

CHAPTER-7  
COORDINATE GEOMETRY

### Exercise 7.4

Q2. Find a relation between x and y if the points  $(x, y)$ ,  $(1, 2)$  and  $(7, 0)$  are collinear.

**Solution:**

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left( \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left( \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is " 1 " then the vectors are in linearly dependent. For  $2 \times 2$  matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \xrightarrow{R_2=R_2-R_1} \begin{pmatrix} x-1 & y-2 \\ -6 & 2 \end{pmatrix} \quad (6)$$

$$\xrightarrow{R_2=\frac{R_2}{-6}(x-1)-R_1} \begin{pmatrix} x-1 & y-2 \\ 0 & -\frac{1}{3}(x-1)-(y-2) \end{pmatrix} \quad (7)$$

If the rank of the matrix has to be 1, then:

$$-\frac{1}{3}(x-1)-(y-2)=0 \quad (8)$$

$$\mathbf{x} + 3\mathbf{y} - 7 = 0 \quad (9)$$

Suppose, if  $x = -2, y = 3$ , then rank of F is equal to one which is collinear as shown in Figure:1

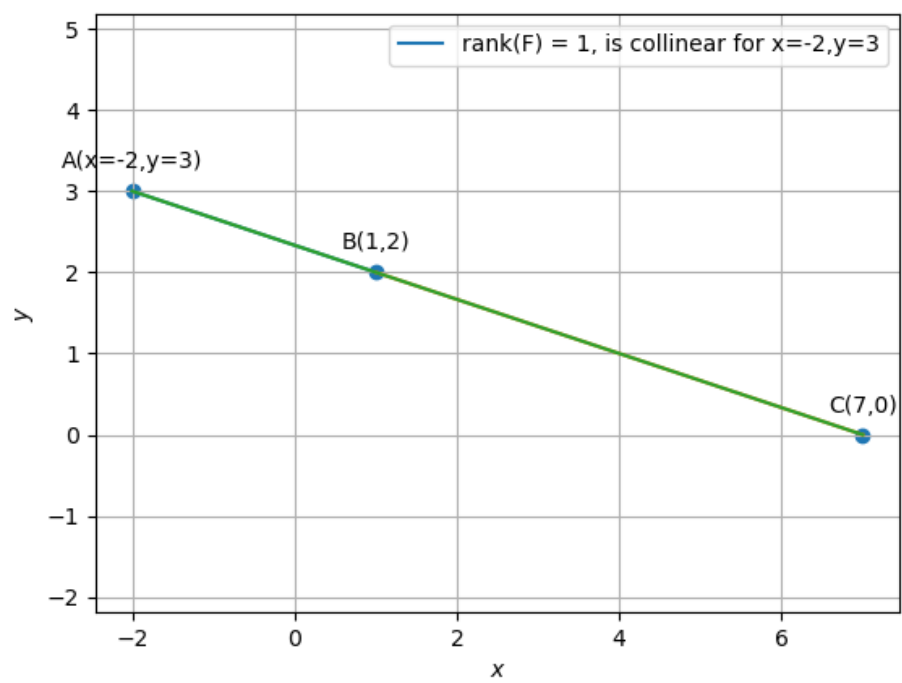


Figure 1: