

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.2

Q3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are $(0, -1)$, $(2, 1)$ and $(0, 3)$. Find the ratio of this area to the area of the given triangle

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (1)$$

Calculating midpoints:

$$\mathbf{P} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{Q} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (3)$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) = \frac{1}{2} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4)$$

Calculating the area of the midpoints:

$$A1 = \frac{1}{2} \|(\mathbf{P} - \mathbf{Q}) \times (\mathbf{Q} - \mathbf{R})\| \quad (5)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (6)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (7)$$

$$= \frac{1}{2} \begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} \quad (8)$$

$$A1 = 1 \quad (9)$$

Calculating the area of given triangle:

$$A_2 = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \quad (10)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \quad (11)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad (12)$$

$$= \frac{1}{2} \begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix} \quad (13)$$

$$A_2 = 4 \quad (14)$$

Resultant ratio of two areas is 1:4.

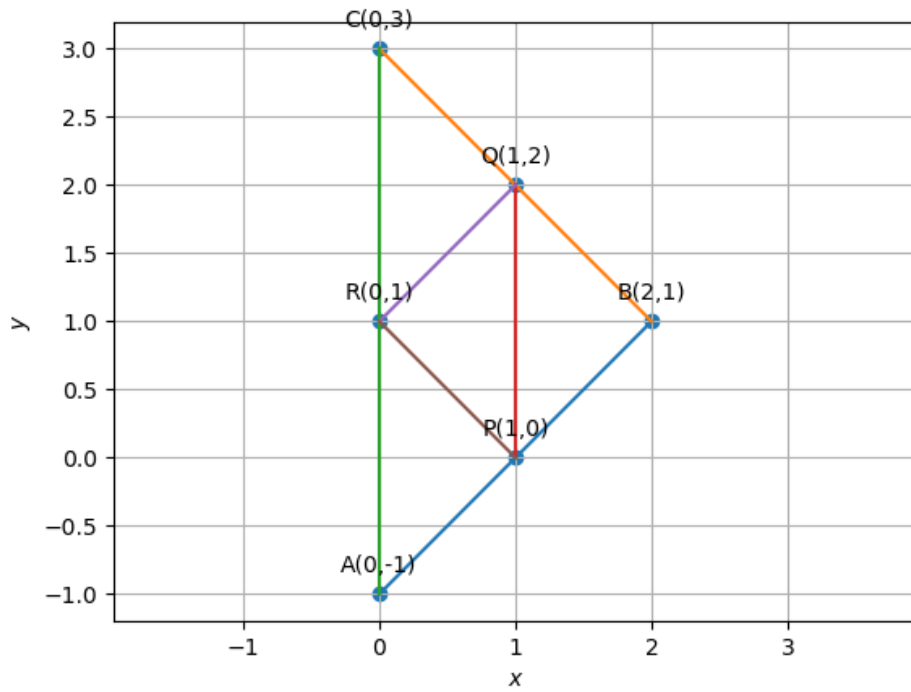


Figure 1: