EE2101 - Control Systems

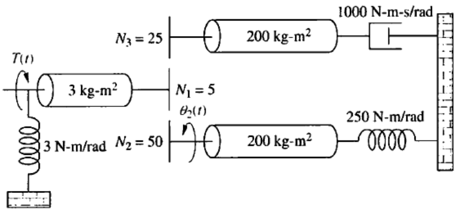
S.LIKHITA
EE19BTECH11032
Electrical Engineering
IIT Hyderabad

September 7, 2020

Problem

Find the transfer function $G(s) = \dfrac{\theta_2(s)}{T(s)}$,

for rotational mechanical system shown in the figure.



Rotational Mechanical Systems

Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. They are moment of inertia, torsional spring and a dashpot.

• If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system.

• Since the applied torque and the opposing torques are in opposite directions, **the algebraic sum of torques acting on the system is zero.**

We know that, for:

•**Spring:**

Torque, T(t) = Kθ(t)

$$Impedance, T(s)/\theta(s) = K \tag{2.1}$$

•**Moment of Inertia:**

Torque, T(t) = J($\frac{d\omega}{dt}$) = $J(\frac{d^2\theta}{dt^2})$

$$Impedance, T(s)/\theta(s) = Js^2 \tag{2.2}$$

•**Dashpot:**

a)One free end (i.e if one end is fixed to a reference):

$\mathrm{T(t)} = \mathrm{D}\omega(t) = D(\frac{d\theta}{dt})$

b)For two free ends,

$\mathrm{T(t)} = \mathrm{D}(\ \frac{d(\theta_1 - \theta_2)}{dt})$

And,

$$Impedance,\ T(s)/\theta(s) = Ds \qquad (2.3)$$

Gears

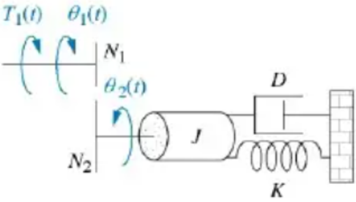
We will assume that connected gears fit perfectly together.
Consider input gear has radius r_1 and is rotated by θ_1; output gear has radius r_2, then θ_2=$(r_1/r_2)\theta_1 = (\mathrm{N}_1/N_2)\theta_1$.
where,N1 and N2 are no.of teeth of gears respectively and radius is proportional to no.of teeth.
No.of teeth is inversely proportional to torque.

Assuming there is no energy loss, we have $T\theta$= constant..i.e. $T_1\theta_1 = T_2\theta_2$.

And, $T_1\mathrm{N}_2 = \mathrm{T}_2\mathrm{N}_1$

Solving the problem

To find the relation between T1 and θ_1:



Consider the torque due to the damper on shaft2: $T_{D2} = Ds\theta_2(s)$

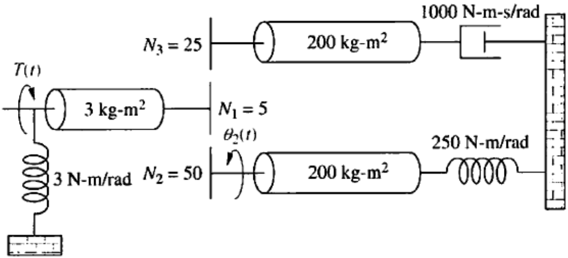
$$T_{D2} = Ds\frac{N_1}{N_2}\theta_1(s)$$

And,

$$T_{D1} = (N_1/N_2)T_{D2}$$

$$T_{D1} = \frac{N_1}{N_2}[Ds\frac{N_1}{N_2}\theta_1(s)] = (\frac{N_1}{N_2})^2 Ds\theta_1(s)$$

Reflecting Impedances to θ_2 on the system:



Due to springs $\implies$ $250 + 3(50/5)^2$.
Due to dashpot $\implies$ $[1000(5/25x50/5^2)]s$.

Due to moment of inertia
$\implies [200 + 3(50/5)^2 + 200(5/25x50/5^2)]s^2.$

Adding up all the effects,
$[[250 + 3(50/5)^2] + [1000(5/25x50/5^2)]s + [200 + 3(50/5)^2 + 200(5/25x50/5^2)]s^2]\theta_2(s) = (50/5)[T(s)] \qquad (2.4)$

$$\implies [(550) + (4000)s + (500 + 800)s^2](\theta_2(s)) = 10[T(s)]$$
$$\implies \frac{(\theta_2(s))}{T(s)} = \frac{10}{1300s^2 + 4000s + 550}$$

Therefore, the required transfer function G(s) is
$$\frac{\theta_2(s)}{T(s)} = \frac{1}{130s^2 + 400s + 55}.$$