

CLASS-12
CHAPTER-11
THREE DIMENSIONAL GEOMETRY

Exercise 11.2

Q1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}, \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Solution:

$$\mathbf{A} = \begin{pmatrix} \frac{12}{13} \\ \frac{-3}{13} \\ \frac{-4}{13} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{4}{13} \\ \frac{12}{13} \\ \frac{3}{13} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{3}{13} \\ \frac{-4}{13} \\ \frac{12}{13} \end{pmatrix} \quad (1)$$

Stack all three vectors into a single matrix \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix}, \mathbf{P}^\top = \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} \quad (2)$$

1. Check whether all three vectors are orthogonal to each other or not using:

$$\mathbf{P}\mathbf{P}^\top = \mathbf{I} \quad (3)$$

$$\begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{I} \quad (4)$$

2. Check whether all three vectors are orthogonal to each other or not using:

$$\mathbf{P}^\top\mathbf{P} = \mathbf{I} \quad (5)$$

$$\begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{I} \quad (6)$$

Hence, it satisfies the above conditions, that all three vectors are orthogonal(perpendicular) to each other.