Excercise 10.4

Q2. Find the values of θ and p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Solution:

From the given line equation:

$$\mathbf{m} = -\sqrt{3} \tag{1}$$

$$c = -2 \tag{2}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{3}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} -\sqrt{3} \\ -1 \end{pmatrix} \tag{4}$$

$$\mathbf{n}^\top = \begin{pmatrix} -\sqrt{3} & -1 \end{pmatrix} \tag{5}$$

Angle between perpendicular and the positive x-axis is given by:

$$\cos\theta = \frac{\mathbf{e}_1^\top \mathbf{n}}{\|\mathbf{e}_1\|\,\|\mathbf{n}\|} \tag{6}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} -\sqrt{3} \\ -1 \end{pmatrix}}{2} \tag{7}$$

$$= -\frac{\sqrt{3}}{2} \tag{8}$$

$$\implies \theta = 210^\circ \tag{9}$$

The perpendicular distance to the line is given by:

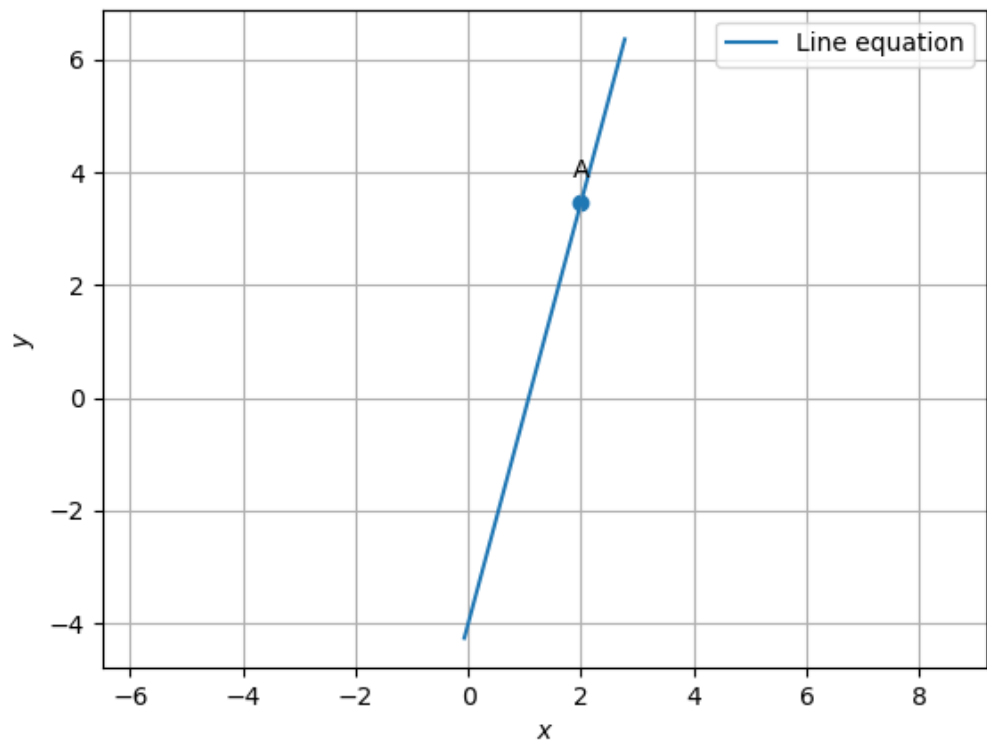$$p = \frac{|c|}{\|\mathbf{n}\|} = \frac{2}{2} = 1 \tag{10}$$

Figure 1: