

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.2

Q2. Find a relation between x and y if the points (x, y) , $(1, 2)$ and $(7, 0)$ are collinear.

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is " 1 " then the vectors are linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} x-1 & x-7 \\ y-2 & y \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} x-1 & x-7 \\ y-2 & y \end{pmatrix} \xrightarrow{C_2=C_2-C_1} \begin{pmatrix} x-1 & -6 \\ y-2 & 2 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} x-1 & -6 \\ y-2 & 2 \end{pmatrix} \xrightarrow{R_2=3R_2+R_1} \begin{pmatrix} x-1 & -6 \\ x+3y-7 & 0 \end{pmatrix} \quad (7)$$

If the rank of the matrix has to be 1, then $x+3y-7=0$.

Suppose, if $x=-2, y=3$, then area of triangle ABC is equal to zero which is collinear as shown in Figure:1

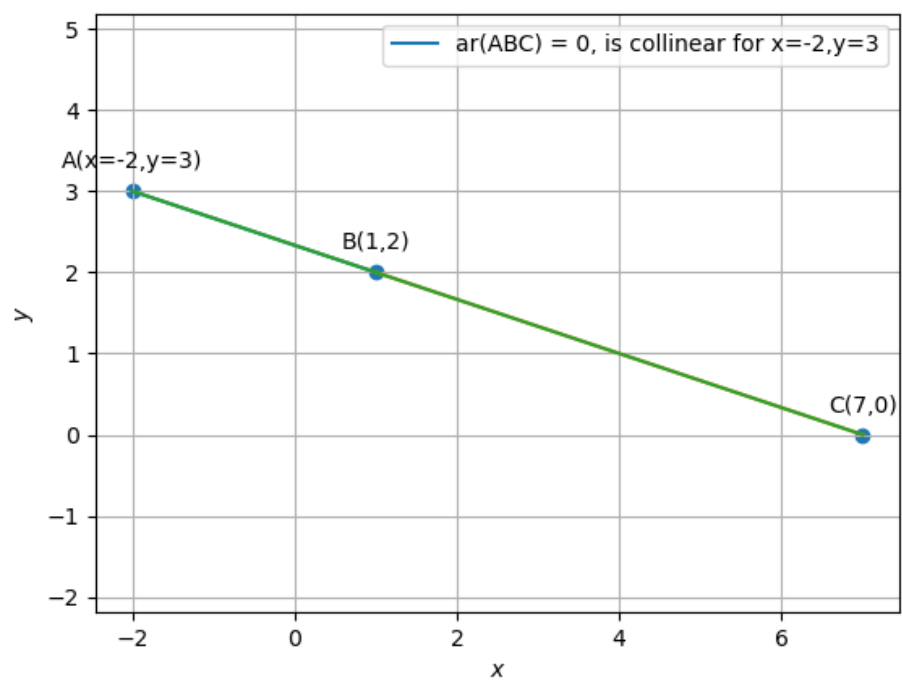


Figure 1: