

Formation of PDE

1. $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, a, b are constants

→ diff w.r.t x , keeping y as constant

$$x \frac{\partial z}{\partial x} = \frac{z x}{a^2} \Rightarrow \boxed{\frac{1}{x} \frac{\partial z}{\partial x} = \frac{1}{a^2}}$$

→ diff w.r.t y , treat x as constant

$$y \frac{\partial z}{\partial y} = \frac{z y}{b^2} \Rightarrow \boxed{\frac{1}{y} \frac{\partial z}{\partial y} = \frac{1}{b^2}}$$

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$= x^2 \cdot \frac{1}{x} \frac{\partial z}{\partial x} + y^2 \cdot \frac{1}{y} \frac{\partial z}{\partial y}$$

$$\boxed{z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}}$$

order 1
degree 1
homogeneous

No. of variables (independent) = No. of constant

PDE eq for which order = 1

→ form PDE

$$z = f(x^2 + y^2)$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} f(x^2 + y^2) = f'(x^2 + y^2)(2x)$$

$$\frac{\partial z}{\partial x} = 2x f'(x^2 + y^2) \rightarrow \textcircled{1}$$

$$\frac{\partial z}{\partial y} = f'(x^2 + y^2)(2y) \rightarrow \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2} \quad \frac{z \frac{\partial z}{\partial x}}{z \frac{\partial z}{\partial y}} = \frac{x f'(x^2 + y^2)}{y f'(x^2 + y^2)} \Rightarrow \boxed{y \frac{\partial z}{\partial x} = x \frac{\partial z}{\partial y}}$$

order 1
degree 1

No of Independent variable \geq No. of constants
order ≥ 1

No of Constant $>$ No. of independent variable
order > 1

