

A study of the spherical coordinates parameterization

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Abstract

This report provides the results of using the spherical coordinates to resolve the constrained problems.

Part I

Introduction

For the resource allocation problem, suppose resource p_i is allocated to i where i is the index of the object which get the resource, p_i is the allocation ratio to the total available resource. Then we can get:

$$\sum_{i=1}^n p_i = 1 \tag{1}$$

We can rewrite the equation 1 as bellow:

$$\sum_{i=1}^n (r_i)^2 = 1 \tag{2}$$

Any feasible allocation vector $r = (r_1, \dots, r_n)$ on the unit ball can be described through $n - 1$ angels denoted by $\theta_i, 1 \leq i \leq n - 1$, in the following way. Indeed, the spherical coordinated parameterization of r via $\theta^T = (\theta_1, \dots, \theta_{n-1})$ is given by:

Part II

Experiments

Cost function is linear

In this experiment, the cost function is a linear function. The optimization problem is described as below:

$$\begin{aligned} \min_{x_1, x_2 \in \mathbb{R}} \quad & J(x_1, x_2) = 2x_1 + x_2 \\ \text{s.t.} \quad & 0 \leq x_1 \leq 1 \\ & 0 \leq x_2 \leq 1 \\ & \sum_{i=1}^2 x_i = 1 \end{aligned} \tag{3}$$

As $\sum_{i=1}^2 x_i = 1$, we can change equation 4 to:

$$\begin{aligned} \min_{x \in \mathbb{R}} \quad & J(x) = x + 1 \\ \text{s.t.} \quad & 0 \leq x \leq 1 \end{aligned} \tag{4}$$

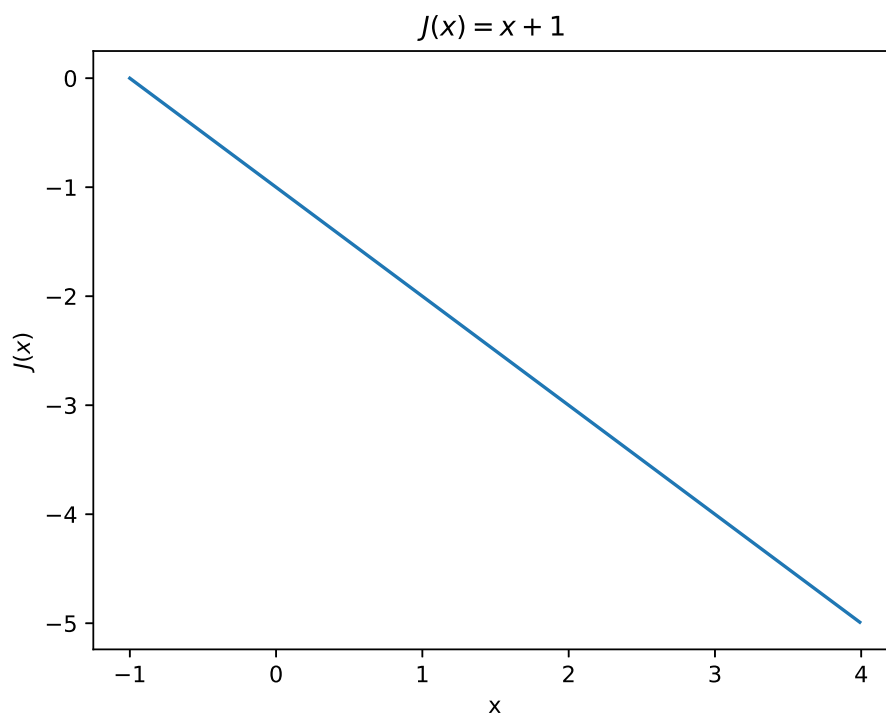


Figure 1: The plot of the cost function: $J(\theta) = \cos(\theta)^2 + 1$.

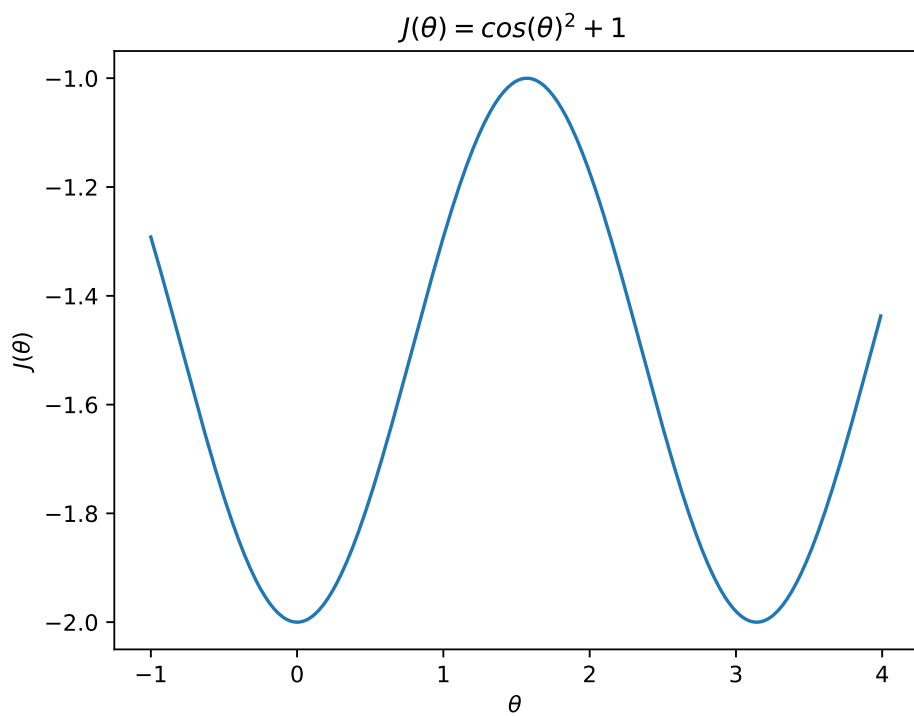


Figure 2: The $J(x) = x+1$.