Conic Sections - Ellipse

1 11^{th} Maths - Chapter 11

This is Problem-3 from Exercise 11.3

1. Find the coordinates of the focii, the vertices, the length of major and minor axes, the eccentricity and the length of the latus rectum of an ellipse whose equation is given by $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

   Solution: The given equation of ellipse can be rearranged as

$$9x^2 + 16y^2 - 144 = 0 \tag{1}$$

   The above equation can be equated to the general equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

   From (1) and (2)

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \mathbf{0} \tag{4}$$

$$f = -144 \tag{5}$$

   From (3) the eigen values λ_1 and λ_2 are given as

$$\lambda_1 = 9 \tag{6}$$

$$\lambda_2 = 16 \tag{7}$$

(a) The eccentricity of the ellipse is given as

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{8}$$

$$= \sqrt{1 - \frac{9}{16}} \tag{9}$$

$$= \frac{\sqrt{7}}{4} \tag{10}$$

(b) Finding the coordinates of Focii

$$\mathbf{F} = \pm e \sqrt{\frac{|f_0|}{\lambda_2 \left(1 - e^2\right)}} \mathbf{e_1} \tag{11}$$

$$\text{Where } f_0 = -f \tag{12}$$

$$\mathbf{F} = \pm \sqrt{7} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{13}$$

$$= \pm \begin{pmatrix} \sqrt{7} \\ 0 \end{pmatrix} \tag{14}$$

(c) The length of the major axis is given by

$$2\sqrt{\left|\frac{f_0}{\lambda_1}\right|} \tag{15}$$

$$2\sqrt{\left|\frac{144}{9}\right|} = 8 \tag{16}$$

(d) The length of minor axis is given by

$$2\sqrt{\left|\frac{f_0}{\lambda_2}\right|} \tag{17}$$

$$2\sqrt{\left|\frac{144}{16}\right|} = 6 \tag{18}$$

(e) The vertices of the ellipse are given by

$$\pm \begin{pmatrix} 0 \\ \sqrt{\left|\frac{f_0}{\lambda_2}\right|} \end{pmatrix} = \pm \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{19}$$

(f) The length of latus rectum is given as

$$2\frac{\sqrt{|f_0\lambda_1|}}{\lambda_2} \tag{20}$$

$$= 2\frac{\sqrt{|144\,(9)|}}{16} \tag{21}$$

$$= \frac{9}{2} \tag{22}$$

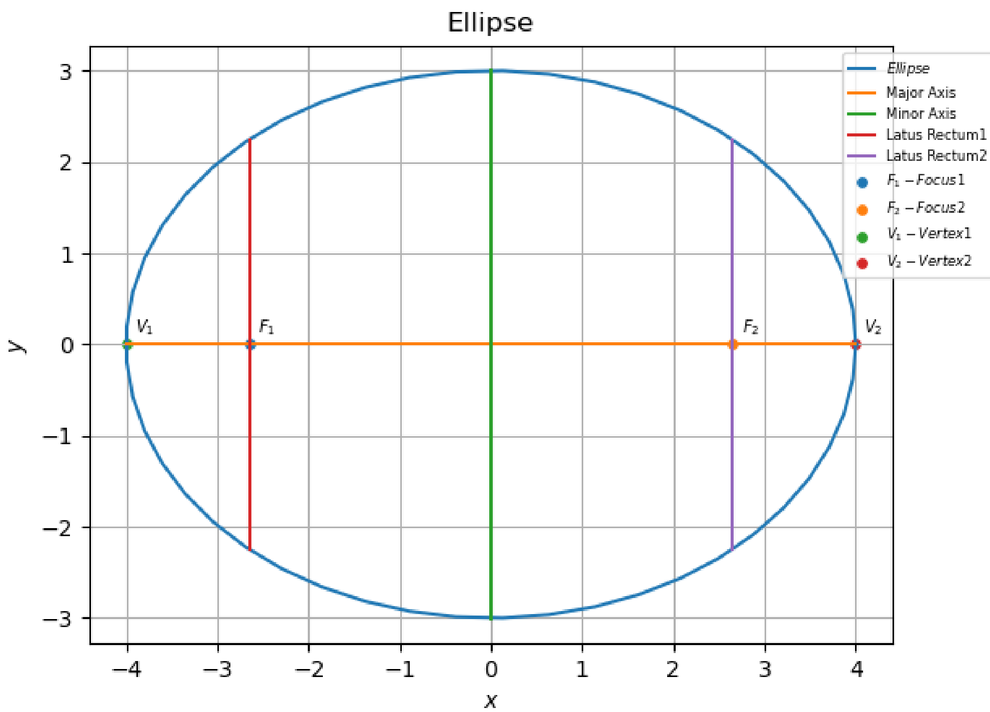

Figure 1