

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.4

Q2. Find a relation between x and y if the points (x, y) , $(1, 2)$ and $(7, 0)$ are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is " 1 " then the vectors are linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \xrightarrow{R_2=R_2-R_1} \begin{pmatrix} x-1 & y-2 \\ -6 & 2 \end{pmatrix} \quad (6)$$

$$\xleftarrow{R_2=\frac{R_2}{-6}(x-1)-R_1} \begin{pmatrix} x-1 & y-2 \\ 0 & -\frac{1}{3}(x-1)-(y-2) \end{pmatrix} \quad (7)$$

If the rank of the matrix has to be 1, then $-\frac{1}{3}(\mathbf{x}-1) - (\mathbf{y}-2) = 0$

$$\implies \mathbf{x} + 3\mathbf{y} - 7 = 0 \quad (8)$$

Suppose, if $x = -2, y = 3$, then rank of \mathbf{F} is equal to one which is collinear as shown in Figure:1

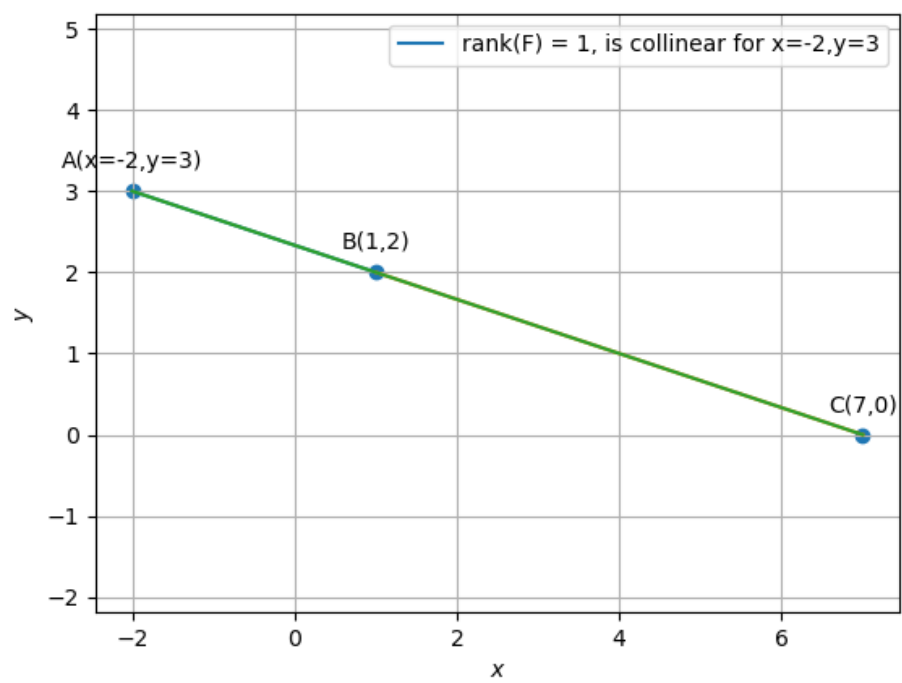


Figure 1: