The Effect of Market Regimes on Style Allocation

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Abstract

We analyse time-varying risk premia and the implications for portfolio choice. Using Markov Chain Monte Carlo (MCMC) methods, we estimate a multivariate regime-switching model for the Carhart (1997) four factor model. We find two clearly separably regimes with different mean returns, volatilities and correlations. In the High-Variance Regime, only value stocks deliver a good performance, whereas in the Low-Variance Regime, the market portfolio and momentum stocks promise high returns. Regime-switching induces investors to change their portfolio style over time depending on the investment horizon, the risk aversion and the prevailing regime, e.g., value investing seems to be a rational strategy in the High-Variance Regime, momentum investing in the Low-Variance Regime. An empirical out-of-sample backtest indicates that this switching strategy can be profitable.

1 Introduction

Equity style classes such as value stocks, growth stocks, small and large caps are popular from an academic and practical perspective. From an academic perspective, there is strong evidence that a portfolio of value stocks, small stocks and momentum stocks has historically earned a return above the return predicted by the CAPM (e.g. Rosenberg, Reid and Lanstein (1985), Banz (1981), Jegadeesh and Titman (1993)). To explain this finding, two different approaches have be suggested, a rational, multifactor asset pricing approach and an irrational approach based on anomalies (see e.g. Fama and French (1993)). From a practitioner's perspective, style factors are usually used to determine asset allocation (see e.g. Kao and Shumaker (1999)).

Overall, style decisions have a large impact on the performance of a portfolio. Carhart (1997), for example, finds that cross-sectional differences of mutual fund performance can almost completely be explained by style factors. Moreover, style premia seem to be, at least partially, predictable. For example, Fama and French (1998) document that the

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value premium (HML) and the size premium (SMB) might be related to economic growth and Chordia and Shivakumar (2002) document this for the momentum factor (UMD). Similarly, Kao and Shumaker (1999) and Lucas, von Dijk and Kloek (2002) show how style rotation based on macroeconomic variables can be implemented.

Time-varying expected returns and therefore, at least partially predictable returns can be regarded as a generally accepted fact in finance (see. e.g. Cochrane (1999)). As shown by Evans (1994), time-varying expected returns are due to two sources of variation, variation in factor loadings and variation in risk premia. A number of approaches have been suggested to model the dynamics of factor loadings, of expected returns, or of the joint dynamics (e.g. Evans (1994), Ferson and Harvey (1999), Ghysels (1998)).

Our goal is to analyze time-varying risk premia and the implications for asset allocation. To quantify the effect, we use a regime-switching model that enables us to allow for time-varying mean returns, volatilities and correlations. More specifically, our contribution consists of two main parts:

First, we formulate a data-generating process for risk premia that allows for asymmetric means, volatility and correlation. A regime-switching model reproduces asymmetric patterns, whereas standard models such as multivariate normal or multivariate GARCH models do not. Therefore, our approach of modelling time-varying risk premia is fundamentally different from traditional approaches (e.g. Evans (1994)) where time variation is captured by a linear function of state variables. In the approach pursued here, expected returns, volatilities, and correlations vary with the regime rather than with state variables. This extension of regime-switching models to multifactor asset pricing models has not been performed up to know. Existing applications solely focus on one factor models (for an comprehensive overview we refer to Kim and Nelson (1999)). Overall, our setup is comparable to the approach proposed by Ang and Bekaert (2002), although, the focus of our analysis is very different. While Ang and Bekaert (2002) focus on time-varying world market integration, i.e., on time-varying correlations, we focus on time-varying means, volatilities and correlations of risk premiums for different style classes.

The empirical results provide interesting new insights into the time series behavior of the market risk premium (MRP), the size premium (SMB), the value premium (HML) and the momentum premium (UMD). We find two clearly separable regimes with different means, volatilities and correlations. Regime 1, occurring approximately 25% of the sample period, is characterized by high volatility, low returns for MRP and UMD, medium returns for SMB, and high returns for HML. In contrast, Regime 2 is characterized by low volatility, high returns for the market portfolio and momentum stocks, and small returns for small caps and value stocks. Regime 2 occurs approximately 75% of the sample period and is more stable than Regime 1, i.e., the likelihood of switching from Regime 2 to Regime 1 is less than vice versa. To check for the stability of the model, we use a rolling scheme

and a recursive approach. In particular, we check whether the regime-switching model is able to classify observations at the end of the sample period. Similarly, the rolling schemes validates previous findings, i.e., in Regime 1 value stocks deliver the highest return and in Regime 2 the market portfolio and momentum stocks. Overall, estimated parameters are reasonably stable for Regime 2 and exhibit larger variation in Regime 1 than in Regime 2.

Second, we analyze the implications for asset allocation from a strategic and tactical point of view. From a strategic perspective, we numerically solve and develop intuition on the style allocation problem in the presence of regime switches for investors with constant relative risk aversion (CRRA) preferences. From a tactical perspective, we empirical test tactical asset allocation strategies based on the regime-switching model. Most existing approaches have focussed on switching within one style class (e.g. switching between growth and value stocks). In contrast, we show that switching between different style classes is a promising strategy. In particular, value investing during bear markets and momentum investing during bull markets has historically earned a superior performance. An out-of-sample analysis indicates that the findings are robust, i.e., the regime-switching model assigns properly the prevailing regime at the end of the sample period, the results are similar for different subperiods, and the trading strategies seem to remain profitable after accounting for transaction costs. To our knowledge, this is the first paper analyzing the use of regime-switching models for tactical style allocation.

The outline of this paper is as follows. We start in section 2 by giving a literature overview about the econometrics of regime-switching models, existing applications in finance, and style investing. Then, we continue by formulating the general asset allocation problem in Section 3, and show how to numerically solve the problem with regime-switching. In Section 4, we present our empirical results and discuss our results in the light of existing literature. Section 5 concludes.

2 Literature Overview

This paper is related to three branches of the existing literature. The first branch is the issue of parameter estimation for regime-switching models. The classical reference for regime-switching models is Hamilton (1989). For an extensive overview concerning the econometrics issues of regime-switching models and an overview about empirical evidence, we refer to Kim and Nelson (1999). One of the first papers in financial econometrics that estimates time-varying integration of single countries to the world market is Bekaert and Harvey (1995).

The second branch of literature, portfolio choice and regime-switching, analyses the effects of regime-switching on asset allocation. Overall, the main findings are that regime-

switching induces a change in the asset allocation depending on the investment horizon and depending on the current regime. One of the main references is Ang and Bekaert (2002). In their paper, they analyze dynamic asset allocation with regime shifts in an international context. Recent contributions include Graffund and Nilsson (2003), Bauer, Haerden and Molenaar (2003), Ang and Bekaert (2004), and Guidolin and Timmermann (2005).

Finally, the third branch analyses the issue of style investing, i.e., the asset allocation in comparison to specific style factors such as momentum stocks, growth and value stocks, and small and large caps. Barberis and Shleifer (2003) study asset prices in an economy where some investors categorize risky assets into different styles and move funds among these styles depending on their relative performance. From an empirical point of view, the contributions by Kao and Shumaker (1999), Arshanapalli, Ouville and Nelson (2004), and Cooper, Gutierrez and Hameed (2004) are related to this paper, because they also analyze time variation in returns for different investment styles.

2.1 Regime Switching Models in Finance

Hamilton (1994) and Kim and Nelson (1999) give an overview about the econometrics of state-space models with regime-switching. From an econometric point of view, the main problem in estimating regime-switching models is the unobservability of the prevailing regime. Two different approaches have been suggested: a classical maximum likelihood (based on filters such as the Kim filter or on the expectation maximization algorithm) and a Bayesian approach (based on numerical Bayesian methods such as the Gibbs sampler and Markov Chain Monte Carlo methods). Bayesian econometrics is for regime-switching particularly well suited because Bayesian approaches require only the specification of conditional distributions and not of the joint distribution. Kim and Nelson (1999) provide an overview of possible applications to finance. Depending on the purpose of an particular analysis, regimes are usually separated by differences in the mean, volatility or different behavior of volatility (such as different factor loadings for an ARCH model). Object of the analysis are usually univariate time series such as a representative stock index or an interest rate. In a multivariate setting, Bekaert and Harvey (1995), for example, estimate a multivariate regime-switching model to explain time varying would market integration.

2.2 Portfolio Choice and Regime Switching

A number of authors analyze the implications of regime-switching in portfolio selection. Ang and Bekaert (2002) analyze international asset allocation with regime shifts. The starting point of their paper is time-varying correlation between different equity markets, i.e., in bad times correlations and volatilities increase in comparison to good times and therefore, the investment opportunity set is stochastic. In the empirical part, they

assume a two state model with Markov switching and constant transition probabilities. For parameter estimation, they use a Bayesian procedure similar to Hamilton (1989) and Gray (1996). Overall, there are always relatively large benefits of international diversification, although the optimality of the home-biased portfolios cannot always be rejected statistically. The costs of ignoring regime-switching are very high if the investor is allowed to switch to a cash position. If the investment universe is limited to equities, costs of ignorance are lower. With respect to hedging demands, they find that intertemporal hedging demands under regime-switching are economically negligible and statistically insignificant. Similar, Ang and Bekaert (2004) find that for a global all-equity portfolio, the regime-switching strategy dominates static strategies in an out-of-sample test. In a persistent high-volatility market, the model tells the investors to switch primarily to cash.

2.3 Style Investing

According to Kao and Shumaker (1999), "Style" is broadly defined as any system of classification by market segment that have distinguishing characteristics. Given the large number of possible criteria which can be used to separate investments strategies, academics and practitioners have developed sets of common characteristics of "factors" to characterize style. Beta, size, value, growth, quality, momentum, leverage, and even sectors are commonly used criteria to differential investment styles. Most frequently used for classification in academic literature is the Fama and French (1993) three-factor model (market risk, size, value vs. growth) and the momentum factor by Carhart (1997). With respect to the equity premium a vast number of studies addressing unconditional and conditional aspects have been published (see e.g. Evans (1994), Cochrane (1999), Fama and French (2002)).

The importance of momentum for stock returns has first been documented by Jegadeesh and Titman (1993) and is still an active research area. Overall, most studies indicate that momentum trading is a robust strategy (see e.g. Jegadeesh and Titman (2001), Korajczyk and Sadka (2004)), also after trading costs. Momentum investing is closely related to the 52-Week High investing (George and Hwang (2004)) and is mainly due to industry effects (Moskowitz and Grinblatt (1999)). While the size effect and the value premium seems to predict future economic growth (Liew and Vassalou (2000)), this is not the case for the momentum premium (Griffin, Ji and Martin (2003)). As shown by Badrinath and Wahal (2004), momentum investing is a very popular investment strategy followed by institutional investors.

Similar to momentum stocks, value stocks have historically shown an abnormal performance in almost any country (Fama and French (1998)). Cohen, Polk and Vuolteenaho (2003) show that the expected return on value stocks depends on the cross-sectional dispersion of the market-to-book ratios. The value spread has shown strong seasonalities,

i.e., value stocks have a higher return in the first quarter and growth stocks in the fourth quarter (Kao and Shumaker (1999)) and analysts are very likely to underestimate earnings of value companies (Doukas, Kim and Pantzalias (2002)). With respect to the size effect originally discovered by Banz (1981) it is doubtful how robust these findings are (see e.g. Berk (1997), Knez and Ready (1997)).

Style rotation and equity style timing has recently been addressed by a number of authors. Kao and Shumaker (1999) analyze the opportunities for equity style timing. Based on the Fama and French (1993) factors, using recursive partitioning (regression and classification trees), and macroeconomic factors (term spread, real bond yield, corporate credit spread, high-yield spread, estimated GDP growth, earnings-yield gap, CPI) they try to predict future differences in style returns. They find that timing strategies in the US market based on asset class and size have historically provided more opportunity for outperformance than a timing strategy based on value and growth. An extended analysis shows that return differences between value and growth stocks have a straightforward, intuitive basis. The key insight, from the point of view of this article, is that there is strong evidence of cyclical time variation of style factors and that the variation seems to be related to macroeconomic factors. Similarly, Arshanapalli et al. (2004) address the question whether size, value and momentum are related to recession risk. Their main finding is that an increase in the conditional variance for HML and UMD coincidence with a higher recession probability within a time horizon of six months.

Levis and Liodakis (1999) investigate the opportunity for style rotation in the United Kingdom. They implement and test a style rotation model based on OLS regressions and a Logit model. As independent variables, they use macroeconomic factors and valuation ratios (such as inflation, short-term interest rate, term spread, exchange rate, market return, and dividend yield spreads). Similar, Lucas et al. (2002) analyze different models for style rotation in the US market and find that business cycle oriented strategies deliver a better performance than pure statistical approaches.

3 Style Investing with Changes in Regimes

In this section, we describe the general portfolio choice problem and the parameter estimation.

3.1 Portfolio Choice

The general asset allocation nesting also mean variance portfolio choice can be stated as follows. A buy-and-hold investor facing at time t a T-month horizon and an investment opportunity set consisting of N assets maximizes his expected end of period utility over terminal wealth $U(W_T)$. Formally,

$$\max_{Q_t} \mathbb{E}_t \left[U(W_T) \right] \tag{1}$$

where α_t denotes the vector of portfolio weights at time t which must sum up to one. Next periods wealth, W_{t+1} , is given by $W_{t+1} = R_{t+1}(\alpha_t)W_t$. The gross return on the portfolio, $R_{t+1}(\alpha_t)$, is

$$R_{t+1}(\alpha_t) = \sum_{j=1}^{N} \exp(y_{t+1}^j) \alpha_t^j$$
 (2)

where y_{t+1}^j is the logarithmic return on asset j from time t to t+1 and a_t^j is the proportion of the jth asset in the investor's portfolio at time t. We use CRRA utility

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma} \tag{3}$$

with γ the investor's coefficient of risk aversion. The CRRA utility function is chosen because it can be regarded as the standard benchmark and the results can be compared to other findings.

We concentrate on the investment problem of an US investor and ignore intermediate consumption and dynamic asset allocation. The investment decision is taken at time 0 for the whole investment horizon. In this paper, we do not address the general market equilibrium, so the investor is not necessarily the representative agent. We also do not consider the asset allocation faced by non-US investors.

The buy-and-hold investor chooses at time t the portfolio weights α_t^* which maximizes his utility:

$$\alpha_t^* = \arg\max_{\alpha_t} \mathbb{E}_t \left[\frac{W_T^{1-\gamma}}{1-\gamma} \right] \tag{4}$$

Up to now, no specific data generating process for the asset returns y has been assumed and therefore, the setup is fairly general. Samuelson (1969) shows that, if the returns are iid and under CRRA utility, portfolio weights are constant. Therefore, it this special case, the multiperiod solution is identical to the myopic solution. If returns are not iid, a hedging component might arise protecting the investor's against unfavorable changes in the investment opportunity set (Merton (1971)).

We introduce K different regimes s into the data generating process. The return in a specific period y_{t+1} depends on the regime s_t prevailing at that time. The regimes s_t follow a Markov Chain where the transition probability of going from regime i at time t to regime j at time t+1 are denoted by $p_{ij,t} = p(s_{t+1} = j|s_t = 1)$. $f(y_{t+1}|s_{t+1})$ denotes the probability density function of y_{t+1} conditional of regime s_{t+1} . In our model, $f(y_{t+1}|s_{t+1})$ is a multivariate normal distribution and transition probabilities are constant. Therefore,

being conditional on the regime in the previous period, s_t , the distribution of the return in period t+1, y_{t+1} , is a mixture of normals. The probability density function of y_{t+1} conditional on s_t , $g(y_{t+1}|s_t)$, is given by

$$g(y_{t+1}|s_t = i) = \sum_{i=1}^{K} p_{ij,t} \cdot f(y_{t+1}|s_{t+1} = j)$$
(5)

To compute optimal portfolio weights, we use standard numerical integration algorithms, i.e., Gaussian quadrature.

