608. Midterm I ($20' \times 5 = 100'$ points)

1. (a) $10'$

$f(x)$ is convex, means $\forall x_1, x_2 \in R$, $\exists \alpha \in [0,1]$ s.t. $f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha) f(x_2)$ --- ①

set $C = \{x \in R \mid f(x) \leq k\}$, k is constant

$\because$ C is subgraph of $f(x)$

$\therefore$ $\forall x_3, x_4 \in C$, $x_3, x_4 \in R$

$\because$ ①

$\therefore$ $\exists \alpha' \in [0,1]$ s.t. $f(\alpha' x_3 + (1-\alpha')x_4) \leq \alpha' f(x_3) + (1-\alpha') f(x_4)$

$\because$ $\alpha' f(x_3) + (1-\alpha') f(x_4) \leq \max(f(x_3), f(x_4)) \leq k$ $5'$

$\therefore$ $\alpha' x_3 + (1-\alpha') x_4$ is also in set C

$\therefore$ C is a convex set.

(b) $10'$

$\because$ $f(x)$ and $g(x)$ are both convex

$\therefore$ $\forall x_1, x_2 \in R$. $\exists \alpha \in [0,1]$, s.t. $\begin{cases} f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha) f(x_2) \\ g(\alpha x_1 + (1-\alpha)x_2) \leq \alpha g(x_1) + (1-\alpha) g(x_2) \end{cases}$

Assume $h(x)$ is convex combination of $f(x)$ and $g(x)$

$h(x) = \beta f(x) + (1-\beta) g(x)$ $\quad \beta \in [0,1]$

let $x = \alpha x_1 + (1-\alpha) x_2$,

$h(x) = h(\alpha x_1 + (1-\alpha)x_2) \overset{5'}{=} \beta f(\alpha x_1 + (1-\alpha)x_2) + (1-\beta) g(\alpha x_1 + (1-\alpha) x_2)$

$\leq \alpha\beta f(x_1) + (1-\alpha)\beta f(x_2) + \alpha(1-\beta) g(x_1) + (1-\alpha)(1-\beta) g(x_2)$

$= \alpha(\beta f(x_1) + (1-\beta) g(x_1)) + (1-\alpha)(\beta f(x_2) + (1-\beta) g(x_2))$

$= \alpha h(x_1) + (1-\alpha) h(x_2)$ which is the definition of convex function

$\therefore$ ......

$\boxed{2. \; 20'}$ objective function : min $2x+3y$ s.t. $\begin{cases} x+y=B \\ x \geq 0 \\ y \geq 0 \end{cases}$ (or $x+y \geq B$ is also OK)

6'

\+

- - - - - - - -
- - - - - - - -

} 4'

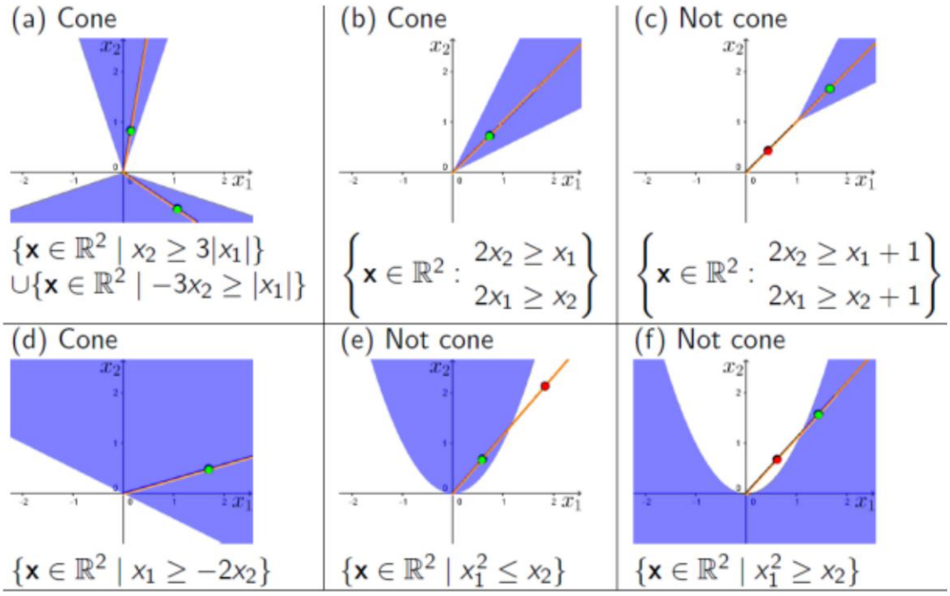
\+

result : $x=B$ $y=0$

10'

$\boxed{3. \; 20'}$

3' each



$\boxed{20' \; 4.}$ objective function : max xy s.t. $\begin{cases} 2x+2y \leq 2 \\ x > 0 \\ y > 0 \end{cases}$

6'

\+

- - - - - -
- - - - - -

} 4'

\+

result : 0.25

10'

$\boxed{5. \; 20'}$

(a) not convex 3' why 3'

(b) Convex 3' why 3'

(c) not convex 3' why 3'