

2R Example (Planar)

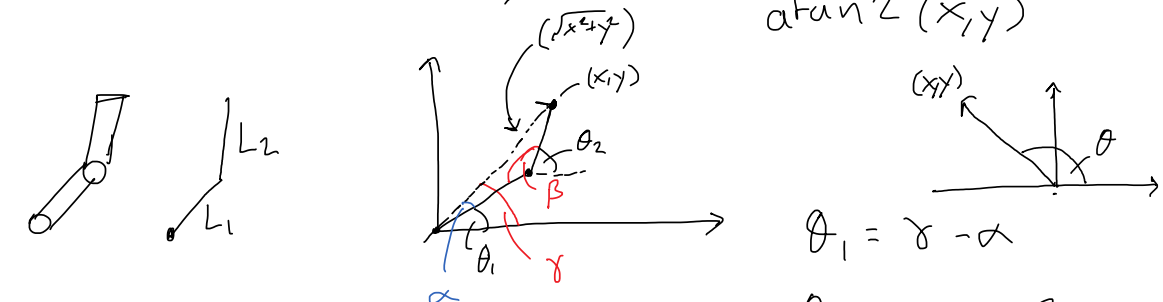


Diagram illustrating the geometry of a 2R planar arm. The arm consists of two links, L_1 and L_2 , with a target point (x, y) . The distance from the base to the target point is $\sqrt{x^2 + y^2}$. The angle between the two links is β . The angle between the first link and the horizontal axis is θ_1 . The angle between the second link and the horizontal axis is θ_2 . The angle between the line from the base to the target point and the horizontal axis is γ . The angle between the first link and the line from the base to the target point is α .

Diagram illustrating the geometry of a triangle with sides a , b , and c , and angle θ between sides a and b .

Equations for the joint angles:

$$c^2 = a^2 + b^2 - 2ab \cos(\theta)$$

$$L_1^2 + L_2^2 - 2L_1 L_2 \cos(\beta) = x^2 + y^2$$

$$\beta = \cos^{-1} \left(\frac{-x^2 - y^2 + L_1^2 + L_2^2}{2L_1 L_2} \right)$$

$$\alpha = \cos^{-1} \left(\frac{x^2 + y^2 + L_1^2 - L_2^2}{2L_1 \sqrt{x^2 + y^2}} \right)$$

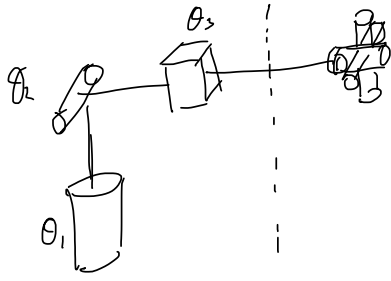
Diagram illustrating the geometry of a point (x, y) in the plane, with the angle θ measured from the horizontal axis to the line connecting the origin to the point. The function $\text{atan2}(x, y)$ is used to calculate the angle θ .

Equations for the joint angles:

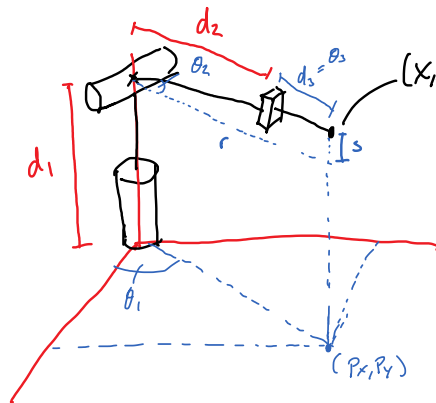
$$\theta_1 = \gamma - \alpha$$

$$\theta_2 = \pi - \beta$$

Stanford Arm Example



RRP
Given (x, y, z) need to find $(\theta_1, \theta_2, \theta_3)$



$$\begin{aligned} & \text{atan2}(p_x, p_y) \\ & r^2 = p_x^2 + p_y^2 \\ & s = p_z - d_1 \\ & \theta_2 = \text{atan2}(s, r) \\ & (\theta_3 + d_2)^2 = r^2 + s^2 \\ & \theta_3 = \sqrt{r^2 + s^2} - d_2 \\ & \theta_3 = \sqrt{p_x^2 + p_y^2 + (p_z - d_1)^2} - d_2 \end{aligned}$$