

CLASS-9
CHAPTER-10
CIRCLES

Exercise 10.5

Q1. In Figure 1. A,B,C are the three points with centre O such that $\angle BOC=30^\circ$ and $\angle AOB=60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$

Solution

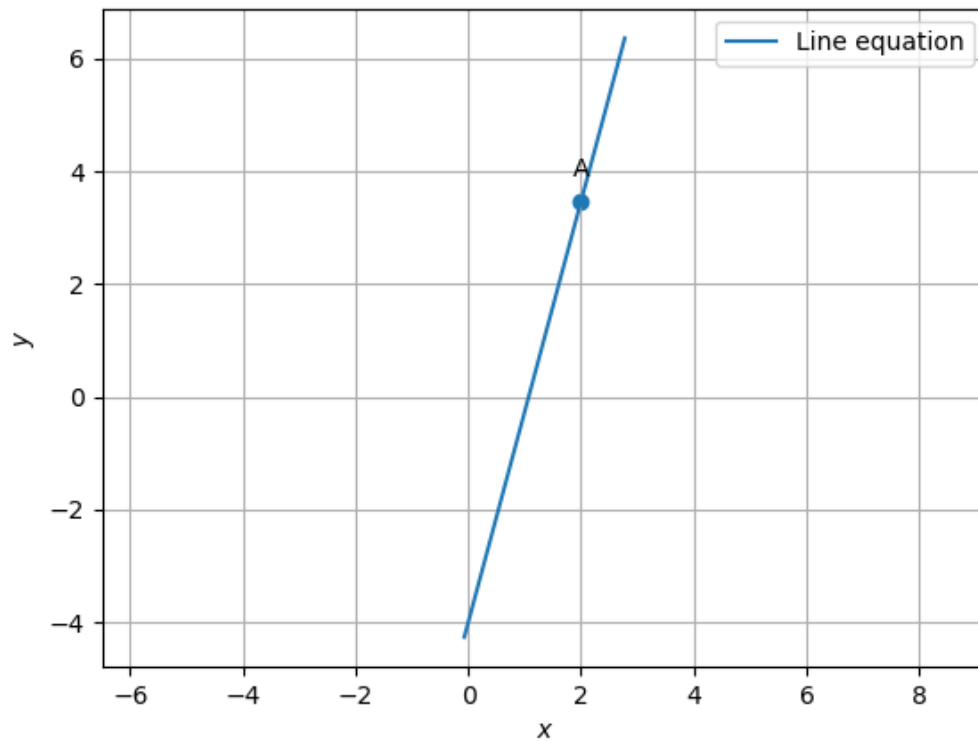


Figure 1:

Construction

The input parameters are the lengths

Symbol	value	Description
O		centre
$\angle BOC$	30°	Angle between vectors B and C
$\angle AOB$	60°	Angle between vectors A and B
$\angle ADC$??	Angle between vectors A and C

Table 1:

Assumptions

1. Let P be a point on the circle such that by expandig OC upto P we get diameter POC.
2. To find $\angle ADC$ let the circle be unit circle and diameter POC on x axis.
3. Take three points C,A,D and α, β, γ be three angles made by the points C,A,D with respect to diameter POC.

From the Figure 2:

$$\alpha = \angle \mathbf{POC} = 180^\circ, \beta = \angle \mathbf{POA} = 90^\circ, \gamma = \angle \mathbf{POD} \quad (1)$$

Proof:

From assumptions the vector points C,A,D be

$$\mathbf{C} = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \cos \gamma \\ \sin \gamma \end{pmatrix} \quad (2)$$

Let AC be the chord that subtends angles at the center O and at point D. The cosine of the angle subtended at point D is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A} - \mathbf{D})^T (\mathbf{C} - \mathbf{D})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{C} - \mathbf{D}\|} \quad (3)$$

Where

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} \cos\beta - \cos\gamma \\ \sin\beta - \sin\gamma \end{pmatrix}, \mathbf{C} - \mathbf{D} = \begin{pmatrix} \cos\alpha - \cos\gamma \\ \sin\alpha - \sin\gamma \end{pmatrix} \quad (4)$$

$$(A - D)^T(C - D) = (\cos\beta - \cos\gamma)(\sin\beta - \sin\gamma) \quad (5)$$

$$= 4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \cos \frac{\alpha - \beta}{2} \quad (6)$$

$$\|A - D\|^2 \|C - D\|^2 = ((\cos \alpha - \cos \gamma)^2 + (\sin \alpha - \sin \gamma)^2) \quad (7)$$

$$= 16 \sin^2 \frac{\alpha - \gamma}{2} \sin^2 \frac{\beta - \gamma}{2} \quad (8)$$

$$\|A - D\| \|C - D\| = 4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \quad (9)$$

Substituting (6) and (9) in (3),

$$\cos(\angle ADC) = \frac{4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \cos \frac{\alpha - \beta}{2}}{4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2}} \quad (10)$$

$$\cos(\angle ADC) = \cos \frac{\alpha - \beta}{2} \quad (11)$$

By substituting α and β values in (11)

$$\angle ADC = \frac{\alpha - \beta}{2} = \frac{(180^\circ - 90^\circ)}{2} = 45^\circ \quad (12)$$