Asymptotic behaviour of functions

$$\Theta(g(n)) = \{f(n) : \exists\, c_1, c_2, n_0 > 0 \text{ such that } c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0\}$$
$$O(g(n)) = \{f(n) : \exists\, c, n_0 > 0 \text{ such that } f(n) \le c g(n) \text{ for all } n \ge n_0\}$$
$$\Omega(g(n)) = \{f(n) : \exists\, c, n_0 > 0 \text{ such that } f(n) \ge c g(n) \text{ for all } n \ge n_0\}$$

Notations $f(n) = O(g(n))$ or $f(n) \in O(g(n))$ can both be used.

Master Theorem

3. 8

The Master Theorem applies to recurrences of the following form:

$$T(n) = aT(n/b) + f(n)$$

where $a \ge 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function (positive for all sufficiently large n).

There are 3 cases:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ with some constant $\epsilon > 0$, and $f(n)$ satisfies the regularity condition, then $T(n) = \Theta(f(n))$. Regularity condition: $af(n/b) \le cf(n)$ for some constant $c < 1$ and all sufficiently large n.

Practice problems for Master method

Give a closed form for the following recurrences!

1. We have seen in Practice:

a) $T(n) = 9T(\frac{n}{3}) + n$

b) $T(n) = T(\frac{2n}{3}) + 1$

c) $T(n) = 3T(\frac{n}{4}) + n \log n$

d) $T(n) = 2T(\frac{n}{2}) + n \log n$

2. Further practice problems:

a) $T(n) = 2T(\frac{n}{2}) + n^3$

b) $T(n) = T(\frac{9n}{10}) + n$

c) $T(n) = 16T(\frac{n}{4}) + n^2$

d) $T(n) = 7T(\frac{n}{3}) + n^2$

e) $T(n) = 7T(\frac{n}{2}) + n^2$

f) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

3. Now let $T_1(n) = 4T(\frac{n}{2}) + f(n)$ and let $T_2(n) = 2T(\frac{n}{4}) + f(n)$ and

a) $f(n) = 1$

b) $f(n) = \sqrt{n}$

c) $f(n) = n$

d) $f(n) = n \log n$

e) $f(n) = n^2$

f) $f(n) = n^3$

If you have questions you can contact me at harrachnora@inf.elte.hu.