Programming theory
midterm test - sample

You are allowed to use the short form $(a_1, \ldots a_n)$ in order to denote the state $\{v_1{:}a_1, \ldots v_n{:}a_n\}$.

1. Let $A = [1..5]$ be a statespace, $S \subseteq A \times (A \cup \{fail\})^{**}$ a program over the statespace A.

$$S = \begin{cases} 1 \to< 1, 2, 5, 1 > & 1 \to< 1, 4, 3, 5, 2 > & 1 \to< 1, 3, 2, 3, \ldots > \\ 2 \to< 2, 1 > & 2 \to< 2, 4 > & 3 \to< 3, 3, 3, \ldots > \\ 4 \to< 4, 1, 5, 4, 2 > & 4 \to< 4, 3, 1, 2, 5, 1 > & 5 \to< 5, 2, 3, 4 > \\ 5 \to< 5, 2, fail > & 5 \to< 5, 3, 4 > \end{cases}$$

Let $F \subseteq A \times A$ denote the following problem: $F = \{ (2,1), (2,4), (4,1), (4,2), (4,5) \}$

   (a) Determine the program function of S and the domain of $p(S)$.

   (b) Determine the following two sets: $S(1)$ and $p(S)(2)$.

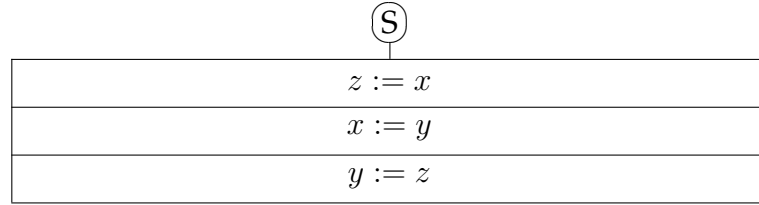   (c) Decide whether S solves the problem F.

(12 points)

2. (a) Consider the statespace A and program S that were given in task 1.   (6 points)
   Let $Q, R : A \to \mathbb{L}$ be logical functions given such that $\lceil R \rceil = \{1, 5\}$ and $\lceil Q \rceil = \{5\}$.

   – Determine the truth-set $\lceil wp(S, R) \rceil$.
   – Decide whether 4 is an element of $\lceil wp(S, Q) \rceil$.

   (b) Find a program S_2 over A so that S is deterministic and solves problem F. Detailed explanation is required.   (6 points)

3. (a) Let $H = \{a \in \mathbb{Z} \mid a \geqslant -5\}$ (6 points)
   $A = (x : H)$
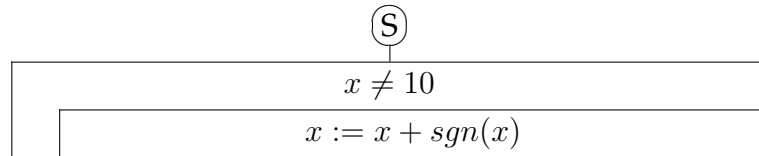
   – Write down the sequences assigned to the states 4, 13, -2, 0 and 10 by the program S.
   – Determine the program function of S.

(b) $A = (x{:}\mathbb{Z}, y{:}\mathbb{Z})$ is the base-statespace and $z{:}\mathbb{Z}$ is an auxiliary variable of the program. (6 points)

$$\boxed{\begin{array}{c} S \\ \hline z := x \\ \hline x := y \\ \hline y := z \end{array}}$$

- Write down the sequences assigned to the state $\{x{:}3, y{:}8\}$ by the program S.
- What does the program function of S addign to the state $\{x{:}3, y{:}8\}$?

4. (a) Let $S_1, S_2 \subseteq A \times (A \cup \{fail\})^{**}$ be any arbitrary programs, such that $S_1 \subseteq S_2$. Decide whether $D_{p(S_1)} \subseteq D_{p(S_2)}$ holds or not. (6 points)

(b) Problem: Given two integer numbers. Find a common divisor of them. (6 points)
   - What is the statespace of the problem?
   - Draw a picture to illustrate the given problem.

5. Problem F is given by its specification: (12 points)

$A = (n{:}\mathbb{N}, p{:}\mathbb{N})$
$B = (n'{:}\mathbb{N})$
$Q = (n = n')$
$R = (Q \wedge prime(p) \wedge \forall i \in \mathbb{N}^+ : prime(i) \rightarrow |n - i| \geqslant |n - p|)$

- Determine the following sets: $\lceil Q_{\{n'{:}9\}}\rceil$ and $\lceil R_{\{n'{:}9\}}\rceil$.
- What does F assign to the following states: $\{n{:}9, p{:}5\}$ and $\{n{:}10, p{:}1\}$?