Problem Set 1

19fmiROBEG - AI Robotics

1 Problems

Problem 1. (0.5 points) Given the two random variables $x = [2, 4, 5, 1, 9, 11, 6]$ and $y = [4, 1, 8, 2, 7, 10, 6]$ calculate the 2×2 covariance matrix for these two random variables.

Problem 2. (0.5 points) In the lectures state estimation using a Kalman Filter was applied to the case of tracking the position and velocity of a vehicle in one-dimension was shown. Given a vehicle in 3D space, modelled with cartesian coordinates x, y, z. Determine the state vector x_k, control vector u_k, the transition matrix A and control-input matrix B.

Problem 3. (2 point) Starting from the covariance update equation

$$P_k = (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R_k K_k^T$$

show that the Kalman Gain $K = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1}$

Note the superscript minus symbol in P_k^- indicates that this is the error covariance obtained in the first prediction stage

Hint: $\frac{\partial Tr(P_k)}{\partial K_k} = 0$

Problem 4. (2 points) The Kalman filter is limited to linear models, however the extension to non-linear models is possible through the use of the Extended Kalman filter. This is achieved by linearising the model around the current state. We replace the state vector x with a new, possibly non-linear function g. Linearisation of the function g about the time-step k is given by computing the Taylor series exansion of g to first order.

In 2D the robot has a state vector $x_k = [x_k, y_k, \theta_k]^T$ and control vector $v_k = [V_{x,k}, V_{y,k}, \omega_k]^T$. Its motion model $g()$ can be described as the following

$$\begin{aligned}
x_k &= x_{k-1} + \Delta t V_{x,k-1} \times cos(\theta_{k-1}) \\
y_k &= y_{k-1} + \Delta t V_{y,k-1} \times sin(\theta_{k-1}) \\
\theta_k &= \theta_{k-1} + \omega_{k-1}
\end{aligned}$$

We can calculate the Jacobian of G to provide the first order differential of the the motion model. Given as

$$G = \begin{pmatrix} \frac{\partial x_k}{\partial x} & \frac{\partial x_k}{\partial y} & \frac{\partial x_k}{\partial \theta} \\ \frac{\partial y_k}{\partial x} & \frac{\partial y_k}{\partial y} & \frac{\partial y_k}{\partial \theta} \\ \frac{\partial \theta_k}{\partial x} & \frac{\partial \theta_k}{\partial y} & \frac{\partial \theta_k}{\partial \theta} \end{pmatrix}$$

Calculate the Jacobian G.