CLASS-12
CHAPTER-11
THREE DIMENSIONAL GEOMETRY

Excercise 11.2

Q1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.
Solution:

$$\mathbf{A} = \begin{pmatrix} \frac{12}{13} \\ \frac{-3}{13} \\ \frac{-4}{13} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{4}{13} \\ \frac{12}{13} \\ \frac{3}{13} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{3}{13} \\ \frac{-4}{13} \\ \frac{12}{13} \end{pmatrix} \tag{1}$$

Stack all three vectors into a single matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix}, \mathbf{P}^\top = \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} \tag{2}$$

$$\mathbf{P}\mathbf{P}^\top = \mathbf{I} \tag{3}$$

$$\begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{I} \tag{4}$$

Hence, all three vectors are mutually orthogonal(perpendicular) to each other.