1. $\lim\limits_{x \to 0} \dfrac{x^2-1}{2x^2-x-1}$.

2. $\lim\limits_{x \to \infty} \dfrac{1}{2^x}$.

3. $\lim\limits_{n\to\infty}(-1)^n\dfrac{1}{n}$.

4. $\lim\limits_{n\to\infty}\dfrac{3n+7}{2n+1}$.

5. $\lim_{x\to 0^+} \frac{|x|}{x}$.

6. $\lim_{x\to \infty} e^{(\frac{1}{x})^2}$.

7. $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$.

8. $\lim\limits_{x\to\infty}\dfrac{x}{2x+1}$.

9. $\lim\limits_{x\to\infty}\dfrac{\sin x}{\sqrt{x}}$.

10. $\lim\limits_{x\to 1}\left(\sin\dfrac{x}{2}\pi+\cos x\pi\right)$.

11. $\lim\limits_{x \to -1} \dfrac{x^3+1}{x+1}$.

12. $\lim\limits_{x \to -1} \dfrac{x-\sqrt{3+2x}}{x^2+1}$.

13. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x}+1}{2x+3}$.

14. $\lim\limits_{x \to \infty} \dfrac{1+2x}{(x-3)^2}$.

15. $\lim\limits_{x\to\pi}\dfrac{\sqrt{1+\sin x}+\cos\frac{x}{2}}{x+1}$.

16. $\lim\limits_{x\to 0}\dfrac{4x^3-2x^2+x}{3x^2+2x}$.

17. $\lim\limits_{x \to \sqrt{2}} \dfrac{x^2 - 3}{x^2 + x + 1}$.

18. $\lim\limits_{x \to 2} \dfrac{x^2 + 4}{x - 3}$.

19. $\lim\limits_{x \to 4} \dfrac{x^2-6x+8}{x^2-5x+4}$.

20. $\lim\limits_{x \to \infty} \dfrac{1}{x}\left(2x - \dfrac{1}{x^2}\right)$.

21. $\lim\limits_{x \to 2}\left(\frac{4}{x^2-4}-\frac{1}{x-2}\right)$.

1. $\lim\limits_{x \to 1} \dfrac{x^2-1}{2x^2-x-1}$.

2. $\lim\limits_{x \to \infty} \dfrac{x^2-1}{2x^2-x-1}$.

3. $\lim\limits_{x \to \infty} \dfrac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(5x-1)^5}$.

4. $\lim\limits_{x \to \infty} \dfrac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}}$.

5. $\lim\limits_{x \to 2} \dfrac{x^2-4x+4}{x^2-4}$.

6. $\lim\limits_{x \to +\infty} \dfrac{\sqrt{6x+\sqrt{x+1}}}{\sqrt{3x+2}}$.

7. $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$.

8. $\lim\limits_{x \to \infty} \dfrac{x+\sin x}{x-\sin x}$.

9. $\lim\limits_{x\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)$.

10. $\lim\limits_{n\to\infty}\dfrac{(n+2)(2n+3)(3n+4)}{n^3}$.

11. $\lim\limits_{x\to+\infty} \arccos(\sqrt{x^2+x}-x)$.

12. $\lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}}}{x^{100}}$.

13. $\lim\limits_{x\to-\infty}(\sqrt{x^2+x}-x)$.

14. $\lim\limits_{x\to+\infty}(\sqrt{x^2+x}-x)$.

15. $\lim\limits_{x \to 1^-} \arctan \dfrac{1}{1-x}$.

16. $\lim\limits_{x \to 1^+} \arctan \dfrac{1}{1-x}$.

17. $\lim\limits_{x \to 0^-} \dfrac{1}{1+e^{\frac{1}{x}}}$.

18. $\lim\limits_{x \to 0^+} \dfrac{1}{1+e^{\frac{1}{x}}}$.

(1992) 当 $x \to 1$ 时，函数 $\frac{x^2-1}{x-1}e^{\frac{1}{x-1}}$ 的极限（　　）.

(A) 等于 2　　(B) 等于 0

(C) 为 ∞　　(D) 不存在但不为 ∞

(2000) 求 $\lim\limits_{x\to 0}\left(\dfrac{2+e^{\frac{1}{x}}}{1+e^{\frac{4}{x}}}+\dfrac{\sin x}{|x|}\right)$.

1. $\lim\limits_{x \to 0} \dfrac{\cos x - e^{-\frac{x^2}{2}}}{x^4}$.

2. $\lim\limits_{x \to 0} \dfrac{x\cos x - \sin x}{x^3}$.

3. $\lim\limits_{x\to 0}\left(\frac{1}{\ln(1+2x)}-\frac{1}{\sin 2x}\right)$.

4. $\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{1}{x}-\cot x\right)$.

5. $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$.

6. $\lim\limits_{x \to 0} \dfrac{e^{x^3} - 1 - x^3}{(\sin 2x)^6}$.

7. $\lim_{x\to\infty}\left(x^2-x^3\tan\frac{1}{x}\right)$.

8. $\lim_{x\to 0^+}\frac{e^x-1-x}{\sqrt{1-x}-\cos\sqrt{x}}$.

9. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-1}{3x}$.

10. $\lim\limits_{x\to 0}\left(\cot x-\dfrac{1}{x}\right)$.

11. $\lim\limits_{x\to 0}\dfrac{\frac{1}{2}x^2+1-\sqrt{1+x^2}}{[x-\ln(1+x)](1-\cos x)}$.

12. $\lim\limits_{x\to 0}\dfrac{x(e^x+1)-2(e^x-1)}{x^2\sin x}$.

13. $\lim\limits_{x \to 0} \dfrac{\cos x - e^{-\frac{x^2}{2}}}{x^2[x + \ln(1-x)]}$.

14. $\lim\limits_{x \to 0} \dfrac{e^x + \ln(1-x) - 1}{x - \arctan x}$.

15. $\lim\limits_{x\to 0}\dfrac{\frac{x^2}{2}+1-\sqrt{1+x^2}}{x^2\sin x^2}$.

16. $\lim\limits_{x\to 0}\dfrac{e^{-\frac{x^3}{2}}-\cos\sqrt{x^3}}{x^6}$.

17. $\lim\limits_{x \to 0} \dfrac{e^x \sin x - x(1+x)}{x^2 \sin x}$.

18. $\lim\limits_{x \to 0} \dfrac{e^x \cos x - 1 - x}{x^3}$.

1. $\lim\limits_{x\to+\infty}(\sqrt[6]{x^6+x^5}-\sqrt[6]{x^6-x^5})$.

2. $\lim\limits_{x\to+\infty}\left[(x^3-x^2+\frac{x}{2})e^{\frac{1}{x}}-\sqrt{x^6+1}\right]$.

3. $\lim\limits_{x\to 0}\dfrac{\ln(1+x+x^2)-x}{x^2}$.

4. $\lim\limits_{x\to 0}\dfrac{\arcsin x-\sin x}{\arctan x-\tan x}$.

5. $\lim\limits_{x\to 0}\dfrac{\int_0^x e^t\cos t\,dt - x - \frac{1}{2}x^2}{(x-\tan x)(\sqrt{1+x}-1)}$.

6. $\lim\limits_{x\to 0^+}\dfrac{e^{x^3}-1}{1-\cos\sqrt{x-\sin x}}$.

7. $\lim\limits_{n\to\infty} n^2\ln\left(n\sin\frac{1}{n}\right)$.

8. $\lim\limits_{x\to 0} \dfrac{\frac{x^2}{2}+1-\sqrt{1+x^2}}{\left(\cos x-e^{x^2}\right)\sin x^2}$.

9. $\lim\limits_{x \to 0} \dfrac{\sin(\tan x) - \tan(\tan x)}{x^3}$.

10. $\lim\limits_{x \to 0} \dfrac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$.

11. $\lim\limits_{x\to 0}\dfrac{\ln\left(\dfrac{\sin x}{x}\right)}{x^2}$.

12. $\lim\limits_{x\to 0}\dfrac{[\sin x-\sin(\sin x)]\sin x}{x^4}$.

13. $\lim\limits_{x\to\infty} x\left[\left(\sin\frac{2}{x}+\cos\frac{1}{x}\right)^{x}-e^{2}\right]$.

14. $\lim\limits_{x\to 0}\frac{\sqrt{1+2\sin x}-x-1}{x\ln(1+x)}$.

15. $\lim\limits_{x\to 0}\dfrac{e^{\frac{2}{x}\ln(1+x)}-e^{2}[1-\ln(1+x)]}{x}$.

16. $\lim\limits_{x\to +\infty}\dfrac{\ln(x+\sqrt{x^{2}+1})-\ln(x+\sqrt{x^{2}-1})}{(e^{\frac{1}{x}}-1)^{2}}$.

17. $\lim\limits_{x\to+\infty} x^2\left(a^{\frac{1}{x}}-a^{\frac{1}{x+1}}\right)$.

18. $\lim\limits_{x\to 0} \dfrac{\cos(\sin x)-\cos x}{(1-\cos x)\sin^2 x}$.

19. $\lim\limits_{x\to 0}\dfrac{\ln^2(x+\sqrt{1+x^2})+e^{-x^2}-1}{x^4}$.

20. $\lim\limits_{x\to 0}\dfrac{\sqrt{2+\tan x}-\sqrt{2+\sin x}}{x^3}$.

21. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{e^{x}-1}$.

22. $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{1}{\sin^2 x}\right)$.

23. $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}-e}{x}$.

24. $\lim\limits_{x\to 0}\dfrac{\int_0^x x\ln(1+t)\,dt}{x-\arcsin x}$.

25. $\lim\limits_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

26. $\lim\limits_{x \to 0} \frac{e^{x^4} - \cos^2 x - x^2}{x^4}$.

(1994) 设 $\lim\limits_{x \to 0} \dfrac{\ln(1+x)-(ax+bx^2)}{x^2}=2$，则（　　）.

(A) $a=1, b=-\frac{5}{2}$　　(B) $a=0, b=-2$

(C) $a=0, b=-\frac{5}{2}$　　(D) $a=1, b=-2$

(1998) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}+\sqrt{1-x}-2}{x^2}$.

(1999) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x\ln(1+x)-x^2}$.

(2000) $\lim\limits_{x \to 0} \dfrac{\arctan x - x}{\ln(1+2x^3)}$.

(1987) $\lim\limits_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$.

(1994) $\lim\limits_{x\to 0} \cot x\left(\frac{1}{\sin x}-\frac{1}{x}\right)$.

(1994) $\lim\limits_{x\to \infty}\left[x-x^2\ln\left(1+\frac{1}{x}\right)\right]$.

(1999) $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{x\tan x}\right)$.

(2004) $\lim\limits_{x\to 0}\left(\frac{1}{\sin^2 x}-\frac{\cos^2 x}{x^2}\right)$.

(2007) $\lim\limits_{x\to 0}\dfrac{\arctan x-\sin x}{x^3}=$ __________.

(2020) $\lim\limits_{x\to 0}\left[\dfrac{1}{e^x-1}-\dfrac{1}{\ln(1+x)}\right]=$ ______.

1. $\lim\limits_{x \to 0} \frac{\sin 5x}{x}$.

2. $\lim\limits_{x \to 0} \frac{1-\cos x}{x^2}$.

3. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^3+x^2}$.

4. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sqrt{1+x^2}-1}$.

5. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos x^2}}{1-\cos x}$.

6. $\lim\limits_{x\to\infty} x\ln\left(1+\dfrac{1}{x}\right)$.

7. $\lim\limits_{x \to 0} \dfrac{\sin x + x}{x}$.

8. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\ln(1+x^2)}$.

9. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}-1}{2x}$.

10. $\lim\limits_{n\to\infty} n\sin\dfrac{\pi}{n}$.

11. $\lim\limits_{x \to 1} \dfrac{\sin(x-1)}{1-x^2}$.

12. $\lim\limits_{x \to 0} \dfrac{1-\cos 4x}{x \sin x}$.

13. $\lim\limits_{x\to\infty} x^2\left(1-\cos\frac{1}{x}\right)$.

14. $\lim\limits_{x\to 0} \frac{x^2\sin x}{\arctan(x^3)}$.

15. $\lim\limits_{x \to 0} \dfrac{\ln(1+x\sin x)}{1-\cos x}$.

16. $\lim\limits_{x \to \infty} x^2 \ln(1+\dfrac{3}{x^2})$.

17. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\tan 3x}$.

18. $\lim\limits_{x\to 0}\dfrac{\ln\cos ax}{\ln\cos bx}$.

19. $\lim\limits_{x \to 0} \dfrac{x^2}{\sqrt{1+x^2}-1}$.

20. $\lim\limits_{x \to 0} \dfrac{1-\sqrt{1-2x}}{x}$.

21. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{e^{2x}-1}$.

22. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x^2}-1}{x\sin 2x}$.

23. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-1}{e^{x^2}-1}$.

1. $\lim\limits_{x \to 0} \dfrac{2x - \sin x}{2x + \sin x}$.

2. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{\pi - 2x}$.

3. $\lim_{n\to\infty} 2^{n+1}\sin\frac{x}{2^n}\ (x\neq 0)$.

4. $\lim_{x\to 0} x\cot x$.

5. $\lim\limits_{x \to 0} \dfrac{\arcsin x - \sin x}{x^2(\sqrt{1+2x}-1)}$.

6. $\lim\limits_{x \to \infty} x \ln \dfrac{x+2}{x}$.

7. $\lim\limits_{x\to 1}\dfrac{\arcsin(1-x)}{\ln x}$.

8. $\lim\limits_{x\to 0}\dfrac{\sin x-\tan x}{(\sqrt[3]{1+x^2}-1)(\sqrt{1+\sin x}-1)}$.

9. $\lim\limits_{x \to 0} \dfrac{\sin(\sin x)}{x}$.

10. $\lim\limits_{x \to \pi} \dfrac{\sin x}{x - \pi}$.

11. $\lim\limits_{x\to 0}\dfrac{\sin 5x-\sin 2x}{\sin 2x}$.

12. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$.

13. $\lim\limits_{x \to e} \dfrac{\ln x - 1}{x - e}$.

14. $\lim\limits_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$.

15. $\lim\limits_{t \to 1}(1-t)\tan\frac{\pi}{2}t$.

16. $\lim\limits_{x \to 0}\frac{\sec^2 x(x-\arcsin x)}{x\ln\cos x}$.

17. $\lim\limits_{x\to 0}\dfrac{x-x\cos x}{\sin x-\tan x}$.

18. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-1}{x(e^{-x}-1)}$.

19. $\lim\limits_{x\to 0}\dfrac{(\sqrt{1+\sin x}-1)\arctan^4 x}{\arcsin 2x(1-\cos x)(x^2+x^4)}$.

20. $\lim\limits_{x\to 5}\dfrac{1-\sqrt{x-4}}{x-5}$.

21. $\lim\limits_{x\to 2}\dfrac{\sqrt[4]{x+14}-2}{x^2-4}$.

22. $\lim\limits_{x\to 0}\dfrac{1}{x^2}\ln\dfrac{\sin x}{x}$.

23. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$.

24. $\lim\limits_{x\to 0}\dfrac{(\arctan x-\tan x)(1-\cos\sqrt{x})}{x^2(1-\sqrt{\cos x})}$.

25. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos x}{x^2}$.

26. $\lim\limits_{x\to 0}\dfrac{\ln(\sin^2 x+e^x)-x}{\ln(x^2+e^{2x})-2x}$.

27. $\lim\limits_{x\to 0}\dfrac{\ln(1+xe^{x})}{\ln(x+\sqrt{1+x^{2}})}$.

28. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}}$.

29. $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$.

30. $\lim\limits_{x\to 0}\dfrac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\sin^2 x}$.

31. $\lim\limits_{h\to 0}\dfrac{a^{x+h}+a^{x-h}-2a^{x}}{h^{2}}$.

32. $\lim\limits_{n\to\infty} n(\sqrt[n]{x}-1)\ (x>0)$.

33. $\lim\limits_{x\to 0}\dfrac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})}$.

34. $\lim\limits_{x\to 0}\dfrac{\ln(x+\sqrt{1+x^2})-\ln(1+x)}{\ln^2(1+x)}$

35. $\lim\limits_{x \to 0} \dfrac{\sin 2x - 2\sin x}{x^3}$.

36. $\lim\limits_{x \to 0} \dfrac{\ln(1+\sin^2 x)}{(1+\cos x)\tan^2 x}$.

37. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+\tan x}-\sqrt{1-\sin x}}{x}$.

38. $\lim\limits_{x \to +\infty} \dfrac{x\sqrt{x}\,\sin\frac{1}{x}}{\sqrt{x}-1}$.

39. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+2x}-1}{\ln(2-\cos x+\sin x)}$.

40. $\lim\limits_{x\to 0}\dfrac{\sqrt{\cos x}-\sqrt[3]{1+\sin^2 x}}{x^2}$.

41. $\lim\limits_{x \to 1} \dfrac{x^x - 1}{\sin \pi x}$.

42. $\lim\limits_{x \to -\infty} \dfrac{\ln(1+3^x)}{\ln(1+2^x)}$.

43. $\lim\limits_{x\to+\infty}\dfrac{\ln(1+3^x)}{\ln(1+2^x)}$.

44. $\lim\limits_{x\to+\infty}\ln(1+2^x)\ln\left(1+\dfrac{3}{x}\right)$.

(1988) $\lim_{x\to 1} \dfrac{x^x-1}{x\ln x}$.

(1991) $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^2(e^x-1)}$.

(1992) $\lim\limits_{x \to 0} \dfrac{1-\sqrt{1-x^2}}{e^x - \cos x}$.

(1992) $\lim\limits_{x \to 0} \dfrac{e^{x} - \sin x - 1}{1 - \sqrt{1 - x^{2}}}$.

(1994) 设 $\lim\limits_{x\to 0}\dfrac{a\tan x+b(1-\cos x)}{c\ln(1-2x)+d(1-e^{-x^2})}=2$，其中 $a^2+c^2\neq 0$，则必有（　　）.

(A) $b=4d$　(B) $b=-4d$　(C) $a=4c$　(D) $a=-4c$

(1995) $\lim\limits_{x\to 0^+}\dfrac{1-\sqrt{\cos x}}{x(1-\cos\sqrt{x})}$.

(1997) $\lim\limits_{x\to 0}\dfrac{3\sin x + x^2\cos\frac{1}{x}}{(1+\cos x)\ln(1+x)}$.

(2005) 极限$\lim\limits_{x\to\infty} x\sin\dfrac{2x}{x^2+1}=$ ________.

(2006) $\lim\limits_{x \to 0} \dfrac{x\ln(1+x)}{1-\cos x} =$ ________.

(2009) 求极限$\lim\limits_{x \to 0} \dfrac{(1-\cos x)[x-\ln(1+\tan x)]}{\sin^4 x}$.

(2009) $\lim_{x \to 0} \dfrac{e - e^{\cos x}}{\sqrt[3]{1+x^2} - 1} =$ ________.

1. $\lim\limits_{x\to 0^+} x^x$.

2. $\lim\limits_{x\to 0}\dfrac{2x-\ln(1+2x)}{x^2}$.

3. $\lim\limits_{x\to 0}\dfrac{e^{x}-e^{-x}}{\sin x}$.

4. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\ln \sin x}{(\pi-2x)^{2}}$.

5. $\lim\limits_{x\to 0}\dfrac{1+\sin x-\cos x}{1+\sin px+\cos px}$.

6. $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}$.

7. $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a}$.

8. $\lim\limits_{x \to a} \dfrac{\cos x - \cos a}{x - a}$.

9. $\lim\limits_{x\to a}\dfrac{\tan x-\tan a}{x-a}$.

10. $\lim\limits_{x\to a}\dfrac{\cot x-\cot a}{x-a}$.

11. $\lim\limits_{x \to a} \dfrac{\sec x - \sec a}{x - a}$.

12. $\lim\limits_{x \to a} \dfrac{\csc x - \csc a}{x - a}$.

13. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\sin(x-\frac{\pi}{3})}{1-2\cos x}$.

14. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\tan^3 x - 3\tan x}{\cos(x+\frac{\pi}{6})}$.

15. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$.

16. $\lim\limits_{x \to 0} \dfrac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$.

17. $\lim\limits_{x\to 0} x\cot 2x$.

18. $\lim\limits_{x\to +\infty} x[\ln(x+1)-\ln x]$.

19. $\lim\limits_{x \to +\infty} \dfrac{\ln(x^2 - x + 1)}{\ln(x^{10} + x + 1)}$.

20. $\lim\limits_{x \to +\infty} \dfrac{\ln(2 + e^{3x})}{\ln(3 + e^{2x})}$.

21. $\lim\limits_{x \to 0} \dfrac{\ln \tan\left(\frac{\pi}{4} + ax\right)}{\sin bx}$.

22. $\lim\limits_{x \to 0} \dfrac{\ln \cos ax}{\ln \cos bx}$.

23. $\lim\limits_{x \to a} \dfrac{x^m - a^m}{x^n - a^n}$.

24. $\lim\limits_{x \to 0} (x+e^x)^{\frac{1}{x}}$.

25. $\lim\limits_{x\to+\infty}\dfrac{x^k}{e^x}\ (k>0)$.

26. $\lim\limits_{x\to\infty}x(a^{\frac{1}{x}}-b^{\frac{1}{x}})\ (a,b>0)$.

(1993) $\lim\limits_{x\to\infty}\dfrac{3x^2+5}{5x+3}\sin\dfrac{2}{x}$.

(1987) $\lim\limits_{x\to+\infty}\dfrac{\ln\left(1+\frac{1}{x}\right)}{\operatorname{arccot} x}$.

(1988) $\lim\limits_{x \to 1} \dfrac{x^x - 1}{x \ln x}$.

(1991) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^2 (e^x - 1)}$.

(1992) $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-x^2}}{e^x-\cos x}$.

(1992) $\lim\limits_{x\to 1}\dfrac{\ln\cos(x-1)}{1-\sin\frac{\pi}{2}x}$.

(1992) $\lim\limits_{x \to 0} \dfrac{e^x - \sin x - 1}{1 - \sqrt{1 - x^2}}$.

(1993) $\lim\limits_{x \to 0^+} x \ln x$.

(1988) 求极限$\lim\limits_{x\to 1}(1-x^2)\tan\dfrac{\pi}{2}x$.

1. $\lim\limits_{x \to 0} (1-x)^{\frac{1}{x}}$.

2. $\lim\limits_{x \to 0} (1+2x)^{\frac{1}{x}}$.

3. $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}\right)^{kx}$.

4. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}+\dfrac{1}{n^2}\right)^{n}$.

5. $\lim\limits_{x \to 0} (1+\tan x)^{3\cot x}$.

6. $\lim\limits_{x \to 0} (1+x)^{\frac{1}{2x}}$.

7. $\lim\limits_{x \to 0} (1+3x)^{\frac{1}{\sin x}}$.

8. $\lim\limits_{x \to 0} (1+3x)^{\frac{2}{x}}$.

9. $\lim\limits_{x \to +\infty} \left(1 - \frac{1}{x}\right)^{\sqrt{x}}$.

10. $\lim\limits_{x \to 0} \left(1 + e^{x} \sin^{2} x\right)^{\frac{1}{1-\cos x}}$.

11. $\lim\limits_{x\to 0} \sqrt[x]{1-2x}$.

12. $\lim\limits_{x\to 0} (1+x^2)^{\cot^2 x}$.

13. $\lim\limits_{x \to 1} (1+\sin \pi x)^{\cot \pi x}$.

14. $\lim\limits_{x \to \infty} \left(\frac{1+x}{x}\right)^{2x}$.

15. $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x+1}\right)^{x+1}$.

16. $\lim\limits_{x\to+\infty}\left(\dfrac{ax+b_1}{ax+b_2}\right)^{x}$.

17. $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$.

18. $\lim\limits_{x\to \frac{\pi}{4}}(\tan x)^{\frac{1}{\cos x-\sin x}}$.

19. $\lim\limits_{x\to 0}(\cos 2x+2x\sin x)^{\frac{1}{x^4}}$.

20. $\lim\limits_{x\to 0}\left(\dfrac{\ln(1+x)}{x}\right)^{\frac{1}{e^x-1}}$.

21. $\lim\limits_{x\to\infty}\left[\dfrac{x^2}{(x-a)(x+b)}\right]^x$.

22. $\lim\limits_{x\to 0}(\cos^2 x)^{\frac{1}{\sin^2 x}}$.

23. $\lim\limits_{n\to\infty}\left(\dfrac{n+\ln n}{n-\ln n}\right)^{\frac{n}{\ln n}}$.

24. $\lim\limits_{x\to 0}\left(\dfrac{1+\tan x}{1+\sin x}\right)^{\frac{1}{\sin x}}$.

25. $\lim\limits_{x \to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$.

1. $\lim\limits_{x\to 0}\left(\dfrac{e^{x}+e^{2x}+\cdots+e^{nx}}{n}\right)^{\frac{1}{x}}$.

2. $\lim\limits_{x\to 0}\left(\dfrac{a_1^{x}+a_2^{x}+\cdots+a_n^{x}}{n}\right)^{\frac{1}{x}}$.

3. $\lim\limits_{x\to+\infty}\left(x^{\frac{1}{x}}-1\right)^{\frac{1}{\ln x}}$.

4. $\lim\limits_{x\to-\infty}\left(\frac{\pi}{2}+\arctan x\right)^{\frac{1}{x}}$.

5. $\lim\limits_{x \to 0^+} (\cot x)^{\sin x}$.

6. $\lim\limits_{x \to +\infty} \left(\frac{\pi}{2} - \arctan x\right)^{\frac{1}{\ln x}}$.

7. $\lim\limits_{x\to 1}(x-1)^2 e^{\frac{1}{x-1}}$.

8. $\lim\limits_{x\to 0}\left(\frac{\arctan x}{x}\right)^{\frac{1}{x^2}}$.

9. $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$.

10. $\lim\limits_{x\to 1} (2-x)^{\tan\frac{\pi}{2}x}$.

11. $\lim\limits_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$.

12. $\lim\limits_{x \to \infty} \left(\tan \dfrac{\pi x}{2x+1}\right)^{\frac{1}{x}}$.

13. $\lim\limits_{x\to a}\left(\frac{\tan x}{\tan a}\right)^{\cot(x-a)}$.

14. $\lim\limits_{x\to 0}\frac{(1+x)^{x}-1}{x^{2}}$.

15. $\lim\limits_{x \to 0^+} (\arcsin x)^{\tan x}$.

16. $\lim\limits_{x \to +\infty} \left(\frac{2}{\pi} \arctan x\right)^x$.

17. $\lim\limits_{x\to 0}\left(\dfrac{\arcsin x}{x}\right)^{\frac{1}{x^2}}$.

18. $\lim\limits_{x\to 0}\left(\dfrac{(1+x)^{\frac{1}{x}}}{e}\right)^{\frac{1}{x}}$.

19. $\lim\limits_{x \to 0} \left(\frac{2}{\pi} \arccos x\right)^{\frac{1}{x}}$.

20. $\lim\limits_{x \to 0} \left(\frac{\cos x}{\cos 2x}\right)^{\frac{1}{x^2}}$.

21. $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

22. $\lim\limits_{x \to 0} \left[\tan\left(\frac{\pi}{4} - x\right)\right]^{\cot x}$.

23. $\lim\limits_{x\to 0^+}\sqrt[x]{\cos\sqrt{x}}$.

24. $\lim\limits_{n\to\infty}\cos^n\dfrac{x}{\sqrt{n}}$.

25. $\lim\limits_{x \to 0}\left(\dfrac{1+x\cdot 2^x}{1+x\cdot 3^x}\right)^{\frac{1}{x^2}}$.

26. $\lim\limits_{n \to \infty} \tan^n\left(\dfrac{\pi}{4}+\dfrac{1}{n}\right)$.

27. $\lim\limits_{x\to\infty}\dfrac{(x+a)^{x+a}(x+b)^{x+b}}{(x+a+b)^{2x+a+b}}$.

28. $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt[n]{a}+\sqrt[n]{b}}{2}\right)^{n}$.

29. $\lim\limits_{x\to 0}\left(\dfrac{a^{x+1}+b^{x+1}+c^{x+1}}{a+b+c}\right)^{\frac{1}{x}}$.

30. $\lim\limits_{x\to 0}\left(\dfrac{a^{x^2}+b^{x^2}}{a^x+b^x}\right)^{\frac{1}{x}}$.

(1987) $\lim\limits_{x \to 0}(1+xe^{x})^{\frac{1}{x}}$.

(1989) $\lim\limits_{x \to \infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^{x}$.

(1989) $\lim\limits_{x \to 0}(2\sin x+\cos x)^{\frac{1}{x}}$.

(1991) $\lim\limits_{x \to 0^+}(\cos\sqrt{x})^{\frac{\pi}{x}}$.

(1990) 设 a 为非零常数，则 $\lim\limits_{x\to\infty}\left(\dfrac{x+a}{x-a}\right)^x=$ ______.

(1992) $\lim\limits_{x\to\infty}\left(\dfrac{3+x}{6+x}\right)^{\frac{x-1}{2}}$.

(1993) $\lim\limits_{x\to\infty}\left(\sin\dfrac{2}{x}+\cos\dfrac{1}{x}\right)^{x}$.

(1996) 设 $\lim\limits_{x\to\infty}\left(\frac{x+2a}{x-a}\right)^{x}=8$，则 $a=$ ______.

(1995) $\lim\limits_{x \to 0}(1+3x)^{\frac{2}{\sin x}}$.

(2003) $\lim\limits_{x \to 0}(\cos x)^{\frac{1}{\ln(1+x^2)}}$.

(1988) $\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{x}}\right)^{\tan x}$.

(1989) $\lim\limits_{x\to +\infty}(x+e^x)^{\frac{1}{x}}$.

(1990) 已知 $\lim\limits_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}=9$，求常数 a.

(1987) $\lim_{n\to\infty}\left(\frac{n-2}{n+1}\right)^{n}=$ ________.

(2011) $\lim\limits_{x \to 0}\left(\frac{1+2^x}{2}\right)^{\frac{1}{x}} =$ ________.

(2013) $\lim\limits_{x\to 0}\left[2-\dfrac{\ln(1+x)}{x}\right]^{\frac{1}{x}} =$ ____________.

(2018) 若 $\lim\limits_{x \to 0}\left(\dfrac{1-\tan x}{1+\tan x}\right)^{\frac{1}{\sin kx}} = e$，则 $k =$ ________.

(2019) $\lim\limits_{x \to 0}(x+2^{x})^{\frac{2}{x}} =$ ________.

(2022) $\lim\limits_{x\to 0}\left(\dfrac{1+e^{x}}{2}\right)^{\cot x} =$ ________.

1. 找出下列函数的间断点，并说明间断点属于哪种类型.

(1) $y=\dfrac{x^2-1}{x^2-3x+2}$.

(2) $y=\dfrac{x}{\tan x}$.

(3) $y=\cos^2\frac{1}{x}$.

(4) $y=\begin{cases} x-1, & x\leq 1, \\ 3-x, & x>1. \end{cases}$

(5) $y=\dfrac{2^{\frac{1}{x}}-1}{2^{\frac{1}{x}}+1}$.

(6) $y=\begin{cases} \cos\dfrac{\pi}{2}x, & |x|\leq 1, \\ |x-1|, & |x|>1. \end{cases}$

2. 函数 $f(x)=\dfrac{(e^{x}+e)\tan x}{x(e^{\frac{1}{x}}-e)}$ 在 $[-\pi,\pi]$ 上的第一间断点

是 $x=$ ______.

3. 设 $f(x)=\lim\limits_{n\to\infty}\dfrac{(n-1)x}{nx^2+1}$，则 $f(x)$ 的间断点 $x=$ ______.

1. 函数 $f(x)=\dfrac{(x^2+x)(\ln|x|)\sin\frac{1}{x}}{x^2-1}$ 的可去间断点的个数为（　）.

（A）0　　（B）1　　（C）2　　（D）3

2. 函数 $f(x)=\dfrac{|x|^{x}-1}{x(1+x)\ln|x|}$ 的可去间断点的个数为（ ）.

(A) 0　　(B) 1　　(C) 2　　(D) 3

(2009) 函数 $f(x)=\dfrac{x-x^3}{\sin \pi x}$ 的可去间断点的个数为（ ）.

（A）1 （B）2 （C）3 （D）无穷多个

(1998) 求函数 $f(x)=(1+x)^{\frac{x}{\tan\left(x-\frac{\pi}{4}\right)}}$ 在区间 $(0,2\pi)$ 内的间断点，并判断其类型.

1. 当$x>0$时，曲线$y=x\sin\frac{1}{x}$（　　）.

(A) 有且仅有水平渐近线

(B) 有且仅有垂直渐近线

(C) 既有水平渐近线，也有垂直渐近线

(D) 既无水平渐近线，也无垂直渐近线

2. 曲线 $y=\dfrac{x^2}{2x+1}$ 的斜渐近线方程为________.

3. 曲线 $y=\dfrac{x+4\sin x}{5x-2\cos x}$ 的水平渐近线为________.

4. $y=(2x-1)e^{\frac{1}{x}}$ 的斜渐近线方程为________

1. 求 $y=\sqrt{\dfrac{x^3}{x-1}}$ 的所有渐近线方程.

2. 求 $y=\sqrt{4x^2+x}\ln\left(2+\frac{1}{x}\right)$ 的斜渐近线方程.

(2017) 曲线 $y=x\left(1+\arcsin\frac{2}{x}\right)$ 的斜渐近线方程为________.

(2016) 曲线 $y=\frac{x^3}{1+x^2}+\arctan(1+x^2)$ 的斜渐近线方程为__________.

1. 设$f(x)=(x-1)(x-2)^2(x-3)^3$，求$f'(1)$，$f'(2)$，$f'(3)$.

2. 设 $f(x)=x^2\sin(x-2)$，求 $f'(2)$.

3. 设 $f(x)=x+(x-1)\arcsin\sqrt{\frac{x}{x+1}}$，求 $f'(1)$.

4. 设$f(x)=(e^x-1)\sqrt[3]{\frac{1-x^2+x^3}{1+x-x^3}}$，求$f'(0)$.

5. 若$f(x)=\left(\tan\frac{\pi x}{4}-1\right)\left(\tan\frac{\pi x^2}{4}-2\right)\cdots\left(\tan\frac{\pi x^{100}}{4}-100\right)$，求$f'(1)$.

(1989) 设$f(x)=x(x+1)(x+2)\cdots(x+n)$，则$f'(0)=$______.

(1995) 设 $f(x)=\begin{cases} x\arctan\frac{1}{x^2}, & x\neq 0, \\ 0, & x=0, \end{cases}$ 试讨论 $f'(x)$ 在 $x=0$ 处的连续性.

(1990) 设函数 $f(x)$ 对任意的 x 均满足等式 $f(1+x)=af(x)$，且有 $f'(0)=b$，其中 a,b 为非零常数，则（　　）.

(A) $f(x)$ 在 $x=1$ 处不可导

(B) $f(x)$ 在 $x=1$ 处可导，且 $f'(1)=a$

(C) $f(x)$ 在 $x=1$ 处可导，且 $f'(1)=b$

(D) $f(x)$ 在 $x=1$ 处可导，且 $f'(1)=ab$

(1993) 设函数 $f(x)=\begin{cases}\sqrt{|x|}\sin\frac{1}{x^2}, & x\neq 0,\\ 0, & x=0,\end{cases}$ 则 $f(x)$ 在 $x=0$ 处( ).

(A) 极限不存在　　　　(B) 极限存在但不连续

(C) 连续但不可导　　　　(D) 可导

(1995) 设 $f(x)=\begin{cases}\dfrac{2}{x^2}(1-\cos x), & x<0, \\ 1, & x=0, \\ \dfrac{1}{x}\displaystyle\int_0^x \cos t^2\,dt, & x>0,\end{cases}$ 讨论 $f(x)$ 在 $x=0$ 的连续性与可导性.

(1996) 设 $f(x)=\begin{cases}\dfrac{g(x)-e^{-x}}{x}, & x\neq 0,\\ 0, & x=0,\end{cases}$ 其中 $g(x)$ 具有二阶连续导数，且 $g(0)=1$，$g'(0)=-1$.

(1) 求 $f'(x)$;

(2) 讨论 $f'(x)$ 在 $(-\infty,+\infty)$ 上的连续性.

(2012) 设函数 $f(x)=(e^{x}-1)(e^{2x}-2)\cdots(e^{nx}-n)$，其中 n 为正整数，则 $f'(0)=(\quad)$.

(A) $(-1)^{n-1}(n-1)!$　　(B) $(-1)^{n}(n-1)!$　　(C) $(-1)^{n-1}n!$　　(D) $(-1)^{n}n!$

1. 设 $f(1)=1, f'(1)=2$, 则 $\lim\limits_{x\to 1}\dfrac{f^2(x)-f^2(1)}{x^2-1}=$ ________.

2. $f'(0)$存在，且$f(0)=0$，则$\lim\limits_{x\to 0}\dfrac{f(1-\cos 2x)}{x\sin x}=$ ________；

$\lim\limits_{x\to 0}\dfrac{f(x^2)}{\sin^2(\frac{x}{3})}=$ ________.

3. 求 $\lim\limits_{x\to a}\dfrac{x^x-a^a}{x-a}$.

4. 求$\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$.

5. 已知α,β为常数，$f(x)$可导，求$\lim\limits_{\Delta x\to 0}\dfrac{f(x+\alpha\Delta x)-f(x-\beta\Delta x)}{\Delta x}$.

1. 若$f(x)$在$x=a$可导，则$\lim\limits_{h\to 0}\dfrac{f(a+mh)-f(a-nh)}{h}=$______.

2. 若$f(x)=\begin{cases} g(x)\cos\frac{1}{x}, & x\neq 0, \\ 0, & x=0, \end{cases}$ 而$g(0)=g'(0)=0$，则$f'(0)=$____.

3. 若$f(x)=\begin{cases} 2\sin x+x^2\sin\frac{1}{x}, & x\neq 0, \\ 0, & x=0, \end{cases}$ 则$f'(0)$______.

4. 设 $f'(x_0)=3$，则 $\lim\limits_{\Delta x \to 0} \dfrac{f(x_0-\Delta x)-f(x_0)}{\Delta x}=$ ________.

5. 设a是自然数，求$\lim\limits_{n\to\infty} n\left[\left(1+\frac{1}{n}\right)^{\frac{1}{a}}-1\right]$.

6. 设 $f(0)=1, f'(0)=-1$，求 $\lim\limits_{x\to 1}\frac{f(\ln x)-1}{1-x}$ 与 $\lim\limits_{x\to 0}\frac{2^x f(x)-1}{x}$.

7. 设 $f''(x)$ 存在，求 $\lim\limits_{h\to 0}\dfrac{f(x+2h)-2f(x+h)+f(x)}{h^2}$.

8. $f(x)$对任何x满足$f(x+1)=2f(x)$，且$f(0)=1$，$f'(0)=C$，

求$f'(1)$.

9. 若 $f(x)$ 在 $x=1$ 处可导，且 $f'(1)=1$，求

$$\lim_{x\to 0}\frac{f(1+x)+f(1+2\sin x)-2f(1-3\tan x)}{x}.$$

10. 设 $y=f(x)$ 由方程 $y-x=e^{x(1-y)}$ 确定，求 $\lim\limits_{n\to\infty}\left[f\left(\frac{1}{n}\right)-1\right]n$.

11. 设$f'(a)$存在，求$\lim\limits_{n\to\infty}\left[\dfrac{f(a+\frac{1}{n})}{f(a)}\right]^n$.

(1987) 设$f(x)$在$x=a$处可导，则$\lim\limits_{x\to 0}\dfrac{f(a+x)-f(a-x)}{x}$等于（ ）.

(A) $f'(a)$ (B) $2f'(a)$ (C) 0 (D) $f'(2a)$

(1989) 已知$f'(3)=2$，则$\lim_{h\to 0}\dfrac{f(3-h)-f(3)}{2h}=$________.

(1994) 计算$\lim\limits_{n\to\infty} \tan^n\left(\frac{\pi}{4}+\frac{2}{n}\right)$.

(2011) 设函数 $f(x)$ 在 $x=0$ 处可导，且 $f(0)=0$，则 $\lim\limits_{x\to 0}\dfrac{x^2f(x)-2f(x^3)}{x^3}=(\quad)$.

(A) $-2f'(0)$　　(B) $-f'(0)$　　(C) $f'(0)$　　(D) 0

(2013) 设曲线 $y=f(x)$ 与 $y=x^2-x$ 在点 $(1,0)$ 处有公共切线，

则 $\lim\limits_{n\to\infty} nf\left(\frac{n}{n+2}\right)=$ __________.

1. $y=\arctan\sqrt{x}$.

2. $y=\sqrt{x^2+a^2}$.

3. $y = \arctan\sqrt{e^{2x}-1}$.

4. $y = \arcsin\frac{x}{a}$.

5. $y=\arctan\frac{x}{a}$.

6. $y=x-\ln(1+e^x)$.

7. $y = \ln(\sec x + \tan x)$.

8. $y=\ln(\csc x-\cot x)$.

9. $y=\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln(\sec x+\tan x)$.

10. $y=-\frac{1}{2}\csc x\cot x+\frac{1}{2}\ln(\csc x-\cot x)$.

11. $y=\ln(x+\sqrt{x^2+a^2})$.

12. $y = \ln(x + \sqrt{x^2 - a^2})$.

13. $y=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right).$

14. $y=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$.

15. $y=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}$.

16. $y=\frac{1}{4(1+x^4)}+\frac{1}{4}\ln\frac{x^4}{1+x^4}$.

17. $y = \ln\sqrt{\dfrac{1-\sin x}{1+\sin x}}$.

18. $y=\frac{1}{x}(\ln^3 x+3\ln^2 x+6\ln x+6)$.

19. $y=\ln\left[\frac{1}{x}+\ln\left(\frac{1}{x}+\ln\frac{1}{x}\right)\right]$

1. $y=\sin(\cos^2 x)\cos(\sin^2 x)$.

2. $y=\sin[\sin(\sin x)]$.

3. $y=\dfrac{\ln 3\cdot\sin x+\cos x}{3^{x}}$.

4. $y=e^{ax}\dfrac{a\sin bx-b\cos bx}{\sqrt{a^2+b^2}}$.

5. $y=\ln[\ln(\ln x)]$.

6. $y=\ln\left[\ln^2(\ln^3 x)\right]$.

7. $y=x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}$.

8. $y=\ln(e^x+\sqrt{1+e^{2x}})$.

9. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$.

10. $y = y = \sqrt[3]{1+\sqrt[3]{1+\sqrt[3]{x}}}$.

11. $y=e^{x}+e^{e^{x}}+e^{e^{e^{x}}}$.

12. $y=x^{a^a}+a^{x^a}+a^{a^x}+a^{a^a}\,(a>0)$.

13. $y = x^{x^a} + x^{a^x} + a^{x^x} \ (a>0, x>0)$.

14. $y=\left(\frac{a}{b}\right)^{x}\left(\frac{b}{x}\right)^{a}\left(\frac{x}{a}\right)^{b}\ (a>0,b>0)$.

15. $y=\operatorname{arccot}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$.

16. 设当$x=0$时$\dfrac{df(\sin x)}{dx}=\dfrac{df^2(\sin x)}{dx}$，且$f'(0)\neq 0$，求$f(0)$.

(1988) 若 $f(t)=\lim\limits_{x\to\infty} t\left(1+\frac{1}{x}\right)^{2tx}$，则 $f'(t)=$ ________.

(1989) 已知 $y=\arcsin e^{-\sqrt{x}}$，求 y'.

(1990) 设 $y=e^{\tan\frac{1}{x}}\sin\frac{1}{x}$，则 $y'=$ ________.

(1991) 设 $y=\ln(1+3^{-x})$，则 $y'=$ ________.

(1993) 设 $y=\sin[f(x^2)]$，其中 f 具有二阶导数，求 $\frac{d^2y}{dx^2}$.

(1993) 已知 $y=f\left(\frac{3x-2}{3x+2}\right)$，$f'(x)=\arctan x^2$，求 $\left.\frac{dy}{dx}\right|_{x=0}$.

(1996) 设 $y=(x+e^{-\frac{x}{2}})^{\frac{2}{3}}$，则 $y'|_{x=0}=$ ________.

(1997) 设 $y=\ln\sqrt{\dfrac{1-x}{1+x^2}}$，则 $y''\Big|_{x=0}=$ ________.

(2006) 设函数 $g(x)$ 可微，$h(x)=e^{1+g(x)}$，$h'(1)=1$，$g'(1)=2$，则 $g(1)$ 等于(   ).

(A) $\ln 3-1$ (B) $-\ln 3-1$ (C) $-\ln 2-1$ (D) $\ln 2-1$

(1987) 已知 $y=\ln\dfrac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}$，求 y'.

(2005) 设 $y=(1+\sin x)^x$，则 $dy\big|_{x=\pi}=$ ________.

(2012) 设函数 $f(x)=\begin{cases}\ln\sqrt{x}, & x\geq 1,\\ 2x-1, & x<1,\end{cases}$ $y=f[f(x)]$，则 $\left.\dfrac{dy}{dx}\right|_{x=e}=$ ________.

(2021) 若 $y=\cos(e^{-\sqrt{x}})$，则 $\left.\frac{dy}{dx}\right|_{x=1}=$ ______.

1. $y^2 = 2px$（抛物线），求 y'.

2. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$（椭圆），求$y'$.

3. $\sqrt{x}+\sqrt{y}=\sqrt{a}$（抛物线），求 y'.

4. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$（星形线），求 y'.

5. $\arctan\frac{y}{x}=\ln\sqrt{x^2+y^2}$（对数螺线），求 y'.

6. $y=y(x)$ 由 $y-\sin x-\cos(x-y)=0$ 确定，求 y'.

7. $y=y(x)$ 由 $\frac{x}{y}=\ln(xy)$ 确定，求 y'.

8. 求椭圆 $\frac{x^2}{16}+\frac{y^2}{9}=1$ 在 $(2, \frac{3}{2}\sqrt{3})$ 处的切线方程.

9. 求由方程$x-y+\frac{1}{2}\sin y=0$所确定的隐函数y的y', y''.

10. 求由方程 $y=\tan(x+y)$ 所确定的函数 $y=y(x)$ 的 y', y''.

1. 设$y=y(x)$是由$xy+e^y=x+1$确定的隐函数，则$\left.\dfrac{d^2y}{dx^2}\right|_{x=0}=$______.

2. 设 $y=y(x)$ 是由 $xy+\int_0^x e^{t^2}dt+1=x+e^y$ 确定的隐函数，

则 $\left.\dfrac{d^2y}{dx^2}\right|_{x=0}=$ ______.

3. 曲线 $\tan\left(x+y+\frac{\pi}{4}\right)=e^{y}$ 在 $(0,0)$ 处的切线方程是______.

4. 曲线 $\sin(xy)+\ln(y-x)=x$ 在点 $x=0, y=1$ 处的切线方程是________.

5. 设 $\ln\frac{x^2}{y} - xy^2 = 1$，求 dy.

6. 设 $x^{2y}+y^{2x}=x$，求 dy.

7. 设$e^{x+y}-xy=1+x$，求$y''(0)$.

8. 求由方程 $e^y - xy = e$ 所确定隐函数 $y = y(x)$ 的 y', y''.

(1988) 已知 $y=1+xe^{xy}$，求 $y'\Big|_{x=0}$ 及 $y''\Big|_{x=0}$.

(1990) 设 $y=y(x)$ 是由 $2y-x=(x-y)\ln(x-y)$ 所确定的隐函数，求 dy.

(1992) 设函数 $y=y(x)$ 由方程 $y-xe^{y}=1$ 所确定，求 $\left.\frac{d^{2}y}{dx^{2}}\right|_{x=0}$.

(1992) 设函数 $y=y(x)$ 由方程 $e^{x+y}+\cos xy=0$ 确定, 求 $\frac{dy}{dx}$.

(1993) 函数 $y=y(x)$ 由方程 $\sin(x^2+y^2)+e^x-xy^2=0$ 所确定,

则 $\dfrac{dy}{dx}=$ ________.

(1994) 设 $y=y(x)$ 由方程 $e^{xy}+y^2=\cos x$ 确定，则 $\frac{dy}{dx}=$ ______.

(1994) 设 $y=f(x+y)$，其中 f 具有二阶导数，且其一阶导数不等于 1，求 $\frac{d^2y}{dx^2}$.

(1995) 设函数 $y=y(x)$ 由方程 $xe^{f(y)}=e^{y}$ 确定，其中 f 具有二阶导数，且 $f'\neq 1$，求 $\frac{d^2y}{dx^2}$.

(1999) 设函数 $y=y(x)$ 由方程 $\ln(x^2+y)=x^3y+\sin x$ 确定，

则 $\left.\dfrac{dy}{dx}\right|_{x=0}=$ ________.

(2000) 设函数 $y=y(x)$ 由方程 $2^{xy}=x+y$ 所确定，求 $dy\Big|_{x=0}$.

(2002) 已知函数 $y=y(x)$ 由方程 $e^y+6xy+x^2-1=0$ 确定，则 $y''(0)=$ ________.

(2007) 已知函数 $f(u)$ 具有二阶导数，且 $f'(0)=1$，函数 $y=y(x)$ 由方程 $y-xe^{y-1}=1$ 所确定，设 $z=f(\ln y-\sin x)$，求 $\left.\frac{dz}{dx}\right|_{x=0}$，$\left.\frac{d^2z}{dx^2}\right|_{x=0}$.

(2012) 设 $y=y(x)$ 是由方程 $x^2-y+1=e^y$ 所确定的隐函数，则 $\left.\dfrac{d^2y}{dx^2}\right|_{x=0}=$ ______.

(2022) 已知函数 $y=y(x)$ 由方程 x^2+xy+y^3 确定，则 $y''(1)=$ ______.

1. $\begin{cases} x = \sin t, \\ y = \cos 2t, \end{cases}$ 求y'.

2. $\begin{cases} x = \dfrac{3at}{1+t^2}, \\ y = \dfrac{3at^2}{1+t^2}, \end{cases}$ 求 y'.

3. $\begin{cases} x = \dfrac{t^2}{2}, \\ y = 1 - t, \end{cases}$ 求 y''.

4. $\begin{cases} x = f'(t), \\ y = tf'(t) - f(t), \end{cases}$ 求y''.

5. $\begin{cases} x=\ln(1+t^2), \\ y=t-\arctan t, \end{cases}$ 求y''.

6. 求由参数方程$\begin{cases} x = \arctan t, \\ y = \ln\sqrt{1+t^2} \end{cases}$所确定的函数 $y = y(x)$的二阶导数$\dfrac{d^2y}{dx^2}$.

7. 求曲线 $\begin{cases} x = t^2 + 7, \\ y = t^2 + 4t + 1 \end{cases}$ 上对应 $t = 1$ 点处的切线方程.

8. 已知函数 $y=f(x)$ 由参数方程 $\begin{cases} x=\sqrt{3}t-1, \\ te^{y}-y=0 \end{cases}$ 确定，求

$\left.\dfrac{dy}{dx}\right|_{x=-1}$ 和 $\left.\dfrac{d^2y}{dx^2}\right|_{x=-1}$.

1. 设函数 $y=y(x)$ 由方程组 $\begin{cases} x=3t^2+2t+3, \\ e^y\sin t-y+1=0 \end{cases}$ 所确定，求 $\dfrac{d^2y}{dx^2}$

在 $t=0$ 时的值.

2. 验证由参数方程 $\begin{cases} x = e^t \sin t, \\ y = e^t \cos t \end{cases}$ 所确定的 $y = y(x)$ 满足关系式 $(x+y)^2 \dfrac{d^2y}{dx^2} = 2\left(x\dfrac{dy}{dx} - y\right)$.

3. 设 $\begin{cases} x=(t^2+1)e^t, \\ y=t^2e^{2t}, \end{cases}$ 求 $\left.\dfrac{d^2y}{dx^2}\right|_{x=1}$.

4. 求心形线 $r=2(1-\cos\theta)$ 在对应点 $\theta=\frac{\pi}{2}$ 处的切线方程.

5. 求由参数方程$\begin{cases} x=a(\cos t+t\sin t), \\ y=a(\sin t-t\cos t) \end{cases}$确定的函数$y=y(x)$的导数$\dfrac{dy}{dx}$及二阶导数$\dfrac{d^2y}{dx^2}$.

6. 设 $\begin{cases} x=\arcsin\dfrac{t}{\sqrt{1+t^2}}, \\ y=\arccos\dfrac{1}{\sqrt{1+t^2}}, \end{cases}$ 求 $y'(t>0)$.

7. 设$y=y(x)$由$\begin{cases} x^x+tx-t^2=0, \\ \arctan(ty)=\ln(1+t^2y) \end{cases}$确定，求$\dfrac{dy}{dx}$.

8. 写出曲线$\begin{cases} x=\dfrac{1+t}{t^3}, \\ y=\dfrac{3}{2t^2}+\dfrac{1}{2t} \end{cases}$在$t=1$处的切线和法线方程.

(1992) 设 $\begin{cases} x=f(t)-\pi, \\ y=f(e^{3t}-1), \end{cases}$ 其中 f 可导，且 $f'(0)\neq 0$，则 $\left.\dfrac{dy}{dx}\right|_{t=0}=$ ______.

(1994) 设 $\begin{cases} x=\cos t^2, \\ y=t\cos t^2-\int_1^{t^2}\frac{1}{2\sqrt{u}}\cos u\,du, \end{cases}$ 求 $\frac{dy}{dx}$，$\frac{d^2y}{dx^2}$ 在 $t=\sqrt{\frac{\pi}{2}}$ 的值.

(1994) 设函数 $y=y(x)$ 由参数方程 $\begin{cases} x=t-\ln(1+t), \\ y=t^3+t^2 \end{cases}$ 所确定，

则 $\dfrac{d^2y}{dx^2}=$ ________.

(1996) 设 $\begin{cases} x=\int_0^t f(u^2)du, \\ y=[f(t^2)]^2, \end{cases}$ 其中 $f(u)$ 具有二阶导数且 $f(u)\neq 0$,

求 $\dfrac{d^2y}{dx^2}$.

(1997) 设函数 $y=y(x)$ 由 $\begin{cases} x=\arctan t, \\ 2y-ty^2+e^t=5 \end{cases}$ 所确定，求 $\dfrac{dy}{dx}$.

(2003) 设函数 $y=y(x)$ 由 $\begin{cases} x=1+2t^2, \\ y=\int_1^{1+2\ln t} \frac{e^u}{u}du \end{cases}$ $(t>1)$ 所确定，求 $\left.\frac{d^2y}{dx^2}\right|_{x=9}$.

(2010) 设 $\begin{cases} x = e^{-t}, \\ y = \int_0^1 \ln(1+u^2)\,du, \end{cases}$ 则 $\left.\dfrac{d^2y}{dx^2}\right|_{t=0} =$ ________.

(2013) 设 $\begin{cases} x=\sin t, \\ y=t\sin t+\cos t \end{cases}$ (t为参数), 则 $\left.\dfrac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}}=$ ________.

(2015) 设 $\begin{cases} x = \arctan t, \\ y = 3t + t^3, \end{cases}$ 则 $\left.\dfrac{d^2y}{dx^2}\right|_{t=1} =$ ______.

(2017) 设函数 $y=y(x)$ 由参数方程 $\begin{cases} x=t+e^t, \\ y=\sin t \end{cases}$ 确定，则 $\left.\dfrac{d^2y}{dx^2}\right|_{t=0}=$ ______

(2020) 设$\begin{cases} x=\sqrt{t^2+1}, \\ y=\ln\left(t+\sqrt{t^2+1}\right) \end{cases}$，则$\left.\dfrac{d^2y}{dx^2}\right|_{t=1}=$ ________.

(2021) 设函数 $y=y(x)$ 由参数方程 $\begin{cases} x=2e^t+t+1, \\ y=4(t-1)e^t+t^2 \end{cases}$ 确定，则 $\left.\dfrac{d^2y}{dx^2}\right|_{t=0}=$ ________.

1. 讨论 $y=e^{|x|}$ 在 $x=0$ 的可导性.

2. 求函数 $f(x)=\begin{cases} \ln(1+x), & x \geq 0, \\ x, & x<0 \end{cases}$ 的导数.

3. 设 $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0, \\ 0, & x=0, \end{cases}$ 讨论 $f(x)$ 在 $x=0$ 处的连续性与可导性.

4. 设 $f(x)=x|x(x-2)|$，求 $f'(x)$.

5. 设 $f(x)=\begin{cases} 1+\ln(1-4x), & x\leqslant 0, \\ a+be^{x}, & x>0, \end{cases}$ 试确定 a,b 的值，

使 $f(x)$ 在 $x=0$ 处可导，并求 $f'(0)$.

6. 设$f(x)$的二阶导数连续，且$f(0)=0$，定义$g(x)=\begin{cases}\dfrac{f(x)}{x}, & x\neq 0,\\ f'(0), & x=0,\end{cases}$ 求$g'(x)$并研究$g'(x)$的连续性.

7. 设 $f(x)=|\sin x|^3 \quad (-1<x<1)$，求 $f''(x)$.

1. 设 $f(x)=\begin{cases} a+x+\sqrt{1-x}, & x<0, \\ 1+b\ln(1+x), & x\geq 0. \end{cases}$

(1) 求常数 a, b, 使 $f(x)$ 在 $x=0$ 处可导，并求 $f'(x)$;

(2) 求 $y=f(x)$ 在 $x=0$ 处的切线方程与法线方程.

2. 设 $f(x)$ 在 $(-\infty,+\infty)$ 内具有一阶连续导数，且 $f''(0)$ 存在，

$f(0)=1$，$f'(0)=1$，试证明

$$F(x)=\begin{cases}\dfrac{f(x)-e^{x}}{x}, & x\neq 0,\\ 0, & x=0\end{cases}$$

在 $(-\infty,+\infty)$ 内连续，且具有一阶连续导数.

3. 设 $f(x)=\begin{cases} x^{\alpha}\cos\dfrac{1}{x^{\beta}}, & x>0, \\ 0, & x\leqslant 0 \end{cases}$ $(\alpha>0,\beta>0)$，若 $f'(x)$ 在 $x=0$ 处连续，则（　　）.

(A) $\alpha-\beta>1$　　(B) $0<\alpha-\beta\leqslant 1$

(C) $\alpha-\beta>2$　　(D) $0<\alpha-\beta\leqslant 2$

4. 设 $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x>0, \\ 0, & x=0, \\ \frac{1-\cos x^2}{x}, & x<0, \end{cases}$ 求 $f'(x)$，并讨论 $f'(x)$ 的连续性.

5. 设 $f(x)=\begin{cases} ax^2+b\sin x+c, & x\leqslant 0, \\ \ln(1+x), & x>0, \end{cases}$ 试问 a,b,c 为何值时，

$f(x)$ 在 $x=0$ 处一阶导数连续，但二阶导数不存在？

6. 设 $f(x)$ 连续，$\varphi(x)=\int_0^1 f(xt)dt$，且 $\lim\limits_{x\to 0}\frac{f(x)}{x}=A$（$A$ 为常数），求 $\varphi'(x)$ 并讨论 $\varphi'(x)$ 在 $x=0$ 的连续性.

7. 设 $f(x)=\left|\ln|x|\right|$，求 $f'(x)$.

1. $f(x)=\int_0^x e^{t^2}dt$，求$f'(x)$.

2. $y=\int_0^{x^2}\sqrt{1+t^2}\,dt$，求$y'$.

3. $y=\int_{x^2}^{x^3}\frac{dt}{\sqrt{1+t^4}}$，求 y'.

4. $y=\int_{x}^{-2} \sqrt[3]{t} \ln(1+t^2)\,dt$，求$y'$.

5. $y=\int_0^x (x-t)g(t)dt$，求 y'.

6. $\frac{d}{db}\int_a^b \sin x^2 \, dx$.

7. $f(x)$连续，且$F(x)=\int_{x}^{e^{-x}} f(t)dt$，求$F'(x)$.

8. $y=\int_{\sin x}^{x} t^2 f(t)\,dt$，求$y'$.

9. $y=\int_0^{x^2} x^2 f(t)\,dt$，求$y'$.

10. $y=\int_0^x (x-t)^2 f(t)dt$，求$y'$.

1\. 求 $\lim\limits_{x\to+\infty}\dfrac{\int_0^x \frac{dt}{\sqrt[3]{t^3+5t^2+2}}}{\ln x}$.

2. 求 $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} te^t \sin t dt}{x^6 e^x}$.

3. 设 $\begin{cases} x=\int_1^{t^2} u\ln u\,du, \\ y=\int_{t^2}^{1} u^2\ln u\,du \end{cases}$ $(t>1)$, 求 $\dfrac{dy}{dx}$.

4. $f(x)=\int_{\sin x}^{\cos x}\cos(\pi t^2)\,dt$，求$f'(x)$.

5. 求 $\lim\limits_{x\to+\infty}\dfrac{e^{-x^2}\int_0^x \sin^2 t e^{t^2}dt}{x}$.

6. 设 $f(x)$ 有连续导数，且 $f(0)=0$，$f'(0)\neq 0$，$F(x)=\int_0^x (x^2-t^2)f(t)dt$. 当 $x\to 0$ 时，$F'(x)$ 与 x^k 是同阶无穷小，则 $k=$ ________.

7. 设 $F(x)=\int_0^x (2t-x)f(t)dt$，$f(x)$ 可导、且 $f'(x)>0$，证明：$(0, F(0))$ 为拐点。

8. $\dfrac{d}{dt}\displaystyle\int_{t^2}^{t} x f(t^2-x^2)\,dx.$

9. 设 $f(x)$ 连续，且 $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=2$，

$$
令 F(x)=\begin{cases} \int_0^1 f(tx)\,dt, & x>0, \\ 0, & x=0, \\ \dfrac{\int_0^x \ln(1+2t)\,dt}{x}, & x<0, \end{cases}
$$

求 $F'(0)$。

(1987) 求正常数 a 与 b，使等式 $\lim\limits_{x\to 0}\frac{1}{bx-\sin x}\int_0^x\frac{t^2}{\sqrt{a+t^2}}dt=1$ 成立.

(1993) 设 $f(x)=\int_0^{\sin x}\sin t^2 dt$, $g(x)=x^3+x^4$, 则当 $x\to 0$ 时, $f(x)$ 是 $g(x)$ 的 (　　).

(A) 等价无穷小量　　(B) 同阶但非等价无穷小量

(C) 高阶无穷小量　　(D) 低阶无穷小量

(1998) 确定常数 a, b, c 的值，使 $\lim\limits_{x \to 0} \dfrac{ax - \sin x}{\int_b^x \frac{\ln(1+t^3)}{t} dt} = c$ $(c \neq 0)$.

(2004) 把 $x\to 0^+$ 时的无穷小量 $\alpha=\int_0^x \cos t^2 dt$, $\beta=\int_0^{x^2}\tan\sqrt{t}\,dt$, $\gamma=\int_0^{\sqrt{x}}\sin t^3 dt$ 阶数由低到高排列起来，正确的排列次序是 ( ).

(A) α,β,γ　　(B) α,γ,β　　(C) β,α,γ　　(D) β,γ,α

(1990) 设 $f(x)$ 是连续函数，且 $F(x)=\int_x^{e^{-x}} f(t)dt$，则 $F'(x)$ 等于（　）.

(A) $-e^{-x}f(e^{-x})-f(x)$　　(B) $-e^{-x}f(e^{-x})+f(x)$

(C) $e^{-x}f(e^{-x})-f(x)$　　(D) $e^{-x}f(e^{-x})+f(x)$

(1992) 求 $\frac{d}{dx}\int_0^{x^2}(x^2-t)f(t)\,dt$，其中 $f(t)$ 为已知的连续函数.

(1995) $\frac{d}{dx}\int_{x^2}^{0} x\cos t^2 dt =$ ________.

(1998) 设$f(x)$连续，则$\frac{d}{dx}\int_0^x tf(x^2-t^2)dt=$(　　).

(A) $xf(x^2)$　　(B) $-xf(x^2)$

(C) $2xf(x^2)$　　(D) $-2xf(x^2)$

(1999) $\frac{d}{dx}\int_0^x \sin(x-t)^2\,dt =$ ________.

1. 设函数 $y=\frac{1}{2x+3}$，则 $y^{(n)}(0)=$ ________.

2. 设 $y=\frac{x}{1-x^2}$，求 $y^{(n)}$.

3. 设 $y=\dfrac{4x^2-1}{x^2-1}$，求 $y^{(n)}$.

4. 设 $f(x)=\sin^4 x+\cos^4 x$，求 $f^{(n)}(x)$.

5. 设 $y=x^3\sin x$，求 $y^{(10)}(0)$.

6. 若 $y=\arctan x$，求 $y^{(n)}(0)$

1. $y = e^{x}\cos x$，求 $y^{(n)}$.

2. 设 $y=\dfrac{x^3-1}{x^2-2x-3}$，求 $y^{(n)}$.

3. 设 $y=\dfrac{x^4}{1+x^3}$，求 $y^{(n)}(0)$.

4. 设$y=\begin{cases}\dfrac{\sin x}{x}, & x\neq 0,\\ 1, & x=0,\end{cases}$ 求$y^{(n)}(0)$.

5. 设 $y=x^2\ln(1+x)$，求 $y^{(n)}(0)$.

6. 设函数 $f(x)=\arctan x-\dfrac{x}{1+ax^2}$，且 $f'''(0)=1$，则 $a=$ ______.

1. $\int(5x^3+4x+1)dx.$

2. $\int\left(\frac{3}{\sqrt{x}}+\frac{5}{x^2}+2x\right)dx$.

3. $\int\left(\frac{a}{x}+\frac{a^2}{x^2}+\frac{a^3}{x^3}+\frac{a^4}{x^4}\right)dx$.

4. $\int(x+x^2+x^3+x^4)dx$。

5. $\int \sqrt[m]{x^n}\,dx$.

6. $\int (\sqrt{x}+1)(\sqrt{x^3}-1)\,dx$.

7. $\int \frac{(1-x)^2}{\sqrt{x}} dx$,

8. $\int (x^2+1)^2 dx$.

9. $\int(\sqrt{x}+x\sqrt{x}+\sqrt{x\sqrt{x}})dx.$

10. $\int x^2 \sqrt[3]{x}\,dx$.

11. $\int \frac{1}{x^2\sqrt{x}} dx$.

12. $\int \sqrt[3]{x^2}\,dx$.

13. $\int \sqrt{x}(x^2-5)\,dx$.

14. $\int \frac{dh}{\sqrt{2gh}}$.

15. $\int (1-x^2)(1-\frac{1}{x})dx$。

16. $\int\left(2e^{x}+\frac{3}{x}\right)dx$.

17. $\int\left(\frac{3}{1+x^2}-\frac{2}{\sqrt{1-x^2}}\right)dx$.

18. $\int 2^x dx$.

19. $\int \sin x dx, \int \cos x dx$.

20. $\int \frac{3}{\sqrt{1-x^2}} dx$.

21. $\int (1+\sin x+\cos x)\,dx$.

22. $\int\left(\frac{1}{x^2+1}-\frac{1}{x^2-1}\right)dx.$

1. $\int (x^2-1)^3 dx$.

2. $\int (x^2+1)^3 dx.$

3. $\int \left(\frac{1+x}{x}\right)^2 dx.$

4. $\int \frac{dx}{x^2+2x+1}$.

5. $\int \frac{1-x^2}{1+x^2} dx$.

6. $\int \frac{x^2}{1+x^2} dx$.

7. $\int \frac{x-1}{x^2-1}dx.$

8. $\int \frac{x-1}{\sqrt{x}+1} dx$.

9. $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$.

10. $\int \frac{x^4}{1+x^2} dx$.

11. $\int\sqrt{\frac{1-x}{1+x}}dx$。

12. $\int \frac{3}{\sqrt{4-4x^2}}\,dx.$

13. $\int \frac{3x^4+2x^2}{x^2+1}dx$。

14. $$\int \frac{1+2x^2}{x^2(1+x^2)}\,dx.$$

15. $$\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx.$$

16. $\int 2^x \cdot e^x \, dx$.

17. $\int \frac{2\cdot 3^x - 5\cdot 2^x}{3^x}\,dx$.

18. $\int (e^{x+1} + 3^x 4^{-x}) dx$.

19. $\int e^{x}\left(1-\frac{e^{-x}}{\sqrt{x}}\right)dx.$

20. $\int \frac{\sqrt{x}-x^3e^x+x^2}{x^3}dx$.

21. $\int \frac{e^{3x}+1}{e^{x}+1}\,dx$。

22. $\int \frac{2^{x+1}-3^{x+1}}{6^{x}}dx$.

23. $\int (2^x+3^x)^2 dx$.

24. $\int 3e^{x+5}\,dx$.

25. $\int \frac{2^{x+1}-5^{x-1}}{10^{x}}dx$.

26. $\int \sin^2 x \, dx$.

27. $\int \cos^2 x\,dx$.

28. $\int \sqrt{1-\sin 2x}\,dx$.

29. $\int \sqrt{1+\cos 2x}\,dx$.

30. $\int \frac{1}{1+\cos 2x} dx$.

31. $\int \tan x \, dx$.

32. $\int \cot x \, dx$.

33. $\int \tan^2 x \, dx$.

34. $\int \cot^2 x \, dx$.

35. $\int \csc x(\csc x+1)\,dx$.

36. $\int \dfrac{\cos 2x}{\cos^2 x \sin^2 x}\,dx$.

37. $\int \frac{\cos 2x}{\cos x - \sin x} dx.$

38. $\int \cos x(\tan x + \sec x) dx.$

39. $\int \frac{1}{16-x^4}\,dx.$

1. $\int \frac{1}{x+a} dx$.

2. $\int (2x-3)^{10} dx$.

3. $\int \sqrt[3]{1-3x}\, dx$.

4. $\int \frac{dx}{\sqrt{2-5x}}$.

5. $\int \frac{dx}{\sqrt[3]{2-3x}}$.

6. $\int \frac{dx}{(5x-2)^{\frac{5}{2}}}$.

7. $\int e^{5x} dx$.

8. $\int \frac{1}{1-2x} dx$.

9. $\int (\sin ax - e^{\frac{x}{b}}) dx$.

10. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$.

11. $\int xe^{-x^2}dx$.

12. $\int \frac{1}{2+3x^2}dx$.

13. $\int \frac{1}{2-3x^2}dx$.

14. $\int \frac{1}{\sqrt{2-3x^2}}dx$.

15. $\int \frac{1}{\sqrt{3x^2-2}}\,dx$.

16. $\int (e^{-x}+e^{-2x})\,dx$.

17. $\int(\sin 5x-\sin 5\alpha)dx$.

18. $\int \sec x dx$.

19. $\int \sin(ax+b)\,dx$.

20. $\int \tan^4 x \sec^2 x\,dx$.

21. $\int \frac{1}{a^2-x^2}dx$.

22. $\int \frac{1}{\sqrt{x^2-a^2}}dx$.

23. $\int \frac{1}{\sqrt{x^2+a^2}} dx$.

24. $\int \frac{1}{\sqrt{a^2-x^2}} dx$.

25. $\int \frac{1}{x^2+a^2} dx$.

26. $\int \frac{1}{x^2-a^2} dx$.

27. $\int (x^2+1)^{2020} \cdot x \, dx.$

28. $\int (2x-1)^{2020} \, dx.$

29. $\int(\sin 2x+\cos 2x)dx$。

30. $\int x\cdot\cos(1+x^2)dx$.

31. $\int \frac{1}{1+2x}dx.$

32. $\int \frac{1+\cos x}{x+\sin x}dx.$

33. $\int \frac{2x+4}{x^2+4x+6} dx$.

34. $\int \frac{2x-1}{\sqrt{1-x^2}} dx$.

35. $\int \frac{2x-1}{\sqrt{x^2-x}}\,dx$.

36. $\int x e^{x^2+1}\,dx$.

37. $\int e^{3-x}\,dx.$

38. $\int e^{x}\sin(e^{x})\,dx.$

39. $\int \frac{1}{x(1+\ln^2 x)} dx.$

40. $\int \frac{(\arctan x)^2}{1+x^2} dx.$

41. $\int \frac{e^x}{1+e^x}\,dx.$

42. $\int \frac{x}{\sqrt{2-3x^2}}\,dx.$

43. $\int \frac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}} dx$.

44. $\int \frac{dx}{(\arcsin x)^2 \sqrt{1-x^2}}$.

45. $\int \frac{10^{2\arccos x}}{\sqrt{1-x^2}}dx.$

1. $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}dx$.

2. $\int \frac{dx}{\sin^2(2x+\frac{\pi}{4})}$.

3. $\int \frac{dx}{1+\cos x}$.

4. $\int \frac{dx}{1-\cos x}$.

5. $\int \frac{dx}{1+\sin x}$.

6. $\int \sin^3 x\, dx$.

7. $\int \cos^3 x dx$.

8. $\int \frac{1}{\cos x \sin x} dx$.

9. $\int \frac{\ln\tan x}{\cos x \sin x} dx$.

10. $\int \frac{dx}{x+\sqrt{1-x^2}}$.

11. $\int x^2\sqrt{2+x^3}\,dx$.

12. $\int \frac{dx}{\sqrt{4-9x^2}}$.

13. $\int \frac{1}{\sqrt{x}} e^{\sqrt[3]{x}} dx.$

14. $\int \frac{(\arctan\sqrt{x})^2}{\sqrt{x}(1+x)} dx.$

15. $\int \sqrt{\frac{\ln(x+\sqrt{1+x^2})}{1+x^2}}\,dx.$

16. $\int \frac{1}{1+e^x}\,dx.$

17. $\int \frac{1}{(1+e^x)^2} dx$.

18. $\int \frac{1}{e^x+e^{-x}} dx$.

19. $\int \frac{1}{e^{x}-e^{-x}}\,dx$.

20. $\int \frac{1}{\sqrt{x(1+x)}}\,dx$.

21. $\int \frac{1}{\sqrt{(x^2+a^2)^3}}\,dx$.

22. $\int \frac{1}{\sqrt{(x^2-a^2)^3}}\,dx$.

23. $\int \frac{1}{\sqrt{(a^2-x^2)^3}}dx.$

24. $\int \frac{\sqrt{x^2-4}}{x}dx.$

25. $\int \frac{1}{\sqrt{x} + \sqrt[4]{x}} dx$.

26. $\int \tan\sqrt{1+x^2} \frac{x}{\sqrt{1+x^2}} dx$.

27. $\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx.$

28. $\int \frac{x}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}\,dx.$

29. $\int \frac{1-x}{\sqrt{9-4x^2}}dx$.

30. $\int \frac{x^2 dx}{\sqrt{a^2-x^2}}$.

31. $\int \frac{dx}{x\sqrt{x^2-1}}$.

32. $\int \frac{dx}{x\sqrt{x^2+1}}$.

33. $\int \frac{dx}{x^2\sqrt{x^2+1}}$.

34. $\int \frac{dx}{x^2\sqrt{x^2-1}}$.

35. $\int \frac{dx}{x^2\sqrt{1-x^2}}$.

36. $\int \frac{x^3}{9+x^2}\,dx$.

37. $\int \frac{dx}{2x^2-1}$.

38. $\int \frac{dx}{(x+1)(x-2)}$.

39. $\int \frac{x}{x^2-x-2}dx$.

40. $\int \frac{x^3+1}{(x^2+1)^2}dx$.

41. $\int \frac{dx}{\sqrt{e^{2x}+1}}$.

42. $\int \frac{dx}{\sqrt{e^{2x}-1}}$。

43. $\int \frac{dx}{\sqrt{1-e^{2x}}}$.

44. $\int \sqrt{\frac{e^x-1}{e^x+1}}\,dx$.

45. $\int \frac{1}{e^{x}(1+e^{2x})}dx$.

46. $\int \sqrt{1+e^{2x}}\,dx$.

47. $\int e^x\sqrt{1+e^{2x}}\,dx$.

48. $\int \frac{1}{\sqrt{e^x+1}}\,dx$.

49. $\int \frac{2^x \cdot 3^x}{9^x + 4^x} dx$.

50. $\int \frac{1 + \ln x}{x^{-x} + x^x} dx$.

51. $\int \frac{\sqrt{\ln \tan x}}{\sin 2x} dx$.

52. $\int \frac{(\ln \tan x)^2}{\cos x \sin x} dx$.

53. $\int \frac{dx}{x\ln x\ln\ln x}$.

54. $\int \frac{1+\ln x}{(x\ln x)^2}\,dx$.

55. $\displaystyle\int \frac{(1+2x^2)e^{x^2}}{2-3xe^{x^2}}\,dx$.

56. $\displaystyle\int e^{e^x\cos x}(\cos x-\sin x)e^x\,dx$.

57. $\int \frac{\cos x + x\sin x}{(x+\cos x)^2}dx$.

58. $\int \frac{x+\sin x\cos x}{(x\sin x+\cos x)^2}dx$.

59. $\int \frac{dx}{(a\sin x+b\cos x)^2}$.

60. $\int \frac{1-\ln x}{(x-\ln x)^2}\,dx$.

61. $\int \frac{1+x}{x(1+xe^x)}dx$.

62. $\int \frac{\ln x+2}{x\ln x(1+x\ln^2 x)}dx$.

63. $\int x\ln(1+x^2)\arctan x\,dx$.

64. $\int \sqrt{(x^2+x)e^x}\,(x^2+3x+1)e^x\,dx$.

65. $\int \left(\frac{f(x)}{f'(x)} - \frac{f^2(x)\, f''(x)}{[f'(x)]^3}\right) dx$.

66. $\int \left(\sin 2x - e^{\frac{x}{3}}\right) dx$.

67. $\int \cos^2(\omega t+\varphi)\sin(\omega t+\varphi)\,dt$.

68. $\int \frac{\sin x}{\cos^3 x}\,dx$.

69. $\int \frac{\arcsin(1-x)}{\sqrt{2x-x^2}}\,dx.$

70. $\int \frac{2^{\tan\frac{1}{x}}}{x^2}\sec^2\frac{1}{x}\,dx.$

71. $\int \tan^3 x \sec x \, dx$.

72. $\int \sin^2 x \cos^3 x \, dx$.

73. $\int \sin 5x \sin 7x \, dx$.

74. $\int \cos x \cos \frac{x}{2} \, dx$.

75. $\int \sin 5x \cos 7x \, dx$.

76. $\int \frac{\cot x}{\sqrt{\sin x}} dx$.

77. $\int \frac{\tan x}{\cos^3 x} dx$.

78. $\int \sec^3 x \, dx$.

79. $\int \sec^4 x\,dx$.

80. $\int \sec^5 x\,dx$.

81. $\int \sec^6 x dx$.

82. $\int \csc^3 x dx$.

83. $\int \csc^4 x \, dx$.

84. $\int \csc^5 x \, dx$.

85. $\int \tan x(\tan x+1)\,dx$.

86. $\int \sqrt{\tan x}\,dx$.

87. $\int \tan^4 x\,dx$.

88. $\int \tan^5 x\,dx$.

89. $\int xe^x dx.$

90. $\int (1+x)e^x dx.$

91. $\int e^x(f(x)+f'(x))\,dx$.

92. $\int e^x \frac{1-x}{x^2}\,dx$.

93. $\int \frac{x^3}{x^8-2} dx$.

94. $\int \frac{dx}{\sqrt{x}(1+x)}$.

P5. $\displaystyle\int \frac{\sin x\cos x}{\sqrt{a^2\sin^2 x+b^2\cos^2 x}}dx\ (|a|\neq|b|)$.

P6. $\displaystyle\int \frac{dx}{\sin^2 x\sqrt[4]{\cot x}}$.

97. $\int \frac{dx}{\sin x + \cos x}$。

98. $\int \frac{2\sin x + 3\cos x}{5\sin x + 7\cos x} dx$.

99. $\int \frac{dx}{\sin^2 x+2\cos^2 x}$.

100. $\int \frac{x^2-1}{x^4+1}\,dx$.

101. $\displaystyle\int \frac{\cos x\,dx}{\sqrt{2+\cos 2x}}$.

102. $\displaystyle\int \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx$.

103. $\int \frac{\arctan x}{1+x^2} dx$.

104. $\int \frac{dx}{\sqrt{1-x^2}\arcsin x}$.

105. $\int \frac{1}{1-x^2} \ln \frac{1+x}{1-x} \, dx$.

106. $\int \frac{\sin x \cos x}{\sin x + \cos x} \, dx$.

107. $\int \frac{\ln x}{(1+x^2)^{\frac{3}{2}}}dx$.

(1989) $\int \frac{dx}{x\ln^2 x}$.

(1992) $\int \frac{x^3}{\sqrt{1+x^2}} dx$.

(1995) 设 $f(x^2-1)=\ln\frac{x^2}{x^2-2}$，且 $f[\varphi(x)]=\ln x$，求 $\int\varphi(x)dx$.

(1997) $\int \frac{dx}{\sqrt{x(4-x)}}$.

(1995) $\int \frac{x+5}{x^2-6x+13}\,dx$.

(2004) 已知 $f'(e^x) = xe^{-x}$，且 $f(1) = 0$，则 $f(x) =$ ______.

(1992) 计算 $I=\int \frac{\operatorname{arccot} e^x}{e^x} dx$.

(1998) $\int \frac{\ln x - 1}{x^2} dx$.

(1996) 设 $\int xf(x)dx = \arcsin x + C$，则 $\int \frac{1}{f(x)}dx =$ ________.

(1993) $\int \frac{xe^x}{\sqrt{e^x-1}}dx$.

(2001) $\dfrac{dx}{(2x^2+1)\sqrt{x^2+1}}$.

(2002) 设 $f(\sin^2 x)=\dfrac{x}{\sin x}$，求 $\displaystyle\int \frac{\sqrt{x}}{\sqrt{1-x}} f(x)\,dx$.

(2003) 计算不定积分 $\int \frac{xe^{\arctan x}}{(1+x^2)^{3/2}} dx$.

(2006) $\int \frac{\arcsin e^x}{e^x} dx$.

1. $\int x\sin x\,dx$.

2. $\int x\cos x\,dx$.

3. $\int xe^{x}dx$.

4. $\int x\ln x\,dx$.

5. $\int x^2 \ln x \, dx$.

6. $\int x e^{-x} \, dx$.

7. $\int e^x \sin x \, dx$.

8. $\int e^x \cos x \, dx$.

9. $\int e^{2x}\sin 3x\,dx$.

10. $\int e^{ax}\sin bx\,dx$.

11. $\int e^{ax}\cos bx\,dx$.

12. $\int x^2\arctan x\,dx$.

13. $\int x^2 \sin x dx$.

14. $\int x^2 \cos x dx$.

15. $\int x^2 e^x dx$.

16. $\int x^2 e^{-2x} dx$.

17. $\int x^2 \sin 3x dx$.

18. $\int \ln^2 x dx$.

19. $\int x\ln(x-1)\,dx$.

20. $\int \frac{\ln^3 x}{x^2}\,dx$.

21. $\int\left(\frac{\ln x}{x}\right)^{3}dx$.

22. $\int x\sin x\cos x\,dx$.

23. $\int (\arcsin x)^2 dx$.

24. $\int e^{\sqrt{3x+9}} dx$.

25. $\int x(\ln x)^2 dx$.

26. $\int \arctan x \, dx$.

27. $\int \arcsin x \, dx$.

28. $\int \frac{\arcsin x}{x^2} \, dx$.

29. $\int x^2 \arcsin x \, dx$.

30. $\int \ln(x+\sqrt{1+x^2}) \, dx$.

31. $\int x^5 e^{x^3} dx$.

32. $\int x(\arctan x)^2 dx$.

33. $\int \sqrt{a^2-x^2}\,dx.$

34. $\int \sqrt{x^2+a^2}\,dx.$

35. $\int \sqrt{x^2-a^2}\,dx$.

36. $\int \sqrt{x^2+a}\,dx$.

37. $\int e^{\sqrt{x}} dx$.

38. $\int x \sin\sqrt{x}\, dx$.

47. $\int x\sin^2 x\,dx.$

1. $\int xe^{x}\cos x\,dx$.

2. $\int e^{\sqrt[3]{x}}\,dx$.

3. $\int \cos \ln x \, dx$.

4. $\int \sin \ln x \, dx$.

5. $\int e^x \sin^2 x\, dx$.

6. $\int x^n \ln x\, dx$.

7. $\int x\ln\frac{1+x}{1-x}dx$.

8. $\int \arctan\sqrt{x}\,dx$.

9. $\int x^2 \ln \frac{1-x}{1+x} dx$.

10. $\int \frac{x\ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$.

11. $\int \frac{x^2}{(1+x^2)^2} dx$.

12. $\int \frac{1}{(1+x^2)^2} dx$.

13. $\int \arctan(x^2)\,dx$.

14. $\int x^2\sqrt{x^2+a^2}\,dx$.

15. $\displaystyle\int \frac{e^{\arctan x}}{(1+x^2)^{3/2}}\,dx.$

16. $\displaystyle\int e^{2x}\sin^2 x\,dx.$

17. $\int (e^x - \cos x)^2 dx.$

18. $\int \frac{\arctan e^x}{e^x} dx.$

19. $\int \frac{x}{\cos^2 x} dx$.

20. $\int \frac{xe^x}{(x+1)^2} dx$.

(1990) 计算 $\int \frac{\ln x}{(1-x)^2} dx$.

(1994) $\int x^3 e^{x^2} dx =$ ______.

(1996) $\int \frac{\arctan x}{x^2(1+x^2)} dx =$ ______.

(1997) $\int e^{2x}(\tan x+1)^2 dx$.

$(2001)\int \frac{\arctan e^{x}}{e^{2x}}dx.$

(2004) 已知 $f'(e^x) = xe^{-x}$，且 $f(1) = 0$，则 $f(x) =$ ______.

(2000) 设 $f(\ln x)=\dfrac{\ln(1+x)}{x}$，计算 $\int f(x)\,dx$.

1. $\int \frac{2x+3}{(x-2)(x+5)}dx$.

2. $\int \frac{x\,dx}{(x+1)(x+2)(x+3)}$.

3. $\int \frac{x^2+1}{(x+1)^2(x-1)}dx.$

4. $\int \frac{dx}{(x+1)(x^2+1)}.$

5. $\int \frac{dx}{x^4-1}$.

6. $\int \frac{dx}{1+\sqrt[3]{x+1}}$.

7. $\int \frac{(\sqrt{x})^3 - 1}{\sqrt{x} + 1} dx.$

1. $\int \frac{1}{1+x^3}dx$.

2. $\int \frac{x\,dx}{x^3-1}$.

3. $\int \frac{dx}{x^4+1}$.

4. $\int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x}$.

5. $\int \frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}$.

6. $\int \frac{1}{x(1+x^2)}dx$.

7. $\int \frac{x^3}{1+x} dx$.

8. $\int \frac{1}{x^4(1+x^2)} dx$.

9. $\int \frac{1}{x(1+x^4)}\,dx.$

10. $\int \frac{x^2+1}{x^4+1}\,dx.$

1. $\int \frac{dx}{3+\sin^2 x}$。

2. $\int \frac{dx}{3+\cos x}$.

3. $\int \frac{dx}{1+\sin x+\cos x}$.

4. $\int \sqrt{1-x^2}\arcsin x\,dx$.

5. $\int \frac{\arccos x}{\sqrt{(1-x^2)^3}}\,dx.$

6. $\int \frac{x\arccos x}{\sqrt{1-x^2}}\,dx.$

7. $\int \frac{\arcsin x}{x^2} \frac{1+x^2}{\sqrt{1-x^2}} dx$.

8. $\int \frac{x^3 \arccos x}{\sqrt{1-x^2}} dx$.

(1987) $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x} dx \quad (a, b \neq 0)$.

(1993) $\int \frac{\tan x}{\sqrt{\cos x}} dx$.

(1994) $\int \frac{dx}{\sin 2x + 2\sin x}$.

(1998) $\int \frac{\ln \sin x}{\sin^2 x} dx$.

1. $I_n = \int \sin^n x \, dx$.

2. $I_n = \int \cos^n x \, dx$.

3. $I_n = \int \tan^n x \, dx$.

4. $I_n = \int \csc^n x \, dx$.

5. $I_n = \int \sec^n x dx$.

6. $I_n = \int \frac{1}{x^n\sqrt{1+x^2}} dx$.