

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.4

Q2. Find a relation between x and y if the points (x, y) , $(1, 2)$ and $(7, 0)$ are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is "1" then the vectors are linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \xrightarrow{R_2 = \frac{R_2}{y} (x-7) - R_1} \begin{pmatrix} x-1 & x-7 \\ \frac{(x-7)}{y} \cdot (y-2) - (x-1) & 0 \end{pmatrix} \quad (6)$$

If the rank of the matrix has to be 1, then $\frac{(y-2)(x-7) - (x-1)y}{y} = 0$

$$\begin{aligned} (y-2)(x-7) - (x-1)y &= 0 \\ \implies x + 3y - 7 &= 0 \end{aligned} \quad (7)$$

Suppose, if $x = -2, y = 3$, then rank of \mathbf{F} is equal to one which is collinear as shown in Figure:1

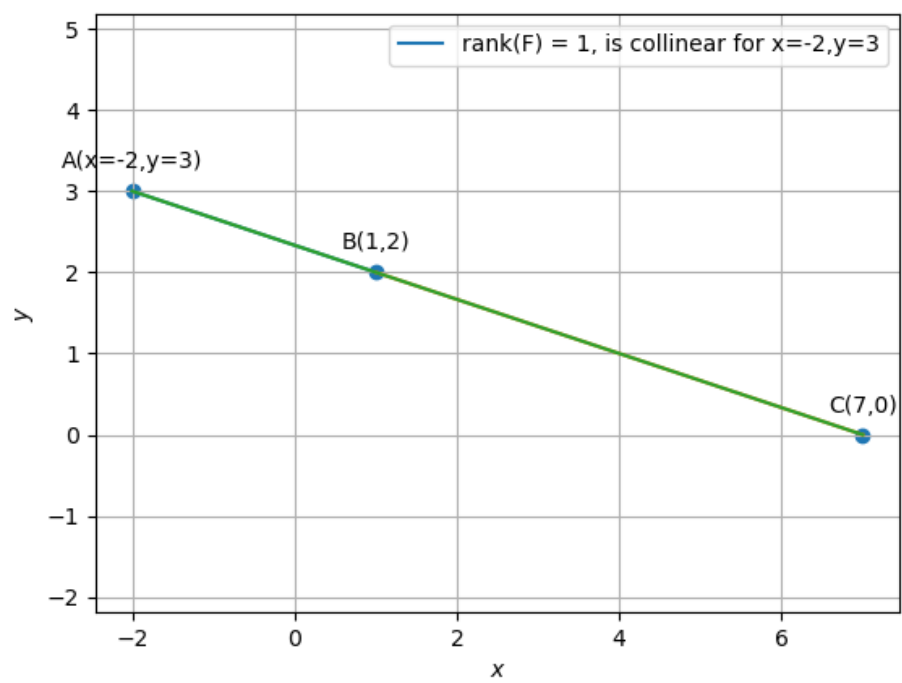


Figure 1: