Regression Methods/Analysis Quiz #1 (released as a Canvas assignment at 9:10 pm) June 7, 2023

Please submit your quiz answers (you do not need to submit this page of questions) into Canvas today by 10:25 pm EST. I will be online for emergency situations (for students who cannot access the quiz, cannot submit their quiz answers, etc.). Assume I am proctoring the quiz and I have no knowledge regarding the questions on the quiz. Just state any assumption(s) you make if you have difficulty understanding a question and/or parts of a question. **You need to show your work and carry through calculations at a sufficient level to show that you are not guessing.** Canvas closes **exactly** at 10:25 pm EST, so leave sufficient time to upload your answers.

The point total of the quiz is 80 points. Your quiz grade will be based on the percent of 80 points you earned.

Reminder: The quiz is open book, open notes. Use of ChatGPT and similar AI resources is not allowed. <u>You are to work alone</u>. <u>Rutgers Honors Pledge is in effect</u>.

1. (10 pts) Prove $\Sigma(Y_i - \overline{Y}) = 0$ where the sum is performed over n data points.

2. (10 pts) Consider the simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ for i=1, . . ., n where β_0 is known. Find the least squares estimator of β_1 for this model.

3. (10 pts) Suppose a researcher has a data set (x_i, y_i) where the data contain 3 observations at x_4 where the responses (y_i) are all different. Provide and explanation to the researcher why the maximum R^2 that will be computed from their results is less than 1.

4. (6 pts) Why did we need to obtain an estimator for σ^2?

5. (10 pts) Given the following 6 data points from the model $Y_i = \beta X_i + \varepsilon_i$ for i=1, 2, . . ., 6.
(-2,-6), (-1,-4), (0,3), (0,0), (1,2), (2,11)
The researcher knows from previous experimental knowledge that β must equal either -1, 0, or 1.
Obtain the Least Squares estimator of β for the researcher.
Calculus is not needed. Show your work so it is clear to me that you are not guessing.

6. (7 pts) In a simple linear regression analysis, the computed R-square is equal to 0.75 and the t-ratio for testing whether a regression relation exists is not significant at $\alpha = 0.05$. What do these results tell you about the analysis?

7. (10 pts) A 95% confidence interval for β_1 in a simple linear model was computed to be (-2.0, -0.5). Circle those statements which **must** be True.
a. The test H0: $\beta_1=0$ was rejected at the $\alpha = 0.05$ level.
b. β_1 is not equal to 0.
c. (-2.0, -0.5) covers β_1 95% of the time.
d. β_1 was estimated to be -1.25.
e. The width of a 90% confidence interval on β_1 computed from these same data is less than 1.5.

8. (7 pts) During one lecture I stated a phrase followed by a question where the response by students was “go”. Provide in your own words using a sentence or two an interpretation of the phrase I stated.

9. (10 pts) **Draw** a scatterplot of approximately n=10 data points for each of the following situations from a simple linear regression. Draw a scatterplot showing situation **a**, and another scatterplot showing situation **b**.
a. SS(Total) = SS(Regression), the least squares estimate of $\beta_1 < 0$.

b. The least squares estimate of β_0 is equal to 3 and the least squares estimate of β_1 is equal to $\overline{Y}$.