$\rm EE/CPE/NIS~608~Exam~2$

For all the following problems upload your source code (Python or Matlab) along with the solution. You have the option to form a group of three students (maximum) per group. Submit one solution per group, write names of all the group members on the solution.

1. Implement the gradient descent algorithm to solve the following problem. Plot the following:
(a) iteration versus the function value for the first few iterations and (b) computed gradient at each iteration. What is the minimizer and the minimum of the function? State the initialization parameters and the stopping criterion you used.

$$\min_{x_1, x_2, x_3} \frac{1}{2} \left([x_1 - 1]^2 + 2[x_2 - 2]^2 + 3[x_3 - 3]^2 \right)$$

2. Implement the penalty function method to solve the following constrained optimization problem. Use the quadratic penalty function, i.e., if constraint is $c(\underline{x}) \leq 0$ penalty function is $\max(0, c(x))^2$. State all the parameters such as initialization, stopping criterion, etc. you used. Plot the iteration vs. the function value for the first few iterations.

$$\min f(x_1, x_2) = [x_1 - 6]^2 + [x_2 - 7]^2,$$

$$c_1(\underline{x}) = -3x_1 - 2x_2 + 6 \le 0$$

$$c_2(\underline{x}) = -x_1 + x_2 - 3 \le 0$$

$$c_3(\underline{x}) = x_1 + x_2 - 7 \le 0$$

$$c_4(\underline{x}) = \frac{2}{3}x_1 - x_2 - \frac{4}{3} \le 0$$

3. Implement the barrier function method to solve the following constrained optimization problem. Use the inverse barrier function, i.e., if a constraint is $c(\underline{x}) \leq 0$ then the barrier function is $-\frac{1}{c(\underline{x})}$. State all the parameters such as initialization, stopping criterion, etc. you used. Plot the iteration vs. the function value for the first few iterations.

$$\min_{x_1, x_2} 2x_1^2 + 9x_2,$$
$$x_1 + x_2 \ge 4$$