CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.1

Q1. Find the distance between the following pairs of points :

1. $(2,3)\,,(4,1)$
2. $(-5,7)\,,(-1,3)$
3. $(a,b)\,,(-a,-b)$

Solution:

1. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{1}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{2}$$

$$(\mathbf{A} - \mathbf{B})^{\top}(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 4 + 4 = 8 \tag{3}$$

$$d = \|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{A} - \mathbf{B})^{\top} (\mathbf{A} - \mathbf{B})} \tag{4}$$

$$d = \sqrt{(\mathbf{A} - \mathbf{B})^{\top} (\mathbf{A} - \mathbf{B})} = \sqrt{8} = 2\sqrt{2} = 2.828 \tag{5}$$

Hence the distance between the two points AB is 2.828 shown in Figure:1

2. The coordinates are given as

$$\mathbf{C} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{6}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{7}$$

$$(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D}) = \begin{pmatrix} -4 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 4 \end{pmatrix} = 16 + 16 = 32 \tag{8}$$

$$d = \|\mathbf{C} - \mathbf{D}\| = \sqrt{(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D})} \tag{9}$$

$$d = \sqrt{(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D})} = \sqrt{32} = 4\sqrt{2} = 5.656 \tag{10}$$

Hence the distance between the two points CD is 5.656 shown in Figure:1

3. The coordinates are given as

$$\mathbf{E} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{F} = \begin{pmatrix} -a \\ -b \end{pmatrix} \tag{11}$$

$$\mathbf{E} - \mathbf{F} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} -a \\ -b \end{pmatrix} = \begin{pmatrix} 2a \\ 2b \end{pmatrix} \tag{12}$$

$$(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F}) = \begin{pmatrix} 2a & 2b \end{pmatrix} \begin{pmatrix} 2a \\ 2b \end{pmatrix} = 4a^2 + 4b^2 \tag{13}$$

$$d = \|\mathbf{E} - \mathbf{F}\| = \sqrt{(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F})} \tag{14}$$

$$d = \sqrt{(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F})} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \tag{15}$$

Suppose, if a=1,b=2 , then d = $2\sqrt{1^2 + 2^2} = 4.472$

Hence the distance between the two points EF is $2\sqrt{a^2 + b^2} = 4.472$ shown in Figure:1

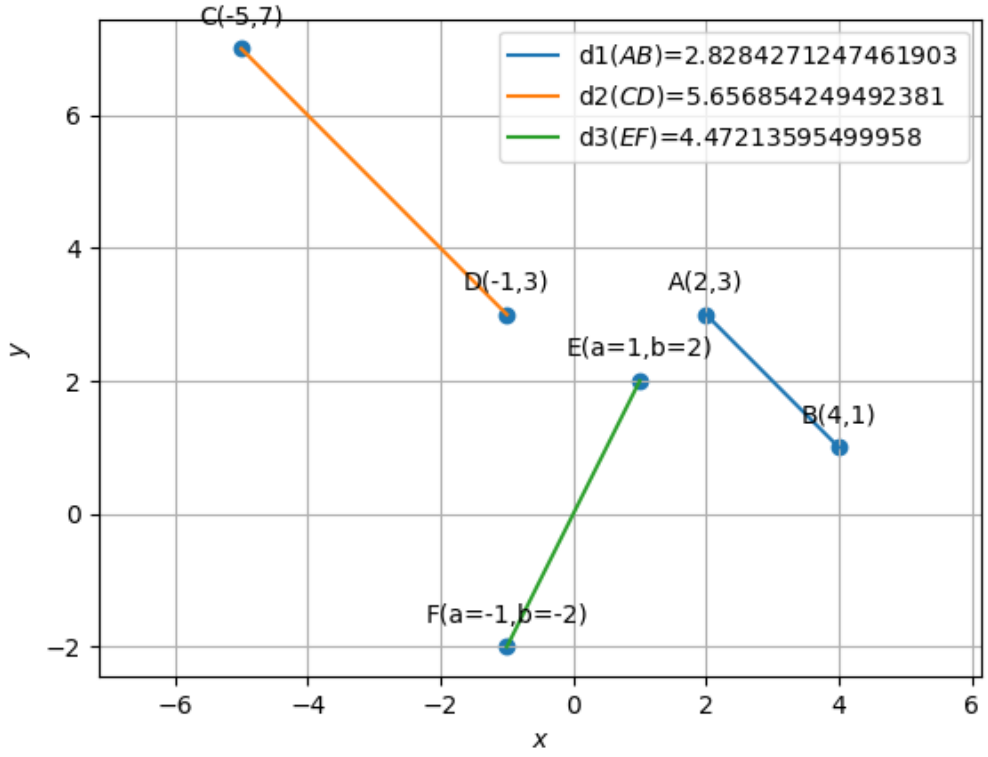
C(-5,7)
d1(AB)=2.8284271247461903
d2(CD)=5.656854249492381
d3(EF)=4.47213595499958
D(-1,3)
A(2,3)
E(a=1,b=2)
B(4,1)
F(a=-1,b=-2)
y
x
6
4
2
0
−2
−6
−4
−2
0
2
4
6

Figure 1: