Definitions, theorems, examples and book sections

Vector spaces

See also: Strang's book Section 2.1 Vector Spaces and Subspaces; Section 2.3 Linear Independence Basis and Dimension; Appendix A1 The Intersection of Two Vector Spaces; Appendix A3 The Cartesian Product of Two Vector Spaces.
See also: Linear Geometry Section 1.2 Vector spaces; Section 1.3 Subspaces; Section 1.4 Dimension.

1. Vector space
2. Subspace, intersection of subspaces
3. The Cartesian product of two vector spaces
4. Linear combination
5. Linear dependence and linear independence of vectors
6. Subspace spanned by vectors
7. Basis of a vector space
8. Dimension of a vector space, dimension of a subspace, codimension, hyperplane

Linear maps between vector spaces and matrices

See also: Strang's book Section 2.6 Linear Transformations; Section 5.1 Eigenvalues and Eigenvectors; Section 4.1 Introduction; Section 4.2 Properties of the Determinant; Section 4.4 Applications of Determinants; Section 5.4 Differential equations and e^{At}; Section 5.2 Diagonalization; Section 5.5 Complex matrices–but only the Hermitian subsection and only when the matrix is real, especially Theorem 5O (the principal axis theorem, also known as the spectral theorem for symmetric matrices); Section 5.6 Similarity Transformations–the subsection Diagonalizing Symmetric and Hermitian Matrices and the subsection The Jordan Form; Section 3.4 Orthogonal Bases and Gram-Schmidt, from here the subsection Orthogonal Matrices.
See also: Linear Geometry Section 4.1 Elementary properties of linear mappings;

Section 4.3 Matrices; Section 4.4 The rank of a linear mapping; Section 5.1 Bilinear maps.

1. Linear maps between vector spaces
2. Endomorphism, automorphism, isomorphism
3. Null space, kernel, image, range of a linear map
4. Rank and nullity. The rank-nullity theorem
5. Invertible linear maps. Characterization with the kernel. Regular and singular linear maps
6. Matrices as linear maps
7. Linear maps as matrices: matrix representations of a linear map
8. Invertibility and the determinant
9. Examples: rotations, reflections, projections, differential and integral operators, shift operators
10. The rank of a matrix as the maximum number of linearly independent row vectors (or column vectors)
11. Eigenvalues and eigenvectors of a linear map. Eigendirection, eigenspace. The spectrum of a matrix, characteristic polynomial
12. The algebraic and geometric multiplicity of an eigenvalue
13. Projections. Eigenvalues of a projection
14. The trace of a matrix. Eigenvalues and the trace
15. The determinant of a matrix. Eigenvalues and the determinant. Transpose and the determinant. Inverse and the determinant
16. Similar matrices. Similar matrices and the change of basis. Invariants: eigenvalues, trace, determinant
17. Similar matrices have the same eigenvalues
18. The Cayley–Hamilton theorem
19. Formula for the inverse matrix with cofactors. The product formula for determinants
20. The definition of $p(A)$ where p is a univariate polynomial and $A \in \mathbb{R}^{n \times n}$
21. Functions of matrices: the definition of $f(A)$ via convergent Taylor series
22. The computation of $f(A)$ via Hermite–Lagrange interpolation. Examples: exp(A), exp(A t), sin(A), cos(A)
23. Elementary properties of $f(A)$ in terms of the commutator and the eigenvalues
24. For any projection in finite dimensions, $e^P = I + (e - 1) P$
25. The relation between the commutativity of A and B, and e^{A+B}, e^A, e^B
26. Application of $e^{A t}$: the solution of 1st-order linear systems of ordinary differential equations with constant coefficients. Liouville’s formula
27. Rotation in $\mathbb{R}^2$ as multiplication with a complex number. The rotation matrix in real form
28. Multilinear maps. Symmetric and antisymmetric maps. The determinant
29. The geometric meaning of the determinant. Elementary properties of the determinant
30. Applications of the determinant: double integrals in polar coordinates; the cross product of vectors
31. Diagonalizability of square matrices. Diagonalization as matrix decomposition
32. Diagonalization of symmetric real square matrices. Eigenvalues and eigenvectors of such matrices. Eigenbasis

33. Some geometric properties of orthogonal matrices
34. The Jordan canonical form of a square matrix. See also: Strang's book Appendix B: The Jordan Form
35. Gaussian elimination as LU decomposition. See also: Strang's book Secion 1.5 Triangular Factors and Row Exchanges
36. Review of matrix factorizations/decompositions: LU, diagonalization, diagonalization of symmetric matrices, the Jordan form, singular-value decomposition (SVD). See also: Strang's book Appendix C: Matrix Factorizations

Abstract spaces and some applications

See also: Strang's book
Section 3.2: Inner products and cosines, Cauchy–Schwarz–Bunyakovsky inequality, Transposes from Inner Products, Problem set 3.2: The triangle inequality
Section 3.4: Orthogonal Bases and Gram–Schmidt, The Gram–Schmidt Process
Section 7.2: Matrix Norm and Condition Number, Definition 7B (the operator norm), A Formula for the Norm (induced by the $\|.\|_2$ norm), Problem Set 7.2: the l_1 norm and the l_∞ norm (Problem 18), Equivalent norms (Problem 19), Triangle inequality (Problem 20),
Section 3.4: Function Spaces and Fourier Series, Hilbert spaces, Lengths and Inner Products, Gram–Schmidt for Functions
Section 3.3: Projections and Least Squares, Least Squares Problems with Several Variables, Least-Squares Fitting of Data
Section 6.2: Positive Definite Matrices. Tests for Positive Definiteness
Section 6.3: Singular Value Decomposition. Application of the SVD

See also: Linear Geometry Section 6.1 Distances and Euclidean Geometries

See also (this is just a sample of links; please use a search engine to find many more relevant references to dig deeper):
https://en.wikipedia.org/wiki/Space_(mathematics)
https://en.wikipedia.org/wiki/Inner_product_space
https://en.wikipedia.org/wiki/Pythagorean_theorem#Inner_product_spaces
https://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality
https://en.wikipedia.org/wiki/Normed_vector_space
https://en.wikipedia.org/wiki/Triangle_inequality#Normed_vector_space
https://en.wikipedia.org/wiki/Triangle_inequality#Reverse_triangle_inequality
https://en.wikipedia.org/wiki/Metric_space

https://en.wikipedia.org/wiki/Triangle_inequality#Metric_space
https://en.wikipedia.org/wiki/Hamming_distance
https://en.wikipedia.org/wiki/Cauchy_sequence
https://en.wikipedia.org/wiki/Cauchy_sequence#Completeness
https://en.wikipedia.org/wiki/Banach_space
https://en.wikipedia.org/wiki/Hilbert_space
https://en.wikipedia.org/wiki/Moore%E2%80%93Penrose_inverse
https://en.wikipedia.org/wiki/Positive-definite_matrix
https://en.wikipedia.org/wiki/Singular_value_decomposition
https://en.wikipedia.org/wiki/Principal_component_analysis

0. Vector spaces (we have been discussing so far in the semester) are also abstract spaces, so they naturally fall into this category
1. Inner-product spaces. Orthogonality, angle. Pythagorean theorem. The Cauchy–Schwarz–Bunyakovsky inequality. The Gram–Schmidt orthonormalization process
2a. Normed vector spaces. The reverse triangle inequality. Equivalent norms. Examples: the p-norms: $\|.\|_1$, $\|.\|_2$, $\|.\|_\infty$. Open and closed balls. An inner-product space is also a normed space: the norm induced by an inner product. The polarization identity.
2b. The operator norm for square matrices. Examples: the 1-norm, the spectral norm, the ∞-norm for matrices
3a. Metric spaces. Example: the discrete metric. Example: the taxicab metric (Manhattan or l_1-distance) in $\mathbb{R}^n$. Example: the Hamming distance. Example: a normed vector space is also a metric space by the induced metric
3b. Cauchy sequences in metric spaces. Complete metric spaces. Example: a non-complete metric space
4a. Banach spaces and Hilbert spaces. Example: C([a,b]) is not a Hilbert space; the L^2 function space as an inner-product space, as a normed space, as a metric space, as a Banach space, as a Hilbert space.
4b. The Banach fixed-point theorem and the Picard–Lindelöf theorem. Fourier series in Hilbert spaces
5. Applications: overdetermined linear systems, the method of least squares. Projections. Generalized inverses: the Moore–Penrose pseudoinverse
6. Positive definite, negative definite, positive semidefinite, negative semidefinite and indefinite matrices
7. Applications: singular-value decomposition (SVD). The Moore–Penrose pseudoinverse and SVD. PCA

Probability spaces, statistics, information theory

See also: J. Rice, *Mathematical Statistics and Data Analysis*, Chapter 1: Probability; Chapter 2: Random variables; Chapter 4: Expected values; Chapter 7.2: Population parameters; Chapter 4.3: Covariance and correlation

See also: W. Feller, *An Introduction to Probability Theory and Its Applications*, Volume 1, Section 1: Sample space; Volume 2, Chapter IV, Section 3: σ-algebras; Section 4: Probability spaces, random variables
See also: T. M. Cover, J. A. Thomas, *Elements of Information Theory*, Chapter 1; Chapter 2, Section 2.1

1. Probability space, σ-algebras, measures. Countable sets. Discrete probability space
2. Conditional probability. Independent events, mutually exclusive events. Law of total probability. Bayes' theorem
3. Discrete and continuous random variables. Probability mass function, probability density function, cumulative distribution function
4. Expected value, variance, and standard deviation of a random variable. Markov's inequality
5. Examples: Bernoulli, Cauchy, and normal distributions. The gamma function. The error function
6. Univariate descriptive statistics: central tendency/location (mean, median, mode) and dispersion (range, variance, standard deviation, quantiles, quartiles)
7. Bivariate descriptive statistics: quantitative measures of dependence (covariance, Pearson's correlation, Spearman's correlation)
8. Some properties of covariance. Correlation and independence
9. Data visualization: histogram, scatter plot, contingency table
10. Entropy of a random variable