3.2 Parameter Estimation

For parameter estimation, we use a Markov Chain Monte Carlo (MCMC) approach. We refer to Kim and Nelson (1999) for an introduction to the estimation of regime-switching models with numerical Bayesian methods. The building blocks of our model are two equations: one for the evolution of the state process across time and one for the return distribution conditional on the prevailing state of nature. The setup and implementation follows the approach described in Congdon (2003). The software package "Bayesian Analysis Using Gibbs-Sampler" (WinBUGS) has been used for parameter estimation.

The state process is assumed to evolve according to a Markov switching process. The transition process depends on the number of regimes K and the transition probability $P(s_{t+1}|s_t=i)$

$$s_{t+1}|s_t \sim Mult(K, P(s_{t+1}|s_t = i))$$
 (6)

The multinomial distribution is denoted by Mult.

The second equation describes the return process in the single states. The return for the four factors (MRP, SMB, HML and UMD) is assumed to follow a multivariate normal distribution \mathcal{N}

$$r_t|s_t \sim \mathcal{N}(\mu_i|\Sigma_i)$$
 (7)

where μ_i denotes the vector of mean returns in state i and Σ_i the variance-covariance matrix for state i.

For the usage of MCMC, we additionally need a number of rather technical definitions for the priors and hyperparameters. The following distributional assumptions have no material impact on the results of the empirical part because all inital values have been choosen to be uninformative.

First, we parameterize the transition matrix. The elements of the transition matrix P are given by

$$P(i,j) = \frac{Px(i,j)}{\sum_{j=1}^{K} Px(i,k)}$$
 (8)

where Px(i,j) is given by

$$Px(i,j) \sim G(\underline{\mathbf{a}},\underline{\mathbf{b}})$$
 (9)

G denotes the gamma distribution with parameters \underline{a} and \underline{b} . It is important to stress that the only purpose of this parameterization is to increase computational efficiency and that it has no effect on the results. The values of \underline{a} and \underline{b} have been chosen to be uninformative. Both value have been set to 1.

Second, we parameterize the return properties in the single states. The mean $\mu_i(k)$ of a factor k in state i is drawn from an uninformative normal distribution N

$$\mu_i(k) \sim N(0, \sigma) \tag{10}$$

where for σ a high value has been chosen to incorporate uninformative prior information, i.e., σ^2 was set to 1000.

For the variance-covariance matrix Σ_i we use an inverse Wishart distribution, denoted as Wishart, as prior to ensure positive definitness as usual in Bayesian multivariate analysis

$$\Sigma_i^{-1} \sim Wishart(\underline{\mathbf{R}}, v)$$
 (11)

where the prior matrix \mathbf{R} was set to a one diagonal matrix and v denotes the degree of freedom and reflects the prior degree of belief in the prior estimate. For computational purposes, v was set to 4, reflecting a very low degree of confidence in the prior estimate. To ensure proper implementation, a Monte Carlo study has been performed to verify that the approach is able to recover the underlying data generating process correctly. Moreover, convergence of the MCMC sampler was ensured.

4 Results

In this section, we present the results of the analysis. After some descriptive statistics, we turn to the estimated parameters for the different regimes and analyze the stability of parameters. Then, we address the implications for portfolio choice.

4.1 Data

For the analysis, we use the common risk factors as introduced by Fama and French (1993) and Carhart (1997). The data for the market portfolio (MRP), the high-minus-low (HML)

factor, the small-minus-big (SMB) factor, the momentum factor (UMD) and the risk-free rate is from the Fama and French data library. The sample period starts in January 1927 and ends in December 2004.

The HML and SMB factors are constructed using six value-weighted portfolios formed on size and book-to-market. These portfolios are the intersections of two portfolios formed on size and three formed on the ratio of book-to-market equity. The break point for dividing stocks between large and small is the median value of the market capitalization on the New York Stock Exchange at mid-year. The book-to-market equity ratios are based on those prevailing at the end of the previous year. The break points are the 30th and the 70th percentiles. The SMB (small minus big) is created by substracting the average return on the three large portfolios from the average return on the three small portfolios: SMB = 1/3*(Small Value + Small Neutral + Small Growth) - 1/3*(Big Value + BigNeutral + Big Growth). HML (high minus low) is created by substracting the average return on the two growth portfolios from the two value portfolios: HML = 1/2*(Small)Value + Big Value) - 1/2* (Small Growth + Big Growth). The momentum portfolio UMD (up minus down) is derived from the six value-weighted portfolios formed on size and past performance during months t-2 through t-12. The portfolios are formed monthly and are the intersections of the size portfolios and the past performance portfolios. The monthly past performance portfolios breakpoints are the 30th and the 70th percentiles. UMD is calculated as the average return of the two high past performance portfolios minus the average return on the two low past return portfolios: UMD = 1/2*(Small High + BigHigh) - 1/2* (Small Low+Big Low).

Table 1 displays the descriptive statistics for the data used. From a risk-return perspective, market and momentum premium are comparable. The mean monthly return for the market risk was 0.65% with a standard deviation of 5.50% and for the momentum premium, 0.75% per monthy with a volatility of 4.73%. In contrast, the returns for SMB and HML have been less volatility (3.38% and 3.57%), but also the average return was much lower (0.18% and 0.48%).

From the viewpoint of this paper, it is important to note that the Jarque-Bera test indicates that all risk factors are not normally distributed. Since regime-switching models account for non-normality by using a mixture-of-normals approach, they deliver a more accurate way of modelling the dynamics and the distribution of the factors than models using only one normal distribution

4.2 Regimes in Style Premia

Table 2 displays the estimated parameters with and without regime-switching. Overall, our analysis shows two clearly separated regimes. In Regime 1, mean expected returns are low for the market risk (MRP), for the size effect (SMB) and for momentum factor (UMD).

Table 1: Descriptive analysis

The table displays the descriptive statistics for the data used in this analysis. The data are from the Fama and French data library. The sample starts in January 1927, ends in December 2004, and has monthly frequency. The momentum premium (UMD) showed the highest average return, closely followed by the market risk premium (MRP).

	MRP	SMB	HML	UMD	RF
Mean	0.647	0.184	0.481	0.751	0.305
Median	0.980	0.025	0.240	0.935	0.270
Maximum	38.180	38.040	35.350	18.380	1.350
Minimum	-29.030	-21.490	-11.480	-50.920	-0.060
Standard deviation	5.489	3.375	3.574	4.731	0.257
Skewness	0.213	1.600	2.037	-2.997	0.999
Kurtosis	10.627	23.897	17.635	30.861	4.112
Observations	936.000	936.000	936.000	936.000	936.000
Percentile (10%)	-5.308	-3.359	-3.140	-3.457	0.020
Percentile (25%)	-2.165	-1.590	-1.350	-0.805	0.090
Percentile (75%)	3.780	1.845	1.840	2.930	0.450
Percentile (90%)	6.028	3.598	4.138	4.926	0.640
Jarque-Bera test for normality	2262.207	17346.256	8955.209	31527.381	202.813
Jarque-Bera test (p-value)	0.000	0.000	0.000	0.000	0.000

Only value stocks (HML) show high returns. Since all risk factors are very volatile, we refer to Regime 1 as the High-Variance Regime. In contrast, in Regime 2, market risk and momentum stocks have a high return while small stocks and value stocks have a low return. Since variance is for all risk factors much smaller than in Regime 1, we refer to Regime 2 as the Low-Variance Regime. The estimated parameter for the unconditional model without regime-switching are between the estimated parameters for Regime 1 and Regime 2.

Over the whole sample, the High-Variance Regime, i.e., Regime 1, occurred approximately 25.2% of the time (235 out of 936 months). Consequently, Regime 2, the Low-Variance Regime, occurred approximately 74.8% of the time (700 out of 936 months). Beside the different frequency of occurrence, the transition probabilities are very different for both regimes. While the Low-Variance Regimes shows a high degree of persistency, the High-Variance Regime is relative unstable. Periods of uncertainty seem to disappear relatively fast and periods of certainty seem to be rather stable. In particular, the Low-Variance Regime has a probability of 91% of persistency. If we are in a particular month in a Low-Variance Regime, there is a 91% chance that the next month is also in the Low-Variance Regime. Consequently, there is 9% chance that the next month is in the High-Variance Regime. In contrast, for the High-Variance Regime, the probability of staying in the same class is with 72% much lower. There is a 28% chance of switching back to the Low-Variance Regime.

Mean returns, volatility and correlations between the risk factors are substantially

Table 2: Estimated parameters with and without regime-switching

The table shows the estimated parameters for the model without regime-switching and with regime-switching. Mean returns and volatility have been annualized. Regime 1 is characterized by high volatility and a low return for the market risk (MRP), for small stocks (SMB) and for momentum stocks (UMD) whereas value stocks (HML) show a high return. In Regime 2, volatility is rather small, and the return for the market portfolio (MRP) and momentum stocks (UMD) are high, whereas small stocks (SMB) and value stocks (HML) stocks display a low return. Regime 1 occurs 25% of the time and Regime 2 occurs 75% of the time. The transition probabilities show that the duration of Regime 1 is rather small, i.e., a regime switch within a couple of months is likely, whereas the duration of regime 2 is rather long. * denotes a value significant on the 95% level and ** a value significant on the 99% level. Standard errors are in parenthesis.

Unconditional MRP SMB HML UMD Mean 7.77** (2.17) 2.21 (1.33) 5.77** (1.41) 9.00** (1.87) Volatility 19.03** (0.44) 11.70** (0.27) 12.38** (0.29) 16.39** (0.38) Correlation MRP 0.32** (0.03) 1 1 1 Correlation HML 0.18*** (0.03) -0.02** (0.03) -0.38** (0.03) 1 Correlation UMD -0.34** (0.03) -0.22** (0.03) -0.38** (0.03) -1 Regime 1 8 0.40 (7.48) 3.09 (4.64) 15.16* (5.09) -1.27 (7.04) Volatility 31.80** (1.69) 19.67** (1.04) 21.25** (1.11) 29.72** (1.61) Correlation MRP 1 0.29** (0.06) 0.14* (0.06) 1 1 Correlation SMB 0.37** (0.06) 0.14* (0.06) -0.46** (0.05) 1 Regime 2 1 1.92 (1.05) 2.62* (1.07) 12.44** (1.14) Volatility 12.01** (0.38) 7.33** (0.25) 7.13** (0.28) 7.78** (0.31) Correlation MRP 1 0.20*					
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation MRP	1			
Correlation UMD -0.34** (0.03) -0.22** (0.03) -0.38** (0.03) 1 Regime 1 Mean 0.40 (7.48) 3.09 (4.64) 15.16* (5.09) -1.27 (7.04) Volatility 31.80** (1.69) 19.67** (1.04) 21.25** (1.11) 29.72** (1.61) Correlation MRP 1 0.37** (0.06) 1 21.25** (1.11) 29.72** (1.61) Correlation SMB 0.37** (0.06) 0.14* (0.06) 1 1 1 Correlation HML 0.29** (0.06) 0.14* (0.06) -0.46** (0.05) 1 1 Regime 2 Mean 10.22** (1.70) 1.92 (1.05) 2.62* (1.07) 12.44** (1.14) 1 Volatility 12.01** (0.38) 7.33** (0.25) 7.13** (0.28) 7.78** (0.31) Correlation MRP 1	Correlation SMB	0.32**(0.03)	1		
Regime 1 Mean 0.40 (7.48) 31.80** (1.69) 3.09 (4.64) 15.16* (5.09) 21.25** (1.11) -1.27 (7.04) 29.72** (1.61) Correlation MRP Correlation SMB Correlation HML Correlation UMD 0.37** (0.06) 1 0.14* (0.06) 1 0.15** (0.08) 1 0.14* (0.06) 1 0.15** (0.08) 1 0.07* (0.04) 1 0.15** (0.05) 1 0.07* (0.04) 1 0.15** (0.05) 1 0.07* (0.04) 1 0.15** (0.05) 1 0.07* (0.06) 1 0.05* (0.05*) 1 0.08* (0.06) 1 0.05* (0.0	Correlation HML	0.18**(0.03)	0.08*(0.03)	1	
$\begin{array}{ c c c c c }\hline \text{Mean} & 0.40 & (7.48) & 3.09 & (4.64) & 15.16* & (5.09) & -1.27 & (7.04) \\ \hline \text{Volatility} & 31.80** & (1.69) & 19.67** & (1.04) & 21.25** & (1.11) & 29.72** & (1.61) \\ \hline \text{Correlation MRP} & 1 & & & & & & & & & \\ \hline \text{Correlation SMB} & 0.37** & (0.06) & 1 & & & & & & & \\ \hline \text{Correlation HML} & 0.29** & (0.06) & 0.14* & (0.06) & 1 & & & & \\ \hline \text{Correlation UMD} & -0.50** & (0.05) & -0.28** & (0.06) & -0.46** & (0.05) & 1 \\ \hline \text{Regime 2} & & & & & & & & & \\ \hline \text{Mean} & 10.22** & (1.70) & 1.92 & (1.05) & 2.62* & (1.07) & 12.44** & (1.14) \\ \hline \text{Volatility} & 12.01** & (0.38) & 7.33** & (0.25) & 7.13** & (0.28) & 7.78** & (0.31) \\ \hline \text{Correlation MRP} & 1 & & & & & & \\ \hline \text{Correlation SMB} & 0.20** & (0.04) & 1 & & & & \\ \hline \text{Correlation HML} & -0.07 & (0.05) & -0.07 & (0.04) & 1 & & & \\ \hline \text{Correlation UMD} & 0.15** & (0.05) & -0.01 & (0.05) & -0.03 & (0.05) & 1 \\ \hline \text{Number of states in} & & & & & & & \\ \hline \text{Regime 1} & 235.50 & (16.77) \\ \hline \text{Regime 2} & 700.50 & (16.77) \\ \hline \text{Transition matrix} & P(,1) & P(,2) \\ \hline P(1,) & 0.72 & (0.05) & 0.28 & (0.05) \\ \hline \end{array}$	Correlation UMD	-0.34** (0.03)	-0.22** (0.03)	-0.38** (0.03)	1
Volatility 31.80** (1.69) 19.67** (1.04) 21.25** (1.11) 29.72** (1.61) Correlation MRP 1 20.37** (0.06) 1 20.37** (0.06) 1 20.37** (0.06) 1 20.28** (0.06) 1 20.28** (0.06) 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 2 2 2 2 1 2 2 2 2 1 2 2 2 1 2 2 2 1 2 3 1 3 2 3 1 3 2 3 1 3<	Regime 1				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Mean	0.40 (7.48)	3.09 (4.64)		-1.27 (7.04)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Volatility	31.80** (1.69)	19.67** (1.04)	21.25** (1.11)	29.72** (1.61)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation MRP	1			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation SMB	0.37**(0.06)	1		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation HML			1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation UMD	-0.50** (0.05)	-0.28** (0.06)	-0.46** (0.05)	1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Regime 2				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Mean	10.22** (1.70)	1.92 (1.05)	2.62* (1.07)	12.44** (1.14)
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Volatility		7.33** (0.25)	7.13** (0.28)	7.78** (0.31)
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation MRP	1			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Correlation SMB	0.20**(0.04)	1		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Correlation HML			1	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Correlation UMD	0.15**(0.05)	-0.01 (0.05)	-0.03 (0.05)	1
Regime 2 $700.50 (16.77)$ Transition matrix $P(,1)$ $P(,2)$ $P(1,)$ $0.72 (0.05)$ $0.28 (0.05)$	Number of states in				
Transition matrix $P(,1)$ $P(,2)$ $P(1,)$ $0.72 (0.05)$ $0.28 (0.05)$	Regime 1				
P(1,) 0.72 (0.05) 0.28 (0.05)	Regime 2	700.50 (16.77)			
	Transition matrix	P(,1)	P(,2)		
P(2,) 0.09 (0.02) 0.91 (0.02)	P(1,)	$0.72 \ (0.05)$	$0.28 \; (0.05)$		
	P(2,)	0.09 (0.02)	$0.91 \ (0.02)$		

different between the two regimes and in comparison to the unconditional model. In Regime 1, the excess return on the market portfolio of 0.4% p.a. close to zero. Similarly, momentum stocks pay a small negative return of -1.27% and small stocks pay a slightly higher return of 3.09%, not statistically different from 0. Only value stocks offer a high return of 15.16%. In comparison to the unconditional model (without regime-switching), only value stocks offer a higher return. However, in Regime 2, the picture is different. Both market risk and momentum stocks offer a high return of 10.2% and 12.4% p.a., respectively. Value stocks pay, in contrast to Regime 1, a low return of 2.6%. Small stocks pay in both regimes a fairly low rate of return, 3.1% in Regime 1 and 1.9% on Regime 2.

Volatility is in Regime 1 approximately 10 percentage points higher than in the unconditional model for each risk factor. In comparison to Regime 2, volatility in Regime 1 is about 2.6 to 3.8 times higher. In particular, for MRP, the volatility increases by a factor of 2.6 from 12.0% p.a. to 31.8% p.a., for SMB by a factor of 2.7 from 7.3% to 19.7%, for HML by a factor of 3 from 7.1% to 21.3%, and for UMD by 3.8 from 7.8% to 29.7%.

Beside mean returns and volatilities, also correlations are affected. In general, Regime 1 is characterized by high correlations and Regime 2 by low correlations. This findings is an indication that in highly volatile market, correlations increase and vice versa. In Regime 1, all correlations are, on a 5% level, statistically different from 0. While the momentum factor is strongly negatively correlated with all other risk factors, all other factors (MRP, SMB, HML) exhibit positive correlations. In Regime 2, only two significant correlations can be found, between MRP and SMB and between MRP and UMD.

Exhibit 1 displays the estimated probability of being in the High-Variance Regime 2. Shaded areas show NBER recessions in the sample period. The time period between 1928 and 1943 is characterized by a dominance of the High-Variance Regime, i.e., Regime 1, and rare regime switches. Between 1943 and 1969, the development was rather smooth. We estimate that the financial markets have been in Regime 1 most of the time, except for four short switches in 1949, 1957, 1962 and 1966. Starting in 1969, financial markets faced a period of instability lasting till 1991, characterized by frequent switches to Regime 1. Between 1991 and 1997, risk premia again became less volatile. This phase of relative stability ended by 1997. By the end of 2004, the analysis indicates being in the Low-Variance Regime.

The robustness of the results against alternative specifications has been tested extensively. Overall, the results are robust against alternative specifications of prior distributions and of the likelihood. Our further analysis focusses on two aspects. First, we analyze the model performance using an iterative approach, i.e., we extent the sample size by one month iteratively to check whether the approach is able to classify the prevailing regime properly. Second, we analyze an univariate version of the model, i.e., we allow for

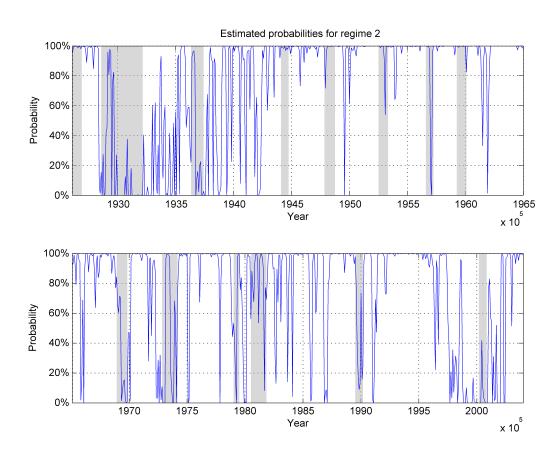


Figure 1: Estimated probabilities for Regime 2

The Figure shows the estimated probabilities for Regime 2. The shaded areas show NBER recessions. We refer to Regime 2 as the Low-Variance Regime and to Regime 1 as the High-Variance Regime. In the sample period, in Regime 1, the mean return for the market risk fator (MRP), the small-size factor (SMB) and the momentum factor (UMD) were statistically not different from 0, whereas the return on value stocks (HML) was high. In Regime 2, the market risk factor and the momentum factor showed a high return, whereas the return on the remaining two risk factors (SMB and HML) was close to 0. In the sample period, the High-Variance Regime occured approximately 25% of the time and the Low-Variance Regime 75% of the time.

independent switching across the factors.

4.3 Out-of-Sample Analysis and Rolling Scheme

In this subsection, we analyze parameter stability by using an iterative and rolling procedure.

Figure 2 shows the estimated parameters. For the out-of-sample analysis, we start with a data sample of 40 years, from 1927 to 1967, estimate the parameters for the regime-switching model, and extend the sample size by one month iteratively until the full sample is included. As previously, Regime 1 is the High-Variance Regime and Regime 2 the Low-Variance Regime.

Overall, the out-of-sample analysis shows a high degree of parameter stability for the market risk premium (MRP), the value premium (HML) and the momentum factor (UMD) for both regimes. The recursive approach shows that the market risk premium in Regime 2 has declined slightly from 1.03% to 0.85%, the value premium (HML) in Regime 2 fluctuated slightly around 0.2% per month. In Regime 1, the MRP has fluctuated around 0.1%, the HML factor around 1.4% with a slight downward trend at the end of the sample period, and the UMD factor fluctuated around -0.3%. In contrast, the size factor (SMB), shows for the High-Variance Regime, i.e., Regime 1, a trend from about 1.03% per month in 1967 to about 0.27% in 2004, while in Regime 2 there is no visible trend.

With respect to the transition probabilities, we find that the persistence probabilities, i.e., the probabilities of staying in the same regime, fluctuated for Regime 2 form 88% to 93% and for Regime 1 from 63% to 77%.

However, for practical applications of regime-switching, the correct classification of the current prevailing regime might be crucial. Therefore, we compare the regime classification obtained by the iterative approach and by the full sample approach. In Figure 2 we show the difference in estimated probabilities for the last month of the iterative approach and the penultimate month of the iterative approach. Our analysis indicates that the regime-switching approach has some slight problems to classify the most current regime properly, but for the previous months only very few observations have been missclassified. Using a threshold of 0.5, i.e., we classify all regimes as Regime 2 with an estimated probabilities larger than 0.5 for Regime 2, 8% of the observations have been classified wrong (37 months out of 456 months). For the penultimate month 3.5% (16 out of 456) of the observations are missclassified and for the prevailing regime two months before, the missclassification rate is 2.4% (11 out of 456).

Figure 3 shows the estimated parameters based on a rolling scheme. The figure shows the estimated risk premia in % per month for each of the four risk factors. The rolling scheme has a window size of 360 observations (30 years).

The rolling scheme validates previous findings. The value premium (HML) is in Regime

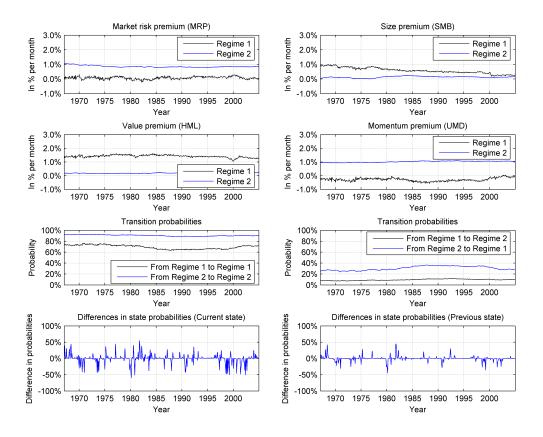


Figure 2: Recursive estimation of parameters

The Figure shows the results from an iterative procedure. We start with a sample length of 40 years, i.e., in 1967 and extend the sample iteratively by one month until we reach the total sample by the end of 2004. The subplots show the estimated risk premia for the four risk factors (in % per month), the estimated transition probabilities, and the difference in estimated state probabilities between a recursive procedure and the full sample analysis to detect miss-classification and to analyse whether Regime-switching models are able to assign correct Regime also in an out-of-sample procedure. As previously, Regime 1 is the High-Variance Regime and Regime 2 is the Low-Variance Regime.

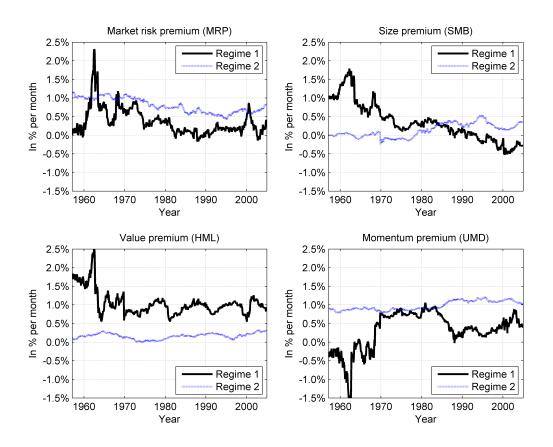


Figure 3: Estimated parameters of the regime-switching model using a rolling window

The figure shows the estimated risk premia in % per month for the four risk factors using a rolling window of 30 years (360 data points). The figure shows that the main findings of the full-sample analysis remain also valid in a rolling scheme. The value premium (HML) in Regime 2, the Low-Variance Regime, is always lower than in Regime 1. In contrast, the market premium (MRP) and the momentum premium (UMD) is in Regime 1, in general, higher than in Regime 2.

1, the High-Variance Regime, always higher than Regime 2. The difference of the value premium in Regime 1 and Regime 2 fluctuates over time with an average of about 0.9% per month, a minimum of 0.3% per month and a maximum of 2.4% per month.

For the momentum premium (UMD), the rolling scheme also validates the full sample analysis. In Regime 1, the value premium is, in general, lower than in Regime 2. Similarly to the value premium in Regime 1, the momentum premium fluctuates in Regime 1 considerable over time. Before 1970, the average difference of momentum premia in Regime 1 and Regime 2 was about 1.2%. Between 1970 and 1985, this difference disappeared and re-emerged by 1985.

For the size premium (SMB), the rolling scheme delivers similar results as the recursive scheme. In Regime 1, the size premium shows a strong downward trend from a positive value of around 1% to a current low of about -0.3%. During market turmoils small stocks seems to have changes their behavior. While in the early sample, small stocks delivered a high performance if the market was volatile, in the late sample, large caps delivered a high performance if the market was volatile. Regime 2, the Low-Variance Regime, shows the reverse pattern. The premium on small stocks increased slightly during the sample period from about 0.0% at the early sample to about 0.3% at the end of the sample.

Similarly, the behavior of the market risk premium has changed strongly over time in Regime 2. During volatility markets, the return on the market portfolio fluctuated between 0% and 2%. In particular, for the sample starting in 1933 and ending in 1963, the sample period after the great depression, and for the sample starting in 1971 and ending in 2001, the sample period including the internet bubble, the payoff in Regime 1 was higher than in Regime 2. For Regime 2, the rolling scheme shows a slight downward trend.

4.4 Univariate Regime Switching

In this section, we allow for univariate regime-switching. In the previous section, we assumed that the switching for all risk factors is governed by one Hidden Markov Model. In this section, we present the results with independent regime-switching, i.e., the evolution of each single risk factor is governed by a separate Markov process. Overall, the results are very similar to the previous findings.

Table 3 shows the estimated parameters for the model with univariate regime-switching. As previously, Regime 1 is the High-Variance Regime and Regime 2 the Low-Variance Regime. In Regime 1, small and value stocks show a strong positive return whereas the return on the market portfolio and for momentum stocks is negative. In contrast, in Regime 2, the return on the market portfolio is positive with 11.42% and for momentum stocks with 10.78%. Small stocks and value stocks show a return of 1.26% and 3.11%, respectively. Overall, this results are similar for the univariate and for the multivariate

Table 3: Estimated parameters for univariate regime-switching

The table shows the estimated parameters for the model with univariate regime-switching. Mean returns and volatilites have been annualized. Regime 1 is characterized by high volatility and a low return of the market risk (MRP), and for momentum stocks (UMD) whereas small stocks (SMB), and value stocks (HML) show a high return. In Regime 2, volatility is rather small, and the returns for the market portfolio (MRP) and momentum stocks (UMD) are high, whereas small stocks (SMB) and value stocks (HML) stocks display a low return. Regime 1 occurs between 10.7 % and 18.7% of the time and Regime 2 occurs 75% of the time. The transition probabilities show that the duration of Regime 1 is small compared to Regime 2. The persistence probabilities are between 0.71 and 0.89 for Regime 1 and between 0.97 to 0.98 for Regime 2. * denotes a value significant on the 95% level and ** a value significant on the 99% level. Standard errors are in parenthesis.

Regime 1	MRP	SMB	$_{ m HML}$	UMD
Mean Volatility	-15.96 (13.24) 39.21** (3.66)	11.93 (14.77) 28.42** (6.55)	17.27* (6.70) 23.82** (1.86)	-0.39 (10.75) 35.62** (2.71)
Regime 2				
Mean Volatility	11.42** (1.89) 13.47** (0.47)	1.26 (1.15) 8.47** (0.56)	3.11* (1.12) 7.57** (0.32)	10.78** (1.24) 9.07** (0.31)
State Counter				
Regime 1 Regime 2	124.43 (21.98) 811.57 (21.98)	175.71 (27.14) 760.29 (27.14)	100.36 (51.22) 835.64 (51.22)	149.22 (16.76) 786.78 (16.76)
Transition matrix				
P(1,1) P(2,2)	0.89 (0.04) 0.98 (0.01)	0.89 (0.03) 0.98 (0.01)	0.71 (0.13) 0.97 (0.01)	0.86 (0.05) 0.97 (0.01)

regime-switching approach.

During the sample period, the analysis shows that the High-Variance Regime occurred about 10.68% of the time for HML and 18.70% for SMB. The transition probabilities show a higher degree of persistence for Regime 2 than for Regime 1. While for Regime 2 the probabilities of remaining in the current regime are between 0.97 and 0.98 for all risk factors, for Regime 1, the transition probabilities are lower with values between 0.71 and 0.89.

Figure 4 shows the estimated probabilities for Regime 2 for the sample period. Overall, there are periods where the risk factors switch jointly and periods of independent switching. For example, during the great depression in the 1930s, MRP, SMB, and UMD apparently switched to Regime 1, while value stocks still remained in Regime 2. During the internet bubble around 2000, small stocks and momentum stocks switched to Regime 1 while value stocks remained in Regime 2 most of the time.

With respect to recessions, we find no clear pattern. Overall, all four risk factors switch to Regime 1 before or during some economic downturns, e.g., in 1970, 1975 and 2001. However, for the period from 1945 to 1970 we find no such pattern.

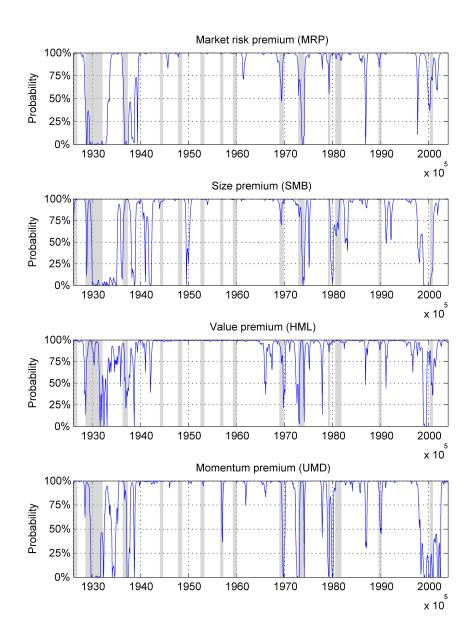


Figure 4: Estimated probabilities for Regime 2 for univariate regime-switching The Figure shows the estimated probabilities of beeing in Regime 2 for the univariate regime-switching model. Shaded areas show NBER recessions.

4.5 Asset Allocation under Regime Switching

In this section, we turn to the question how regimes affect asset allocation.

Asset allocation decisions in the regime-switching model are affected by a least three factors, the time horizon, the current regime and the degree of risk aversion as shown by Ang and Bekaert (2002). Figure 5 displays the results for an investor with a degree of relative risk aversion (RRA) of 3. The results are qualitatively similar for different degrees of risk aversion and therefore, this figure has representative character.

A comparison of the allocation in Regime 1 and Regime 2 stresses the importance of rebalancing. If stock markets are volatile, investors should overallocate value stocks and underallocate momentum stocks and small stocks. For a one year investment horizon, investors should increase their value stock holdings in Regime 1 by about 11% and reduces momentum positions by 7% and small stocks by 4%.

Table 4 shows the optimal asset allocation strategies for different degrees of risk aversion and different maturities. A risk averse investor with a degree of relative risk relative of 7, i.e., a very risk averse investor, should allocate between 4.62% (for an investment horizon of 48 months) and 15.46% (for an investment horizon of 3 months) more to value stocks in Regime 1 than in Regime 2. The amount allocated to small stocks should be reduced in Regime 1 by values between 1.80% (for 48 months) and 2.76% (for 3 months) and the remaining amount should be deducted from momentum stocks.

For a less risk averse investor, the optimal rebalancing amount is smaller. An investor with a relative risk aversion of 3 should allocate between 1.85% (for an investment horizon of 3 months) and 5.13% more to value stocks in Regime 1 than in Regime 2. In Regime 2, the allocation to momentum stocks should be increased by 2.26% (for 3 months) and 4.15% (for 12 months).

The table shows that the main results and findings are very robust with regard to different settings. Although the exact allocation changes, the results can be summarized as follows:

First, the higher the risk aversion, the higher the allocation to small stocks and the lower the allocation to value and momentum stocks.

Second, the longer the investment horizon, the higher the allocation to small stocks and the lower the allocation to value and momentum stocks.

Third, in Regime 1 the investor should focus on value stocks and decrease the amount allocated to small and momentum stocks. Consequently, the contrarian result holds for Regime 2.

Standard critiques towards portfolios choice (such as parameter instability, estimation risk) also apply in this case. However, one strong result holds in any case: In Regime 1 value stocks should be overallocated whereas in Regime 2 the exposure towards momentum stocks should increased. Therefore, our further investigations focus on the empirical

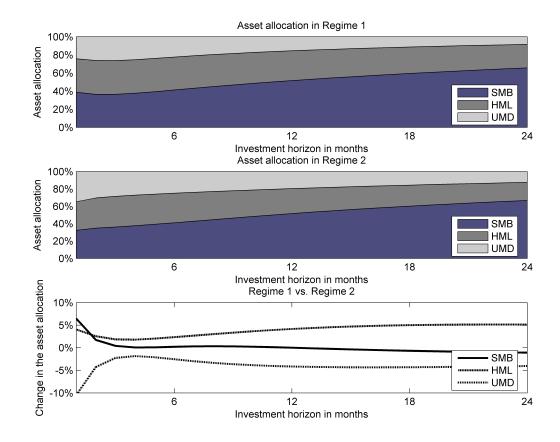


Figure 5: Asset allocation for $\gamma = 3$

The graph shows in the upper and middle part the asset allocation to the style factors small stocks (SMB), value stocks (HML) and momentum stocks (UMD) for a degree of relative risk aversion of $\gamma=3$ and in relation to the investment horizon depended on the prevailing Regime. In the lower part, the graph shows the changes in portfolio weights. In both settings, investors should allocate a substantial amount to small and value stocks. A comparision of the allocation in the regimes shows that in the High-Variance Regime (Regime 1) investors should allocate more to small and value stocks and less to momentum stocks. The findings for different degrees of risk aversion are qualitatively similar. Allocation to MRP is not shown because the portfolio optimization indicates that the optimal weight is 0.

Table 4: Asset allocation under regime-switching

The table shows the asset allocation for different degrees of relative risk aversion, investment horizons dependent on the prevailing regime. Overall, the portfolio optimization shows that a strategy switching between the allocation between value stocks in Regime 1 (High-Variance Regime) and momentum stocks in Regime 2 (Low-Variance Regime) is rational. Allocation to MRP is 0 in all cases displayed in the table.

Investment			RRA = 3			RRA = 5			RRA = 7	
Horizon	Regime	SMB	$_{ m HML}$	UMD	SMB	$_{ m HML}$	UMD	SMB	$_{ m HML}$	UMD
3	Regime 1	36.50%	37.16%	26.33%	45.53%	37.52%	16.95%	49.15%	37.63%	13.22%
	Regime 2	36.09%	35.31%	28.59%	46.22%	27.25%	26.52%	51.91%	22.17%	25.92%
	Difference	0.41%	1.85%	-2.26%	-0.70%	10.27%	-9.57%	-2.76%	15.46%	-12.70%
6	Regime 1	41.23%	36.33%	22.43%	48.75%	35.42%	15.83%	50.94%	35.13%	13.93%
	Regime 2	41.03%	33.99%	24.98%	49.81%	27.15%	23.04%	53.07%	24.56%	22.37%
	Difference	0.21%	2.34%	-2.55%	-1.06%	8.27%	-7.21%	-2.13%	10.57%	-8.43%
12	Regime 1	51.64%	32.84%	15.52%	57.27%	31.08%	11.65%	58.75%	30.62%	10.63%
	Regime 2	51.64%	28.69%	19.67%	59.10%	23.07%	17.83%	61.22%	21.50%	17.28%
	Difference	0.00%	4.15%	-4.15%	-1.84%	8.01%	-6.17%	-2.47%	9.12%	-6.65%
24	Regime 1	65.66%	25.94%	8.41%	69.67%	24.10%	6.24%	70.71%	23.62%	5.67%
	Regime 2	66.74%	20.81%	12.45%	71.78%	17.24%	10.98%	73.11%	16.31%	10.58%
	Difference	-1.08%	5.13%	-4.04%	-2.11%	6.86%	-4.75%	-2.40%	7.31%	-4.91%
36	Regime 1	74.94%	20.64%	4.41%	78.16%	18.94%	2.90%	78.99%	18.50%	2.51%
	Regime 2	76.29%	16.04%	7.67%	80.10%	13.40%	6.50%	81.09%	12.72%	6.19%
	Difference	-1.35%	4.60%	-3.25%	-1.94%	5.54%	-3.60%	-2.10%	5.77%	-3.68%
48	Regime 1	81.79%	16.47%	1.74%	84.51%	14.90%	0.59%	85.21%	14.49%	0.29%
	Regime 2	83.13%	12.57%	4.30%	86.22%	10.43%	3.36%	87.02%	9.88%	3.11%
	Difference	-1.34%	3.90%	-2.56%	-1.71%	4.47%	-2.76%	-1.80%	4.62%	-2.81%

performance of trading strategies taking this insight into account.

4.6 Tactical Asset Allocation under Regime Switching

Table 5 shows the empirical performance of a number of different trading strategies based on the regime-switching model. For each risk factor, we test a buy-and-hold strategy, and two strategies switching to cash in Regime 1 and Regime 2, respectively. Based on the results from the portfolio optimization, we tested a trading strategy mixing value and momentum investing. For each strategy, we report the mean return, its standard deviation and the Sharpe ratio. Using a sign test, we test for the equivalence of the median return of the buy-and-hold strategy and the switching strategy.

The results displayed in table 5 have been generated in a fully out-of-sample procedure, i.e., we estimated the prevailing regime at a certain point in time by using a data sample ending at this point in time. If the estimated probability for Regime 1 was higher than 0.5, the prevailing regime was assumed to be Regime 1 and vice versa. Based on this prevailing regime, the investment decisions are taken and successive return is computed. The sample period for the out-of-sample analysis started in 1967 and was iteratively extended until the full sample ending in 2004 was included.

Tactical asset allocation based on a regime-switching model seems to offer profitable trading strategies. As indicated by the portfolio optimization, switching between value and momentum stocks indeed seems to be a reasonable strategy. A pure buy-and-hold strategy consisting of 50% value stocks and 50% momentum stock was inferior to a strategy switching to 100% value stocks in Regime 1 and to 100% momentum stock in Regime 2, both from a mean return and a risk-adjusted Sharpe ratio point of view. The mean return increases from 7.25% to 9.85% and the Sharpe ratio from 0.04 to 0.09. As expected, the "wrong" switching strategy, i.e., switching to value stocks in Regime 2 and to momentum stocks in Regime 1 was inferior to a simple buy-and-hold and to the correct switching strategy. A return of 4.64% for the incorrect switching stategy and a standard deviation of 13.05% lead to a Sharpe ratio of -0.03.

Similar results hold for all other four risk factors. The regime-switching model indicated that for MRP and UMD overallocation in Regime 2 might be optimal and for SMB and HML overallocation in Regime 1. And indeed, these findings entirely are confirmed by the out-of-sample analysis. From a risk-adjusted perspective, switching temporarily to cash delivered a better performance than a buy-and-hold strategy for SMB, HML, UMD and a comparable performance for MRP.

An investor only holding the market portfolio can implement a simple market timing strategy based on the regime-swichting model. If the investor switches to cash if the model signalizes Regime 1, the average return is reduced slightly by 0.5% and volatility by 2.5%. From a Sharpe ratio perspective, both strategies are equivalent.

Table 5: Out-of-sample backtesting of switching strategies

and momentum stocks in the Low-Variance Regime (Regime 2) shows the highest Sharpe ratio. The results are stable for sub-Returns have been annualized. The p-value is based on signtest for the median and compares the performance of the buy-andhold strategy and the switching strategy. A strategy switching between value stocks in the High-Variance Regime (Regime 1) The table shows the out-of-sample performance for a number of different switching strategies (sample period from 1967 to 2004). 99%-level.

samples. Iransaction costs of 0.2	270 per round trip reduc	of 0.2% per round trip reduce the performance by 0.44%.	7.4470.	aei	cenores significant on the	IL OU UI
Strategy	Regime 1	Regime 2	Mean	Std.	Sharpe Ratio	P-value
Buy-and-Hold for MRP Switch to Cash in Regime 1	MRP (100%) Cash (100%)	MRP (100%) MRP (100%)	10.16	15.98 13.55	0.08	0.08
Switch to Cash in Regime 2	MRP (100%)	Cash (100%)	6.40	8.53	0.02	0.01**
Buy-and-Hold for SMB	SMB (100%)	SMB (100%)	0.81	11.65	-0.13	
Switch to Cash in Regime 1	Cash (100%)	SMB (100%)	2.20	8.47	-0.13	0.56
Switch to Cash in Regime 2	SMB (100%)	$Cash\ (100\%)$	4.50	8.09	-0.05	0.01**
Buy-and-Hold for HML	HML (100%)	HML (100%)	5.40	10.46	-0.01	
Switch to Cash in Regime 1	Cash (100%)	HML (100%)	4.41	7.55	-0.06	80.0
Switch to Cash in Regime 2 \mid	HML $(100%)$	Cash (100%)	88.9	7.26	0.04	0.08
Buy-and-Hold for UMD	UMD (100%)	UMD (100%)	60.6	14.64	90.0	
Switch to Cash in Regime 1	Cash (100%)	UMD (100%)	8.86	10.06	0.09	0.56
Switch to Cash in Regime 2 \mid	UMD (100%)	Cash (100%)	6.12	10.67	0.01	0.00**
Buy-and-Hold for HML and UMD	HML (50%), UMD (50%)	HML (50%), UMD (50%)	7.25	8.82	0.04	
HML in Regime 1, UMD in Regime 2	HML (100%)	UMD (100%)	9.85	12.36	0.09	0.02
UMD in Regime 1, HML in Regime 2 \mid	UMD $(100%)$	HML (100%)	4.64	13.05	-0.03	0.02

For the size premium both switching strategies show a superior performance compared to the buy-and-hold strategy. This findings is due to the fact that the average risk-free rate in the backtesting period was higher than the size premium. Switching to cash in Regime 1 increases the return by 1.4% and decreases the volatility by 3.2% and switching to cash in Regime 2 increases the average return by 3.7% and reduced volatility by 3.5%.

For the value premium, a strategy switching to cash in Regime 2 increases the return by 1.5% and decreases the volatility by 4.3% compared to the buy-and-hold strategy. A strategy switching to cash in Regime 1 and to value stocks in Regime 2 decreases the return by 1.0% compared to the buy-and-hold strategy.

Historically, a buy-and-hold momentum investor has earned 9.1% with an annualized volatility of 14.6% in the backtesting period. Switching to cash in Regime 1, reduces his return slightly by 0.2% and decreases the risk of the portfolio strongly by 4.6%. The incorrect switching strategy, i.e., switching to cash in Regime 2, reduces the return of the portfolio substantially by 3.0%.

However, the results for MRP on the one side and SMB, HML and UMD on the other side cannot be compared directly. The three style factors have been calculated, as shown by Fama and French (1993), based on long–short portfolios while MRP is based on a long-only portfolio. Long-short portfolio require no initial investment in contrast to long-only portfolios and therefore, depending on margin requirements, the interest rate earned on the margin must be added to make these positions comparable.

The robustness of the results has been investigated intensively. The results are stable across sub-periods. Also, after accounting for transaction costs, the results remain stable. In the period 1967-2004, the switching approach leads to 101 rebalancing actions in 456 months. If transaction costs of 0.2% for a full round-trip are taken into account, the performance for each switching strategy is reduced by 0.44%.

5 Conclusion

In this paper, we analyze time-varying risk premia and the implications for portfolio choice.

In the first part, we estimate a multivariate regime-switching model for the Carhart (1997) four factor model. We find two clearly separable regimes, a High-Variance Regime and a Low Variance Regime. In the High-Variance Regime, only value stocks deliver a good performance. In the Low-Variance Regime the market portfolio and momentum stocks promise high returns. The transition probabilities show that the High-Variance Regime has a rather small duration and the Low-Variance Regime a longer duration. Therefore, the High-Variance Regime is less stable than the Low-Variance Regime. Moreover, we intensively validated the out-of-sample performance and robustness of the regime-

switching model. Overall, estimated regimes and parameters are stable.

In the second part, we analyze the implications of regime-switching for portfolio choice. Using a utility maximization framework, we analyze portfolio selection for an buy-and-hold investor with different degrees of relative risk aversion and different investment horizons. We find that in the High-Variance Regime value investing seems to be a rational strategy and in the Low-Variance Regime momentum following. An out-of-sample backtest of the switching strategy shows that tactical asset allocation based on the regime-swichting model would have superior performance. Switching between value stocks during bear markets and momentum stocks in bull markets holds the potential for a promising strategy.

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