CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.4

Q2. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \tag{3}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting,we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \xleftrightarrow{R_2 = R_2 - R_1} \begin{pmatrix} x-1 & x-7 \\ -6 & 2 \end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_2 = \frac{R_2}{2}(y-2) - R_1} \begin{pmatrix} x-1 & y-2 \\ -3(y-2) - (x-1) & 0 \end{pmatrix} \tag{7}$$

If the rank of the matrix has to be 1 , then $-\mathbf{3}(\mathbf{y}-\mathbf{2}) - (\mathbf{x}-\mathbf{1}) = \mathbf{0}$

$$\implies x + 3y - 7 = 0 \tag{8}$$

Suppose, if $x = -2, y = 3$, then rank of F is equal to one which is collinear as shown in Figure:1

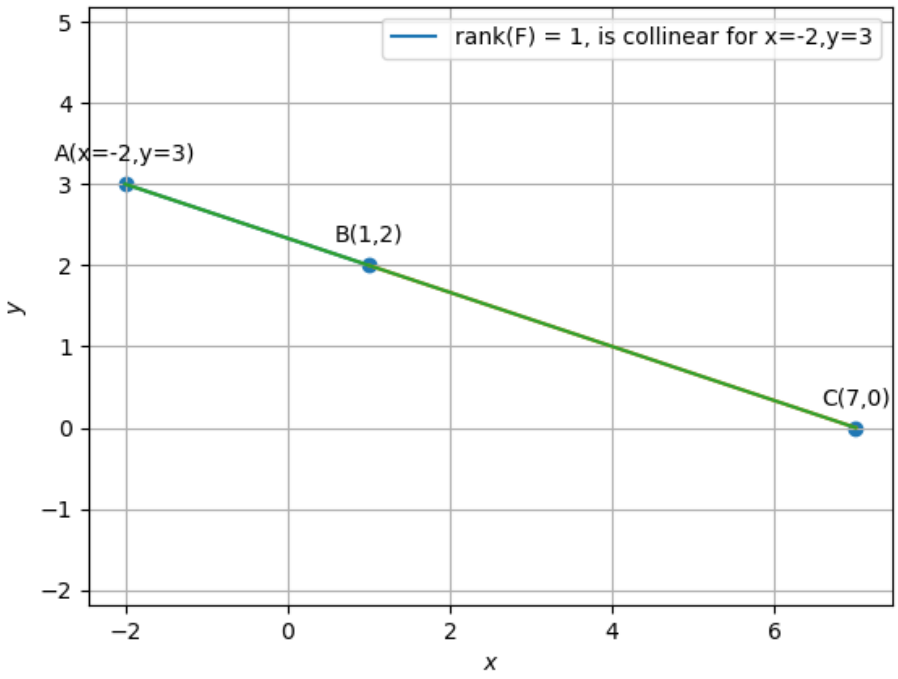
rank(F) = 1, is collinear for x=-2,y=3
A(x=-2,y=3)
B(1,2)
C(7,0)
y
x
5
4
3
2
1
0
−1
−2
−2
0
2
4
6

Figure 1: