CLASS-9
CHAPTER-10
CIRCLES

Excercise 10.5

Q1. In Figure 1. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are the three points with centre **O** such that $\angle$BOC=30°and $\angle$AOB=60°. If **D** is a point on the circle other than the arc ABC, find $\angle$ADC

Solution

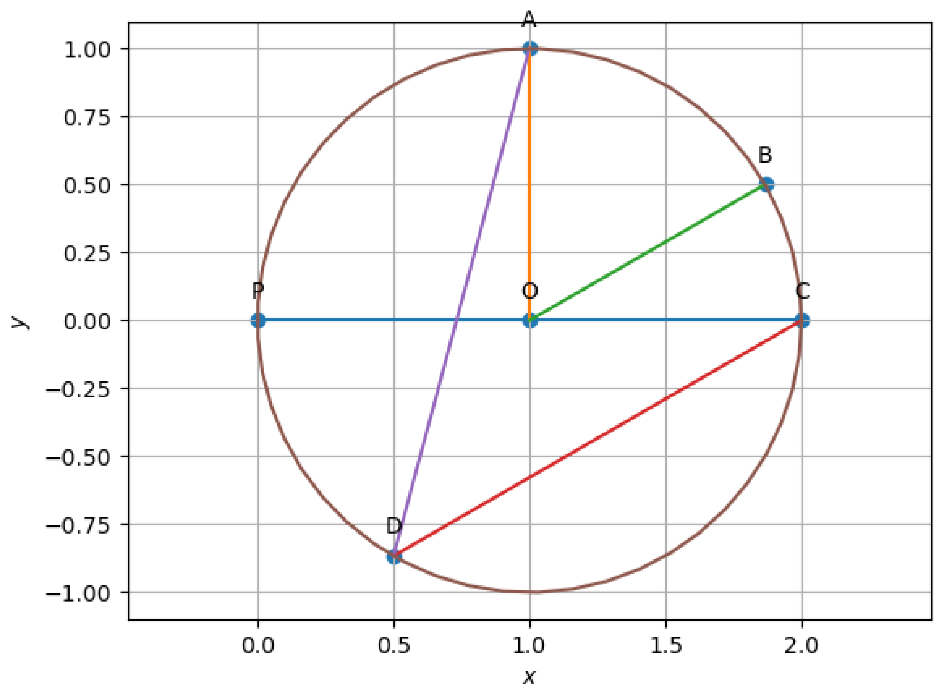

Figure 1:

Construction

Symbol	**Values**	**Description**
r	1 unit	Radius of OA and OB
O	$\begin{pmatrix} 0 \\ r \end{pmatrix}$	Center of the circle
C	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Standard basis vector $\mathbf{e}_1$
α	30°	∠BOC
β	60°	∠AOB
γ	??	∠ADC

Assumptions

1. Let **P** be a point on the circle such that by expandig OC upto **P** we get diameter POC.
2. To find ∠ADC let the circle be unit circle and diameter POC on x axis.
3. Take three points **C**, **A**, **D** and α,β,γ be three angles made by the points **C**, **A**, **D** with respect to diameter POC.

From the Figure 1:

$$\alpha = \angle POC = 180°, \beta = \angle POA = 90°, \gamma = \angle POD = 300° \tag{1}$$

Verification:

From assumptions the vector points **C**, **A**, **D** be

$$\mathbf{C} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} \tag{2}$$

Let AC be the chord that subtends angles at the center $\mathbf{O}$ and at point $\mathbf{D}$. The cosine of the angle subtended at point $\mathbf{D}$ is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D})}{\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\|} \tag{3}$$

Where

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix} \cos\beta - \cos\gamma \\ \sin\beta - \sin\gamma \end{pmatrix}, \mathbf{C}-\mathbf{D} = \begin{pmatrix} \cos\alpha - \cos\gamma \\ \sin\alpha - \sin\gamma \end{pmatrix} \tag{4}$$

$$(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D}) = 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2} \tag{5}$$

$$\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\| = 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2} \tag{6}$$

Substituting (5) and (6) in (3),

$$\cos(\angle ADC) = \frac{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2}}{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}} \tag{7}$$

$$\cos(\angle ADC) = \cos\frac{\alpha-\beta}{2} \tag{8}$$

By substituting α and β values in (8)

$$\angle ADC = \frac{\alpha-\beta}{2} = \frac{(180^\circ - 90^\circ)}{2} = 45^\circ \tag{9}$$