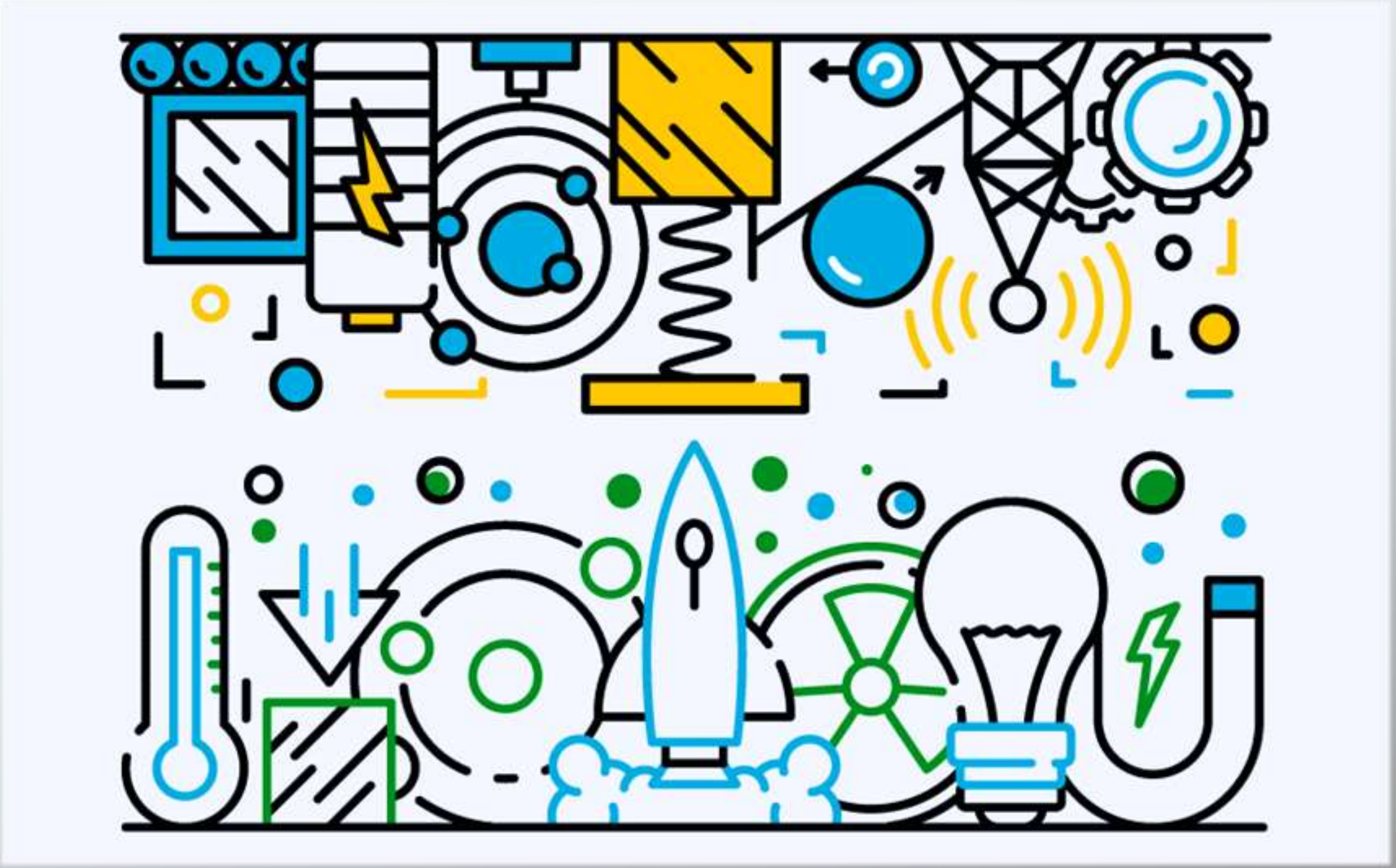
Engineering Physics (FIC 102)

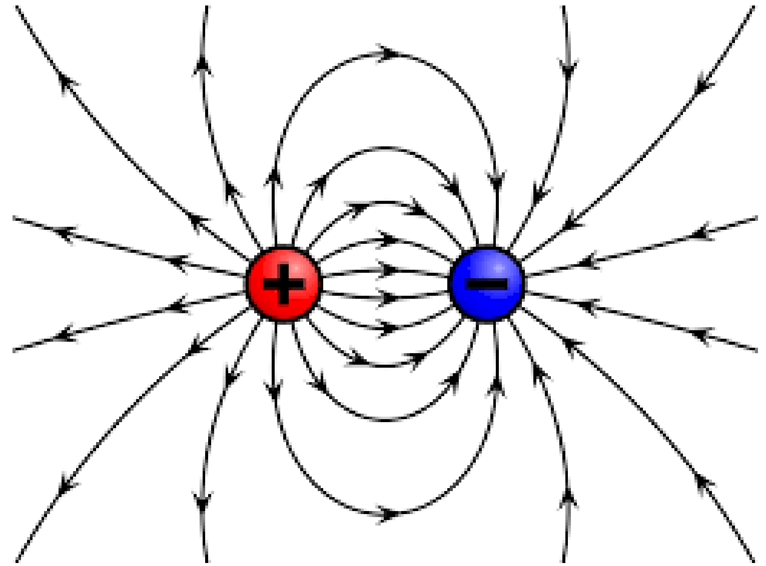
UNIT-IV



CONTENT

LECTURE-01

Electrostatic Field, Flux and Gauss Law



Electric Charge

Two types of electric charge: positive and negative

Unit of charge is the ***coulomb*** [C]

Charge of electron (negative) or proton (positive) is

$$\pm e, \qquad e = 1.602 \times 10^{-19}\, C$$

Charge is quantized

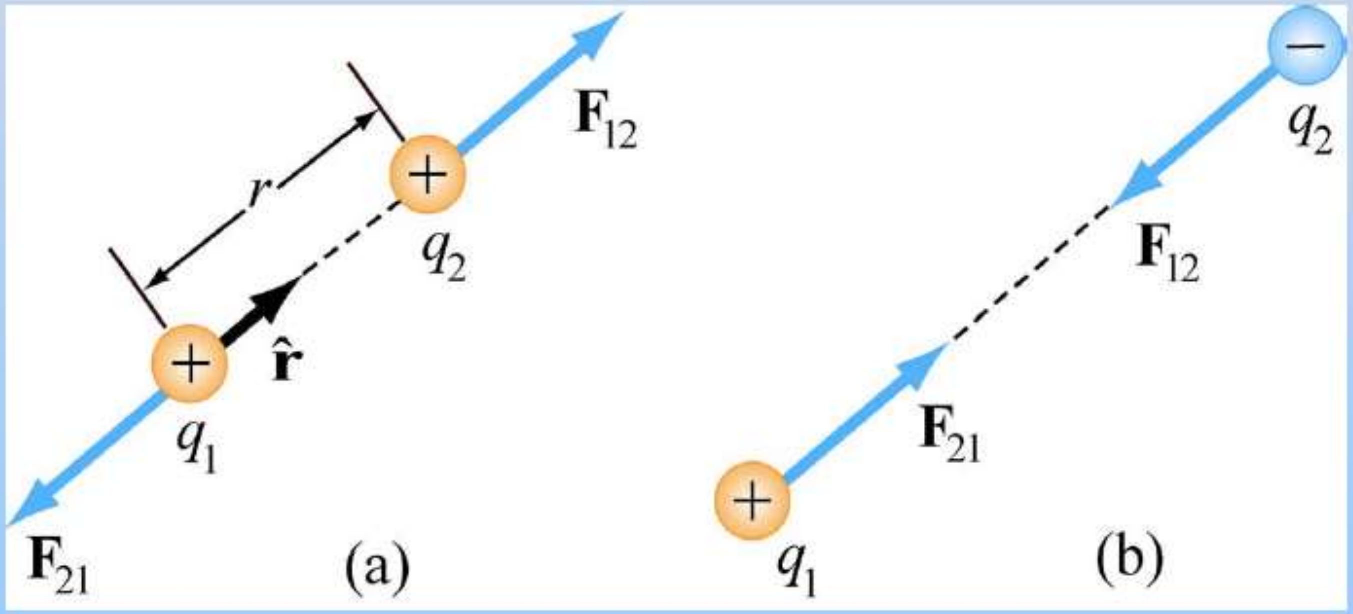
$$Q = \pm N e$$

Electric Force

The electric force between charges q_1 and q_2 is

(a) repulsive if charges have same signs

(b) attractive if charges have opposite signs



Like charges repel and opposites attract !!

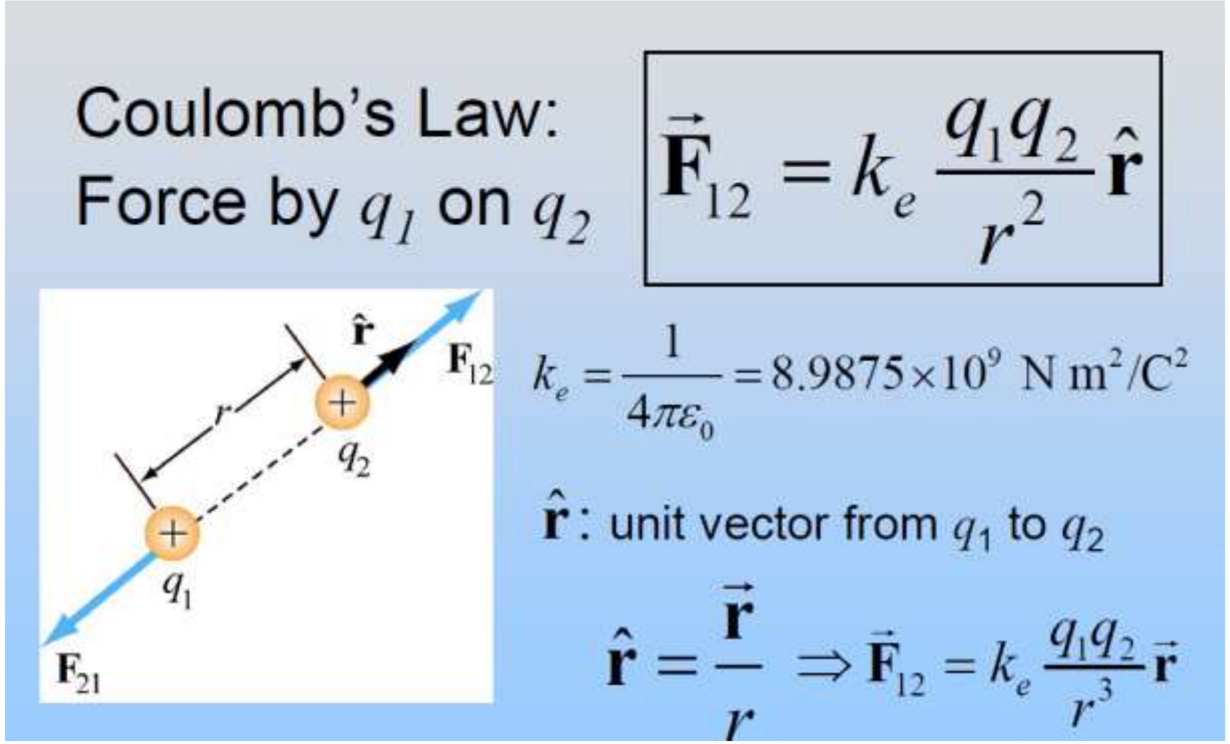
Coulomb's law

The magnitude of the electric force between two point charges q_1 and q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F \propto q_1\, q_2$$

$$F \propto \frac{1}{r^2}$$

$$F = k\frac{q_1\, q_2}{r^2}$$



Here r is the distance between the two charges

k is the proportionality constant and In SI units,

$$k = \frac{1}{4\pi\epsilon_0} = 8.9875 \times 10^9\ N\ m^2/C^2$$

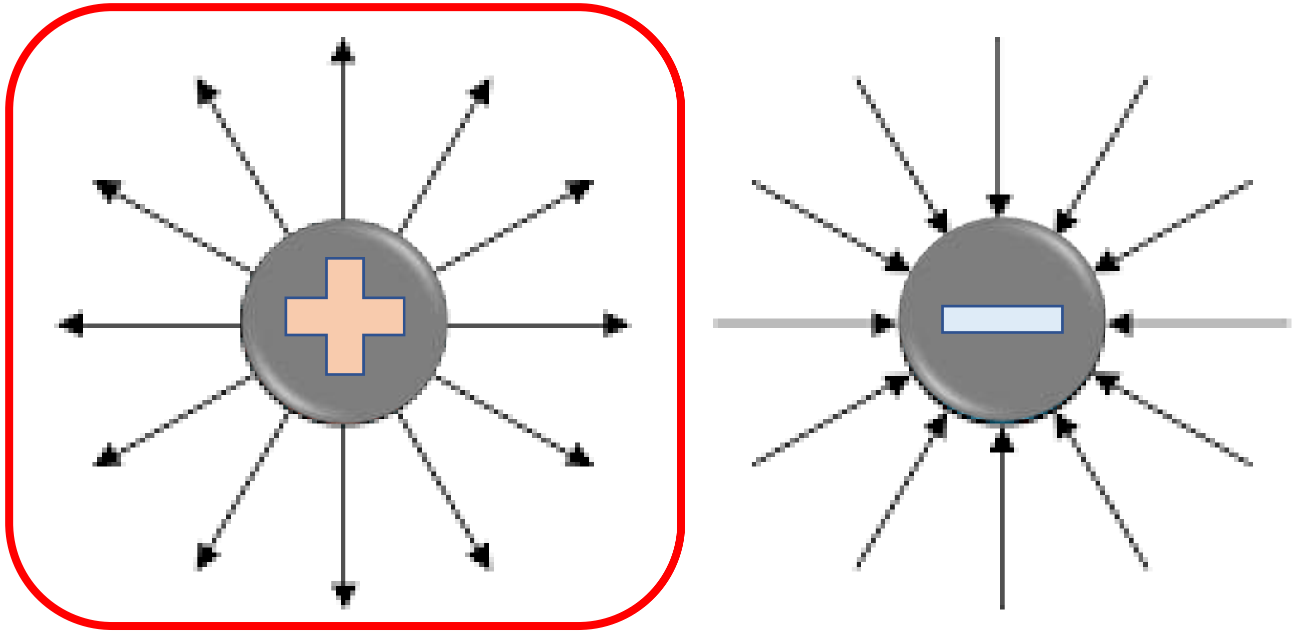
ε_0 is the permittivity of the free space

$= 8.85 \times 10^{-12}\ C^2/\ N.\ m^2$ Thus in SI units

The force acts along the line joining the two charges.

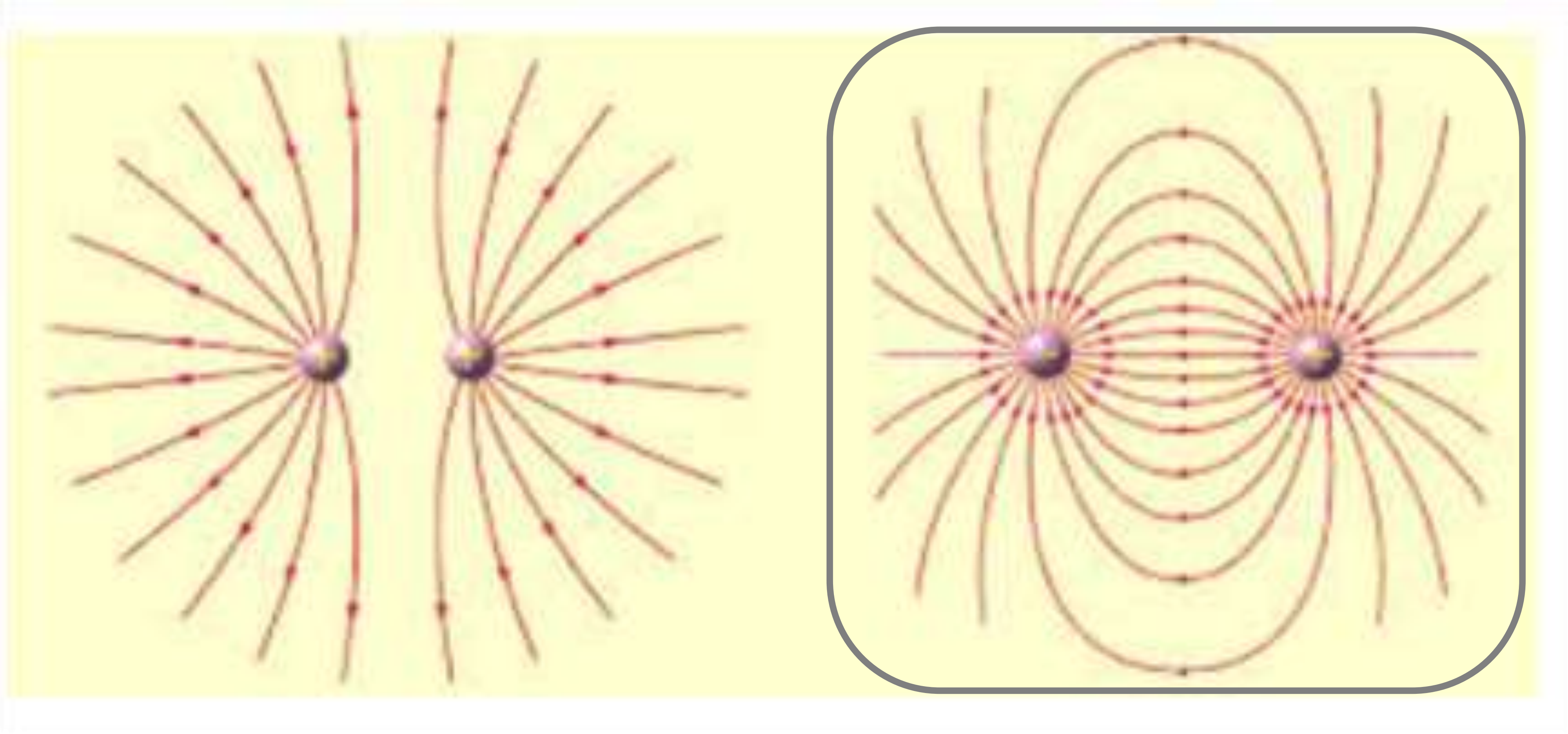
CONCEPT QUESTION

Identify the electro-static field lines representing a positive electrical monopole



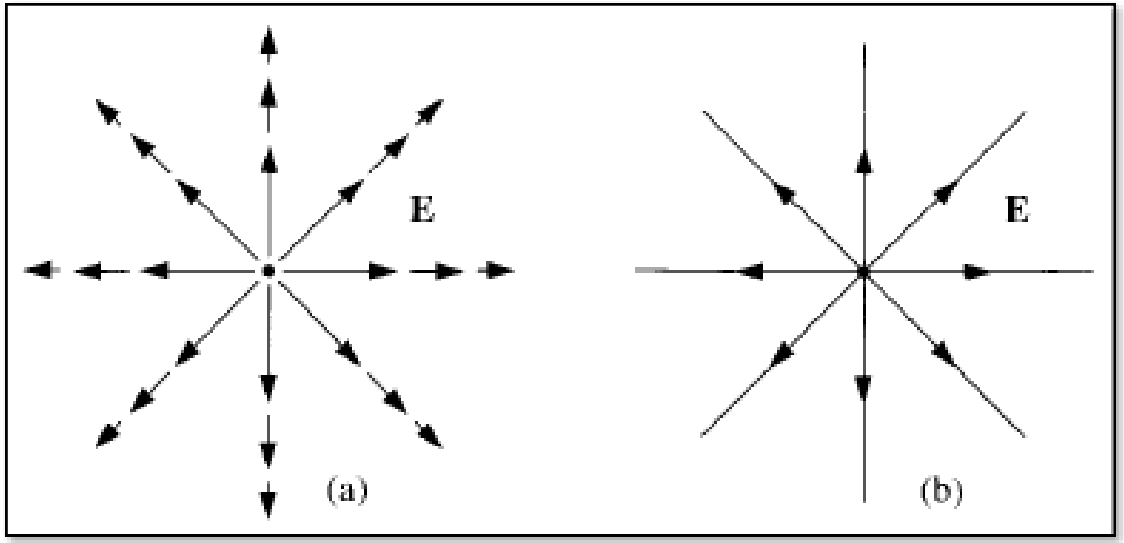
CONCEPT QUESTION

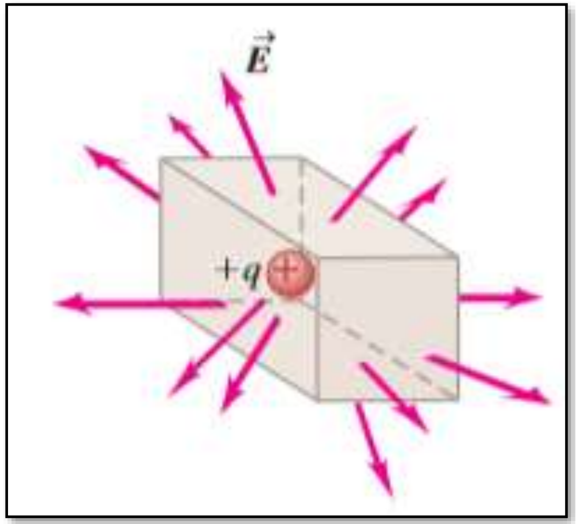
Identify field lines representing an electrostatic di-pole?

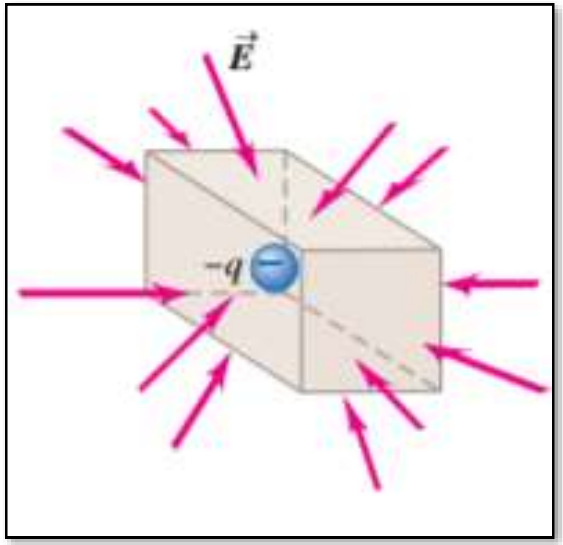


Lines Of Electric Force

- The electric field in a region can be graphically represented by drawing certain curves known as lines of electric force or electric field lines.
- The tangent to the line of force gives the direction of the resultant electric field.
- The electric field due to a positive point charge is represented by straight lines originating from the charge and the electric field due to a negative point charge is represented by straight lines terminating the charge.
- The field lines can never cross.
- The field lines begins on positive charge and end on negative charges.

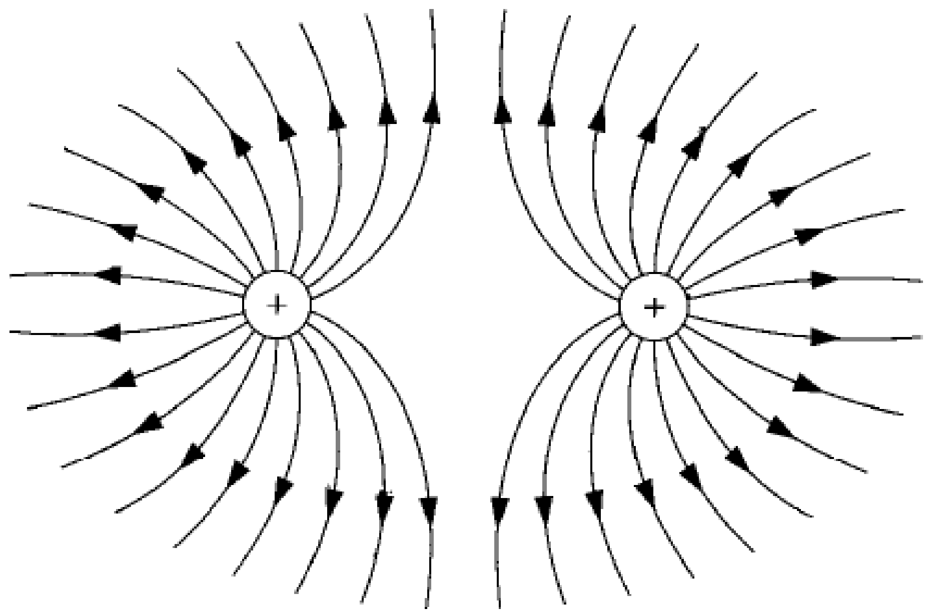




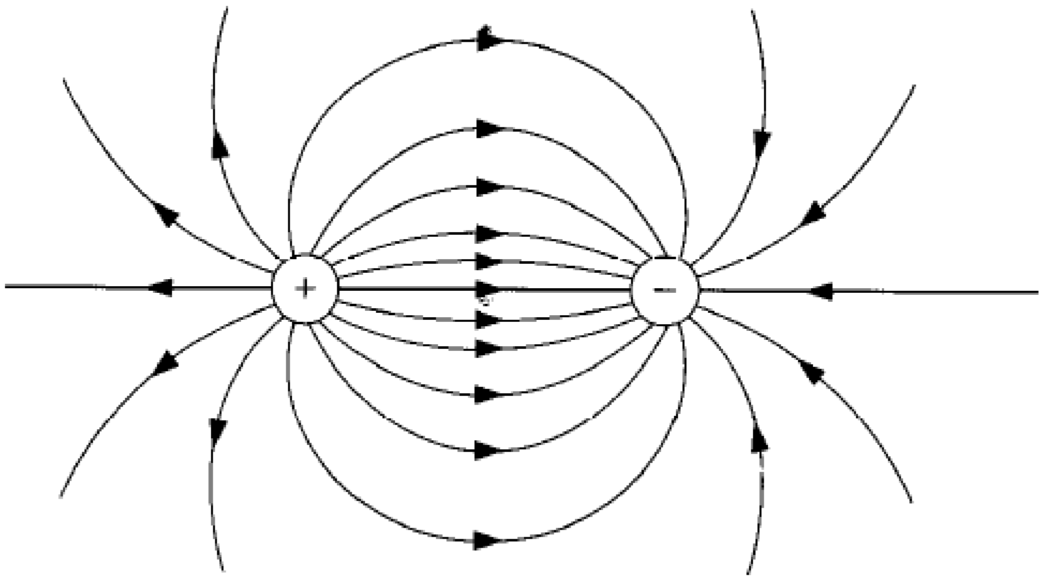


Lines of electric force

- Electric field lines for different combination of charges

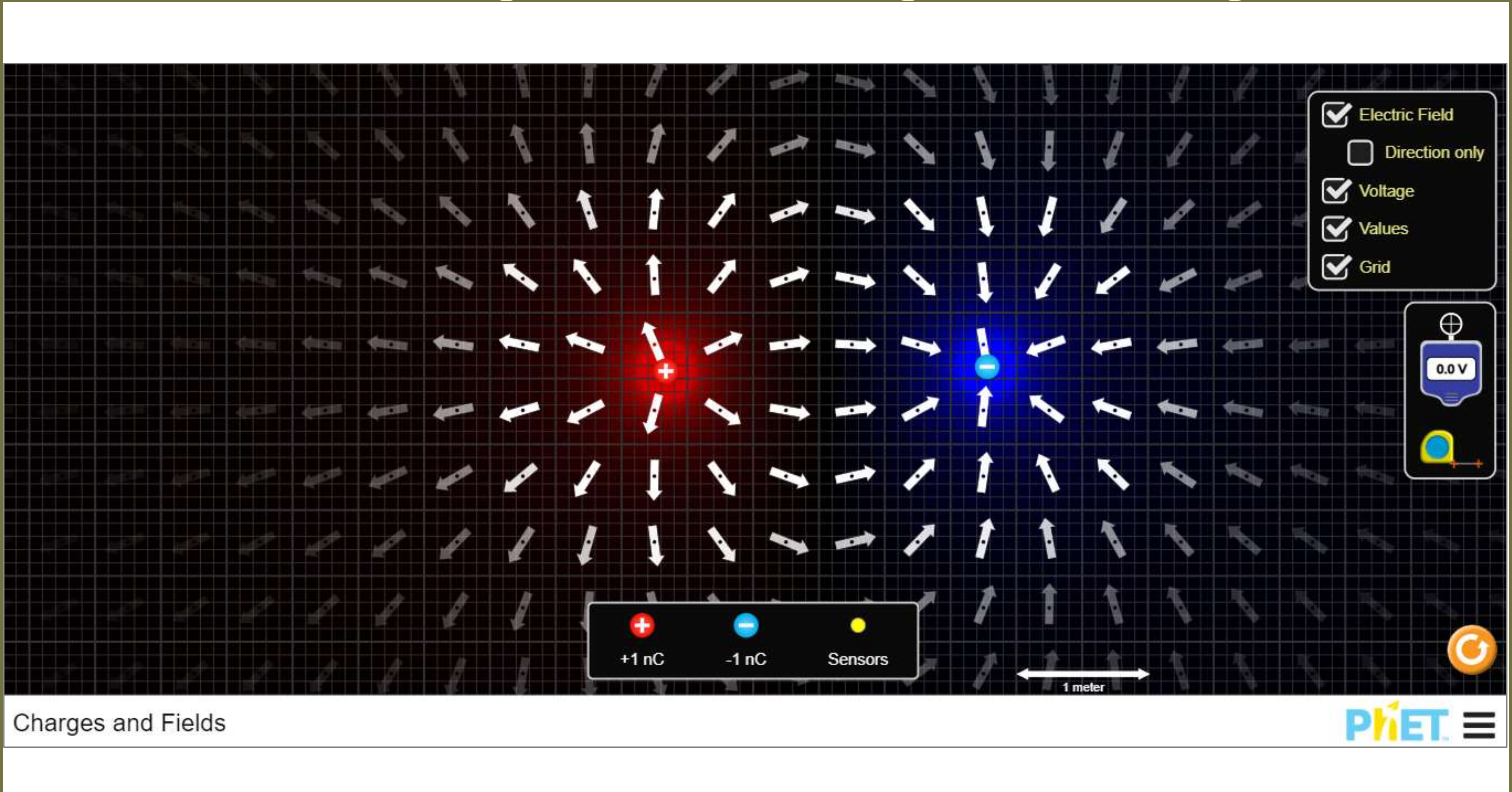


Equal charges



Equal but opposite charges

INTERACTIVE PRESENTATION

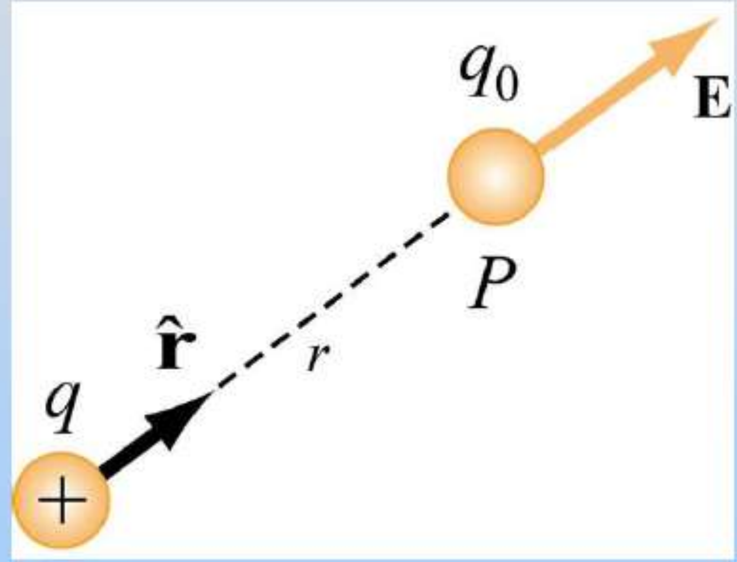


Electric Field

- The electric field at a point is the *force* experienced by *unit positive charge* due to the *source charge*.

The electric field at a point is the force acting on a test charge q_0 at that point, divided by the charge q_0 :

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_0}$$

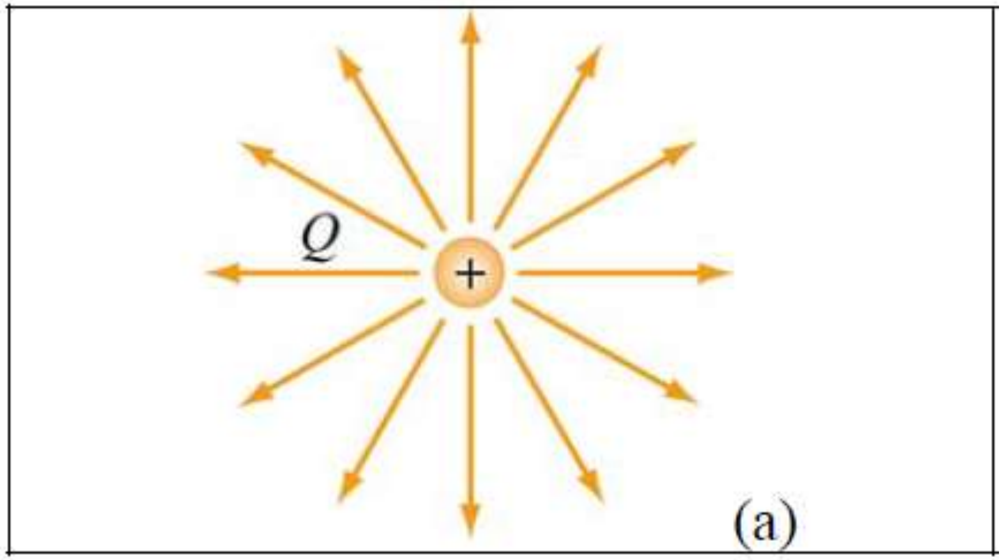


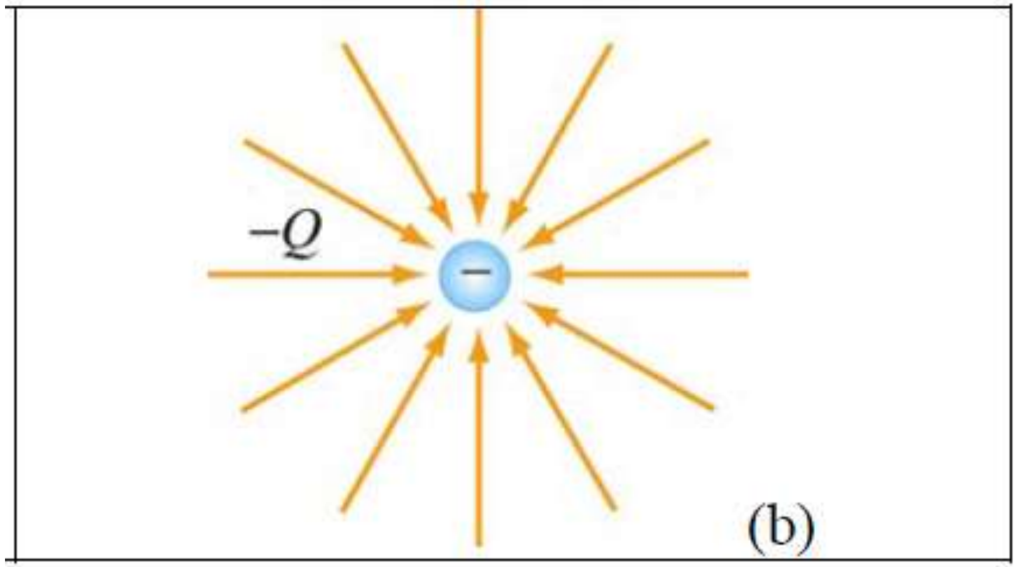
For a point charge q: $$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}$$

Electric Field

The electric field due to a collection of *N* point charges is the vector sum of the individual electric fields due to each charge

$$\vec{\mathbf{E}}_{total} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \ldots\ldots = \sum_{i=1}^{N} \vec{\mathbf{E}}_i$$





Mass and Charge Analogy

	Mass M	Charge q (±)
CREATE:	$\vec{\mathbf{g}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\frac{q}{r^2}\hat{\mathbf{r}}$
FEEL:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to picture field

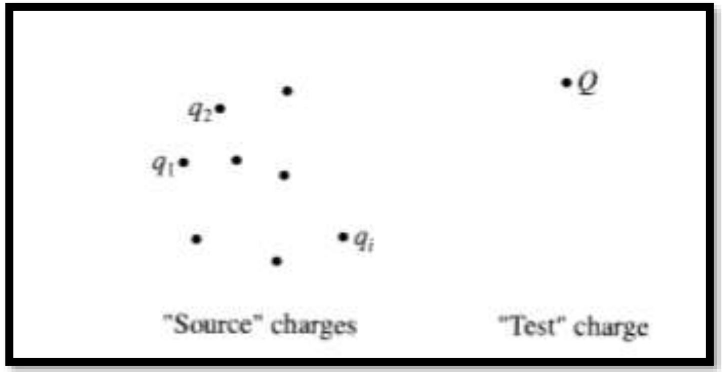
Electric Field

If we have several point charges $q_1, q_2 \ldots, q_n$ at distances $r_1, r_2, \ldots r_n$ from Q.

The total force on Q is

$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{{r_1}^2} + \frac{q_2 Q}{{r_2}^2} + \ldots\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{{r_1}^2} + \frac{q_2}{{r_2}^2} + \ldots\right) = QE$$

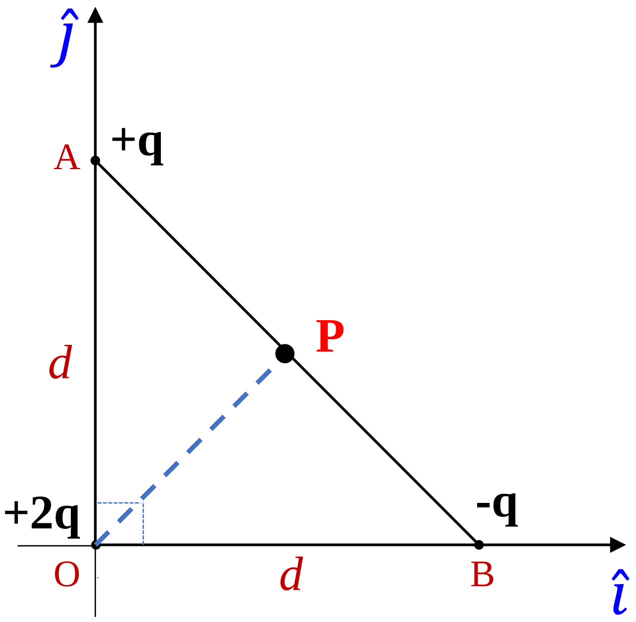
$$E = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{{r_i}^2}\hat{r}_i$$



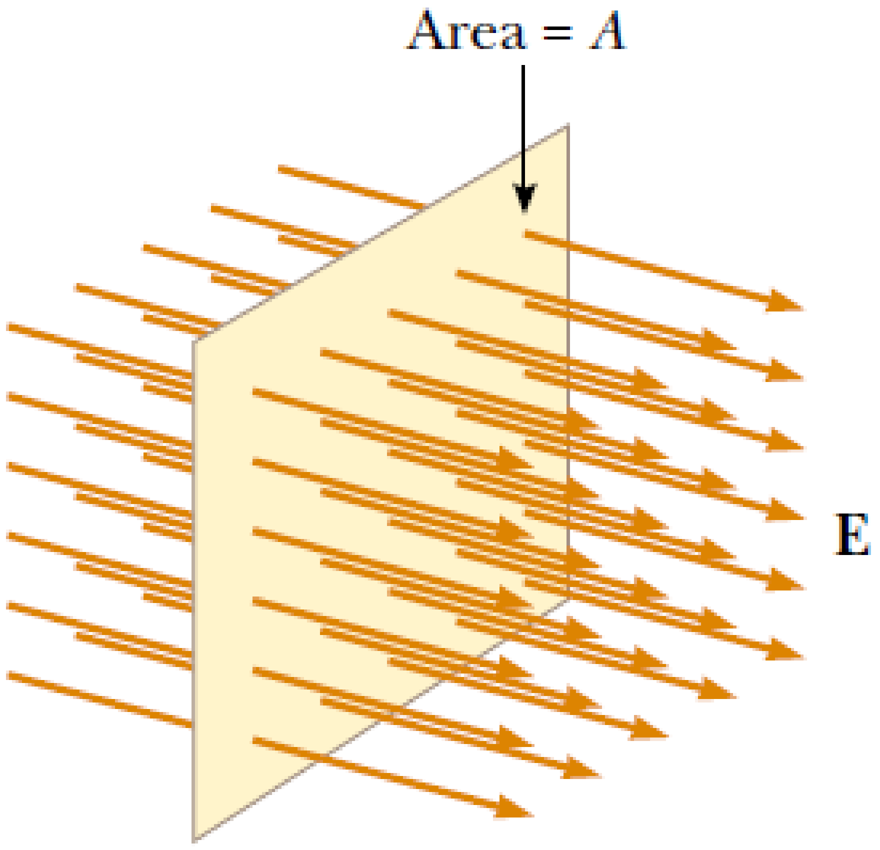
- ✓ The electric field is a vector quantity that varies from point to point.
- ✓ Physically the electric field is the force per unit charge that would be exerted on a test charge if you were to place at P

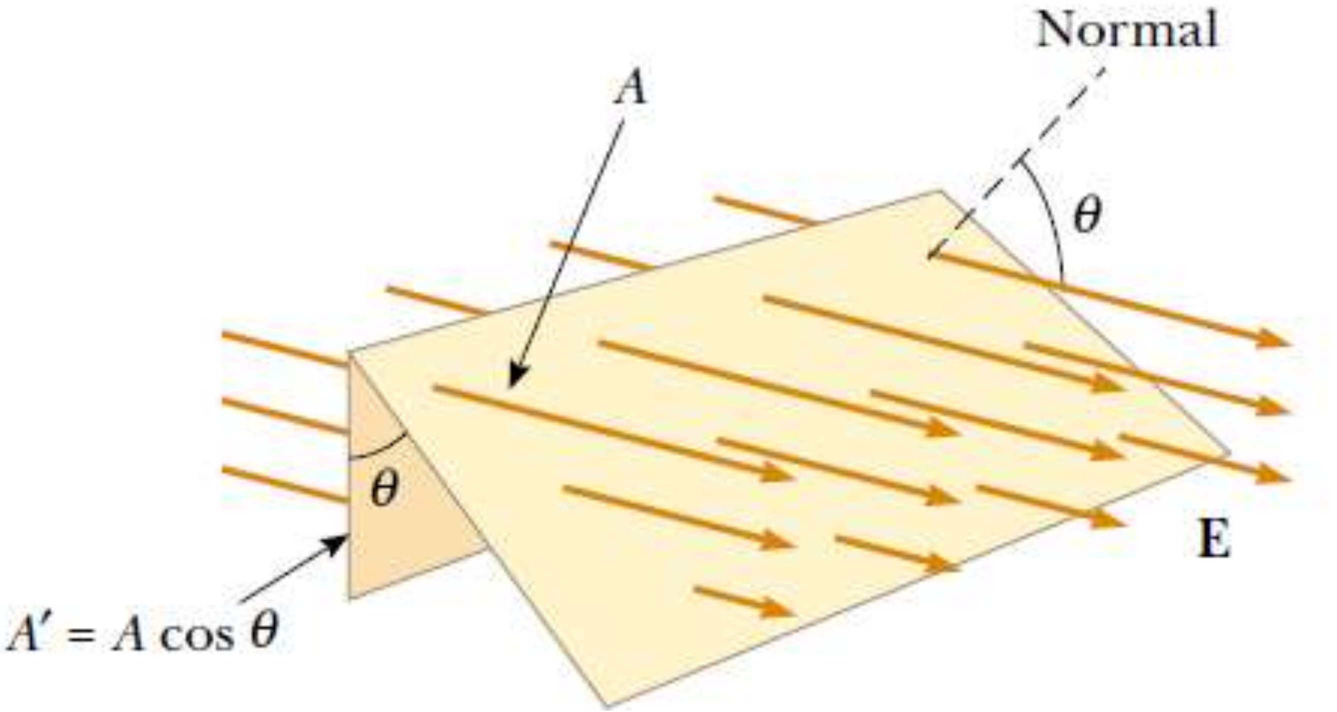
Problem on Electric Field:

Charges +q, -q and +2q are placed as as shown in the figure. Find the *Electric Field* $\vec{E}$ at point **P**, *which is the mid point of the line AB.*



Flux of an electric field through a surface





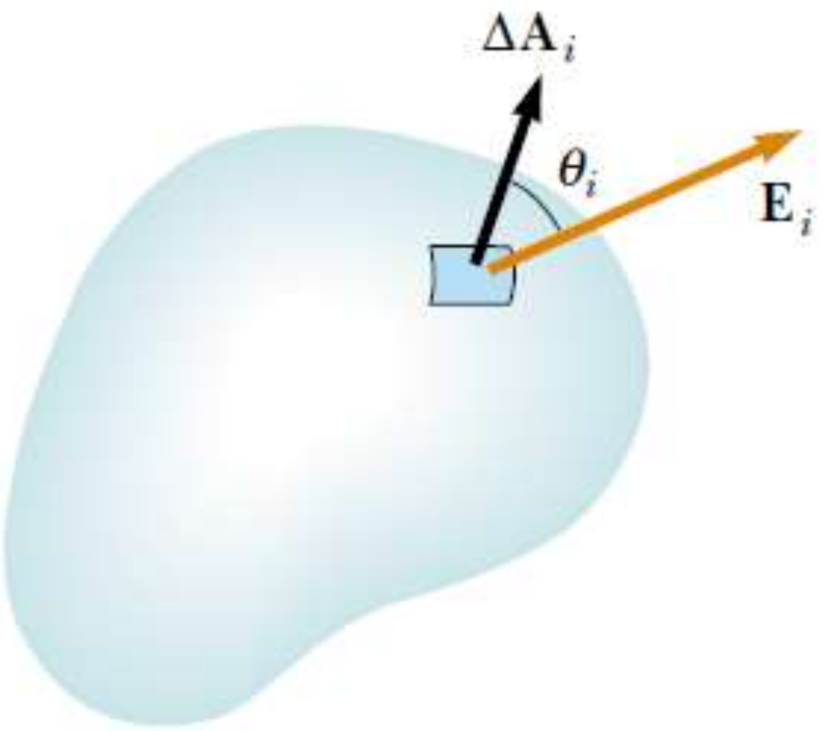
$$\Phi_E = EA$$

$$\Phi_E = EAcos(\theta)$$

Field lines ⊥ surface area

Field lines not ⊥ surface area

Flux of an electric field through a surface

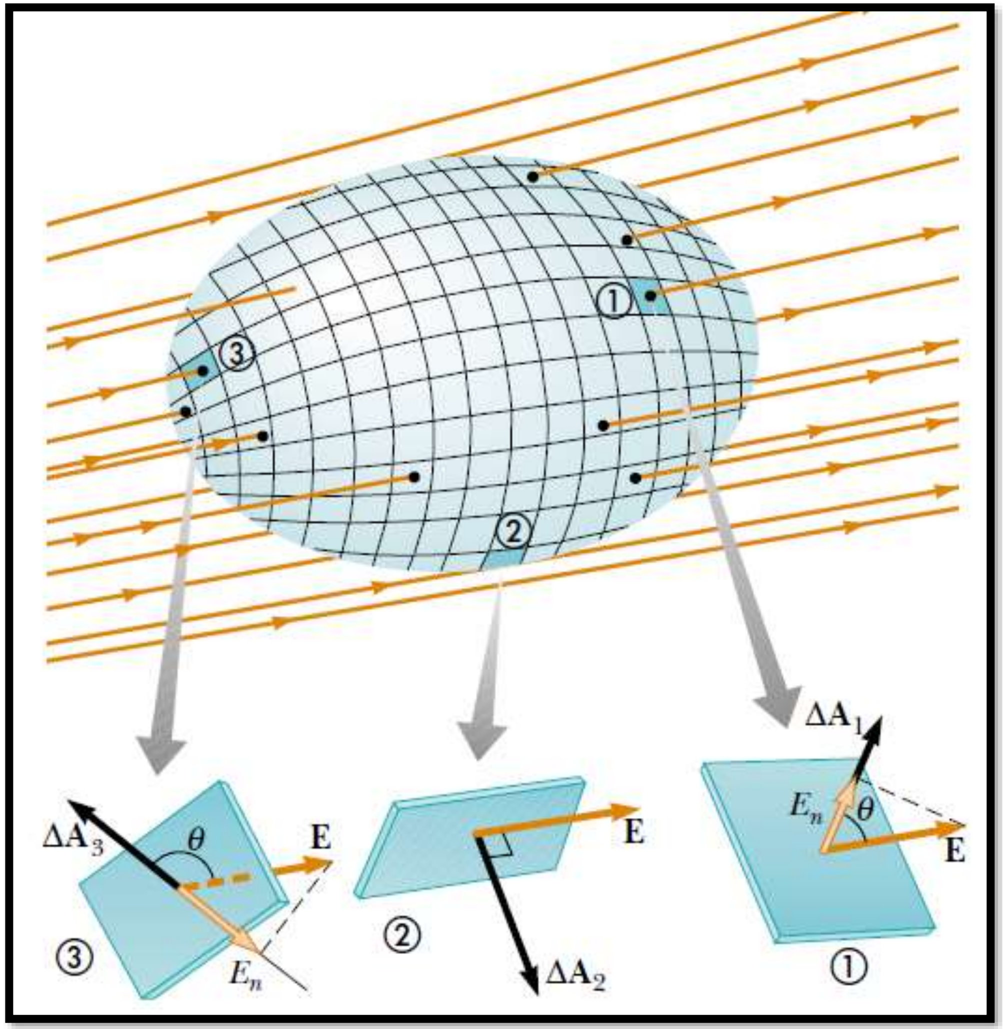


$$\Delta\Phi_E = E_i * \Delta A_i * cos\theta_i = \overrightarrow{E_i} \cdot \overrightarrow{\Delta A_i}$$

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \overrightarrow{E_i} \cdot \overrightarrow{\Delta A_i} = \int \vec{E} \cdot \overrightarrow{dA}$$

- ❑ By summing the contributions of all elements, the total flux through the surface can be obtained.
- ❑ If we let the area of each element approach zero, then the number of elements approaches infinity and the sum is replaced by an integral.

Electrostatic Flux through a closed surface



The flux through an area element can be

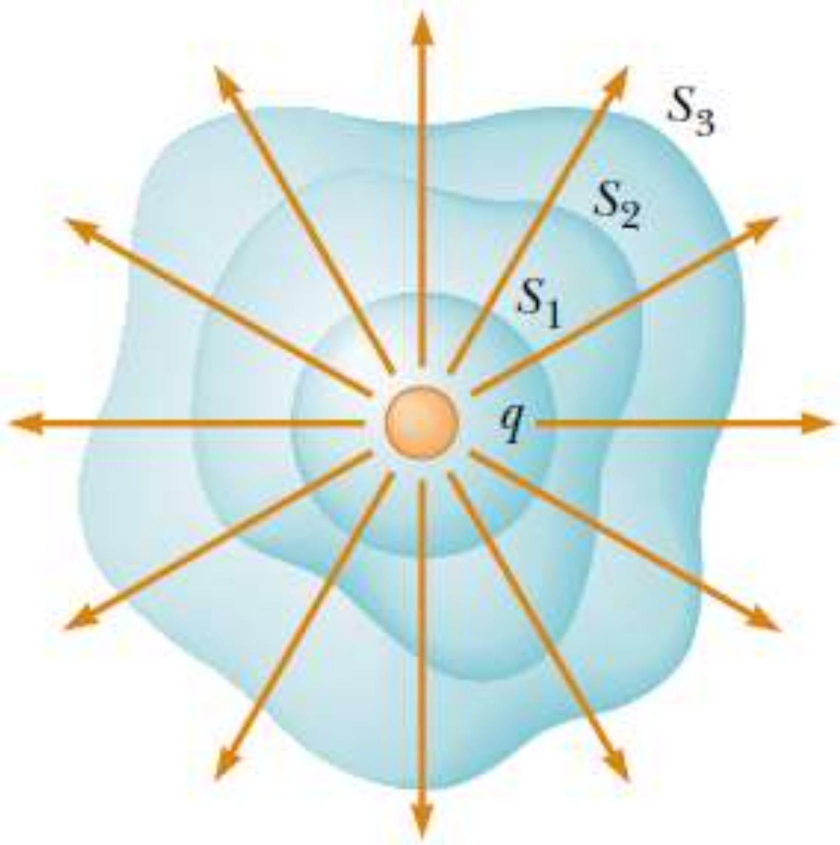
- ✓ Positive (element ①),
- ✓ Zero (element ②), or
- ✓ Negative (element ③).

