CHAPTER-10
STRAIGHT LINES

Excercise 10.3

Q3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

1. $x - \sqrt{3}y + 8 = 0$
2. $y - 2 = 0$
3. $x - y = 4$

Solution:

1. From the given equation:

$$\mathbf{m} = \frac{1}{\sqrt{3}} \tag{1}$$

$$c = \frac{8}{\sqrt{3}} \tag{2}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \tag{3}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{n}^\top = \begin{pmatrix} -\frac{1}{\sqrt{3}} & 1 \end{pmatrix} \tag{5}$$

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top.\mathbf{n}} \tag{6}$$

$$= \frac{2}{\sqrt{3}} \tag{7}$$

Slope of normal is given by:

$$\tan\theta = -\frac{1}{\mathbf{m}} = -\frac{1}{\frac{1}{\sqrt{3}}} = -\sqrt{3} \tag{8}$$

$$\theta = 120^\circ \tag{9}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{8}{2} \tag{10}$$

$$= 4 \tag{11}$$

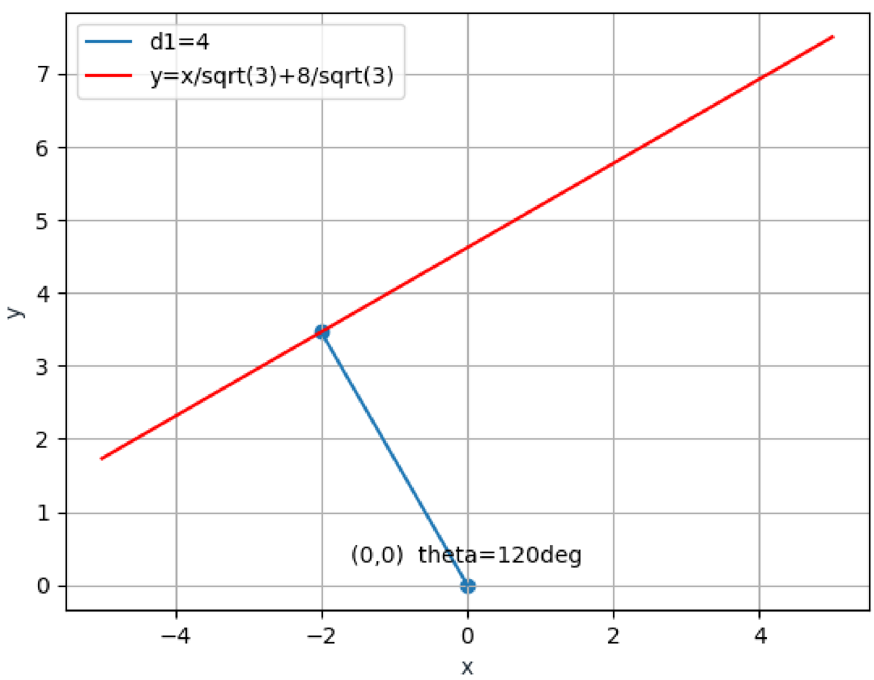

Figure 1:

2. From the given equation:

$$\mathbf{m} = 0 \tag{12}$$
$$c = 2 \tag{13}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{15}$$
$$\mathbf{n}^\top = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{16}$$
$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top . \mathbf{n}} \tag{17}$$
$$= 1 \tag{18}$$

Slope of normal is given by:

$$\tan\theta = -\frac{1}{\mathbf{m}} = -\frac{1}{0} = \infty \tag{19}$$
$$\theta = 90° \tag{20}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{2}{1} \tag{21}$$
$$= 2 \tag{22}$$

3. From the given equation:

$$\mathbf{m} = 1 \tag{23}$$
$$c = -4 \tag{24}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{25}$$

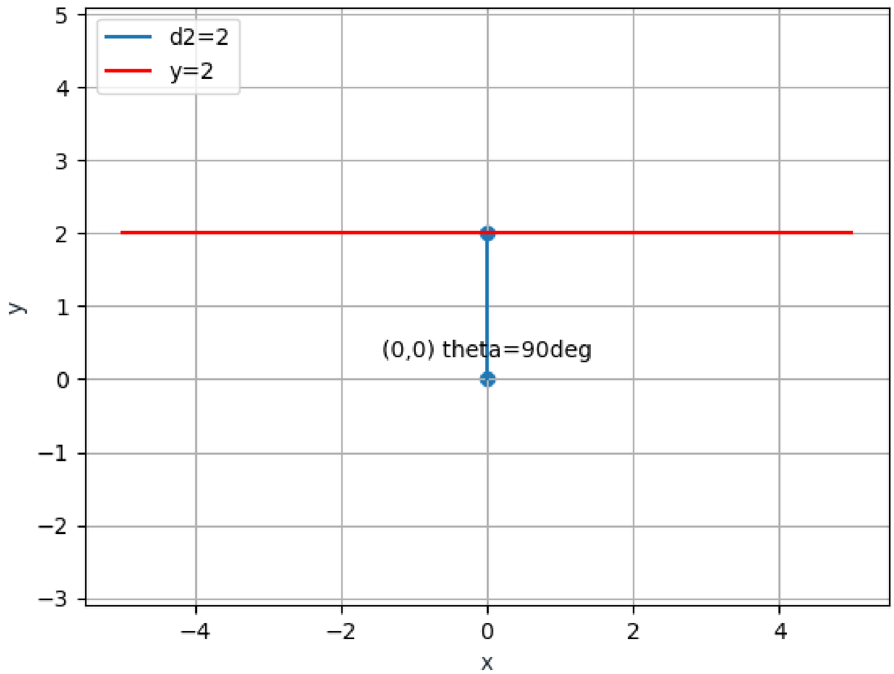

Figure 2:

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{26}$$

$$\mathbf{n}^\top = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{27}$$

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top . \mathbf{n}} \tag{28}$$

$$= \sqrt{2} \tag{29}$$

Slope of normal is given by:

$$\tan\theta = -\frac{1}{\mathbf{m}} = -\frac{1}{1} = -1 \tag{30}$$

$$\theta = 315^\circ \tag{31}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{4}{\sqrt{2}} \tag{32}$$
$$= 2\sqrt{2} \tag{33}$$

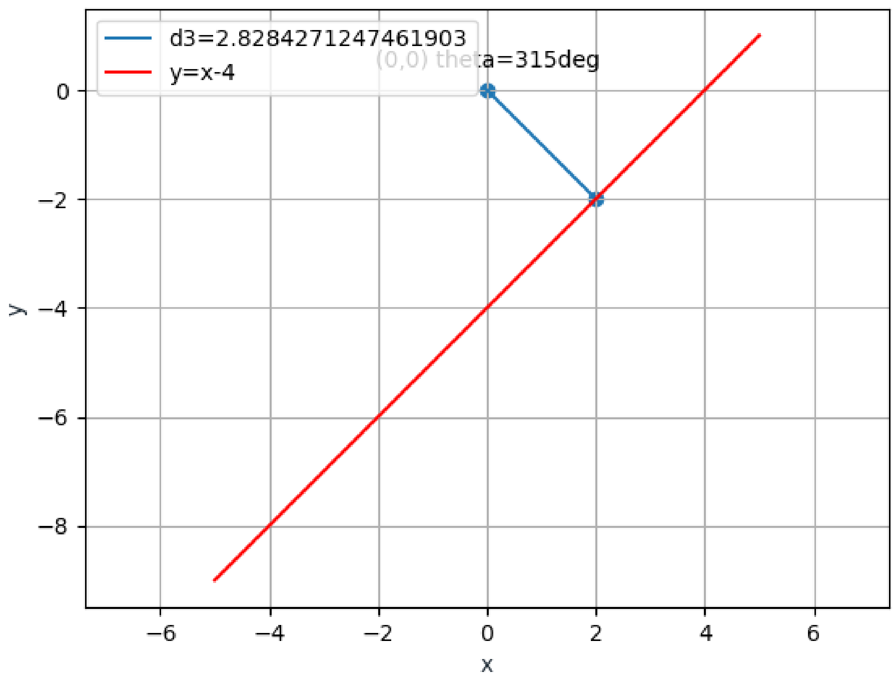

Figure 3: