Notations and mathematical background

1 Sets

Informally, a set is a collection of objects, which are called the elements of the set. The elements of a set do not certainly have a common property, except that they are elements of the same set. For example, here is a set:

$$\{Budapest, \alpha, \emptyset, \sqrt[3]{2}\}$$

One way to define a set is by listing the elements of the set inside curly-brackets (for example, the set of logical values:)

$$\mathbb{L} ::= \{igaz, hamis\}$$

or by providing a condition (for example, the set of integer numbers that can be divided by 5 and are not greater than 100:)

$$\{\, x \in \mathbb{Z} \mid x \leqslant 100 \wedge 5|x \,\}$$

Definition: *The set $[a..b] ::= \{\, x \in \mathbb{Z} \mid a \leqslant x \wedge x \leqslant b \,\}$ is called interval (where a and b are integer numbers). It is empty, if $b < a$.*

Notation: *The cardinality of a set H is denoted by $|H|$. The fact, that set H is finite, can be expressed in this way: $|H| < \infty$.*

Special sets:

$\mathbb{N}$	– the set of all natural numbers (including 0)
$\mathbb{N}^+$	– the set of all positive integers
$\mathbb{Z}$	– the set of all integers
$\mathbb{L}$	– the set of logical values
$\emptyset$	– the empty set

2 Sequences

Notation: *Let H^{**} denote the set of all finite and infinite sequences of the elements of set H. H^∞ includes the infinite sequences; H^* contains the finite ones. So, $H^{**} = H^* \cup H^\infty$ and $H^* \cap H^\infty = \emptyset$. The length of the sequence $\alpha \in H^{**}$ is $|\alpha|$, in case of infinite sequence this value is denoted by ∞.*

3 Relations

Definition: *Let A and B be arbitrary not empty sets. The Cartesian product $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$:*

$$A \times B ::= \{ (a, b) \mid a \in A \wedge b \in B \}$$

Definition: *Let A and B be arbitrary not empty sets. Any subset R (including the empty set) of $A \times B$ is called a relation.*

A relation can be considered as a mapping from the set A to B. In case $(x, y) \in R$, then we say that R assigns (or associates) y to x.

Definition: *Let A and B be arbitrary not empty sets and let $R \subseteq A \times B$ be any relation. The domain of relation R:*

$$\mathcal{D}_R ::= \{ a \in A | \exists b \in B : (a, b) \in R \}$$

the range of relation R:

$$\mathcal{R}_R ::= \{ b \in B | \exists a \in A : (a, b) \in R \}$$

the image of a (where $a \in A$) by R:

$$R(a) ::= \{ b \in B | (a, b) \in R \}$$

Definition: *Let A and B arbitrary not empty sets and let $R \subseteq A \times B$ be any relation. We say that R is deterministic, if*

$$\forall a \in A : |R(a)| \leqslant 1$$

Deterministic relations are called functions. The function $R \subseteq A \times B$, as a special relation, has a particular notation: $R \in A \to B$.

Notation: *In case for the function $f \in A \to B$ it also holds, that its domain equals to set A, then the following notation is used: $f : A \to B$. Notice that in this case the following holds:*

$$\forall a \in A : |f(a)| = 1$$

Remark: *Such $f \in A \to B$ functions, which does not associate exactly one single element to every element of A (in other words, their domain is not equal to A), are called partial functions.*

Remark: *Let $f : A \to B$ be an arbitrary function (now we know that f is a special relation that assigns a single element to every element of set A. Let $a \in A$ and $f(a) = \{b\}$ where $b \in B$ denotes the only element assigned to a. In this case, for the sake of simplicity, the image $f(a)$ can be written as the value b instead of the set $\{b\}$.*