CHAPTER-11
STRAIGHT LINES

Excercise 11.3

Q3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

1. $x - \sqrt{3}y + 8 = 0$
2. $y - 2 = 0$
3. $x - y = 4$

Solution:

1. The given equation is represented as:

$$\mathbf{n} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{1}$$

$$\mathbf{n}^\top = \begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \tag{2}$$

$$\mathbf{n}^\top \mathbf{X} = c \tag{3}$$

$$\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{X} = -8 \tag{4}$$

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top . \mathbf{n}} \tag{5}$$

$$= \sqrt{\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} . \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}} \tag{6}$$

$$= 2. \tag{7}$$

Divide the above equation(4) by 2 on both sides, we get:

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \mathbf{X} = -4 \tag{8}$$

The normal form of straight line is given by:

$$\mathbf{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{9}$$

$$\mathbf{n}^\top \mathbf{X} = p \tag{10}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix} \mathbf{X} = p \tag{11}$$

By equating (8) and (11) equation, we get:

$$\theta = 120° \tag{12}$$
$$p = 4 \tag{13}$$

2. The coordinates are given as

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$
$$\mathbf{n}^\top = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{15}$$
$$\mathbf{n}^\top \mathbf{X} = c \tag{16}$$
$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{X} = 1 \tag{17}$$
$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top . \mathbf{n}} \tag{18}$$
$$= \sqrt{\begin{pmatrix} 0 & 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \tag{19}$$
$$= 1. \tag{20}$$

Divide the above equation(17) by 1 on both sides, we get:

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{X} = 1 \tag{21}$$

The normal form of straight line is given by:

$$\mathbf{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{22}$$
$$\mathbf{n}^\top \mathbf{X} = p \tag{23}$$
$$\begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix} \mathbf{X} = p \tag{24}$$

By equating (21) and (24) equations, we get:

$$\theta = 90° \tag{25}$$
$$p = 1 \tag{26}$$

3. The coordinates are given as

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{27}$$

$$\mathbf{n}^\top = \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{28}$$

$$\mathbf{n}^\top \mathbf{X} = c \tag{29}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = 4 \tag{30}$$

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top . \mathbf{n}} \tag{31}$$

$$= \sqrt{\begin{pmatrix} 1 & -1 \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \end{pmatrix}} \tag{32}$$

$$= \sqrt{2}. \tag{33}$$

Divide the above equation(30) by $\sqrt{2}$ on both sides, we get:

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{X} = 2\sqrt{2} \tag{34}$$

The normal form of straight line is given by:

$$\mathbf{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{35}$$

$$\mathbf{n}^\top \mathbf{X} = p \tag{36}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix} \mathbf{X} = p \tag{37}$$

By equating (34) and (37) equations, we get:

$$\theta = 45^\circ \tag{38}$$

$$p = 2\sqrt{2} \tag{39}$$

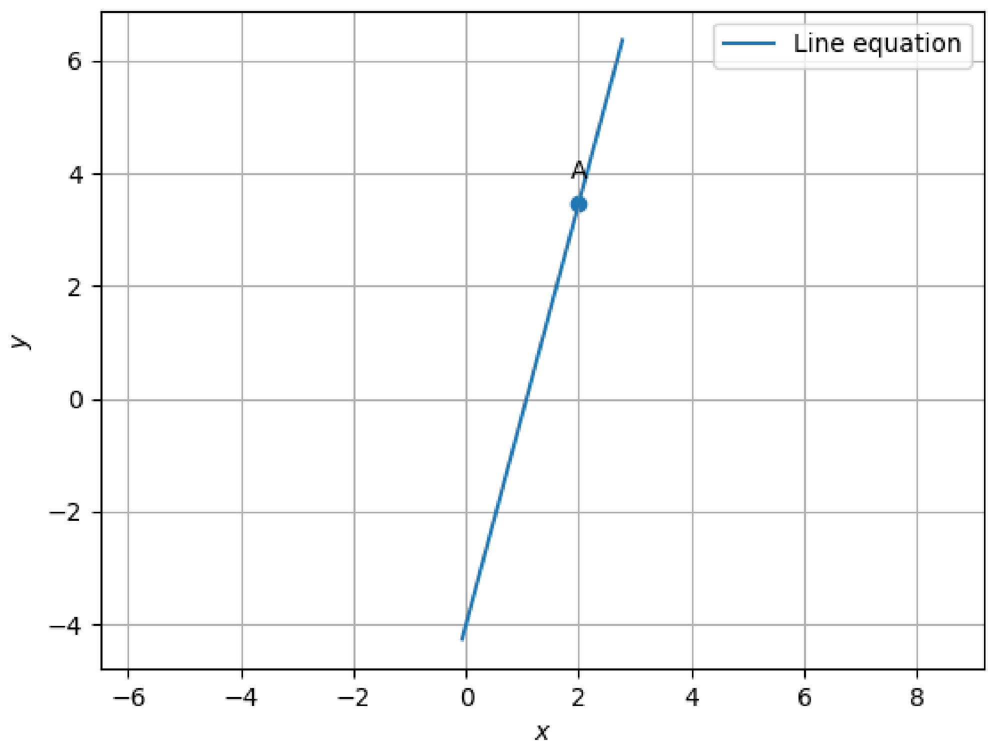

Figure 1: