CLASS-12
CHAPTER-11
THREE DIMENSIONAL GEOMETRY

Excercise 11.3

Q1. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

1. $z = 2$
2. $x + y + z = 1$
3. $2x + 3y–z = 5$
4. $5y + 8 = 0$

Solution:

1. From the given equation:

$$\mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, c = 2 \tag{1}$$

The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

$$\tag{3}$$

The magnitudes for directional vectors $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are

$$\|\mathbf{e}_1\| = 1, \|\mathbf{e}_2\| = 1, \|\mathbf{e}_3\| = 1 \tag{4}$$

Let:

$$\cos\theta_i = 1, 2, 3 \tag{5}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{n}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{1} = 0 \tag{6}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{1} = 0 \tag{7}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{1} = 1 \tag{8}$$

The distance from the origin is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{2}{1} = 2 \tag{9}$$

2. From the given equation:

$$\mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix}, c = 1 \tag{10}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{n}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \tag{11}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \tag{12}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \tag{13}$$

The distance from the origin is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{1}{\sqrt{3}} \tag{14}$$

3. From the given equation:

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, c = 5 \tag{15}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{n}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{14}} = \frac{2}{\sqrt{14}} \tag{16}$$

$$\cos\theta_2 = \frac{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{14}} = \frac{3}{\sqrt{14}} \tag{17}$$

$$\cos\theta_3 = \frac{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{14}} = \frac{-1}{\sqrt{14}} \tag{18}$$

The distance from the origin is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{5}{\sqrt{14}} \tag{19}$$

4. From the given equation:

$$\mathbf{n} = \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix}, c = 8 \tag{20}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{n}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}0\\-5\\0\end{pmatrix}}{5} = 0 \tag{21}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\-5\\0\end{pmatrix}}{5} = -1 \tag{22}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\-5\\0\end{pmatrix}}{\sqrt{3}} = 0 \tag{23}$$

The distance from the origin is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{8}{5} \tag{24}$$