CLASS-9
CHAPTER-10
CIRCLES

Excercise 10.5

Q1. In Figure 1. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are the three points with centre **O** such that $\angle$BOC=30°and $\angle$AOB=60°. If **D** is a point on the circle other than the arc ABC, find $\angle$ADC

Solution

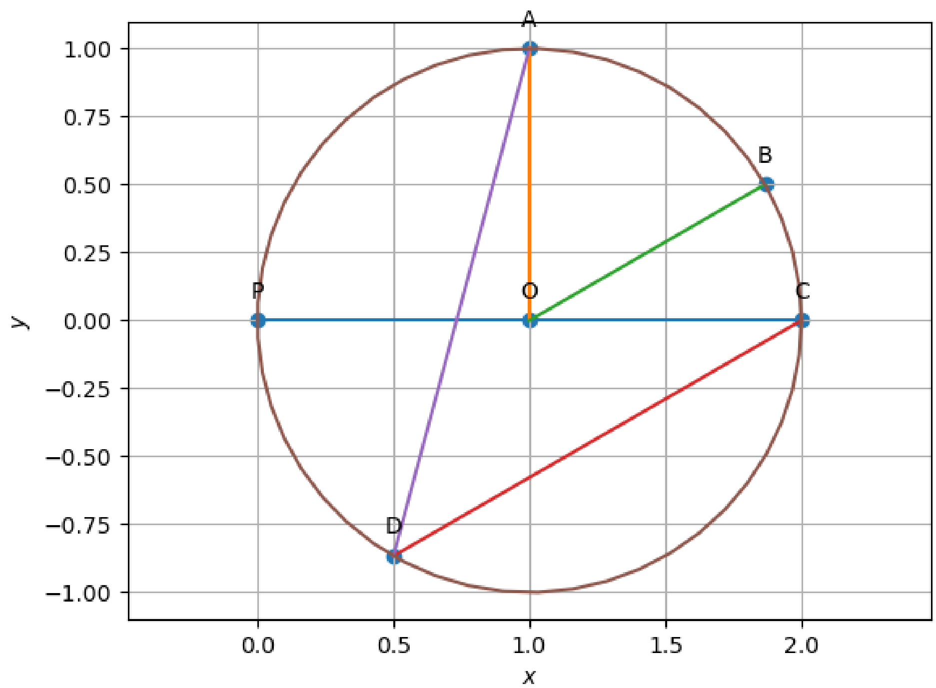

Figure 1:

Construction

Symbol	**value**	**Description**
r	1 unit	Radius of OA and OB
O	(1,0)	Centre of the circle
P	(0,0)	It is a point on the circle forms a diameter
α	180°	Angle between vectors **P** and **C**
β	90°	Angle between vectors **P** and **A**
γ	300°	Angle between vectors **P** and **D**
ϕ	150°	Angle between vectors **P** and **B**
$\mathbf{C}, \mathbf{A}, \mathbf{D}, \mathbf{B}$	(2,0), (1,1), (0.5,-0.86), (1.86,0.5)	Points obtained from α,β,γ,ϕ

Table 1:

Assumptions

1. Let **P** be a point on the circle such that by expandig OC upto **P** we get diameter POC.
2. To find $\angle$ADC let the circle be unit circle and diameter POC on x axis.
3. Take three points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ and α,β,γ be three angles made by the points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ with respect to diameter POC.

From the Figure 1:

$$\alpha = \angle POC = 180°, \beta = \angle POA = 90°, \gamma = \angle POD = 300° \tag{1}$$

Verification:

From assumptions the vector points $\mathbf{C}, \mathbf{A}, \mathbf{D}$ be

$$\mathbf{C} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} \tag{2}$$

Let AC be the chord that subtends angles at the center **O** and at point **D**. The cosine of the angle subtended at point **D** is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D})}{\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\|} \tag{3}$$

Where

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix}\cos\beta-\cos\gamma\\ \sin\beta-\sin\gamma\end{pmatrix}, \mathbf{C}-\mathbf{D} = \begin{pmatrix}\cos\alpha-\cos\gamma\\ \sin\alpha-\sin\gamma\end{pmatrix} \tag{4}$$

$$(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D}) = 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2} \tag{5}$$

$$\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\| = 4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2} \tag{6}$$

Substituting (5) and (6) in (3),

$$\cos(\angle ADC) = \frac{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\alpha-\beta}{2}}{4\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}} \tag{7}$$

$$\cos(\angle ADC) = \cos\frac{\alpha-\beta}{2} \tag{8}$$

By substituting α and β values in (8)

$$\angle ADC = \frac{\alpha-\beta}{2} = \frac{(180^\circ-90^\circ)}{2} = 45^\circ \tag{9}$$