Assignment 1

ECE 712: Matrix Computations for Signal Processing

Name: Dai, Jiatao

Student ID: Student ID: 400548587

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McMaster University

Department of Electrical and Computer Engineering

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1. Question 1: PCA Compression and Reconstruction Error

1.1 Objective

Perform PCA compression on the 1000×5 data matrix X provided in `X.mat`. Explain the method, comment on the effect of varying r, and evaluate the reconstruction error.

1.2 Methodology

- Mean-center the data: $X_c = X - \bar{X}$
- Compute covariance matrix: $C = \frac{1}{N-1} X_c^T X_c$
- Perform eigen-decomposition or SVD.
- Select top r components and reconstruct $\hat{X}_r = X_c V_r V_r^T$.
- Compute reconstruction error $E = \|X_c - \hat{X}_r\|_F^2$.

Discuss how the reconstruction error decreases as r increases, and what it implies about data dimensionality.

2. Question 2: Proof of Minimum Reconstruction Error

2.1 Objective

Prove that PCA yields the minimum reconstruction error among all possible orthonormal bases.

2.2 Theoretical Proof

Use the Eckart–Young–Mirsky theorem:

$$\min_{\text{rank}(B)=r} \|X - B\|_F = \sqrt{\sum_{i=r+1}^{n} \sigma_i^2}$$

where σ_i are singular values of X. Therefore, truncating the SVD at r components yields the optimal low-rank approximation.

2.3 Numerical Verification (Optional)

Insert your result comparison:

```
Truncated SVD error: 0.0023
Random basis projection error: 0.0974
```

Discuss how this confirms the theoretical result.

3. Question 3: Minimum-Variance FIR Filter Design

3.1 Objective

Given a mean-centered random sequence $x[n]$ from `x.mat`, find an FIR filter $h[n]$ (with constrained 2-norm) such that the output $y[n] = (h * x)[n]$ has minimum variance.

3.2 Methodology

- Compute the sample autocorrelation $r_x[l]$ of $x[n]$.
- Construct the Toeplitz covariance matrix R_x.
- Formulate the optimization problem:

$$\min_h \; h^T R_x h \quad \text{s.t. } \|h\|_2 = 1$$

- The solution is the eigenvector of R_x corresponding to its smallest eigenvalue.

Discuss the observed minimum variance and characteristics of the resulting filter.

References

- S. Haykin and B. Van Veen, *Fundamentals of Linear Algebra for Signal Processing*, 3rd Ed., Springer, 2023.
- Lecture Notes, ECE 712, McMaster University.