CLASS-9
CHAPTER-10
CIRCLES

Excercise 10.5

Q1. In Figure 1. A,B,C are the three points with centre O such that $\angle$BOC=30° and $\angle$AOB=60°.If D is a point on the circle other than the arc ABC, find $\angle$ADC

Solution

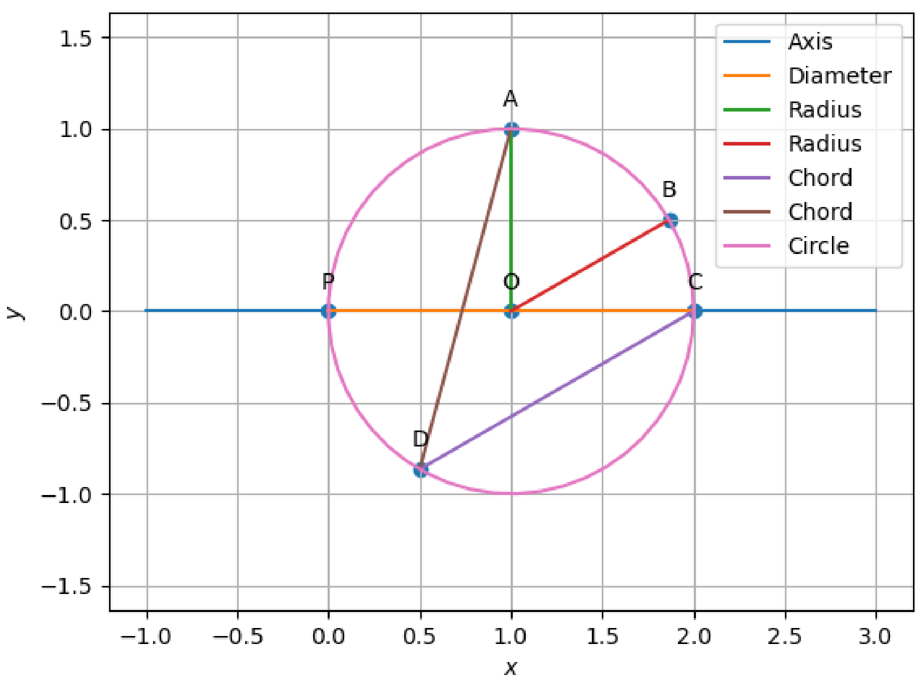

Figure 1:

Construction

The input parameters are the lengths

Symbol	value	Description
O		centre
$\angle$BOC	30°	Angle between vectors B and C
$\angle$AOB	60°	Angle between vectors A and B
$\angle$ADC	??	Angle between vectors A and C

Table 1:

Assumptions

1. Let P be a point on the circle such that by expandig OC upto P we get diameter POC.
2. To find $\angle$ADC let the circle be unit circle and diameter POC on x axis.
3. Take three points C,A,D and α,β,γ be three angles made by the points C,A,D with respect to diameter POC.

From the Figure 2:

$$\alpha = \angle\mathbf{POC} = 180°, \beta = \angle\mathbf{POA} = 90°, \gamma = \angle\mathbf{POD} \tag{1}$$

Proof:

From assumptions the vector points C,A,D be

$$\mathbf{C} = \begin{pmatrix} cos\alpha \\ sin\alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} cos\beta \\ sin\beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} cos\gamma \\ sin\gamma \end{pmatrix} \tag{2}$$

Let AC be the chord that subtends angles at the center O and at point D. The cosine of the angle subtended at point D is given by

$$cos(\angle ADC) = \frac{(A-D)^T(C-D)}{\|A-D\|\,\|C-D\|} \tag{3}$$

Where

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix} cos\beta - cos\gamma \\ sin\beta - sin\gamma \end{pmatrix}, \mathbf{C}-\mathbf{D}=\begin{pmatrix} cos\alpha - cos\gamma \\ sin\alpha - sin\gamma \end{pmatrix} \tag{4}$$

$$(A-D)^T(C-D) = \begin{pmatrix} cos\beta - cos\gamma sin\beta - sin\gamma \end{pmatrix} \tag{5}$$

$$= 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2} \tag{6}$$

$$\|A-D\|^2\|C-D\|^2 = ((\cos\alpha-\cos\gamma)^2+(\sin\alpha-\sin\gamma)^2) \tag{7}$$

$$= 16\sin^2\frac{\alpha-\gamma}{2}\sin^2\frac{\beta-\gamma}{2} \tag{8}$$

$$\|A-D\|\,\|C-D\| = 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2} \tag{9}$$

Substituting (6) and (9) in (3),

$$cos(\angle ADC) = \frac{4sin\frac{\alpha-\gamma}{2}sin\frac{\beta-\gamma}{2}cos\frac{\alpha-\beta}{2}}{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}} \tag{10}$$

$$cos(\angle ADC) = cos\frac{\alpha-\beta}{2} \tag{11}$$

By substituting α and β values in (11)

$$\angle ADC = \frac{\alpha-\beta}{2} = \frac{(180^\circ-90^\circ)}{2} = 45^\circ \tag{12}$$