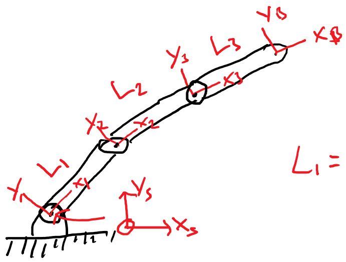
Planar 3R Example

Wednesday, October 14, 2020 12:34 PM

Example: Planar 3R



$L_1 = L_2 = L_3$

$$T = \begin{bmatrix} R(\hat{\omega},\theta) & P \\ 0 & 1 \end{bmatrix}, \quad Rot(\hat{z},\theta)$$

$$T_{sB} = T_{s1}T_{12}T_{23}T_{3B}$$

DH method

$c = \cos$
$s = \sin$

1. Define reference frames

$$\overset{4\times4}{T_{s1}} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{12} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{3B} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{sB} =$$

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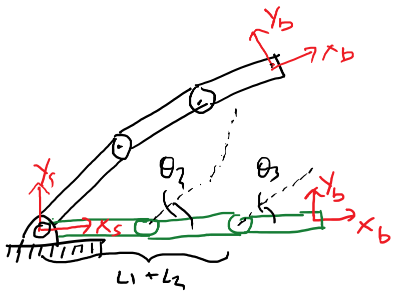
1. PoE We only define fixed {s} and end-effector {b} reference frames

2. Define for home position M, $\theta_1=\theta_2=\theta_3=\phi$



Transformation matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & L_1+L_2+L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{[S]\theta} M$$

$$S_3, \theta_3 \downarrow S_3 = \begin{bmatrix} \omega_3 \\ v_3 \end{bmatrix}$$

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = -\omega_3 \times q_3$$

$$q_3 = \begin{bmatrix} L_1+L_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -(L_1+L_2) \\ 0 \end{bmatrix}$$

$$e^{[S_3]\theta_3} M \qquad S_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -(L_1+L_2) \\ 0 \end{bmatrix}$$

$$\downarrow$$

$$\cdots \; e^{[S_2]\theta_2} e^{[S_3]\theta_3} M$$