Excercise 11.4

Q1. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1), (4, 3, -1)$.
Solution:
Given:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \tag{1}$$

From the given points $\mathbf{A}$ and $\mathbf{B}$, the directional vector is given by:

$$\mathbf{m} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \tag{2}$$

check whether two vectors are perpendicular to each other or not using:

$$\mathbf{m}^\top \mathbf{P} = 0 \tag{3}$$

$$\begin{pmatrix} -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \tag{4}$$

Hence, it satisfies the above condition, shows that the line passing through origin is perpendiclar to the line passing through points $\mathbf{A}$ and $\mathbf{B}$.