_2 g y_2 = m_2 g (L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2))$$

$$\begin{aligned} \mathcal{L} = &\frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2 - m_1 g L_1 \sin\theta_1 \\ &+ \frac{1}{2} m_2 \Big[(L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2)\dot{\theta}_1 \\ &\quad + 2(L_2^2 + L_1L_2\cos\theta_2)\dot{\theta}_1\dot{\theta}_2 \\ &\quad + L_2^2 \dot{\theta}_2^2 \Big] \\ &- m_2 g (L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2)) \end{aligned}$$

$$\tau_i = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i}, \quad i = 1, 2$$

$$\begin{aligned} \tau_1 = &(m_1 L_1^2 + m_2 (L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2))\ddot{\theta}_1 \\ &+ m_2 (L_1L_2\cos\theta_2 + L_2^2)\ddot{\theta}_2 \\ &- m_2 L_1 L_2 \sin\theta_2 (2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) \\ &+ (m_1 + m_2) L_1 g \cos(\theta_1) + m_2 g L_2 \cos(\theta_1 + \theta_2) \end{aligned}$$

$\tau_2 = \ldots$