CLASS-9
CHAPTER-10
CIRCLES

Excercise 10.6

Q1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution

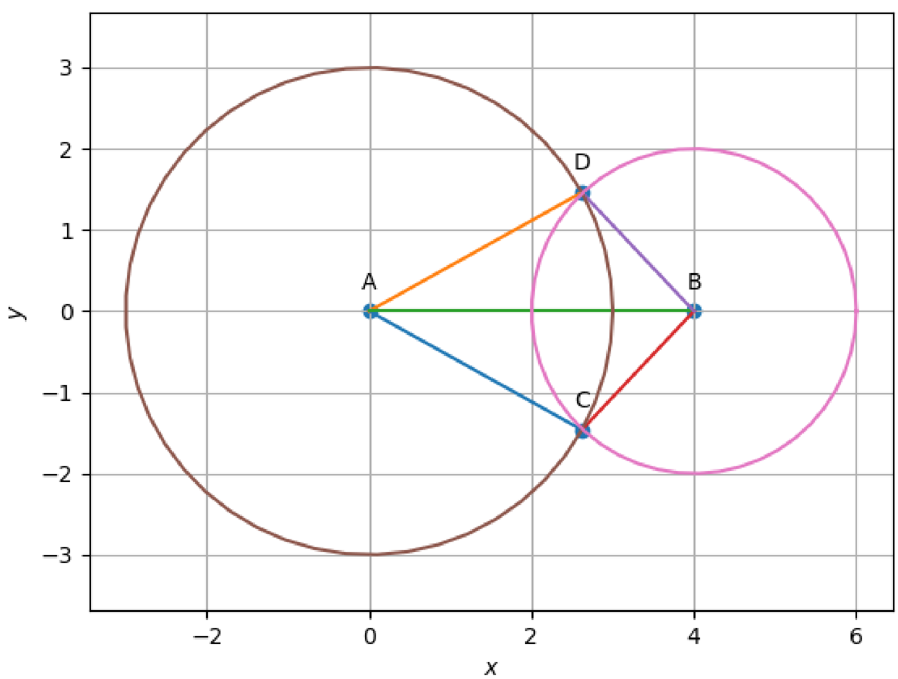

Figure 1:

Construction

Symbol	**Values**	**Description**
$\mathbf{A}$	$\mathbf{0}$	Center of circle 1
r_1	3 units	Radius of the circle 1
$\mathbf{B}$	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Center of circle 2
r_2	2 units	Radius of circle 2
$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Standard basis vector 1
$\mathbf{e}_2$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Standard basis vector 2

Verification:

The two circle equations given by:

$$\|x\|^2 - 9 = 0 \tag{1}$$

$$\|x\|^2 - 8\mathbf{e}_1 + 12 = 0 \tag{2}$$

It is easy to verify that

$$\mathbf{q} = 2.62\mathbf{e_1} \tag{3}$$

By substituting the below values, we get intersecting points:

$$\mathbf{m} = \mathbf{e}_2, \mathbf{q} = 2.62\mathbf{e}_1, \mathbf{V} = \mathbf{I}, \mathbf{u} = \mathbf{0}, f = -9 \tag{4}$$

The intersecing points $\mathbf{C}$ and $\mathbf{D}$

$$\mathbf{C} = \begin{pmatrix} 2.62 \\ -1.46 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2.62 \\ 1.46 \end{pmatrix} \tag{5}$$

Check whether the intersection angles $\angle$ADB and $\angle$ACB are equal or not:

1. Finding ∠ADB:

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix} -2.62 \\ -1.46 \end{pmatrix}, \mathbf{B}-\mathbf{D} = \begin{pmatrix} 1.3 \\ -1.46 \end{pmatrix} \tag{6}$$

$$(\mathbf{A}-\mathbf{D})^\top(\mathbf{B}-\mathbf{D}) = -0.22 \tag{7}$$

$$\|\mathbf{A}-\mathbf{D}\| \, \|\mathbf{C}-\mathbf{D}\| = 5.8 \tag{8}$$

$$\cos(\angle ADB) = \frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{B}-\mathbf{D})}{\|\mathbf{A}-\mathbf{D}\| \, \|\mathbf{B}-\mathbf{D}\|} \tag{9}$$

$$\angle ADB = 104.2^\circ \tag{10}$$

2. Finding ∠ACB:

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} -2.62 \\ 1.46 \end{pmatrix}, \mathbf{B}-\mathbf{C} = \begin{pmatrix} 1.3 \\ 1.46 \end{pmatrix} \tag{11}$$

$$(\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) = -0.22 \tag{12}$$

$$\|\mathbf{A}-\mathbf{C}\| \, \|\mathbf{B}-\mathbf{C}\| = 5.8 \tag{13}$$

$$\cos(\angle ACB) = \frac{(\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\| \, \|\mathbf{B}-\mathbf{C}\|} \tag{14}$$

$$\angle ACB = 104.2^\circ \tag{15}$$

Hence, both the intersecting angles are equal to each other, which satisfies the above condition.