

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.1

Q1. Find the distance between the following pairs of points :

1. $(2, 3), (4, 1)$
2. $(-5, 7), (-1, 3)$
3. $(a, b), (-a, -b)$

Solution:

1. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (2)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 4 + 4 = 8 \quad (3)$$

$$d = \|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})} \quad (4)$$

$$d = \sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})} = \sqrt{8} = 2\sqrt{2} = 2.828 \quad (5)$$

Hence the distance between the two points AB is 2.828 shown in Figure:1

2. The coordinates are given as

$$\mathbf{C} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (6)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \quad (7)$$

$$(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D}) = (-4 \ 4) \begin{pmatrix} -4 \\ 4 \end{pmatrix} = 16 + 16 = 32 \quad (8)$$

$$d = \|\mathbf{C} - \mathbf{D}\| = \sqrt{(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D})} \quad (9)$$

$$d = \sqrt{(\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D})} = \sqrt{32} = 4\sqrt{2} = 5.656 \quad (10)$$

Hence the distance between the two points CD is 5.656 shown in Figure:1

3. The coordinates are given as

$$\mathbf{E} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{F} = \begin{pmatrix} -a \\ -b \end{pmatrix} \quad (11)$$

$$\mathbf{E} - \mathbf{F} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} -a \\ -b \end{pmatrix} = \begin{pmatrix} 2a \\ 2b \end{pmatrix} \quad (12)$$

$$(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F}) = (2a \ 2b) \begin{pmatrix} 2a \\ 2b \end{pmatrix} = 4a^2 + 4b^2 \quad (13)$$

$$d = \|\mathbf{E} - \mathbf{F}\| = \sqrt{(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F})} \quad (14)$$

$$d = \sqrt{(\mathbf{E} - \mathbf{F})^\top (\mathbf{E} - \mathbf{F})} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \quad (15)$$

Suppose, if $a=1, b=2$, then $d = 2\sqrt{1^2 + 2^2} = 4.472$

Hence the distance between the two points EF is $2\sqrt{a^2 + b^2} = 4.472$ shown in Figure:1

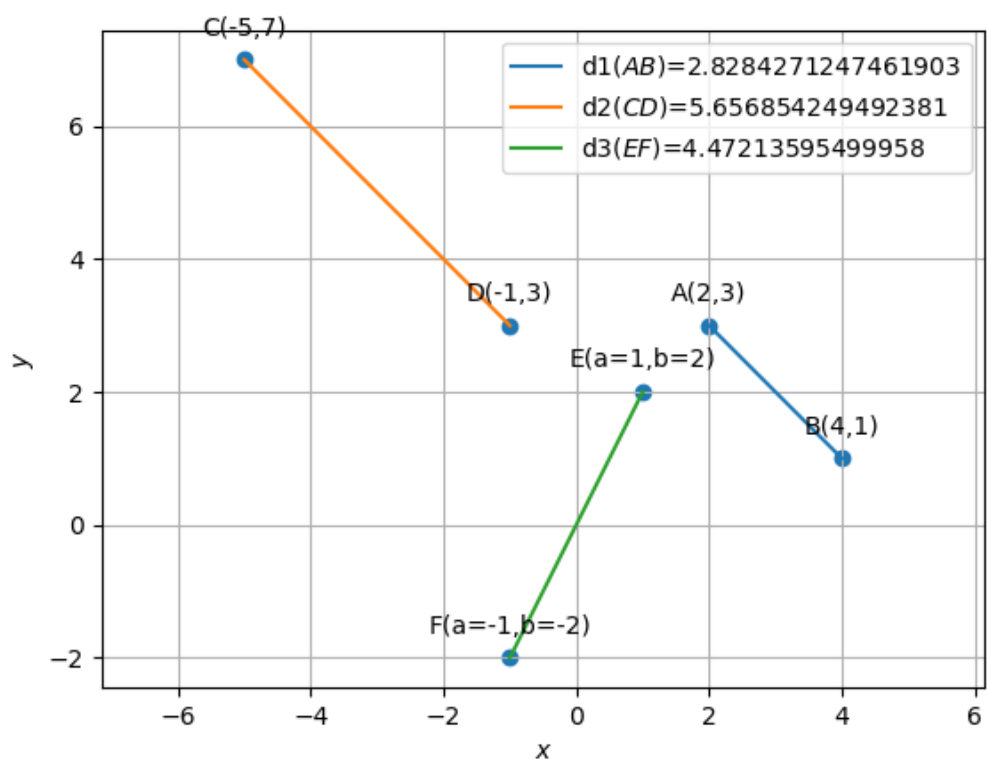


Figure 1: