

CLASS-9
CHAPTER-10
CIRCLES

Exercise 10.5

Q1. In Figure 1. **A, B, C** are the three points with centre **O** such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If **D** is a point on the circle other than the arc ABC, find $\angle ADC$

Solution

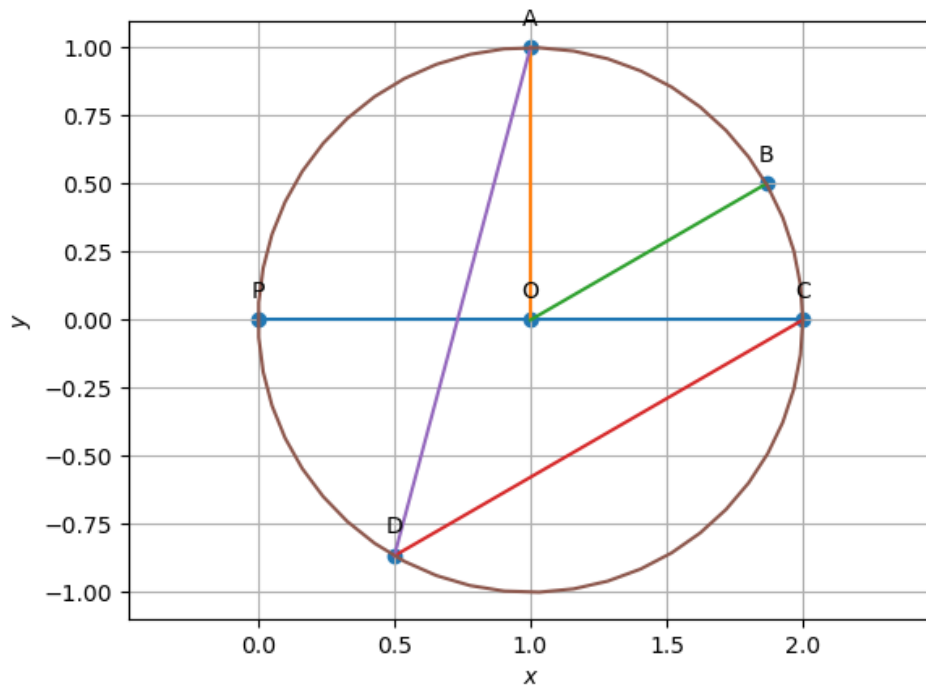


Figure 1:

Construction

Symbol	Values	Description
r	1 unit	Radius of OA and OB
\mathbf{O}	$\begin{pmatrix} 0 \\ r \end{pmatrix}$	Center of the circle
\mathbf{C}	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Standard basis vector \mathbf{e}_1
α	30°	Angle between vectors \mathbf{B} and \mathbf{C} w.r.t \mathbf{O}
β	60°	Angle between vectors \mathbf{A} and \mathbf{B} w.r.t \mathbf{O}
γ	??	Angle between vectors \mathbf{A} and \mathbf{C} w.r.t \mathbf{D}

Assumptions

1. Let \mathbf{P} be a point on the circle such that by expandig OC upto \mathbf{P} we get diameter POC.
2. To find $\angle ADC$ let the circle be unit circle and diameter POC on x axis.
3. Take three points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ and α, β, γ be three angles made by the points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ with respect to diameter POC.

From the Figure 1:

$$\alpha = \angle POC = 180^\circ, \beta = \angle POA = 90^\circ, \gamma = \angle POD = 300^\circ \quad (1)$$

Verification:

From assumptions the vector points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ be

$$\mathbf{C} = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \cos \gamma \\ \sin \gamma \end{pmatrix} \quad (2)$$

Let AC be the chord that subtends angles at the center \mathbf{O} and at point \mathbf{D} . The cosine of the angle subtended at point \mathbf{D} is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{C} - \mathbf{D}\|} \quad (3)$$

Where

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} \cos \beta - \cos \gamma \\ \sin \beta - \sin \gamma \end{pmatrix}, \mathbf{C} - \mathbf{D} = \begin{pmatrix} \cos \alpha - \cos \gamma \\ \sin \alpha - \sin \gamma \end{pmatrix} \quad (4)$$

$$(\mathbf{A} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D}) = 4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \cos \frac{\alpha - \beta}{2} \quad (5)$$

$$\|\mathbf{A} - \mathbf{D}\| \|\mathbf{C} - \mathbf{D}\| = 4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \quad (6)$$

Substituting (5) and (6) in (3),

$$\cos(\angle ADC) = \frac{4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \cos \frac{\alpha - \beta}{2}}{4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2}} \quad (7)$$

$$\cos(\angle ADC) = \cos \frac{\alpha - \beta}{2} \quad (8)$$

By substituting α and β values in (8)

$$\angle ADC = \frac{\alpha - \beta}{2} = \frac{(180^\circ - 90^\circ)}{2} = 45^\circ \quad (9)$$