Excercise 7.2

Q2. Find a relation between x and y if the points $(x, y), (1, 2) (and)(7, 0)$ are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{1}$$

For collinear, ar(ABC) should be equal to zero

$$ar(ABC) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| = 0 \tag{2}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} x - 1 \\ y - 2 \end{pmatrix} \tag{3}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} = \begin{pmatrix} x - 7 \\ y \end{pmatrix} \tag{4}$$

$$= \frac{1}{2} \begin{vmatrix} x - 1 & x - 7 \\ y - 2 & y \end{vmatrix} = 0 \tag{5}$$

$$\implies x + 3y - 7 = 0 \tag{6}$$

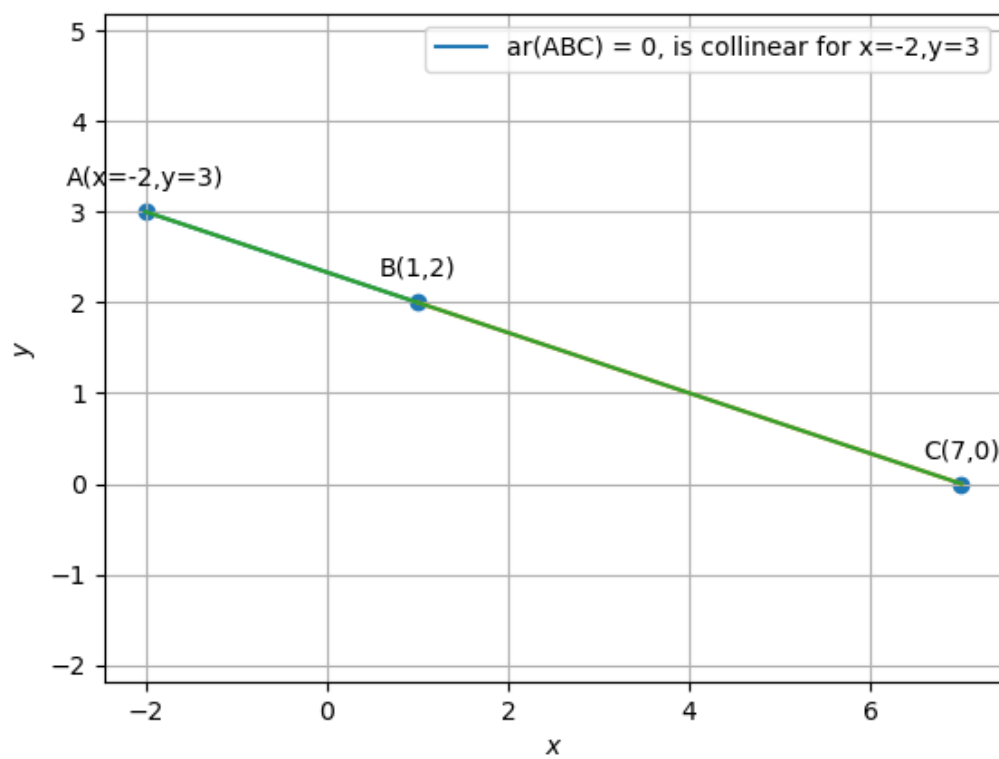Suppose, if x=-2,y=3 , then area of triangle ABC is equal to zero which is collinear as shown in Figure:1

Figure 1: