ECE 712: Matrix Computations for Signal Processing
mini Assignment 2

Due: Tues Oct 28, 2025

Consider the *Approximation Theorem* that we discussed in class (p.83 in the book). If $\mathbf{A}$ is a full–rank $m \times n$ matrix, then the matrix $\mathbf{A}_r$ of rank $r < \min(m, n)$ is defined as

$$\mathbf{A}_r = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \tag{1}$$

where all the above variables are defined in the book. Then the Approximation Theorem says that $\mathbf{A}_r$ is the closest matrix of rank r to $\mathbf{A}$ in the Frobenius norm sense, i.e. that $||\mathbf{A} - \mathbf{A}_r||_F$ is minimum over all $m \times n$ matrices of rank r.

1. Prove that $\mathbf{A}_r = \mathbf{P}_c\mathbf{A}$, where $\mathbf{P}_c$ is the projector onto the subspace defined by $\mathbf{U}_1$. What is this subspace called?
2. Prove that $\mathbf{A}_r^T = \mathbf{P}_r\mathbf{A}^T$, where $\mathbf{P}_r$ is the projector onto the subspace defined by $\mathbf{V}_1$. What is this subspace called?