Risk Measure Calculator User Guide

GR5320 Project Report

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I. Software Introduction

This software is designed to calculate and plot Value at Risk (VaR) and Expected Shortfall (ES) for investments, both of which are key measures of the risk of loss.

There are three methods of calculation provided in this software: the historical method, the parametric method, and the Monte Carlo simulation method.

Users are allowed to construct their investment portfolios by longing single or multiple stocks and are also given an option to liquidate part of the portfolio to purchase the corresponding put options (if the chosen stocks have listed options available for trading) to hedge.

II. Software Design Documentation (GUI)

2.1 General Description

The software contains two parts: GUI.py and Methods.py. The Methods.py file contains all the calculations and plotting functions as we did in homework and the purpose of each function is commented inline. The GUI.py provides a Graphical User Interface, which is constructed using the Python Package Tkinter, which composes a bunch of different functionalities. When users click different buttons on the interface, it will call different plot functions based on the users' choices. The software includes three VaR & ES calculation methods and in a total of four ways of calculations:

1. Parametric VaR & ES (assuming the underlying portfolio follows GBM)
2. Historical VaR & ES (using relative changes and assuming historical distribution maintains)
3. Monte Carlo VaR & ES (assuming portfolio follows GBM)
4. Monte Carlo VaR and reduction after liquidating (assuming underlying securities follows GBM)

and also allows users to conduct backtesting if they are using one of the first three ways of calculation.

2.2 GUI Workflow

To start, user needs to fill in the following inputs (if he/she doesn't like the default settings): Initial Investment (default is $1000000), Invest Date (default is 2000-01-03), Time Horizon (default is 5 days), VaR Confidence Level (default is 99%), ES Confidence Level (default is 97.5%) and a list of tickers (separated by commas).

2.2.1 To calculate the Parametric VaR & ES:

- Make sure all the above information is filled.
- Select "Parametric" on the Methods box and choose the Weighting method (default is "Equal Weighting"). Here, we do not allow the user to input his/her own exponential factor (lambda) for

exponential weighting, instead, we directly use the lambda that is equivalent to the equal weighting method with the selected window length to calibrate and calculate.

- Click “Plot Mu” or “Plot Sigma” to plot the calibrated parameters or click “Plot VaR” or “Plot ES” to plot the desired risk measures.

2.2.2 To calculate Historical VaR & ES:

- Fill in the same boxes as above.
- Select “Historical” on the Methods box. Weighting method, Monte Carlo simulation and Liquidity percentage can be ignored.
- Click “Plot VaR” or “Plot ES” to plot the desired risk measures.

2.2.3 To calculate Monte Carlo VaR & ES:

- Fill in the same boxes as above.
- Select “Monte Carlo” on the Methods box. Specify the number of Monte Carlo simulations. It is worth noticing that a large number of simulations will lead to a really long runtime and potential system failure.
- Select “Stock Only” to calculate the VaR & ES of the stock only portfolio.
- Select “Stock + Option” and specify the percent of liquidation if the user wants to consider long options along with underlying assets. In this case, only VaR is being provided.
- Click “Plot VaR” or “Plot ES” to plot the desired risk measures.

2.2.4 To Back test：

- Fill in the same boxes as above.
- Simply click “BackTest” button to plot the number of exceptions that occur in each one-year window corresponding to the user-selected method.

2.3 Basic Requirements

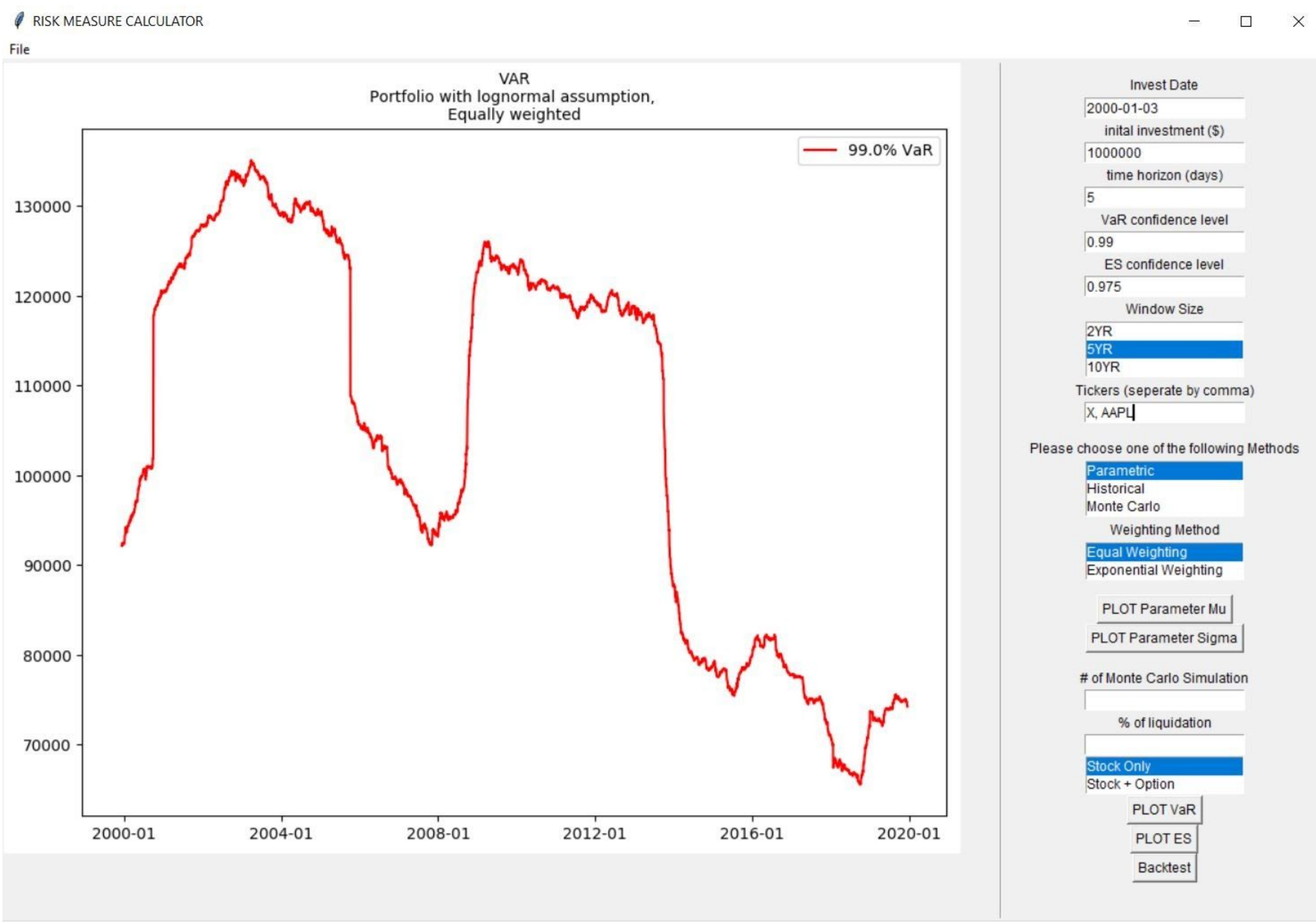
To make the software run, users should make sure:

- They downloaded the GUI.py and Methods.py documents and saved them under the same directory.
- They have installed Python 3 on their computers and they have all the desired packages successfully installed. (Namely, Numpy, Pandas, Matplotlib, Scipy, Pandas_datareader, and WallStreet)
- They input the tickers that are consistent with the Yahoo finance website and multiple tickers are separated by commas (spaces are fine).
- They fill in a valid number or make a required choice for each corresponding method (please refer to the “GUI Workflow” section above).
- The stock listing date should be at least 10 years earlier from the current day. Therefore, it will return enough historical data if using 10 years of rolling windows.

2.4 Interface

The plot below shows the appearance of our GUI, and a more detailed demonstration can be viewed at:

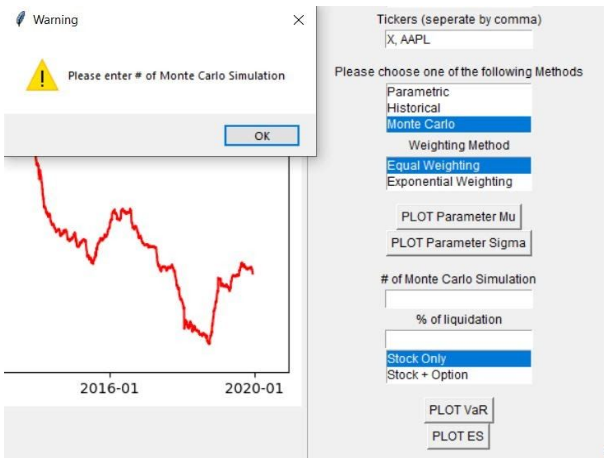
- For the single stock: https://youtu.be/gIzJXQGGTGE
- For multiple stocks: https://youtu.be/bx8ddgR_Elk



Notice that, when there's any misoperation done by the user, a warning box will show up to alert and direct him/her to do the right things as below.

Three quick options are initiated above under the "File" menu that includes a Quit function, a Clear function, and a Refresh function. The Quit function will quit the system; the Clear function will clear the current plot on the canvas, and the Refresh function will clear all the numerical default values.

All of the images shown on the canvas are automatically saved in the same directory as the python file, allowing users to make further use.



III. Model Description

The following material document will cover the risk calculation process for different methods.

3.1 Introduction of VaR

Value at Risk, or VaR, measures the risk of potential losses in an investment over a given time horizon. A p% VaR is a value such that for p percent of the time, the losses are greater than or equal to that value. A general formula for VaR is:

$$VaR(V, T, p) = G^{-1}(p)$$

where $G(X) = P\left[V_0 - V_T \le X\right] = E^P[1_{V_0 - V_T \le X}]$.

Expected Shortfall, or ES, is defined as the average of all losses greater than or equal to VaR. In other words, the p% ES is the expected value of the worst p% of the losses. This is represented as

$$ES(V, T, p) = E^P[V_0 - V_T | V_0 - V_T > VaR(V, T, p)]$$

3.2 Model Assumptions and Advantages & Disadvantages

In order to calculate the VaR of our portfolio using the historical data, we used several mathematical models depending on which type of VaR specified by the user. There are three ways that can be chosen to calculate the VaR: Parametric method, Historical method, and the Monte Carlo method.

3.2.1 Parametric method

- For individual stock, it assumes that stock prices follow a Geometric Brownian Motion (GBM).
- For a portfolio of multiple stocks, the portfolio itself is taken as a whole and assumes it follows GBM.

- Disadvantages: it is only accurate when positions and payoffs are linear, which requires hacks (like quadratic approximation) to improve it.

3.2.2 Monte Carlo method

- It has the same assumptions as the parametric method in terms of the price distribution and the same calibration procedure before the risk calculation part.
- Advantage: Straightforward to compute and correct for nonlinear payoffs.
- Disadvantage: Runs very slow if generates a large number of paths.

3.2.3 Historical method

- Assume today's distribution of market changes is consistent with the historical distribution of market changes and the asset prices follow GBM.
- Choose the relative changes method, that is to say, all calculations are based on the log-returns of the portfolio.
- Advantages: No need for assumptions on the distribution of historical changes or estimation of the drifts and volatilities as well as correlations between the different assets.
- Disadvantages: Extreme past events will have a huge impact on estimations.

3.3 Mathematical description and calculation process

3.3.1 Parametric Method

The daily prices are assumed to follow the GBM. GBM is a stochastic process which satisfies the following SDE:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where W_t is a Brownian motion, μ and σ are constants. Solving this equation gives us:

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right)$$

The historical prices of the assets are utilized to calibrate the mean and variance of the portfolio. There are two ways to do this: using the unweighted windows (equal weighting) or the weighted windows (exponential weighting). We use the daily log-returns of the prices.

The sample mean and variance of the data are calculated as the following:

$$\overline{\mu} = \text{ Average of the log returns}$$

$$\overline{\sigma} = \text{average of the square of log returns } - \overline{\mu}^2$$

Assuming the historical data is daily, then the drift μ and the volatility σ are:

$$\sigma = \overline{\sigma}\sqrt{252}$$

$$\mu = 252\overline{\mu} + \frac{\sigma^2}{2}$$

The other method is to weigh historical data so that recent history counts more. The weighting scheme used in this system is called exponential weighting.

Let λ be the weighting factor, so the current returns are given weight by 1, the previous returns by λ, λ^2 … λ^i , with i=1, 2, 3…n. And the sum of the weights would be $\sum_{i=0}^{n} \lambda^i$.

Then each log return l_i is weighted by $p_i = \lambda^i / \sum_{i=0}^{n} \lambda^i$. Then the mean and variance are calculated as:

$$\overline{\mu} = \sum p_i l_i$$

$$\overline{\sigma} = \sqrt{\sum p_i l_i^2 - \overline{\mu}^2}$$

$$\sigma = \overline{\sigma}\sqrt{252}$$

$$\mu = 252\overline{\mu} + \frac{\sigma^2}{2}$$

With these parameters, we can use the following formula for the GBM VaR:

$$VaR(S, T, p) = S_0 - S_0 e^{(\sigma\sqrt{T}\Phi^{-1}(1-p)+(\mu-\sigma^2/2)T)}$$

For Expected Shortfall, the equation is

$$ES(S, T, p) = S_0 - \left(\frac{1}{1-p}\right)e^{(\mu T)S_0(1-\Phi(d_1))}$$

where $d_1 = \frac{1}{\sigma\sqrt{T}}(\log(\frac{S_0}{X}) + (\mu + \sigma^2/2)T)$, and $X = S_0 - VaR(S, T, p)$.

3.3.2 Monte Carlo Method

For Monte Carlo VaR, the parameters μ and σ are calculated in the same way as the parametric VaR. And for each day, let V_0 represent the current price of the portfolio. Use the parameters μ and σ to generate a large number of Brownian Motion sample paths (correlated sample paths if more than one stocks are selected), and use them to create random prices, V_t, where t is the time horizon specified for the VaR. For one stock, for example, the process would look like this:

$$V_t = aS_t = aS_0 e^{(\mu - \sigma^2/2)t + \sigma W_t}$$

From this, we will get a list of losses $V_0 - V_t$, and then the pth percentile of the losses is considered as the Monte Carlo VaR.

3.3.3 Historical Method

For Historical VaR, there is no fitting of historical data to a model. Instead, the resulting risk measure relies entirely upon the past distributions of the market. For each day in a user-specified time window, apply the daily log returns to that day's price and compile a list of historical samples. Then use the pth percentile of the samples, and the resulting loss is the difference between it and the current price.

IV. Validation

4.1 Use of calculation system

This system is designed for calculating the risk associated with a specific investment, which can be in one of the following:

- Long a single stock
- Long multiple stocks
- Adding 1-year at the money (ATM) puts to reduce the current VaR

4.2 Review of data

The system allows investors to choose stocks by inputting tickers, and it will extract Adjusted Close Prices from Yahoo finance as well as the implied volatility for at-the-money one-year put option from Wallstreet API. If there's no such put option listing, the system will use a close one to replace. The date ranges from the initial investment date specified by the user (or 2000-01-03 as default) to the current date (i.e. the date when the user opens this system).

4.3 Assumptions

For Monte Carlo VaR, it is assumed that stocks (or the portfolio itself) follow Geometric Brownian Motion (GBM), and the same parameter estimation in the rolling window method.

For put options, the following assumptions are made:

- The one-year risk-free rate is 2%*. (* 2% is getting by averaging one-year daily treasury yield rates starting from 2019/01/02 to 2019/12/13).
- The implied volatility surface is flat (all options on the underlying stocks are traded at same volatility, regardless of strike and maturity).
- The only risk factor is the stock price (ignore implied volatility risk).
- The puts are ATM, i.e. strike prices are equal to the current spot prices of the underlying securities.

4.4 Data Scope

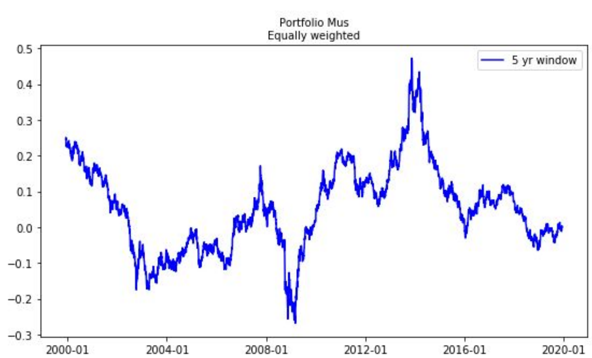
To test the efficiency and stability of the calculation system, we select two stocks, Ford and Xerox and their put options accordingly. If stocks only, each stock is invested equally in the portfolio. If part of the portfolio is liquidated to purchase 1-year ATM puts, then each put options are also equally weighted.

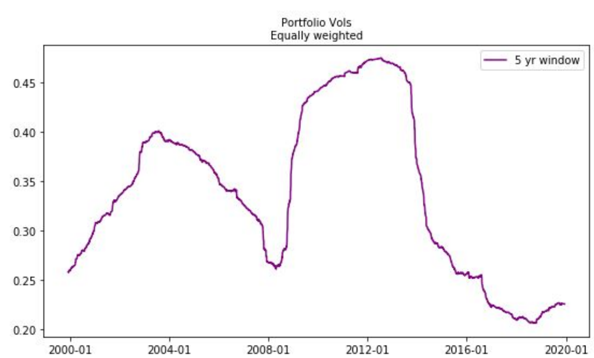
We set the parameters as follows:

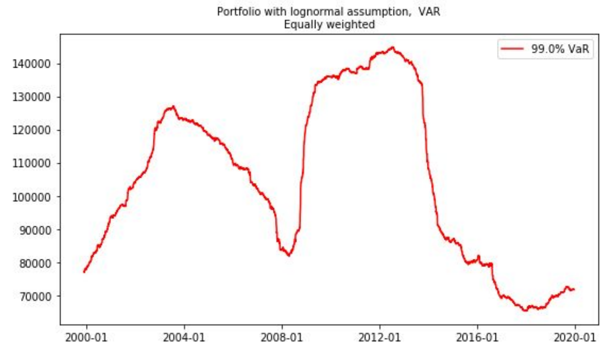
- Initial Investment: 100000
- Investment date: 1997-09-05
- Window size: 5 year
- Time Horizon: 5 days
- VaR confidence level: 99%
- ES confidence level: 97.5%
- Liquidate: 1%
- Tickers: F, XRX
- Weights:
    - o If no liquidation: 0.5*F + 0.5*XRX
    - o If with liquidation: 0.99*0.5*F stocks + 0.99*0.5* XRX stocks +0.01*0.5*F puts +0.01*0.5*XRX puts

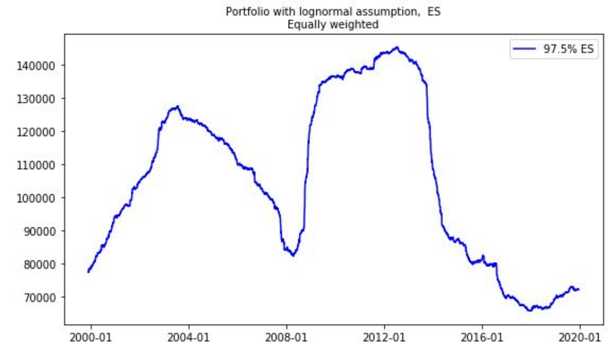
4.5 Validation Results

a) VaR and ES with the lognormal assumption, windowed data.

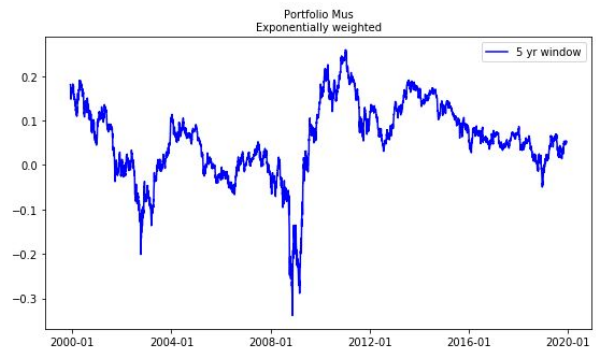


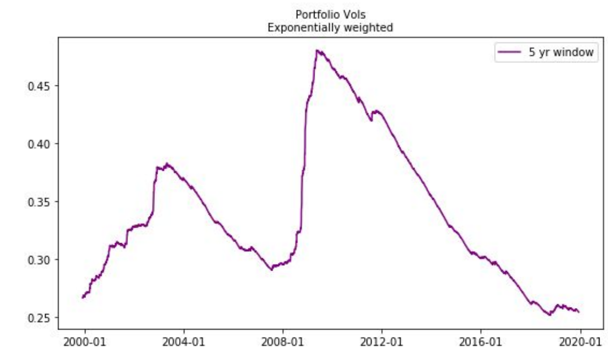


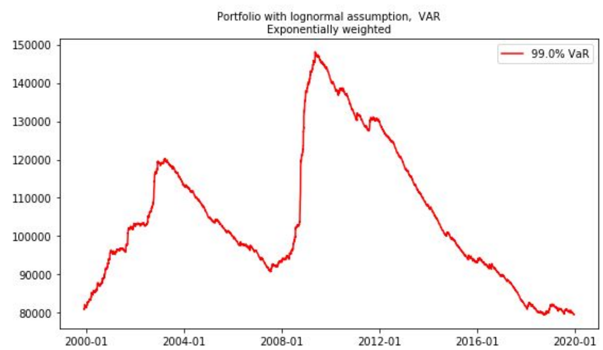


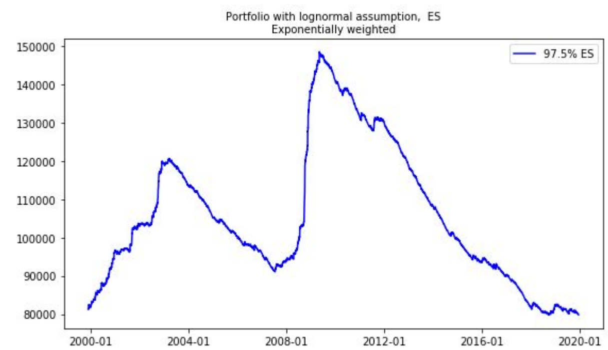


b) VaR and ES with the lognormal assumption, exponentially weighted

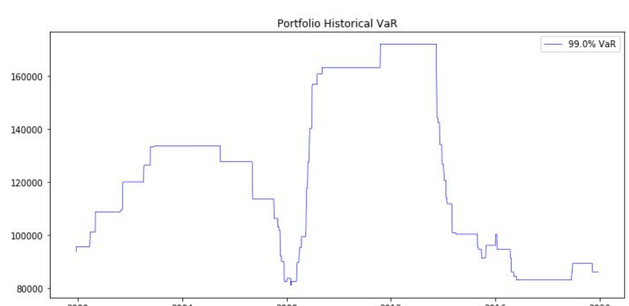


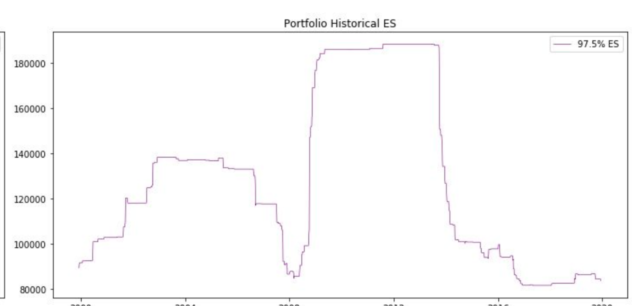




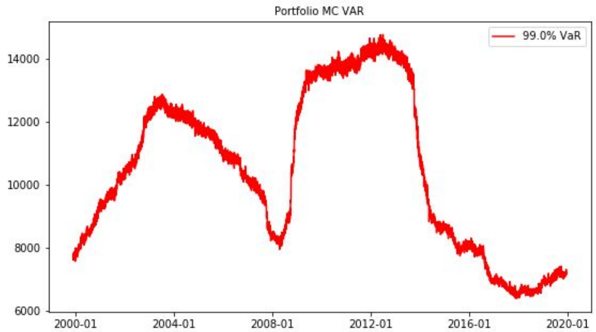


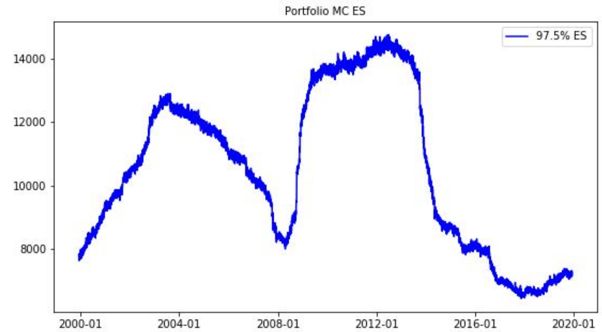
c) Historical VaR and ES, absolute returns and log returns



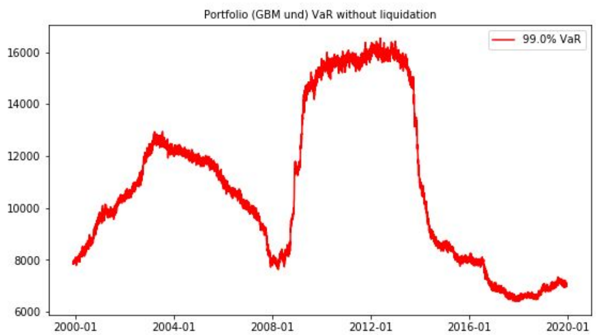


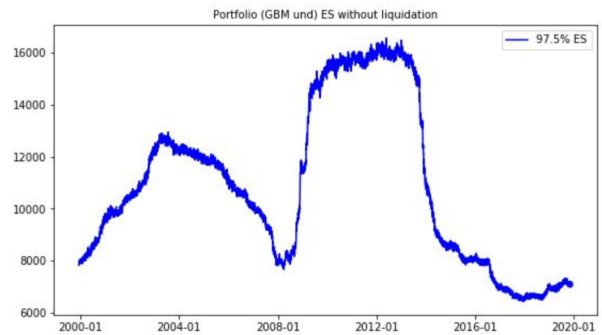
d) MC VaR and ES assuming portfolio follows GBM





e) MC VAR and reduction percentage if liquidation, assuming underlying stocks follows GBM



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Stock price: 9.2, 36.8
Stock shares: 6160.0, 602.0
Put price on one share: 0.79, 3.91
Put shares: 1266.99, 255.73
VaR without options: 7084
VaR with options: 4922
VaR reduction (percentage): 30.52
```

4.6 Outputs Review

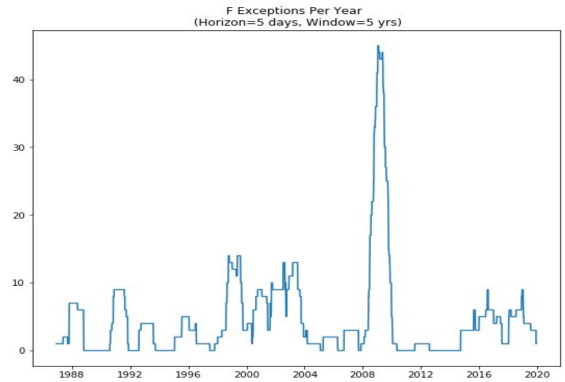
- 97.5% ES is virtually identical to 99% VaR.
- The GBM VaR and Monte Carlo VaR are really similar, while the historical VaR differs from the other two methods.
- We also found that exponential weighting rolls faster as compared with windowed versions and it is also smoother than windowed data, presumably because of the effect beyond the corresponding window.
- The Historical VaR is similar to GBM VaR in general, but there are some periods in which historical VaR is substantially higher, which indicates that using GBM may underestimate VaR.
- Historical VaR and ES have a long flat period, which is because that when extreme events happen, there is a jump in VaR when enters or exits the sliding window.

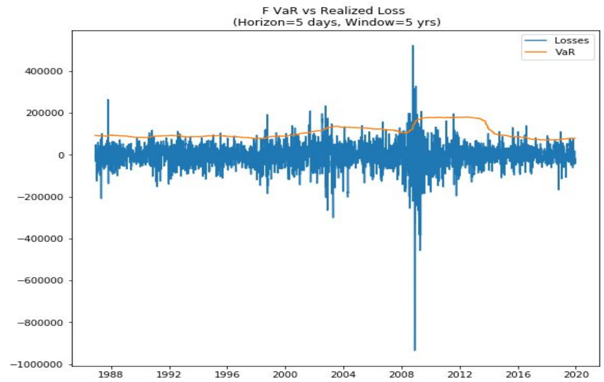
4.7 Back Test

For the backtest, we count the number of times that VaR on each day is exceeded by the subsequent 5-day changes per year during the investment period.

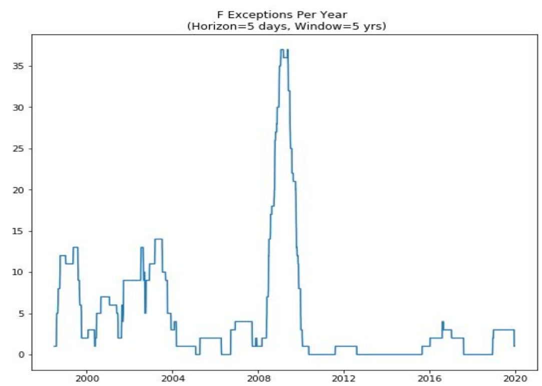
a) Single stock

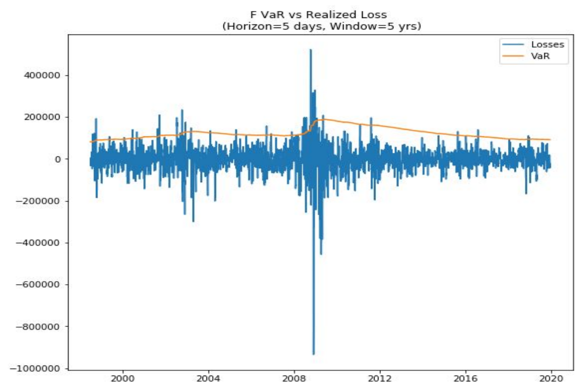
- Equally weighted GBM



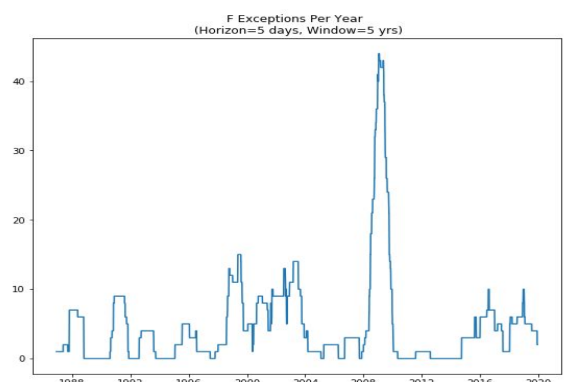


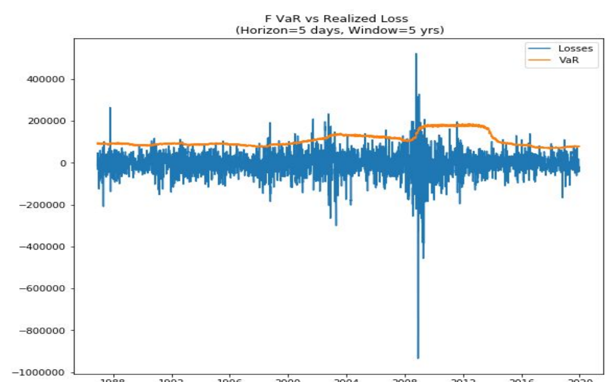
- Exponentially weighted GBM





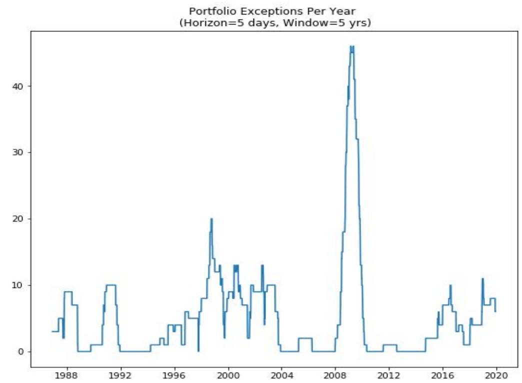
- Monte Carlo

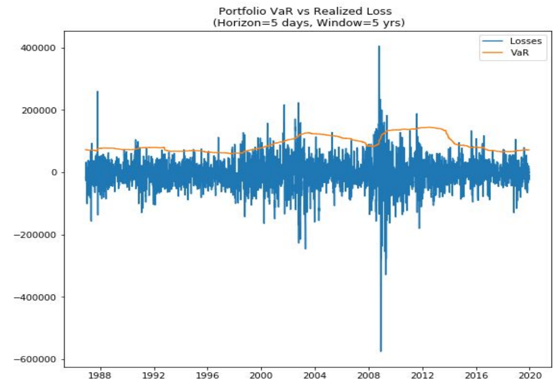




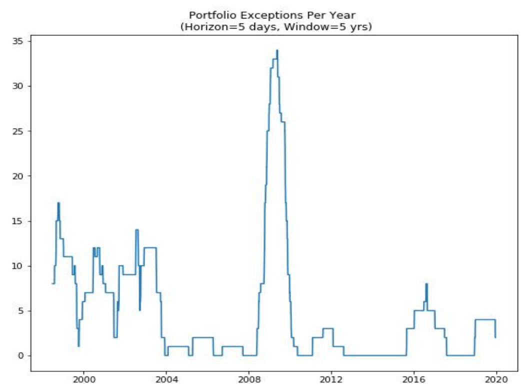
b) Portfolio

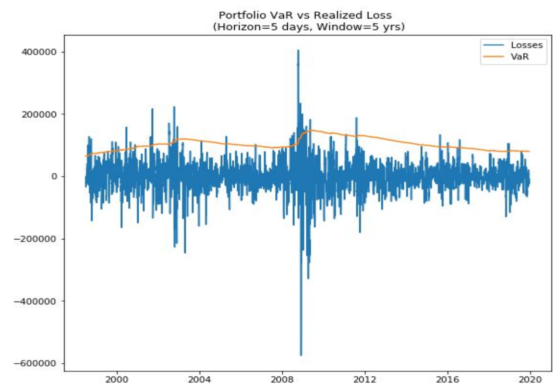
- Portfolio follow GBM with the lognormal assumption, Equally weighted



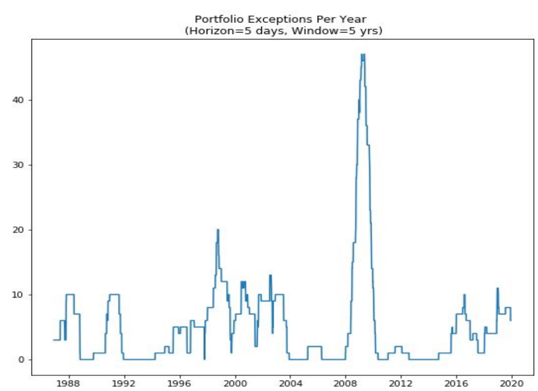


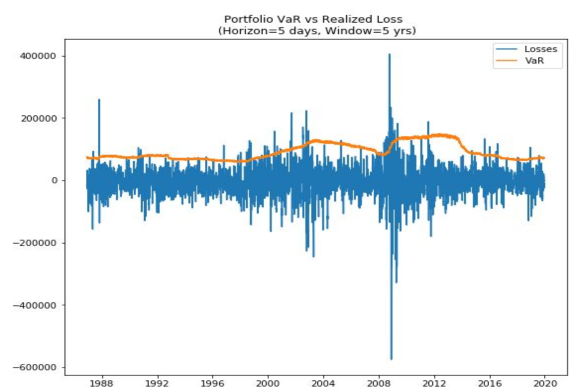
- Portfolio follow GBM with the lognormal assumption, Exponentially weighted





- Monte Carlo VaR portfolio follows GBM





4.8 Some Findings

- There are some periods with no exceptions, while some periods have a large number of exceptions.
- The exceptions occur when volatility begins to increase. This is because although increased volatility increases VaR, VaR doesn't rise fast enough to account for the increased market volatility.
- Besides, the VaR falls when the volatility drops, but it takes a long time to deflate to the new market behavior.

- We also suspect that the overlapping periods contribute to a large number of exceptions. Since we are looking at the 5-day VaR here, and there is a high correlation between the 5-day loss on a given day and the 5-day loss on the following day, when an exception occurs on a given date, it tends to cause VaR exceptions for all 5 days. So, using 1-day VaR may generate better results.
- The exponential weighting, compared with equal weighting, tends to yield fewer exceptions. This is because of exponential weighting reacts to changes in volatility faster, and hence the VaR is more realistic than using equally weighting.

V. References

Harvey J. Stein, Lecture Slides for Fall Math GR5320.

[Ste14a] Harvey J. Stein. Model Validation Municipal Bonds. 2014.

[Ste14b] Harvey J. Stein. Model Validation Report Template. 2014.