CHAPTER-7 COORDINATE GEOMETRY

Excercise 7.2

Q2. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} x - 1 \\ y - 2 \end{pmatrix}$$
 (2)

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} = \begin{pmatrix} x - 7 \\ y \end{pmatrix}$$
 (3)

If points on a line are collinear, rank of matrix is "1" then the vectors are in linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} x-1 & x-7 \\ y-2 & y \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} x-1 & x-7 \\ y-2 & y \end{pmatrix} \xrightarrow{C_2=C_2-C_1} \begin{pmatrix} x-1 & -6 \\ y-2 & 2 \end{pmatrix}$$
 (6)

$$\stackrel{R_2=3R_2+R_1}{\longleftrightarrow} \begin{pmatrix} x-1 & -6\\ x+3y-7 & 0 \end{pmatrix} \tag{7}$$

If the rank of the matrix has to be 1, then x+3y-7=0.

Suppose, if x=-2,y=3 , then rank of F is equal to one which is collinear as shown in Figure:1

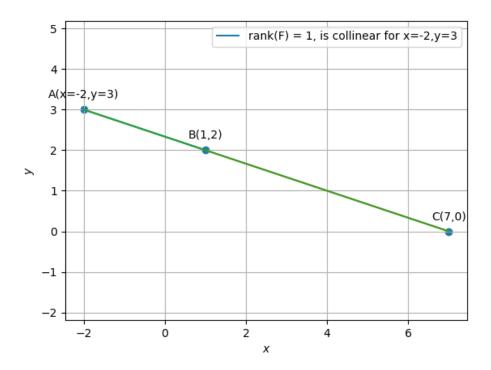


Figure 1: