A study of the spherical coordinates parameterization

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Abstract

This report provides the results of using the spherical coordinates to resolve the constrained problems.

Part I
Introduction

For the resource allocation problem, suppose resource p_i is allocated to i where i is the index of the object which get the resource, p_i is the allocation ratio to the total available resource. Then we can get:

$$\sum_{i=1}^{n} p_i = 1 \tag{1}$$

We can rewrite the equation 1 as bellow:

$$\sum_{i=1}^{n} (r_i)^2 = 1 \tag{2}$$

Any feasible allocation vector $r = (r_1, ..., r_n)$ on the unit ball can be described through $n-1$ angels denoted by $\theta_i, 1 \le i \le n-1$, in the following way. Indeed, the spherical coordinated parameterization of r via $\theta^T = (\theta_1, ..., \theta_{n-1})$ is given by:

Part II
Experiments

Cost function is linear

In this experiment, the cost function is a linear function. The optimization problem is described as below:

$$\begin{aligned} \min_{x_1, x_2 \in \mathbb{R}} \quad & J(x_1, x_2) = -2x_1 - x_2 \\ \text{s.t.} \quad & 0 \leq x_1 \leq 1 \\ & 0 \leq x_2 \leq 1 \\ & \sum_{i=1}^{2} x_i = 1 \end{aligned} \tag{3}$$

As $\sum_{i=1}^{2} x_i = 1$, we can change equation 3 to:

$$\begin{aligned} \min_{x \in \mathbb{R}} \quad & J(x) = -x - 1 \\ \text{s.t.} \quad & 0 \leq x \leq 1 \end{aligned} \tag{4}$$

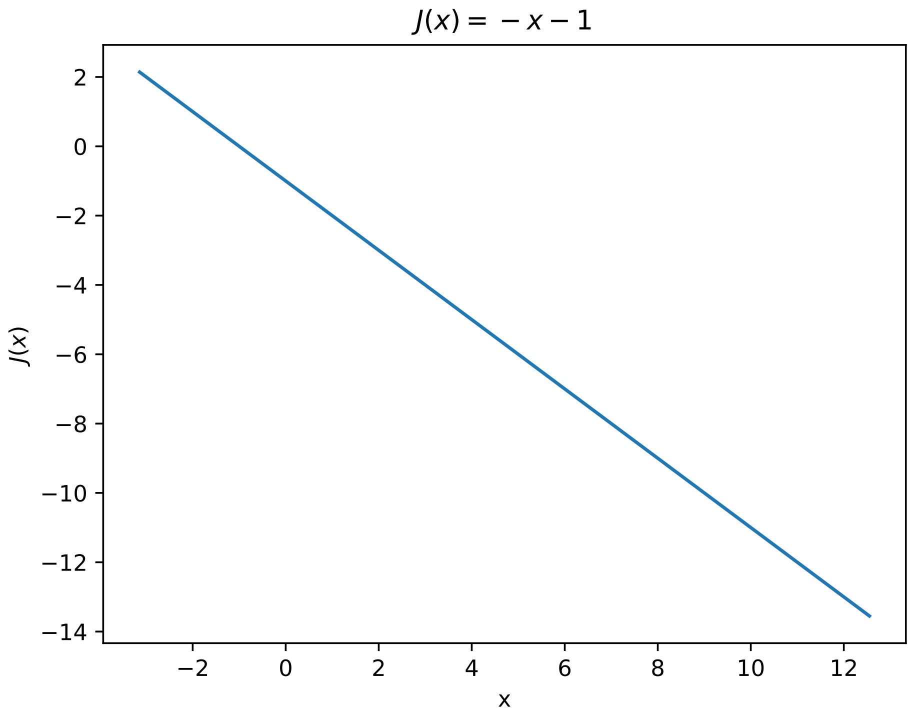

Figure 1: The plot of the cost function: $J(x) = -x - 1$.

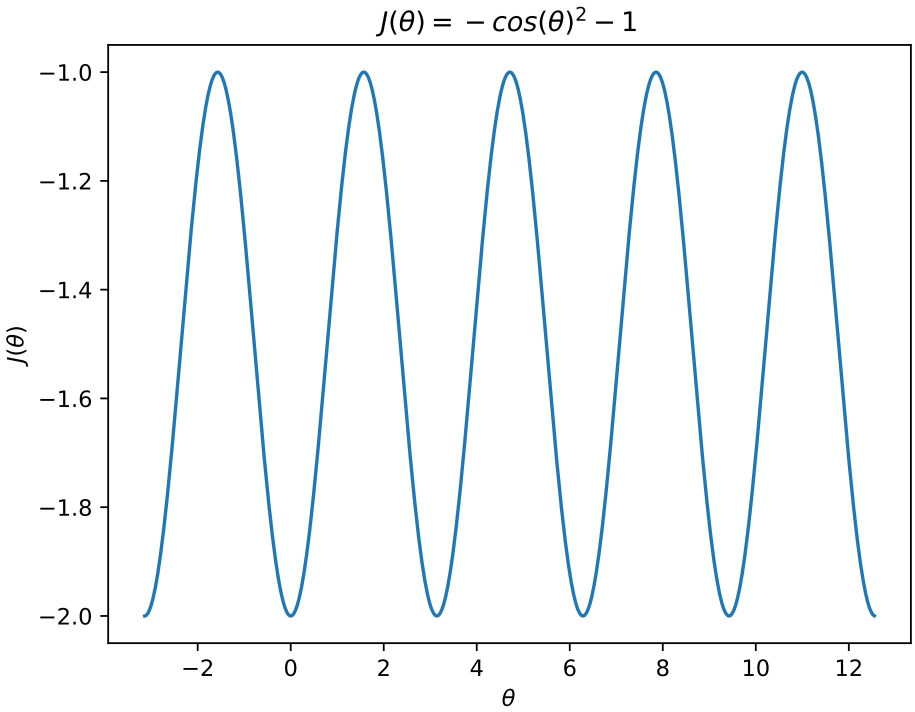

Figure 2: The plot of the cost function: $J(\theta) = -cos(\theta)^2 - 1$.