What is a differential Equation?

- An equation expressing a relation between functions, their derivatives and the variables is called a differential equation

$y \to x$

- Classification of differential equations

$\frac{dy}{dx}, \frac{d^2y}{dx^2}$

➢ Ordinary differential equations

(x, y)

➢ Partial differential equations

$$\boxed{\frac{dy}{dx} = 1 + \frac{y}{x}}$$

Ordinary differential equations

- An ordinary differential equation is a differential equation in which the dependent variable (say y) depends only on one independent variable(say x)
- F(x, y , y ', y'' , y''' ....)= 0
- <u>Example :</u>
- $\frac{dy}{dx}$= sinx+cosx

Partial differential equation

- A partial differential equation is the differential equation in which y depends on two or more independent variables x,t…

$y = xt^2$

- $F\left(x,t,y,\frac{\partial y}{\partial x}, \frac{\partial y}{\partial t}, \frac{\partial^2 y}{\partial x^2}, \frac{\partial^2 y}{\partial t^2}, \frac{\partial^2 y}{\partial x \partial t} \ldots\ldots\ldots\ldots\ldots \right)=0$
- Example :

$$\boxed{\frac{\partial y}{\partial x} + t\frac{\partial y}{\partial t} = 2y}$$

- **<u>Order of the differential equation</u>**
- The order of the highest derivative involved in a differential equation is called the order of the differential equation. $F(x, y, y', \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3} \ldots)$

- **<u>Degree of the differential equation</u>**- The degree of a differential equation is the degree or power of the highest ordered derivative present in the equation, after the equation is made free from radicals and fractions in respect of derivatives $\left(\frac{d^3y}{dx^3}\right)^1 + \left(\frac{dy}{dx}\right)^2 + xy = 0$

Find order and degree of the following differential equations:

1. $\left(\frac{d^2y}{dx^2}\right)^1 = 0$ order = 2, degree = 1

2. $y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ O = 1, d = 2

$y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + a$

3. $\frac{d^2y}{dx^2} + m^2y = 0$ $\to$ (2, 1)

4. $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = y^2 \log y$ (2, 1)

Square root

5. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = a\frac{d^2y}{dx^2}$ (2, 2)

6. $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ (1, 2)

Practical approach to differential equations

Differential equations arise in many engineering and physical problems. The approach of an engineer to the study of differential equations has to be practical, and so, it consists of

(1) formation of a differential equation from the physical conditions, called modelling;

(2) solution of a differential equation under the initial/boundary conditions; and

(3) the physical interpretation of the results.

Formation of a differential equation

we form a DE inorder to eliminate the constants.

* - order of the equation formed = number of constants to be eliminated.

Example-

$y = ax^2 + bx$

$y \rightarrow$ dependent Variable

$x \rightarrow$ independent Variable

$a, b \rightarrow$ Constants

$$\frac{dy}{dx} = 2ax + b$$

$$\frac{d^2y}{dx^2} = 2a \implies \boxed{a = \frac{1}{2}\frac{d^2y}{dx^2}}$$

$$b = \frac{dy}{dx} - 2ax = \frac{dy}{dx} - 2 \cdot \frac{1}{2}\left(\frac{d^2y}{dx^2}\right) \cdot x$$

$$b = \frac{dy}{dx} - x\frac{d^2y}{dx^2}$$

$$y = ax^2 + bx$$

$$y = \frac{1}{2}\left(\frac{d^2y}{dx^2}\right)x^2 + \left(\frac{dy}{dx} - x\frac{d^2y}{dx^2}\right)x$$

$$y = -\frac{1}{2}\Big(x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx}$$

$\downarrow$

order = 2

degree = 1

- 2)y= $Ae^{2x} + Be^{-2x}$
- Y'=2$Ae^{2x} - 2Be^{-2x}$
- Y''= 4$Ae^{2x} + 4Be^{-2x}$=4($Ae^{2x} + Be^{-2x}$)=4y
- Y''-4y=0

Solution of a differential equation:

- A solution of differential equation is the relation between the variables which satisfies the given differential equation.
- Example:
- The solution of the differential equation $\frac{dy}{dx} - 2\mathrm{y} = 0$ is given by $\mathrm{y} = \mathrm{c}e^{2x}$

- **General solution**: The general solution of the differential equation is that in which the number of arbitrary constants is equal to the order of the given differential equation. y= c e^2x
- **Particular solution:** A particular solution is that solution which can be obtained from the general solution by giving particular values to the arbitrary constants. Ex- ---y= 2e^2x
- **Singular solution:** A singular solution is the solution of the differential equation which is neither a general solution nor a particular solution. Only some equations have singular solution.

- Example: The solution of the DE

  $\left(\frac{dy}{dx}\right)^2 - 4\mathrm{y} = 0$ is

- y=$(x+a)^2$ *is the general solution*
- y=x^2 *is the particular solution*
- singular solution ???
- Y=0 is a singular solution

First order first degree differential equations

- The first order first degree differential equations are in the form of
- $F(x, y(x), \frac{dy}{dx}) = 0$

- <u>Classification of first order first degree differential equations</u>

1. Variable separable equations
2. Reducible to variable separable
3. Homogeneous equations
4. Non homogeneous but reducible to homogeneous equations
5. Linear DE
6. Non Linear but reducible to linear DE
7. Exact equations
8. Inexact equations but reducible to exact equation

I. Variable Separable Equations

- If in an equation it is possible to separate all the functions of x and dx to one side and all the functions of y and dy to the other side then the variables are said to be separable. The general form of such equations is
- $f(x)dx = g(y)dy$
- Solution of variable separable equations is obtainable by integrating on both sides of equation

$$\int f(x)dx = \int g(y)dy + c \text{ is the solution}$$

- Example: solve (1-x)dy+(1-y)dx=0
- (1-x)dy=(y-1)dx
- $\frac{1}{(1-x)}$dx=$\frac{1}{(y-1)}$dy→ variable and separable equation
- $\int \frac{1}{(1-x)}$dx =$\int \frac{1}{(y-1)}$dy
- -log(1-x)=log(y-1)+c
- log(y-1)+log(1-x)+c=0
- log(1-x)(y-1)=-c
- (1-x)(y-1)=e^{-c}
- (1-x)(1-y)=A where A=-e^{-c}→ general solution

II.Equations reducible to variable separable

- By choosing a suitable substitution some first order first degree differential equations are reducible to variable separable equation.Then we follow the same solution method of variable separable equations(i.e by integrating the equation)
- Example: Solve $\frac{dy}{dx}=(4x+y+1)^2$