CLASS-12
CHAPTER-11
THREE DIMENSIONAL GEOMETRY

Excercise 11.2

Q1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.
Solution:

1. Check whether angle between the **A** and **B** are mutually perpendicular or not:

$$\mathbf{A} = \begin{pmatrix} \frac{12}{13} \\ \frac{-3}{13} \\ \frac{-4}{13} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{4}{13} \\ \frac{12}{13} \\ \frac{3}{13} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{3}{13} \\ \frac{-4}{13} \\ \frac{12}{13} \end{pmatrix} \tag{1}$$

$$\mathbf{P} = \begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix}, \mathbf{P}^\top = \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} \tag{2}$$

Check whether all three vectors are orthogonal to each other or not using:

$$\mathbf{P}.\mathbf{P}^\top = \mathbf{I} \tag{3}$$

$$\begin{pmatrix} \frac{12}{13} & \frac{4}{13} & \frac{3}{13} \\ \frac{-3}{13} & \frac{12}{13} & \frac{-4}{13} \\ \frac{-4}{13} & \frac{3}{13} & \frac{12}{13} \end{pmatrix} . \begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \\ \frac{4}{13} & \frac{12}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{-4}{13} & \frac{12}{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

Hence, all three lines are perpendicular to each other. Angle between the vectors is given by:

$$\cos\theta_1 = \frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{A}\| \, \|\mathbf{B}\|} \tag{5}$$

$$= \frac{\begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \end{pmatrix} \begin{pmatrix} \frac{4}{13} \\ \frac{-3}{13} \\ \frac{-4}{13} \end{pmatrix}}{1} \tag{6}$$

$$= 0 \tag{7}$$

$$\implies \theta_1 = 90^\circ \tag{8}$$

2. Check whether angle between the **B** and **C** are mutually perpendicular or not:

$$\mathbf{B} = \begin{pmatrix} \frac{4}{13} \\ \frac{-3}{13} \\ \frac{-4}{13} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{3}{13} \\ \frac{-4}{13} \\ \frac{-12}{13} \end{pmatrix} \tag{9}$$

   Angle between the vectors is given by:

$$\cos\theta_2 = \frac{\mathbf{B}^\top \mathbf{C}}{\|\mathbf{B}\| \, \|\mathbf{C}\|} \tag{10}$$

$$= \frac{\begin{pmatrix} \frac{4}{13} & \frac{-3}{13} & \frac{-4}{13} \end{pmatrix} \begin{pmatrix} \frac{3}{13} \\ \frac{-4}{13} \\ \frac{-12}{13} \end{pmatrix}}{1} \tag{11}$$

$$= 0 \tag{12}$$

$$\implies \theta_2 = 90^\circ \tag{13}$$

3. Check whether angle between the **A** and **C** are mutually perpendicular

or not:

$$\mathbf{A} = \begin{pmatrix} \frac{12}{13} \\ \\ \frac{-3}{13} \\ \\ \frac{-4}{13} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{3}{13} \\ \\ \frac{-4}{13} \\ \\ \frac{-12}{13} \end{pmatrix} \tag{14}$$

Angle between the vectors is given by:

$$\cos\theta_3 = \frac{\mathbf{A}^\top \mathbf{C}}{\|\mathbf{A}\| \, \|\mathbf{C}\|} \tag{15}$$

$$= \frac{\begin{pmatrix} \frac{12}{13} & \frac{-3}{13} & \frac{-4}{13} \end{pmatrix} \begin{pmatrix} \frac{3}{13} \\ \\ \frac{-4}{13} \\ \\ \frac{-12}{13} \end{pmatrix}}{1} \tag{16}$$

$$= 0 \tag{17}$$

$$\implies \theta_3 = 90^\circ \tag{18}$$

Hence, all three lines are mutually perpendicualr to each other.