Are Social Networks Naturally Hierarchical?

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June 1, 2025

1 Summary

After a reading group session on *Neither Vertical Nor Horizontal* by Rodrigo Nunes, our group had a discussion about whether hierarchy was "natural" and what hierarchy is. I wanted to study if it was true that hierarchies naturally formed in social networks. To do this, I wrote a program to randomly generate vertex-edge graphs and test for the appearance of hierarchy. In this simple experiment, a model of a growing social network does tend toward hierarchy. We don't have to take this fatalistically. It may show that if we value equality, it takes conscious effort and planning to distribute influence and counteract the inequalities in hierarchy.

2 Methods

2.1 Modeling Social Networks

There are several types of graphs used as models of social networks. I used one of the oldest, **Price's Model** [3], which was intended to understand the growth of the number of academic citations of published papers. The idea of the model is that existing papers will be cited by new papers in proportion to how much they've already been cited. This is called *preferential attachment*. The growth of citations in the model is similar to the actual distribution of paper citations. To apply this model to the growth of a volunteer organization, imagine that the relationships are influence instead of citations. Person A influences person B, $A \to B$, and that if a new person C enters an organization, they'll be influenced by people who are already influential, so the social graph will grow by preferential attachment.

Formally, such a network is a mathematical graph [2] of vertices (nodes) and edges, $G = (V, E)$ where the edges have a direction. In Price's Model, the probability of a new edge

connecting to any node with a degree k is $\frac{(k+1)p_k}{m+1}$, where p_k is the fraction of nodes with degree k and m is the mean number of attachments.

2.2 What is Hierarchy?

To test for hierarchy, I use a measure called **graph agony** [1]. In a hierarchical ranking of nodes, the graph agony is the weighted count of the edges that have a direction that is counter to the hierarchy. So, a ranking assignment that is consistent with hierarchy should have a low graph agony.

Given a directed graph $G = (V, E)$ and a ranking function $r : V \to \mathbb{Z}$ assigning integer ranks to nodes, the agony of an edge (u, v) is defined as: $agony(u, v) = max(0, r(u) - r(v) + 1)$. and

$TotalAgony(G, r) = \sum_{(u,v)\in E} max(0, r(u) - r(v) + 1)$.

Hierarchies can be found in graphs by finding the ranking of nodes that minimizes graph agony:

$\min_{r:V\to\mathbb{Z}} \sum_{(u,v)\in E} max(0, r(u) - r(v) + 1)$

2.3 Experiment

Minimizing graph agony is computationally complex. There are optimized methods for it, but I only wanted small networks that I could visualize, so I used a brute-force method of randomly selecting rankings and keeping the one that created the lowest graph agony. I ran 12 trials of 12 node graphs, starting with 2 nodes and a mean attachment of 2, and picked some representative results. Graphs are shown with nodes arranged in a circle rather than trying to arrange them for the hierarchy.

The brute-force search finds hierarchies in these small graphs, with only a few edges being directed against the ranking hierarchy. Often, graphs have two top-level nodes (rank 0), the first two in the graph (A and B), as shown in Figure 1.

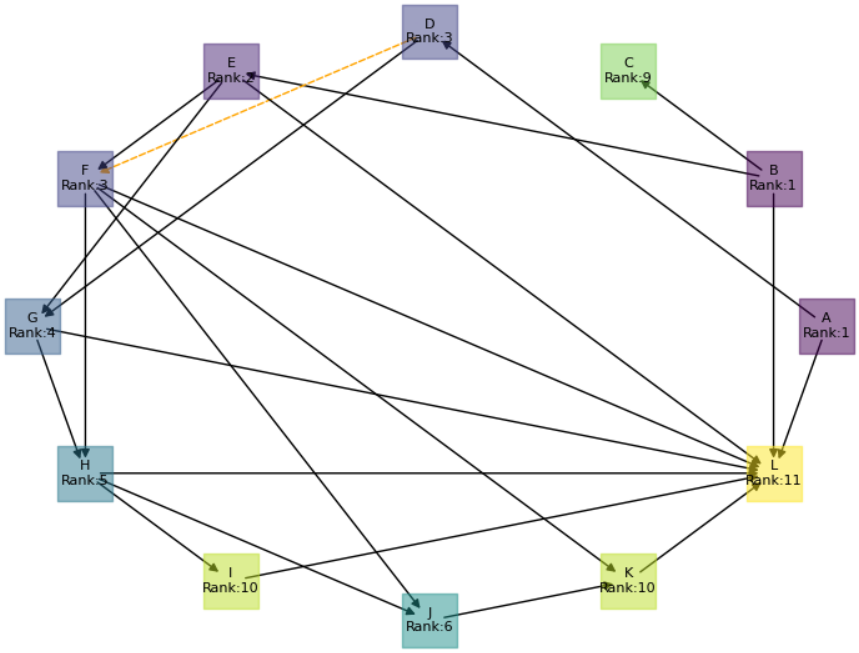

Figure 1: Two Highest Nodes (Rank 1)

More rarely, graphs have one top node, as shown in Figure 2.

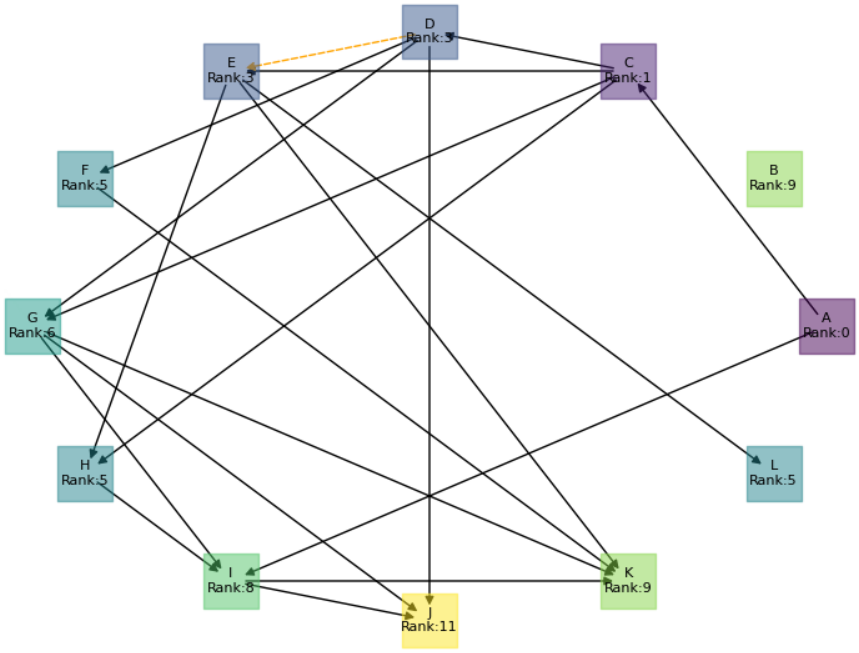

Figure 2: One Top-Most Node (Rank 0)

Both graphs shown have one edge that is directed against the hierarchy, shown as an orange dotted line.

3 Conclusions

Under the simple assumptions of Price's Model, growing social networks tend to organize in hierarchies. Humans are adaptable and creative, and we can't make a survey of all social arrangement throughout human existence, so we shouldn't make many conclusions about the essential features of social networks, but given the ubiquity of hierarchies in contemporary organizations, it is reasonable to conclude that under many conditions, people tend to organize in hierarchies. This shouldn't mean that we are fated to act under hierarchies. If members of an organization value equality, they must counter tendencies to hierarchy through education, training, and formal organizing techniques such as term limits and distributed leadership.

References

[1] Nikolaj Tatti. Faster way to agony: Discovering hierarchies in directed graphs. In *Machine Learning and Knowledge Discovery in Databases: European Conference, ECML PKDD 2014, Nancy, France, September 15-19, 2014. Proceedings, Part III 14*, pages 163–178. Springer, 2014.

[2] Wikipedia contributors. Graph theory — wikipedia, the free encyclopedia. `https://en.wikipedia.org/wiki/Graph_theory`.

[3] Wikipedia contributors. Price's model — wikipedia, the free encyclopedia. `https://en.wikipedia.org/wiki/Price%27s_model`.