### CLASS-9 CHAPTER-10 CIRCLES

# Excercise 10.5

Q1. In Figure 1. A, B, C are the three points with centre O such that  $\angle BOC=30^{\circ}$  and  $\angle AOB=60^{\circ}$ . If D is a point on the circle other than the arc ABC, find  $\angle ADC$ 

### Solution

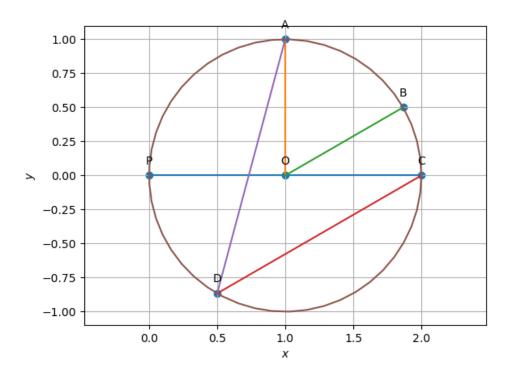


Figure 1:

#### Construction

Symbol	value	Description
0	(1,0)	Centre of the circle
POC	2 units	Diameter of the circle
r	1 unit	Radius of OA and OB
chord1	A(1,1) and $D(0.5, -0.86)$	Chord of AD
chord2	C(2,0) and $D(0.5,-0.86)$	Chord of CD
α	180°	Angle between vectors P and C
β	90°	Angle between vectors P and A
$\gamma$	300°	Angle between vectors P and D
∠BOC	30°	Angle between vectors B and C
∠AOB	60°	Angle between vectors A and B
∠ADC	??	Angle between vectors A and C

Table 1:

### Assumptions

- 1. Let  $\mathbf{P}$  be a point on the circle such that by expandig OC upto  $\mathbf{P}$  we get diameter POC.
- 2. To find  $\angle ADC$  let the circle be unit circle and diameter POC on x axis.
- 3. Take three points C, A, D and  $\alpha$ ,  $\beta$ ,  $\gamma$  be three angles made by the points C, A, D with respect to diameter POC.

From the Figure 1:

$$\alpha = \angle POC = 180^{\circ}, \beta = \angle POA = 90^{\circ}, \gamma = \angle POD = 300^{\circ}$$
 (1)

## Verification:

From assumptions the vector points C, A, D be

$$\mathbf{C} = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \cos \gamma \\ \sin \gamma \end{pmatrix}$$
 (2)

Let AC be the chord that subtends angles at the center  $\mathbf{O}$  and at point  $\mathbf{D}$ . The cosine of the angle subtended at point  $\mathbf{D}$  is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A} - \mathbf{D})^{\top}(\mathbf{C} - \mathbf{D})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{C} - \mathbf{D}\|}$$
(3)

Where

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} \cos \beta - \cos \gamma \\ \sin \beta - \sin \gamma \end{pmatrix}, \mathbf{C} - \mathbf{D} = \begin{pmatrix} \cos \alpha - \cos \gamma \\ \sin \alpha - \sin \gamma \end{pmatrix}$$
(4)

$$(\mathbf{A} - \mathbf{D})^{\top} (\mathbf{C} - \mathbf{D}) = 4 \sin \frac{\alpha - \gamma}{2} \sin \frac{\beta - \gamma}{2} \cos \frac{\alpha - \beta}{2}$$
 (5)

$$\|\mathbf{A} - \mathbf{D}\| \|\mathbf{C} - \mathbf{D}\| = 4\sin\frac{\alpha - \gamma}{2}\sin\frac{\beta - \gamma}{2}$$
(6)

Substituting (5) and (6) in (3),

$$\cos(\angle ADC) = \frac{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2}}{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}}$$
(7)

$$\cos(\angle ADC) = \cos\frac{\alpha - \beta}{2} \tag{8}$$

By substituting  $\alpha$  and  $\beta$  values in (8)

$$\angle ADC = \frac{\alpha - \beta}{2} = \frac{(180^{\circ} - 90^{\circ})}{2} = 45^{\circ}$$
 (9)