$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA}$$

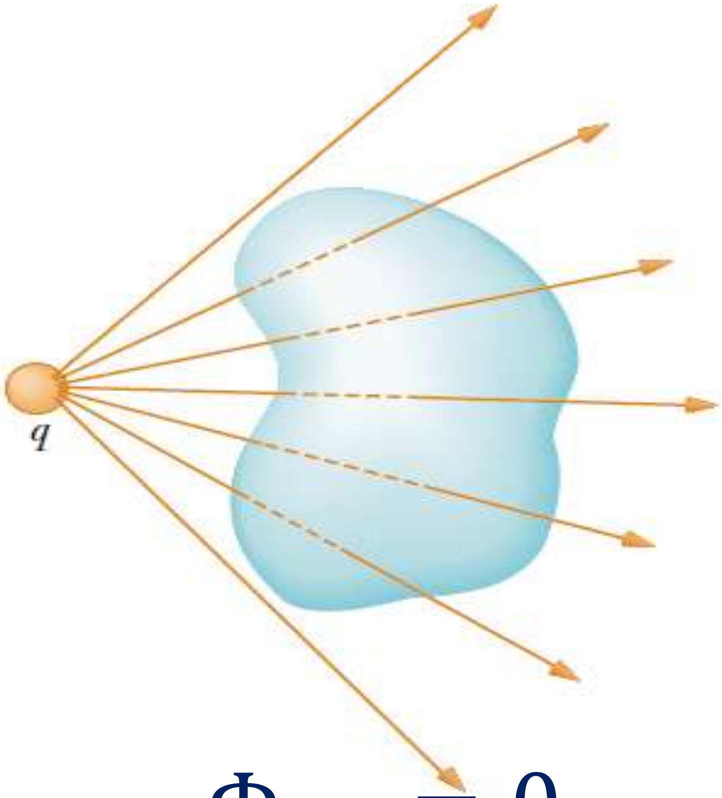
Net flux over closed surface

Integral over closed surface

Electrostatic Flux through a closed surface



$\Phi_{EN} \neq 0$



$\Phi_{EN} = 0$

$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA} \Rightarrow Net\ field\ lines\ passing\ through\ closed\ surface$$

Gauss's Law

The flux through any surface enclosing the charge is $\frac{Q_{enc}}{\epsilon_0}$

In other words, the flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0. If Q is the total charge enclosed by the surface through which the flux is calculated

$$\Phi_E = \underset{\text{closed surface S}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0}$$

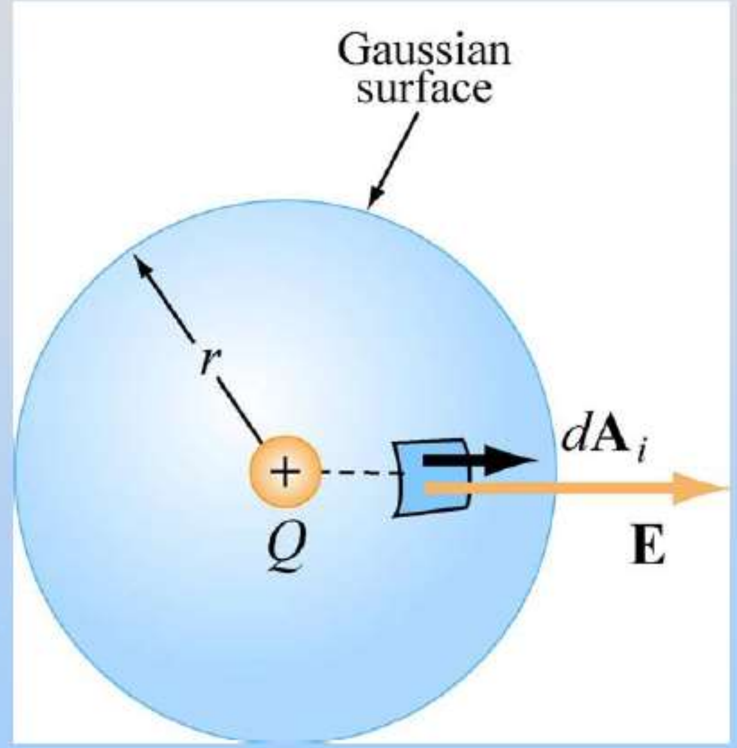
Electric flux Φ_E (the surface integral of E over closed surface *S*) is proportional to charge inside the volume enclosed by *S*

Electric Flux: Point charge

Point charge Q at center of sphere, radius r

E field at surface:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}$$



Electric flux through sphere:

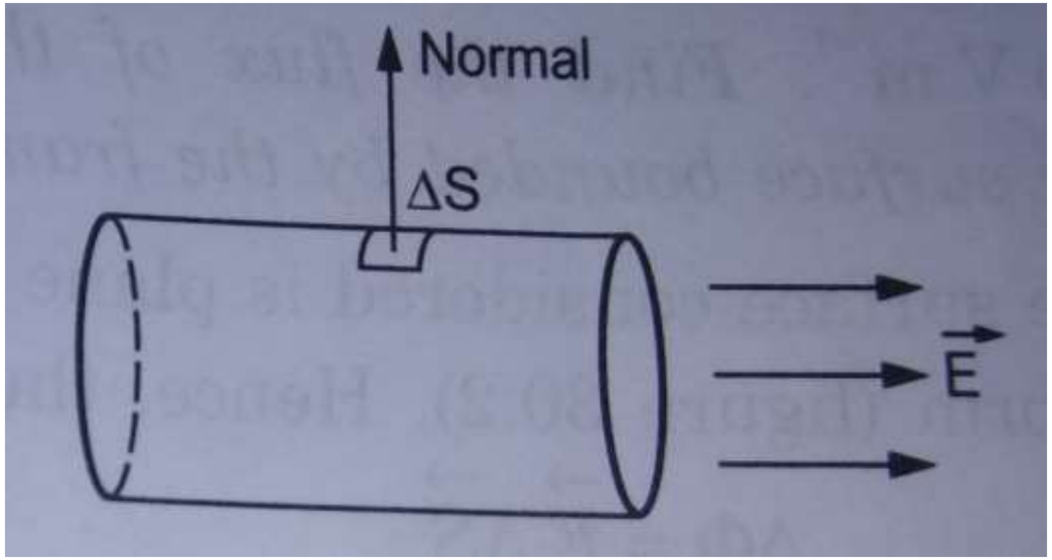
$$\Phi_E = \oiint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oiint_S \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}\cdot dA\,\hat{\mathbf{r}}$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}\oiint_S dA = \frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{d\vec{\mathbf{A}} = dA\,\hat{\mathbf{r}}}$$

SOLVED EXAMPLE

A uniform electric field exists in space. Find the flux of this field through a cylindrical surface with the axis parallel to the field.



Ans.: $On\ the\ curved\ surface \vec{E} \perp \overrightarrow{ds} \Rightarrow Final\ answer = 0$

POLL QUESTION

A spherical gaussian surface surrounds a point charge q.

Describe what happens to the total flux through the surface if

(A) the charge is tripled,

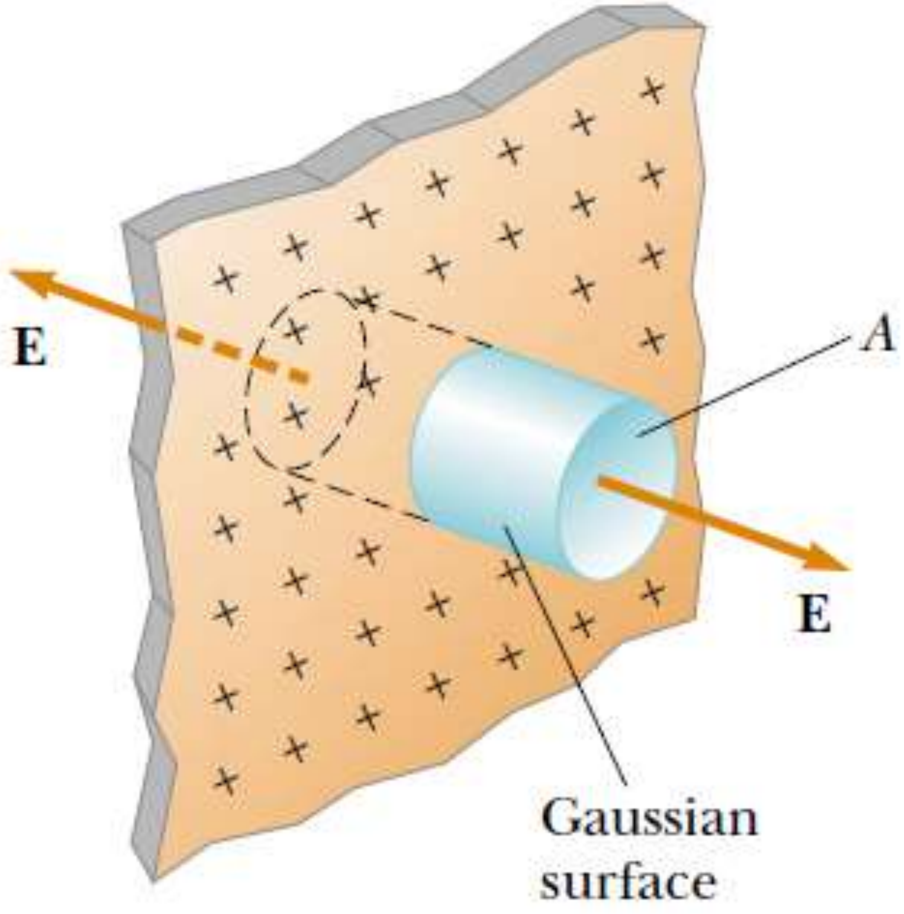
(B) the radius of the sphere is doubled,

(C) the surface is changed to a cube, and

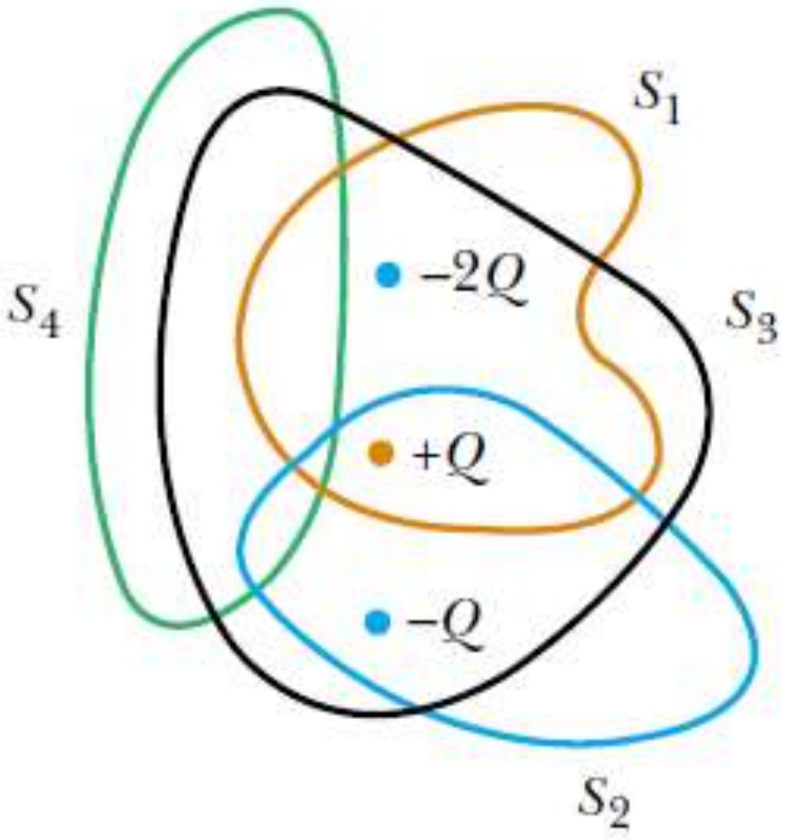
(D) the charge is moved to another location inside the surface.

LECTURE-02

Application of Gauss Law: ES field due to infinite wire and sheet



CONCEPT QUESTION



Find the electric flux through each surface:

A. $\Phi_{S1} = \frac{-Q}{\epsilon_0}; \Phi_{S2} = \frac{+Q}{\epsilon_0}; \Phi_{S3} = \frac{-2Q}{\epsilon_0}; \Phi_{S4} = \frac{+2Q}{\epsilon_0}$

B. $\Phi_{S1} = \frac{-Q}{\epsilon_0}; \Phi_{S2} = 0; \Phi_{S3} = \frac{-2Q}{\epsilon_0}; \Phi_{S4} = \frac{+2Q}{\epsilon_0}$

C. $\Phi_{S1} = \frac{-Q}{\epsilon_0}; \Phi_{S2} = 0; \Phi_{S3} = \frac{-2Q}{\epsilon_0}; \Phi_{S4} = 0$

D. $\Phi_{S1} = \frac{-Q}{\epsilon_0}; \Phi_{S2} = \frac{+Q}{\epsilon_0}; \Phi_{S3} = \frac{-2Q}{\epsilon_0}; \Phi_{S4} = 0$

$$\oint_{s} \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$$

Applications of Gauss Law

1. Identify regions in which to calculate E field.
2. Choose Gaussian surfaces S: Symmetry
3. Calculate $\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
4. Calculate q_{in}, charge enclosed by surface S
5. Apply Gauss's Law to calculate E:

$$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$$

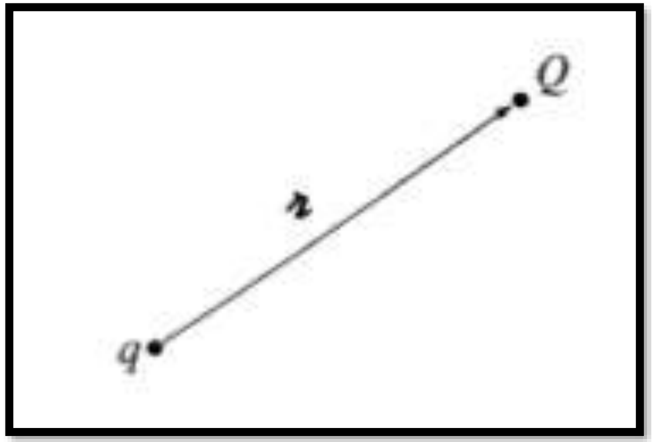
Applications of Gauss Law

Use Gauss's Law to calculate E field from highly symmetric sources

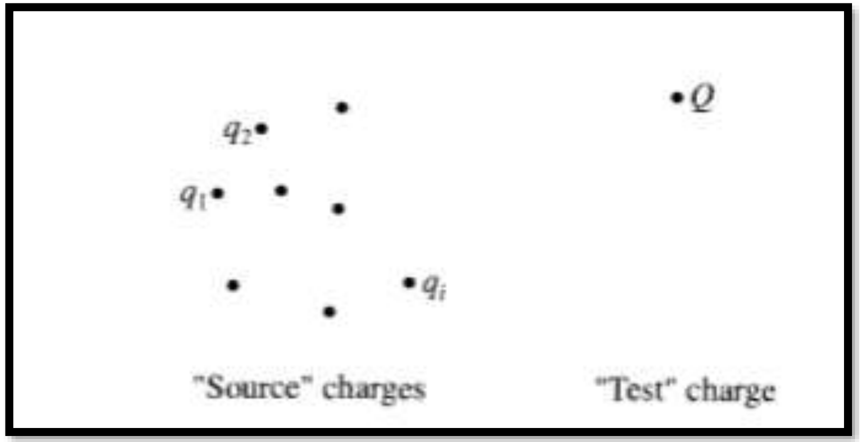
Symmetry	Gaussian Surface
Spherical	Concentric Sphere
Cylindrical	Coaxial Cylinder
Planar	Gaussian "Pillbox"

Point Charge Distribution

Coulomb's Law



Super-position Principle



$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}$$

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \ldots.$$

$$E = \frac{F}{Q} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{{r_i}^2}\widehat{r_i}$$

Continuous Charge Distribution

When the charge is distributed continuously in some region, the sum becomes integral

$$E = \frac{F}{Q} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{{r_i}^2}\widehat{r}_i \quad \Rightarrow \quad E(r) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{r2}\hat{r}\,dq$$

$$Infinitessimal\ charge\ dq \Rightarrow \lambda dl \equiv \sigma da \equiv \rho dv$$

Continuous Charge Distribution

$$Infinitessimal\ charge\ dq = \lambda dl \equiv \sigma da \equiv \rho dv$$

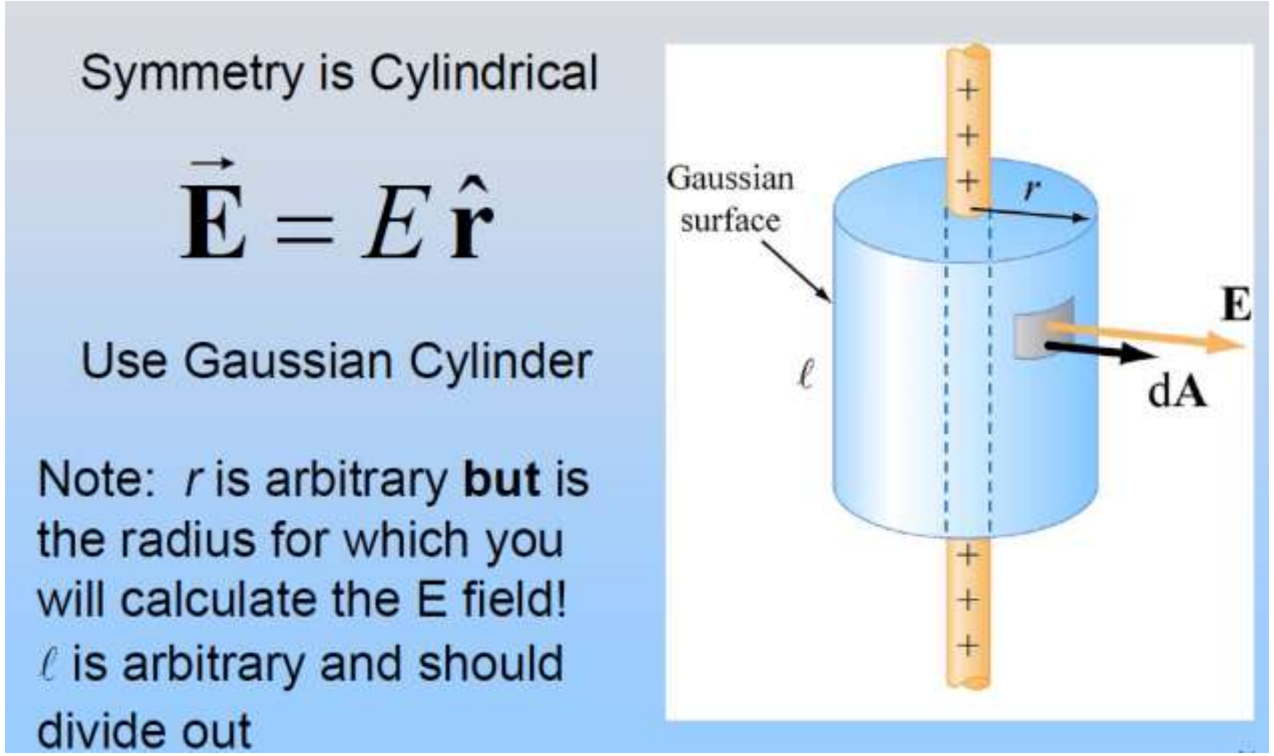
$$\lambda \Rightarrow linear\ charge\ density\ [{}^{C}/_{m}]$$

$$\sigma \Rightarrow surface\ charge\ density\ [C/m^2]$$

$$\rho \Rightarrow volume\ charge\ density\ [C/m^3]$$

Electrostatic Field due to infinite wire

- Electric charge is distributed uniformly along an infinitely long thin wire. The charge per unit length is **λ** (assumed positive). Find the electric field at a distance **r** from the wire by using Gauss's law.

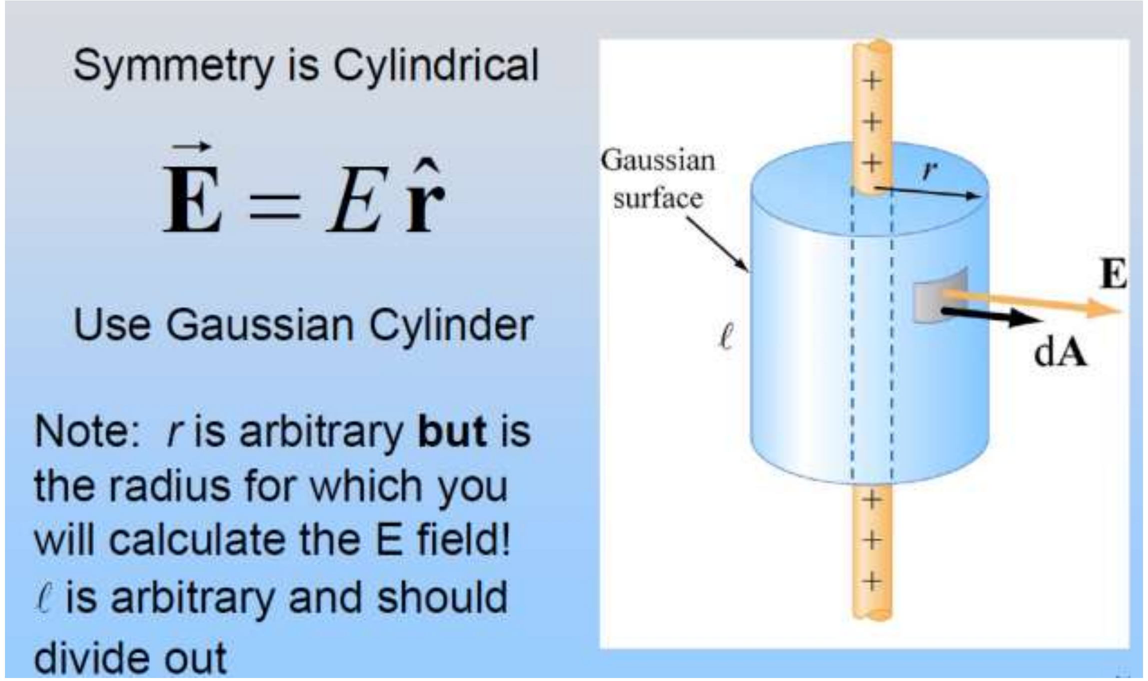


Electrostatic Field due to infinite wire

Electric charge is distributed uniformly along an infinitely long thin wire. The charge per unit length is λ (assumed positive). Find the electric field at a distance **r** from the wire by using Gauss's law.

$$\varphi = \oint_s \vec{E}.\overrightarrow{ds} \Rightarrow \oint_{Curved\ Surf} E\ ds + \oint_{Flat\ Ends} \cancel{E\ ds} = \boldsymbol{E\ 2\pi r l}.\ \vec{E} \perp \overrightarrow{ds}$$

$$\boxed{Q_{enc} = \lambda l}$$

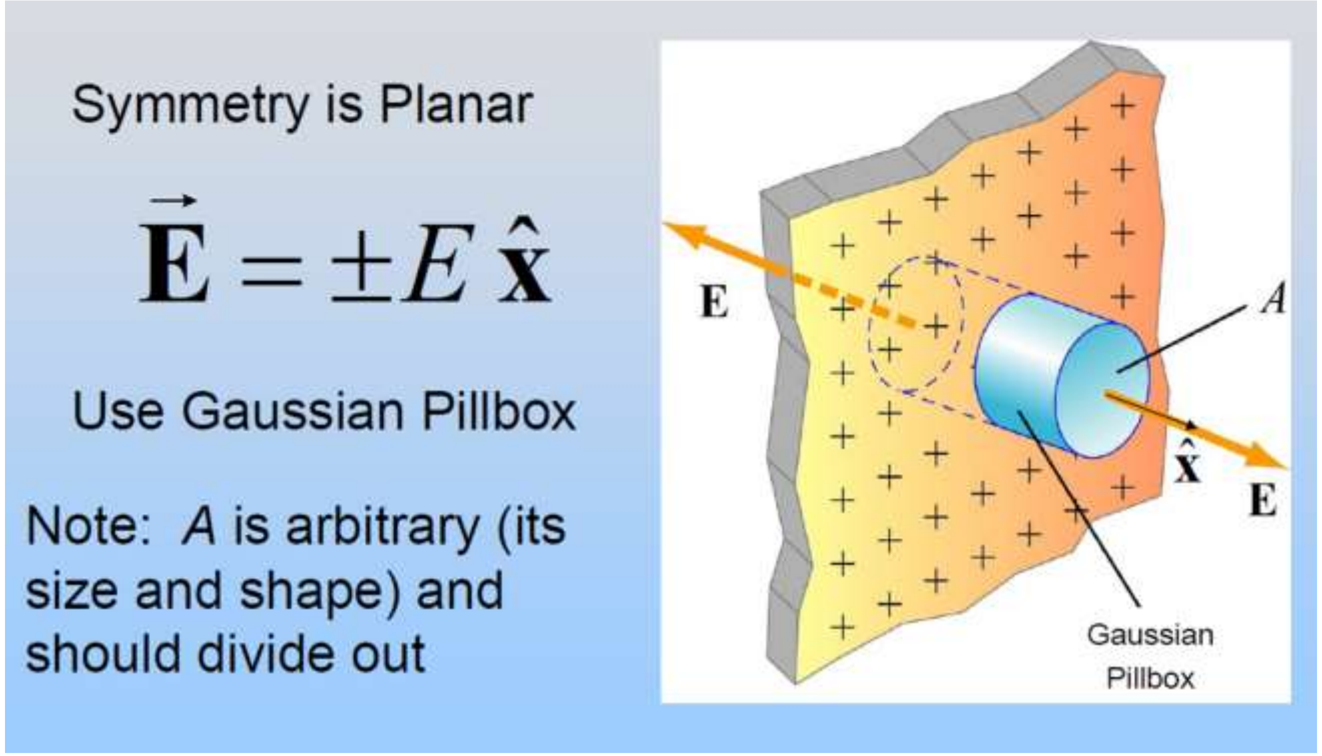


From Gauss law $\varphi = E\ 2\pi r l = \dfrac{\lambda l}{\epsilon_0}$,

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}$$

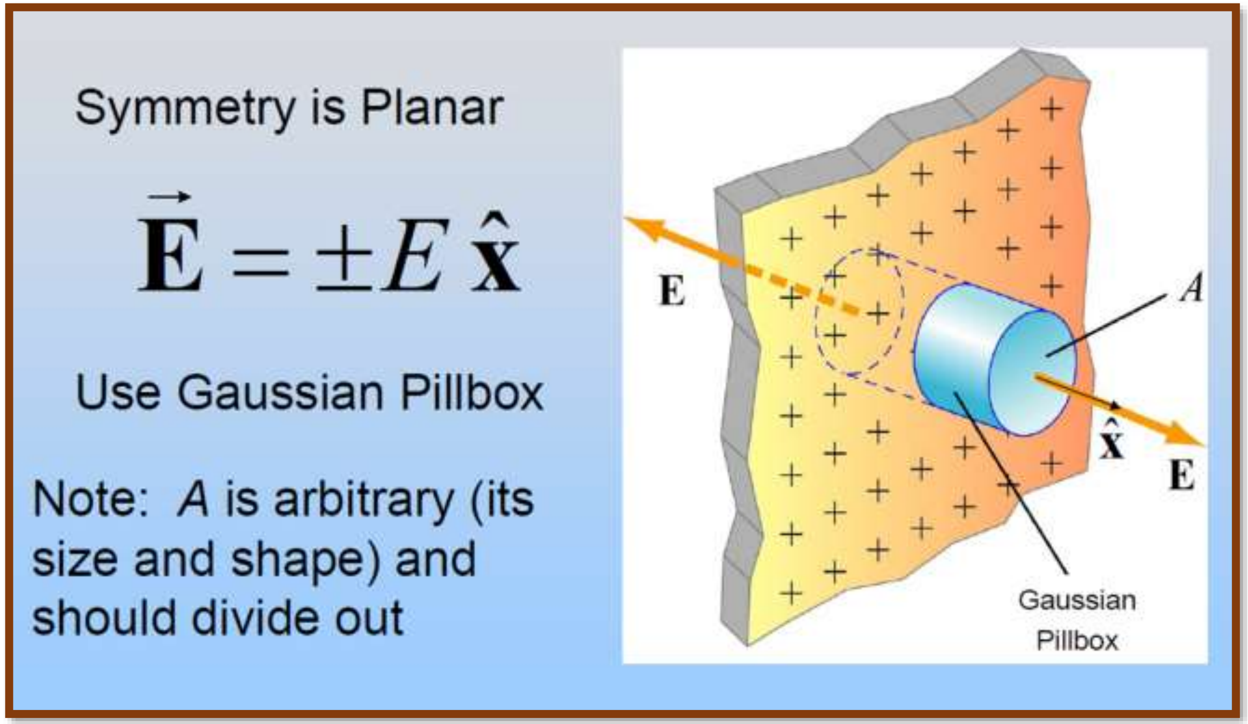
Electrostatic Field due to infinite sheet

Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with uniform positive surface charge density σ.



Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with uniform positive surface charge density σ.

$$\varphi = \oint_s \vec{E}.\overrightarrow{ds} \Rightarrow \oint_{Curved\ Surf} \cancel{E\,ds} + \oint_{Flat\ Ends} E\ ds = 2EA.\ \vec{E} \perp \overrightarrow{ds}$$

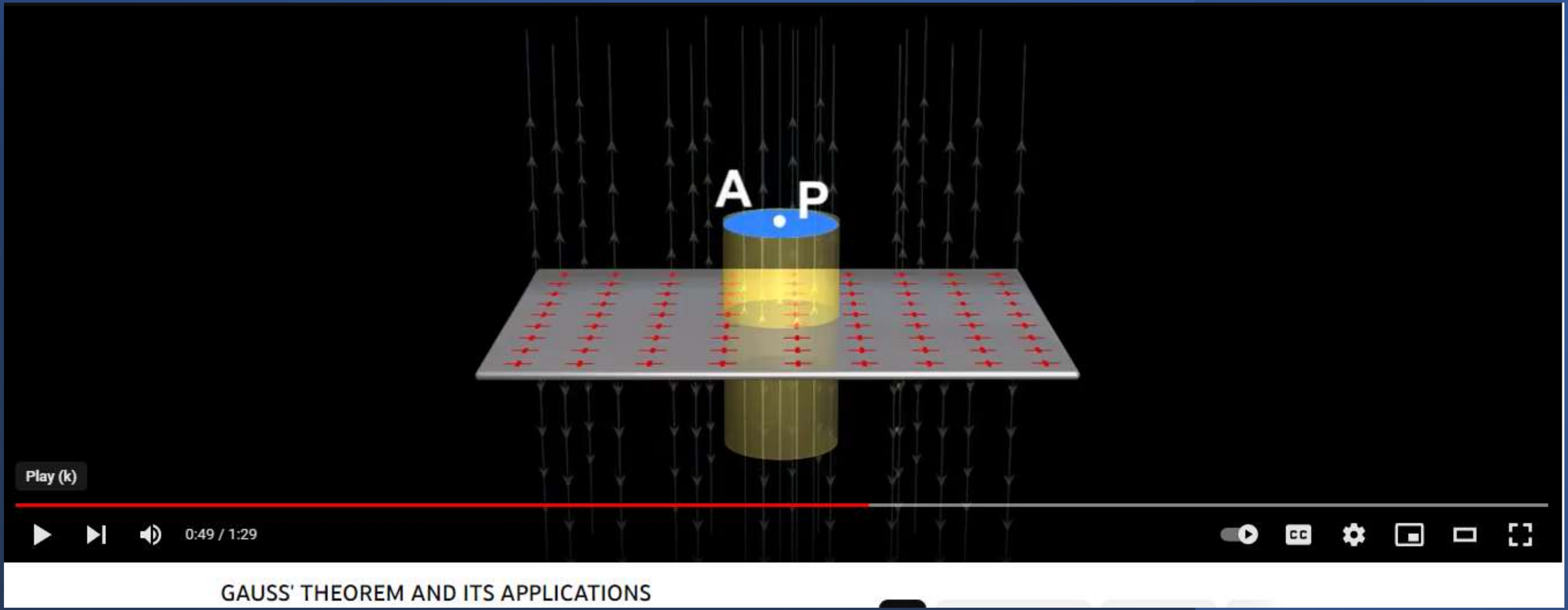


$$\boxed{Q_{enc} = \sigma A}$$

From Gauss's law, $2EA = \dfrac{\sigma A}{\epsilon_0}$

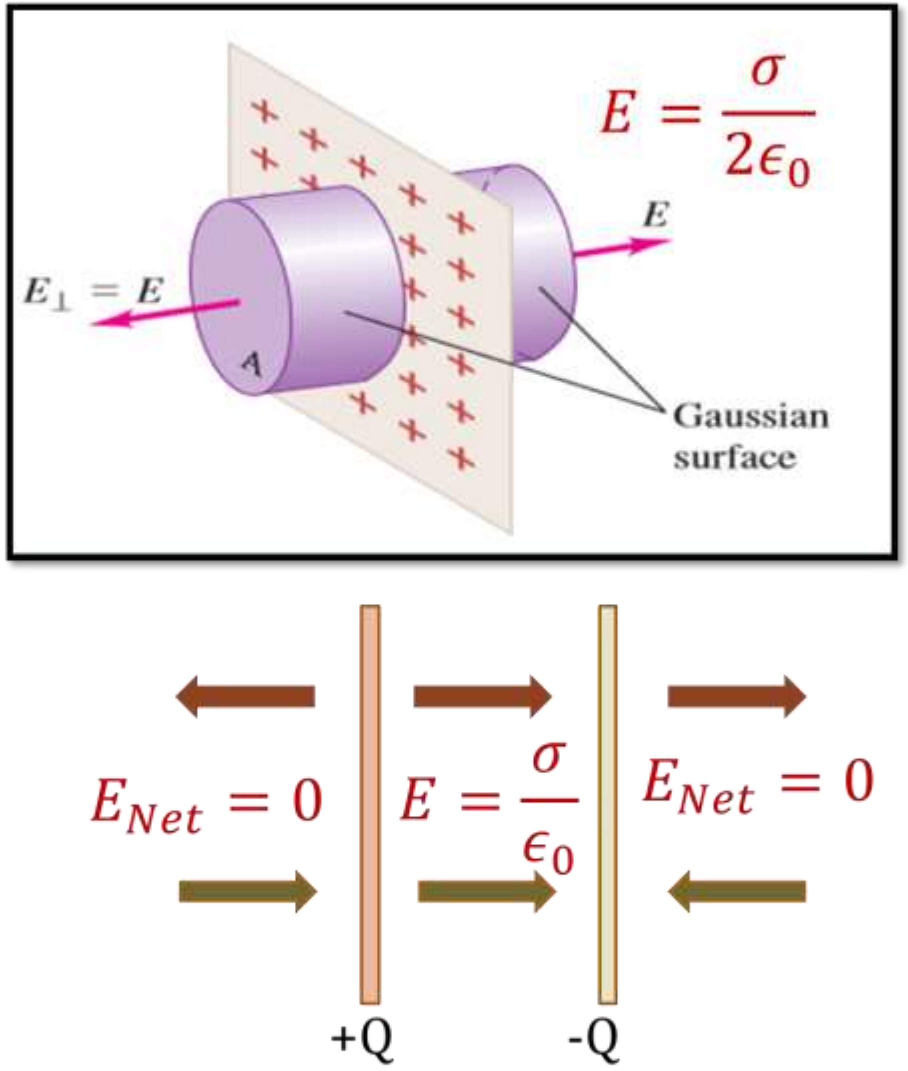
and $E = \dfrac{\sigma}{2\epsilon_0}$

INTERACTIVE PRESENTATION



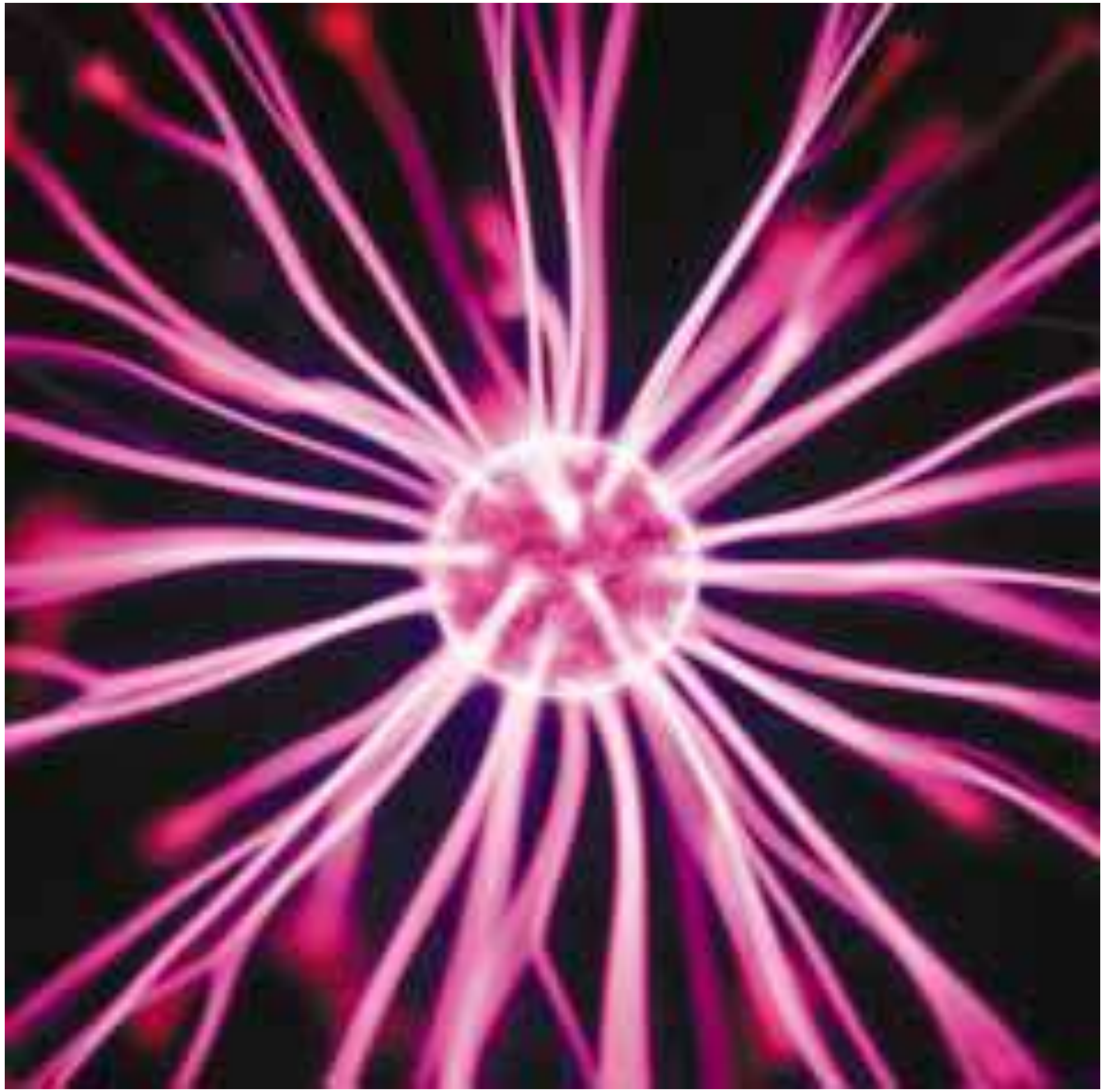
SOLVED EXAMPLE

Two infinite parallel planes carry equal but opposite charge densities **±σ** (see the figure). Find the field in each of the three regions (i) to the left of both (ii) between them, (iii) to the right of both.

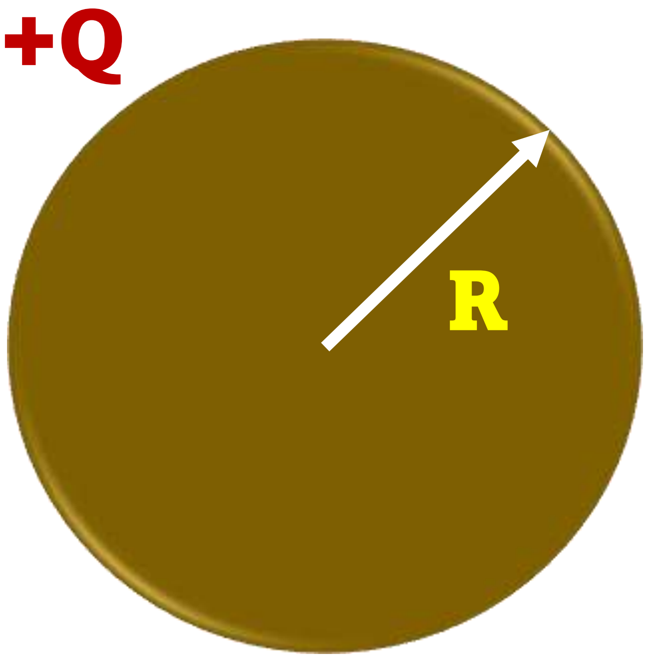


LECTURE-03

Electrostatic field due to conducting and insulating sphere.



CONCEPT QUESTION



If +Q charge is distributed on a conducting sphere of radius R, where can we locate the charge:

A. Outside the sphere

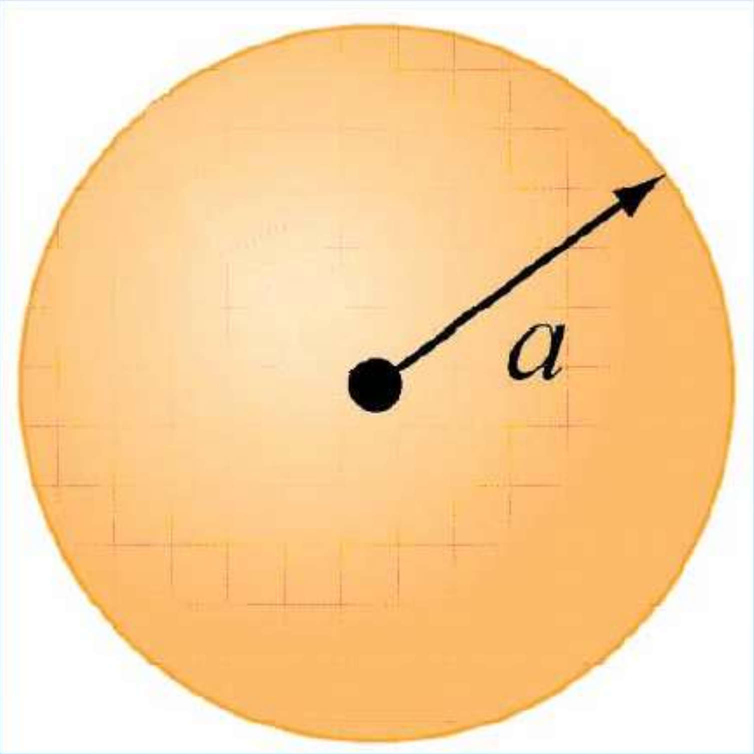
B. inside the sphere

C. on the surface of the sphere

D. None of the above

Application of Gauss Law: Electrostatic Field due to Insulating Sphere

+Q uniformly distributed throughout non-conducting solid sphere of radius a. Find **E** everywhere

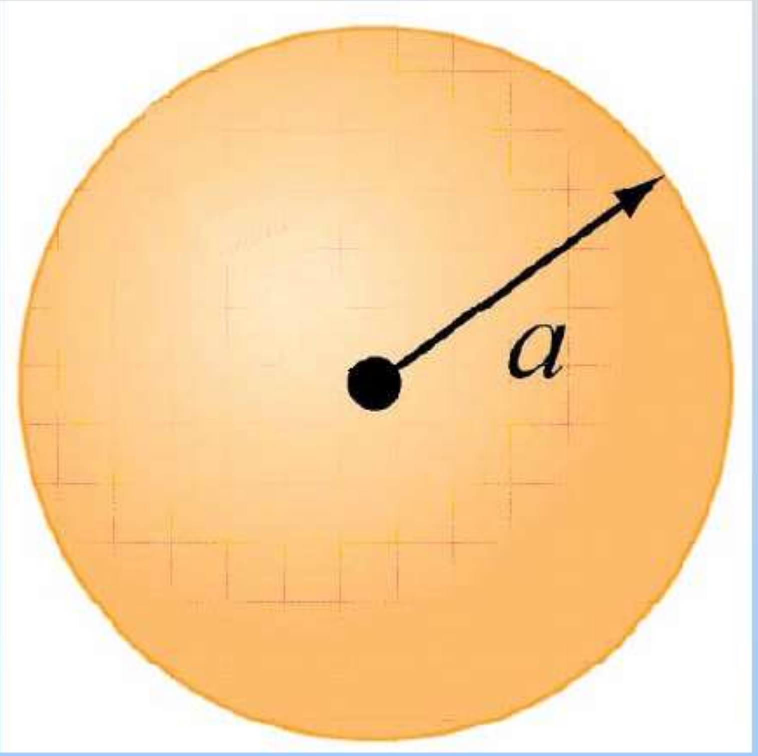


Application of Gauss Law:
Electrostatic Field due to Insulating Sphere

Symmetry is Spherical

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{r}}$$

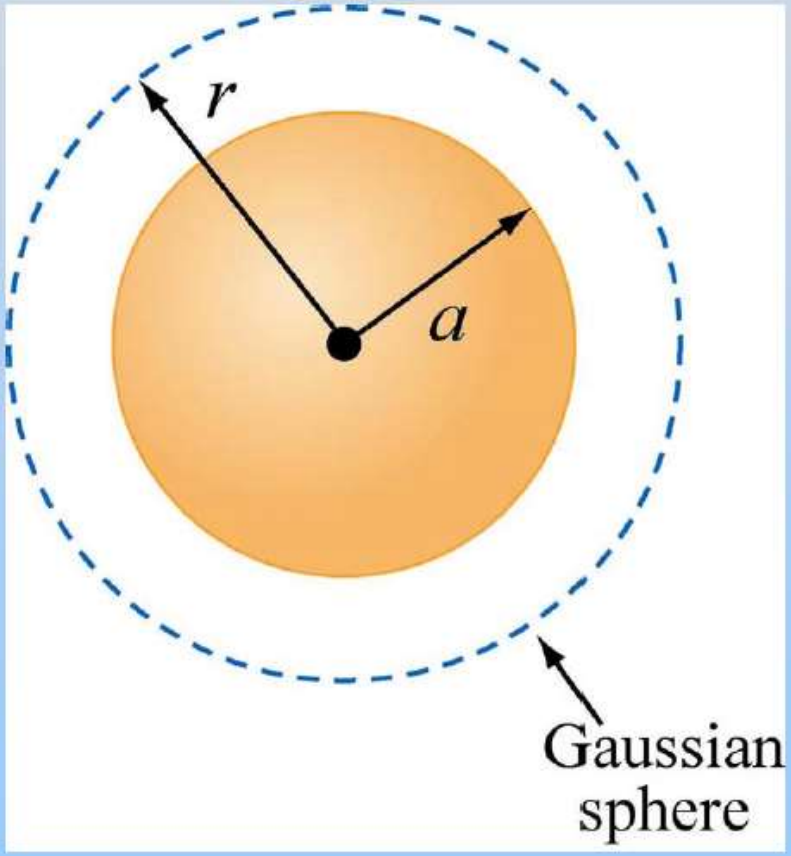
Use Gaussian Spheres



Application of Gauss Law:
Electrostatic Field due to Insulating Sphere

Region 1: $r > a$

Draw Gaussian Sphere in Region 1 ($r > a$)



Note: r is arbitrary **but** is the radius for which you will calculate the E field!

Application of Gauss Law:
Electrostatic Field due to Insulating Sphere

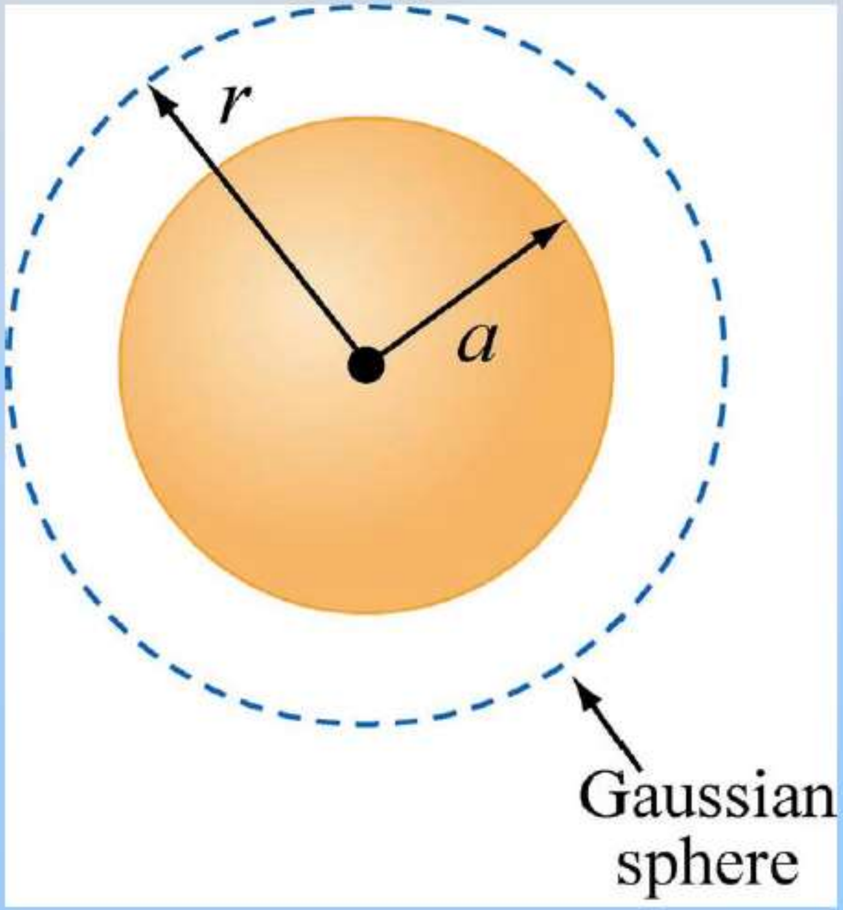
Total charge enclosed $q_{in} = +Q$

$$\Phi_E = \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint\!\!\oint_S dA = EA$$

$$= E\left(4\pi r^2\right)$$

$$\Phi_E = 4\pi r^2 E = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}}$$

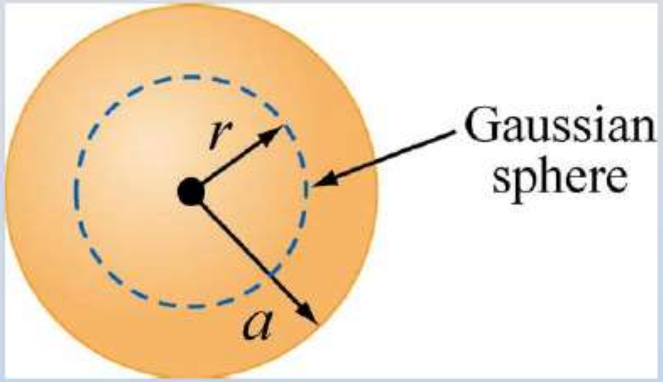


Application of Gauss Law:
Electrostatic Field due to Insulating Sphere

Region 2: $r < a$

Total charge enclosed:

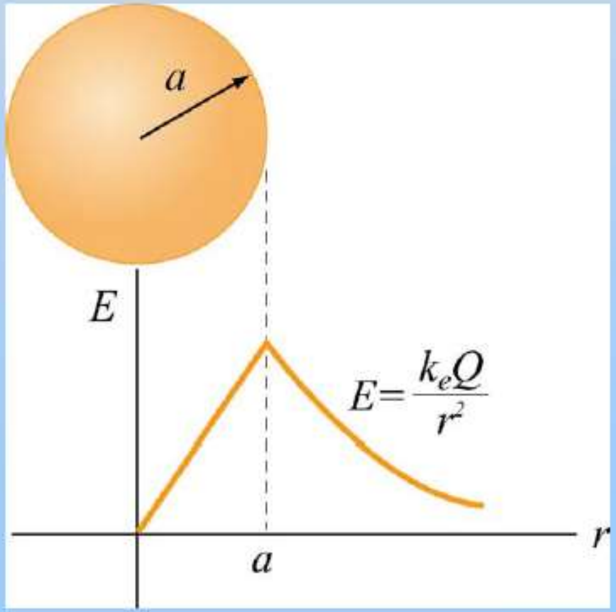
$$q_{in} = \left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} \right) Q = \left(\frac{r^3}{a^3} \right) Q \quad \text{OR} \quad q_{in} = \rho V$$



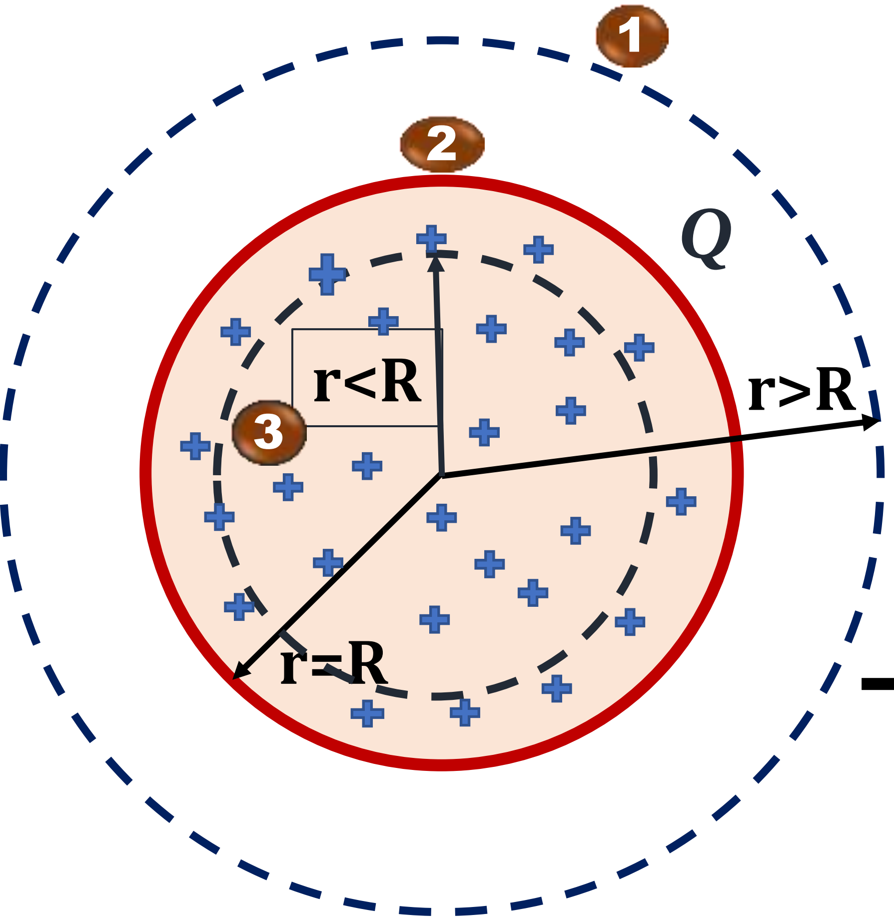
Gauss's law:

$$\Phi_E = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0} = \left(\frac{r^3}{a^3} \right) \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3} \quad \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3} \hat{\mathbf{r}}}$$



SUMMARY SLIDE

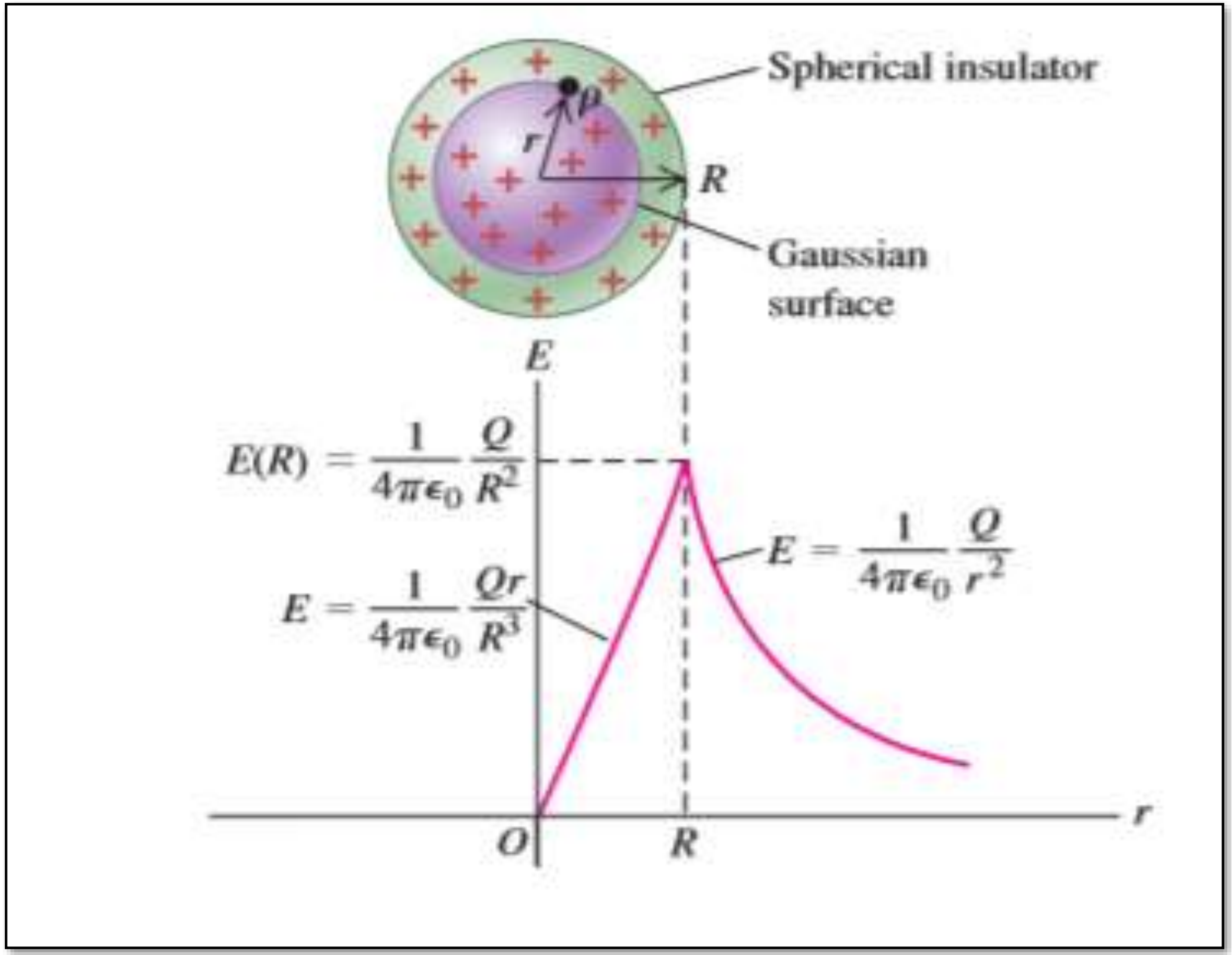


$$\varphi = \oint_s \vec{E}.\overrightarrow{ds} = \oint_s E\, ds = \mathrm{E}\oint_s ds = E\,4\pi r^2$$

$$Charge\ density\ \rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$

1	2	3
$Q_{enc} = Q$	$Q_{enc} = Q$	$Q_{enc} = \rho v = \frac{Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$
$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$	$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 R^2}$	$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = \frac{Qr}{4\pi\epsilon_0 R^3}$

Radial distribution of Electrostatic Field due to Insulating Sphere

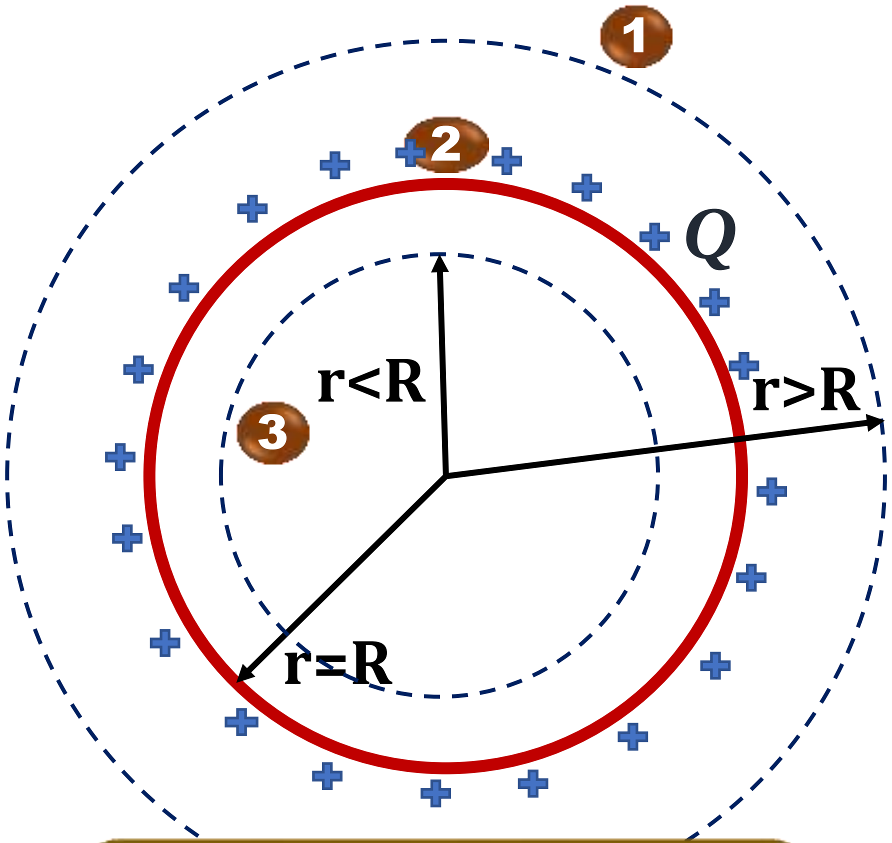


$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} \; FOR\; r > R$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 R^2} \; FOR\; r = R$$

$$\Rightarrow E = \frac{Qr}{4\pi\varepsilon_0 R^3} \; FOR\; r < R$$

Application of Gauss Law:
Electrostatic Field due to Conducting Sphere

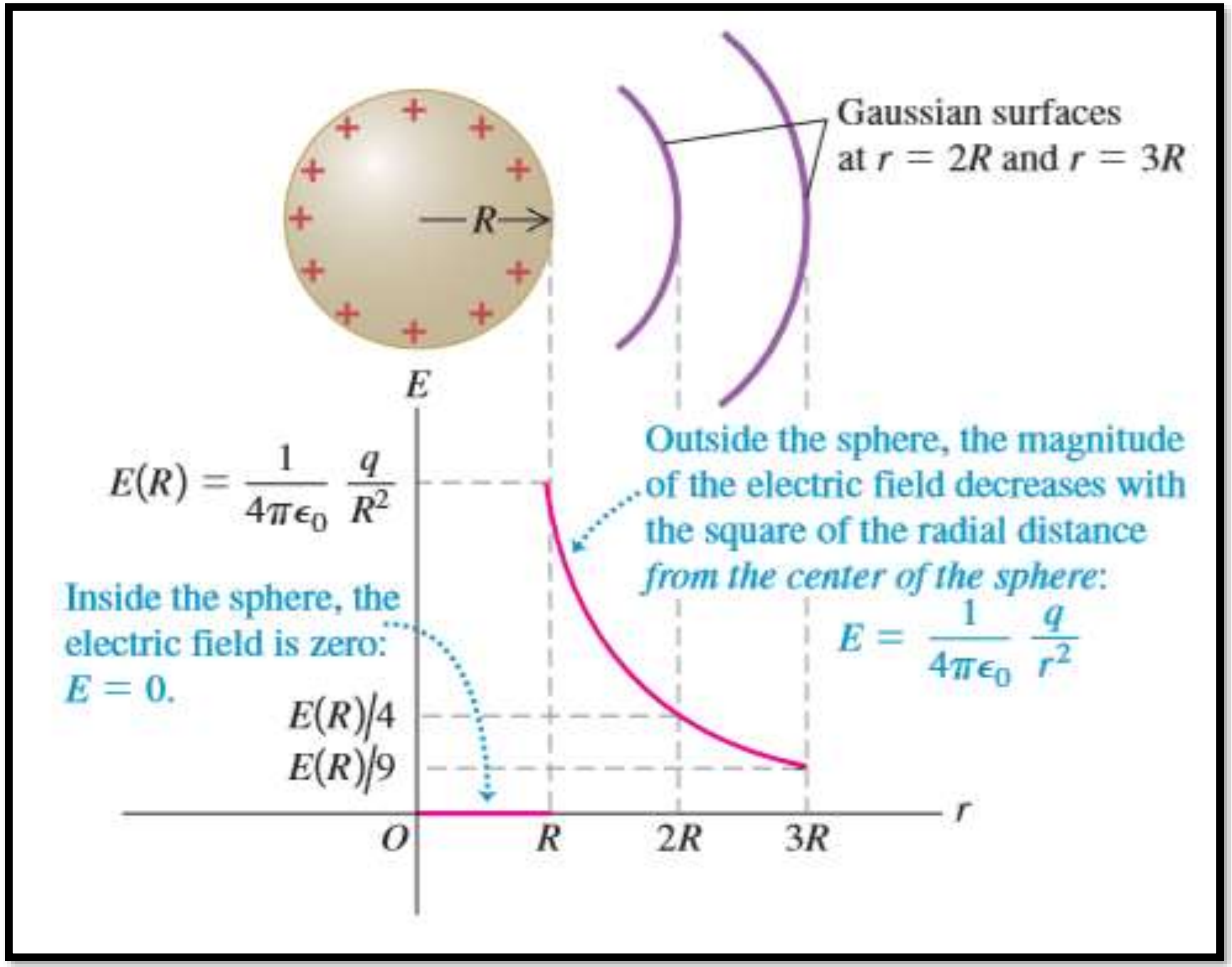


SAME FOR INSULATING SPHERICAL SHELL

$$\varphi = \oint_s \vec{E}.\overrightarrow{ds} = \oint_s E\,ds = \mathrm{E}\oint_s ds = E\,4\pi r^2$$

1	2	3
$Q_{enc} = Q$	$Q_{enc} = Q$	$Q_{enc} = 0$
$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$	$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 R^2}$	$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$ $\Rightarrow E = 0$

Radial distribution of Electrostatic Field due to Conducting Sphere



$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad r > R$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} \quad r = R$$

$$E = 0 \quad r < R$$

For a conductor

- The electric field is zero inside a conductor
- Any net charge must reside on the surface

INTERACTIVE PRESENTATION

POLL QUESTION

Consider a solid ball of radius R whose charge density depends on the radial coordinate as $\rho(r) = Ar^2$, for some constant A. Find the electric field at a point inside (r < R) and outside the ball (r > R).

$$HINT: \oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0} = \frac{\int \rho dv}{\epsilon_0}$$

A. $E_{outside}(r > R) = \frac{AR^2}{4\pi\epsilon_0 r^2}; E_{inside}(r < R) = \frac{A}{4\pi\epsilon_0}$

B. $E_{outside}(r > R) = \frac{AR^3}{\epsilon_0 r^2}; E_{inside}(r < R) = \frac{AR}{4\pi\epsilon_0}$

C. $E_{outside}(r > R) = \frac{AR^5}{5\epsilon_0 r^2}; E_{inside}(r < R) = \frac{Ar^3}{5\epsilon_0}$

D. $E_{outside}(r > R) = \frac{AR^5}{5\epsilon_0 r^2}; E_{inside}(r < R) = \frac{AR^3}{5\epsilon_0 r^2}$

CONCEPT QUESTION

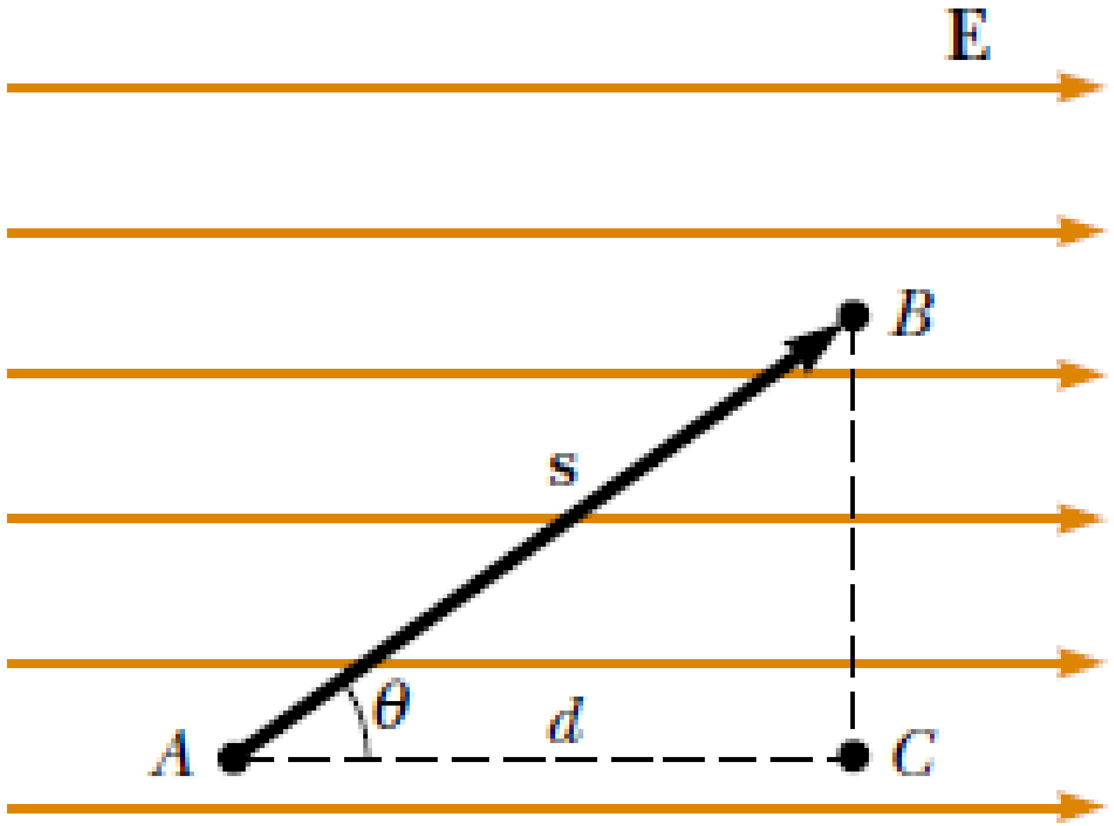
Which is the only correct relationship here:

$A. V_A > V_B > V_C$

$B. V_A < V_B < V_C$

$C. V_A = V_B > V_C$

$\boxed{D. V_A > V_B = V_C}$



CONCEPT QUESTION

For the equipotential surfaces in adjacent figure what is the approximate direction of the electric field?

(a) Out of the page

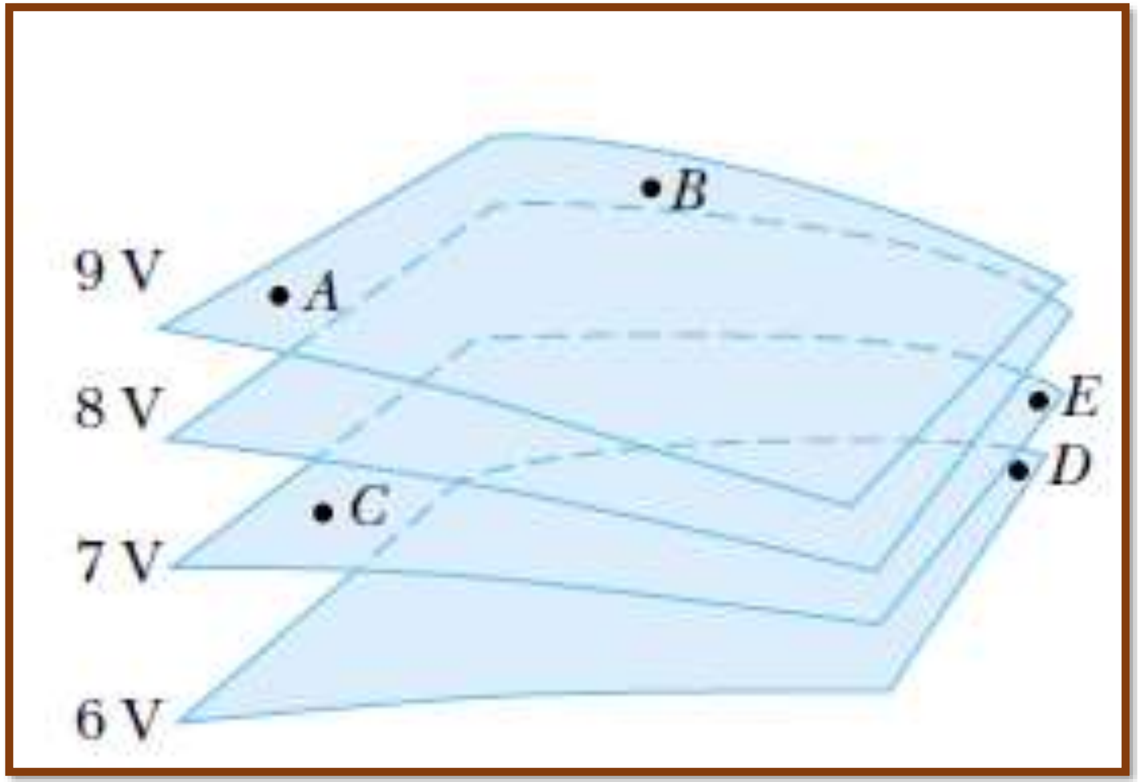
(b) Into the page

(c) Toward the right edge of the page

(d) Toward the left edge of the page

(e) Toward the top of the page

(f) Toward the bottom of the page.



Electric potential energy

- Electric potential energy of two point charges
- $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$, *where* r is the distance between two charges.
- We know that the work done by a conservative force is negative of change in potential energy

$$W_{a\to b} = U_a - U_b = -\Delta U.$$

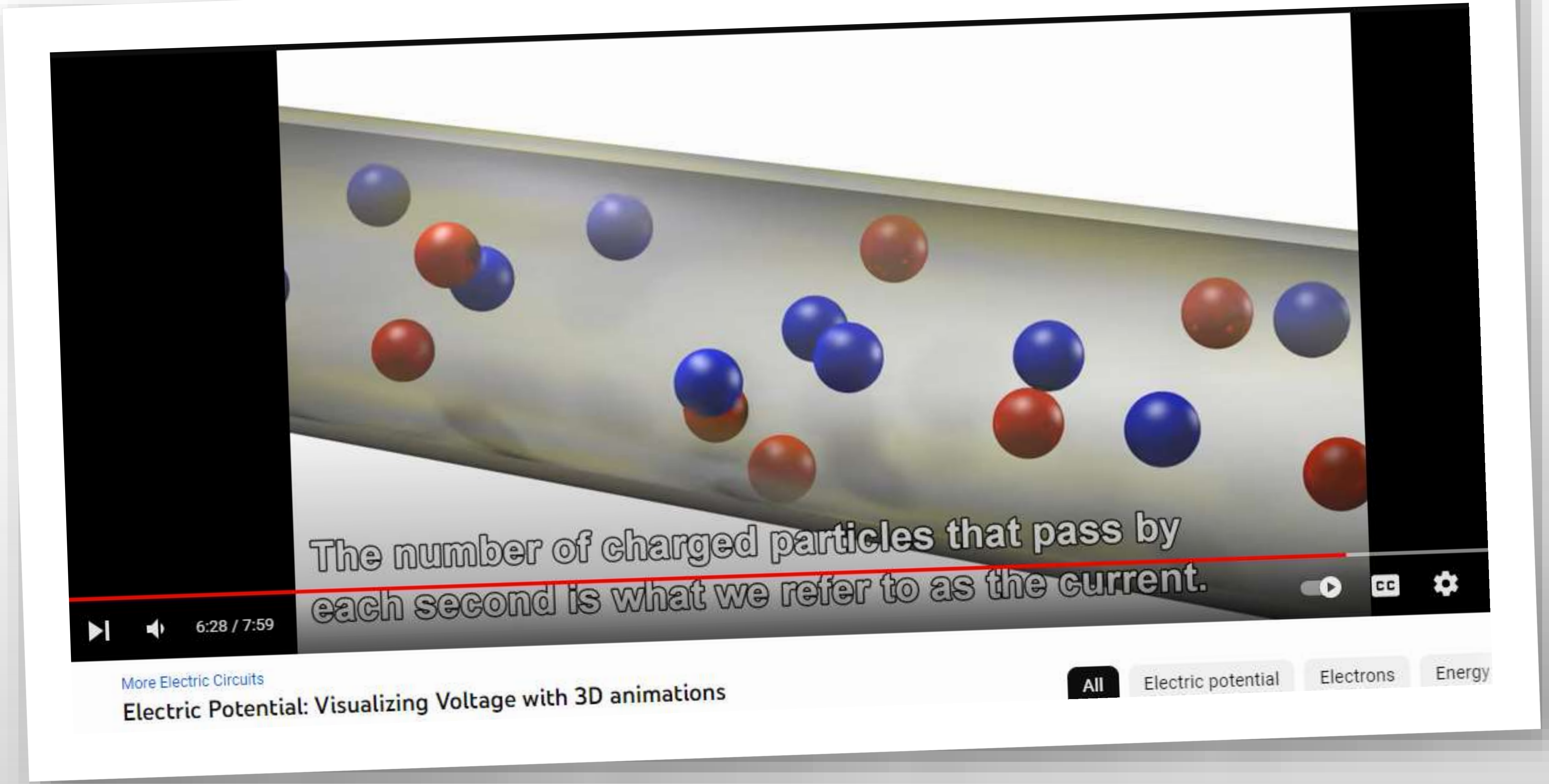
- Potential energy is always defined relative to some reference point where $U = 0$. In the above equation $U = 0$, when the charges are at *infinite separation*. Thus U represents the work done on the charge q_2 by the field of q_1 if q_2 moved from an initial distance r to infinite.
- For a system of charges q_1, q_2, q_3 etc., The total potential energy U is the sum of the potential energies of interaction for each pair of charges.

$$U = \sum_{i<j} \frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$$

Electric potential

- Electric potential is defined as the electrical potential energy per unit charge. It is a scalar quantity.
- Electric potential due to a point charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$, where r is the distance from point charge where potential is measured.
- Electric potential due to a collection of point charges: $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$, $where$ q_i is the value of i^{th} point charge and r_i is the distance from i^{th} point charge to where the potential is measured.
- Electric field due to continuous distribution of charges = $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$
- Electric field and potential are closely related by $V(r) = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$
- Electric potential energy U=$qV(r) = -q\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$
- Electric field vector from potential: $\vec{E} = -\left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V = -\vec{\nabla}V = -Grad\ V$
- In components, $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

INTERACTIVE PRESENTATION

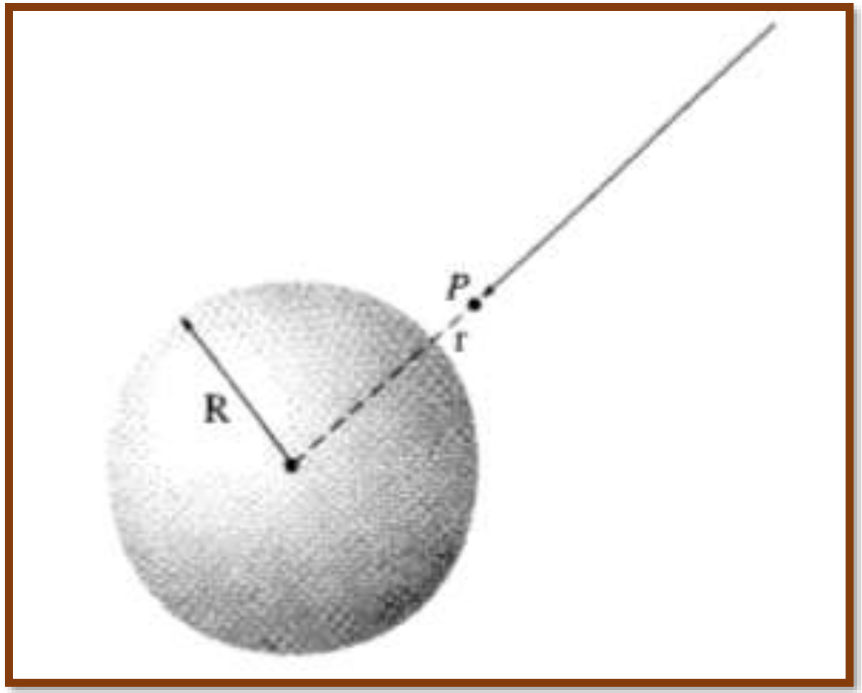


Electric potential of a conducting sphere

Consider a conducting sphere of radius R and charge Q, as shown in Figure.

Find the electric potential everywhere.

From Gauss's law, the electric field outside the metallic spherical shell is given by $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,\hat{r}$, where Q is the total charge on the sphere. The field inside the shell is zero.



For points outside the sphere (***r > R***), $V(r) = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$,

$$V(r) = -\int_{\infty}^{r} \frac{Q}{4\pi r'^2}\,dr' = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Electric potential of a conducting sphere

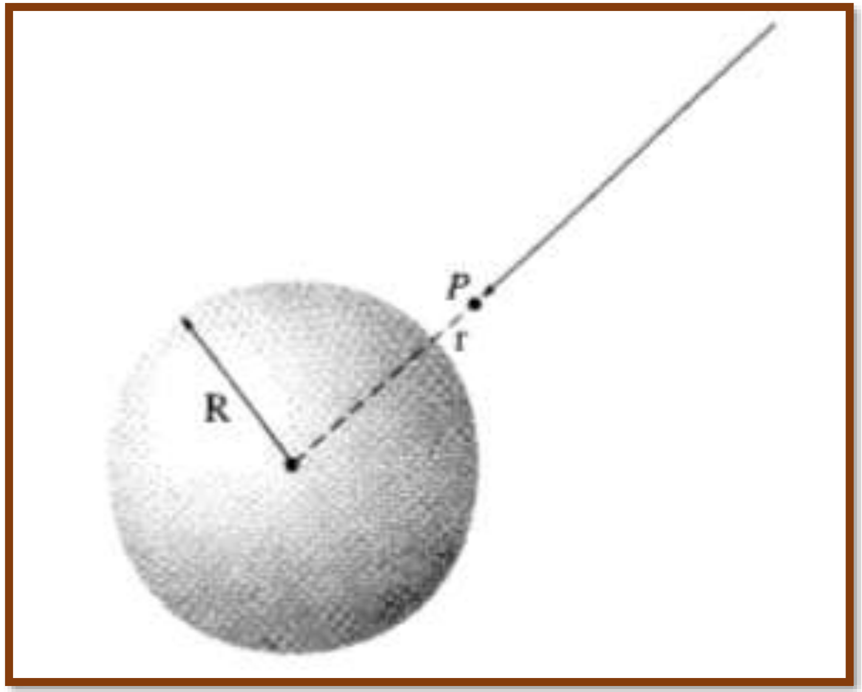
Consider a conducting sphere of radius ***R*** and charge ***Q***, as shown in Figure.

Find the electric potential everywhere.

On the other hand, to find the potential inside the sphere (***r*** < ***R***), we must break the integral into two pieces, using in each region the field prevails there

$$V(r) = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$$

$$= -\int_{\infty}^{R} \frac{Q}{4\pi r'^2} dr' - \int_{R}^{r} 0.dr' = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$



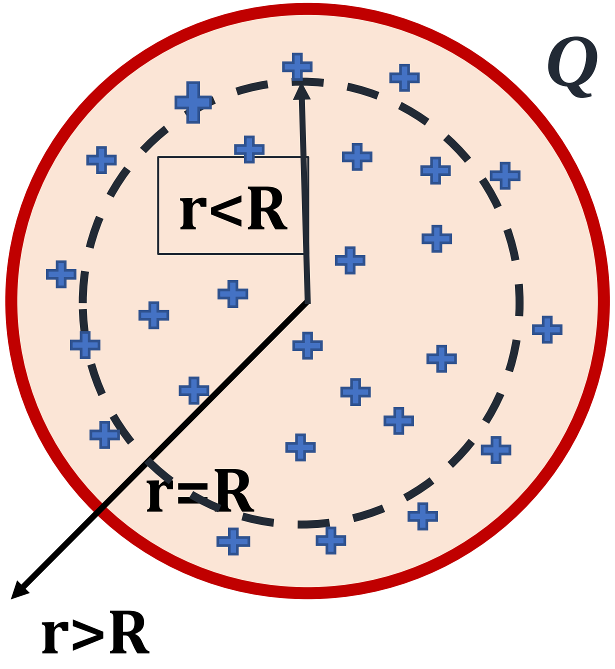
Electric potential of an insulating sphere

Find the potential inside and out-side a uniformly charged solid insulating sphere whose radius is ***R*** and whose total charge is ***Q***.

Solution:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,\hat{r}, outside\ the\ sphere, r > R$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}\,\hat{r}, inside\ the\ sphere, r < R$$



For $r > R$, $V(r) = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$, $V(r) = -\int_{\infty}^{r} \frac{Q}{4\pi r'^2} dr' = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ (r > R)

$$For\ r < R, V(r) = -\int_{\infty}^{R} \frac{Q}{4\pi r'^2} dr' - \int_{R}^{r} \frac{1}{4\pi\epsilon_0}\frac{Qr'}{R^3}.dr' = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2 - R^2}{2}\right)\right] = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

POLL QUESTION

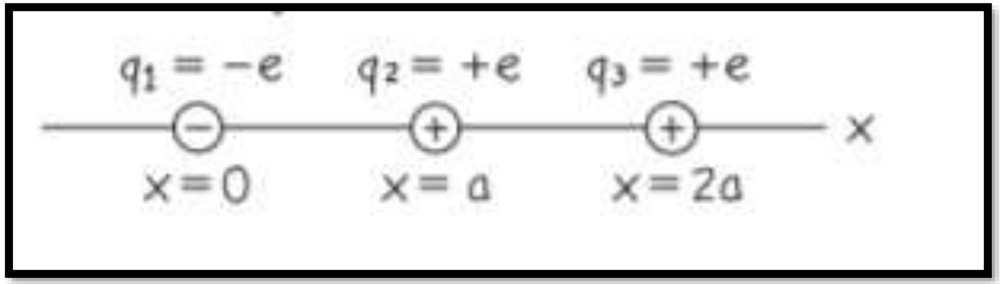
Two point charges are located on the x-axis, q_1= -e at x = 0 and q_2 = +e at x = a (a) Find the work that must be done by an external force to bring a third point charge q_3 =+e from infinity to x = $2a$ (b) Find the total potential energy of the system of three charges.

A. $W = \frac{e^2}{\pi\epsilon_0 a}, U = \frac{-e^2}{\pi\epsilon_0 a}$

B. $W = \frac{e^2}{4\pi\epsilon_0 a}, U = \frac{-e^2}{8\pi\epsilon_0 a}$

C. $W = \frac{e^2}{8\pi\epsilon_0 a}, U = \frac{-e^2}{8\pi\epsilon_0 a}$

D. $W = \frac{-e^2}{4\pi\epsilon_0 a}, U = \frac{-e^2}{4\pi\epsilon_0 a}$

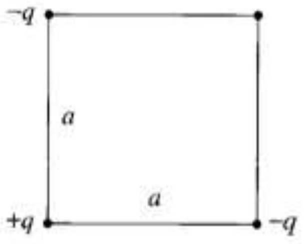


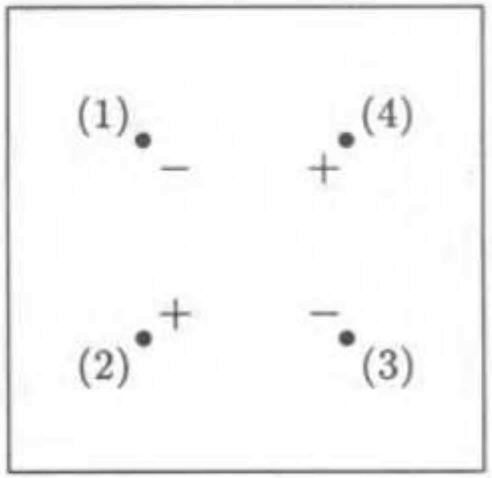
SOLVED EXAMPLE

(a) Three charges are situated at the corners of a square (side a), as shown in the following figure. How much work does it take to bring in another charge, $+q$ from far away and place it in the fourth corner.

b. How much work does it take to assemble the whole configuration of four charges?

Solution:





a. $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\left(\frac{-q}{a} + \frac{q}{\sqrt{2}\,a} + \frac{-q}{a}\right) = \frac{q}{4\pi\epsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$

$$W = qV = \frac{q^2}{4\pi\epsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

b. $W = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right) = \frac{1}{4\pi\epsilon_0}\frac{2q^2}{a}\left(-2 + \frac{1}{\sqrt{2}}\right)$

POLL QUESTION

The electric field in a region is given by $\vec{E} = \frac{A}{x^3}\hat{\imath}$. Write an expression for the potential in the region assuming the potential at infinity to be zero.

A. $V = A/x$

B. $V = A/x^4$

C. $V = A/x^2$

D. $V = A/2x^2$

SOLVED EXAMPLE

Suppose the electric potential due to a certain charge distribution can be written in Cartesian coordinates as $\boldsymbol{V(x,y,z) = Ax^2y^2 + Bxyz}$, where A and B are constants. What is the associated electric field?

Solution: $E_x = -\frac{\partial V}{\partial x} = -2Axy^2 - Byz$

$$E_y = -\frac{\partial V}{\partial y} = -2Ax^2y - Bxz$$

$$E_z = -\frac{\partial V}{\partial z} = -Bxy$$

Thus $\vec{E} = (-2Axy^2 - Byz)\hat{x} - (2Ax^2y + Bxz)\hat{y} - Bxy\hat{z}$

CONCEPT QUESTION

A capacitor gets a charge of $60\mu C$ when it is connected to a battery of emf $12V$.

Calculate the capacitance of the capacitor.

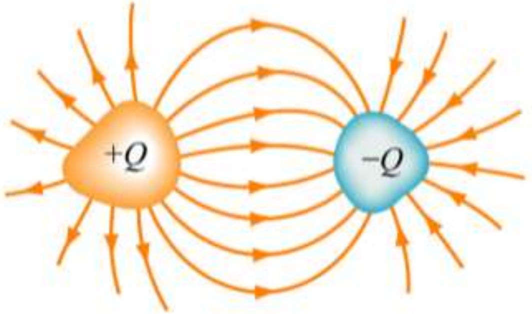
$A.$ $5\mu F$

$B.$ $5mF$

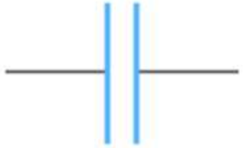
$C.$ $5nF$

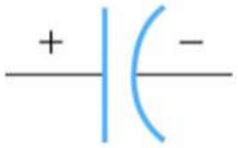
$D.$ $5pF$

Capacitor and Capacitance



- A capacitor is a device which stores electric charge.
- They find many applications in electronics such as in pulsed lasers, air bag sensors for cars, television receivers etc.





- Capacitors vary in shape and size
- *Any two conductors separated by a insulator (or vacuum) form a capacitor*.
- In the uncharged state, the charge on either one of the conductors in the capacitor is zero.
- During the charging process, a charge **+*Q*** is moved from one conductor to the other one, giving one conductor a charge **+*Q*** (positive plate) and the other one an equal negative charge **–*Q*** (negative plate).
- Thus a potential difference V is created with the positive charged conductor at higher potential than the negatively charged conductor.

- For a given capacitor, the charge Q on the capacitor is proportional to the potential difference, V

Thus $\boldsymbol{Q \propto V \text{ or } Q = CV}$ **or** $\boldsymbol{C = \frac{Q}{V}}$

where the proportionality constant C is called the *capacitance* of the capacitor

- The SI unit of capacitance is called farad (F).

$$1F = 1\,farad = \frac{1C}{V} = 1coulomb/volt$$

- The greater the capacitance C of a capacitor, the greater the magnitude Q of charge on either conductor for a given potential difference V and hence *the capacitance is a measure of the ability of the capacitor to store energy*.

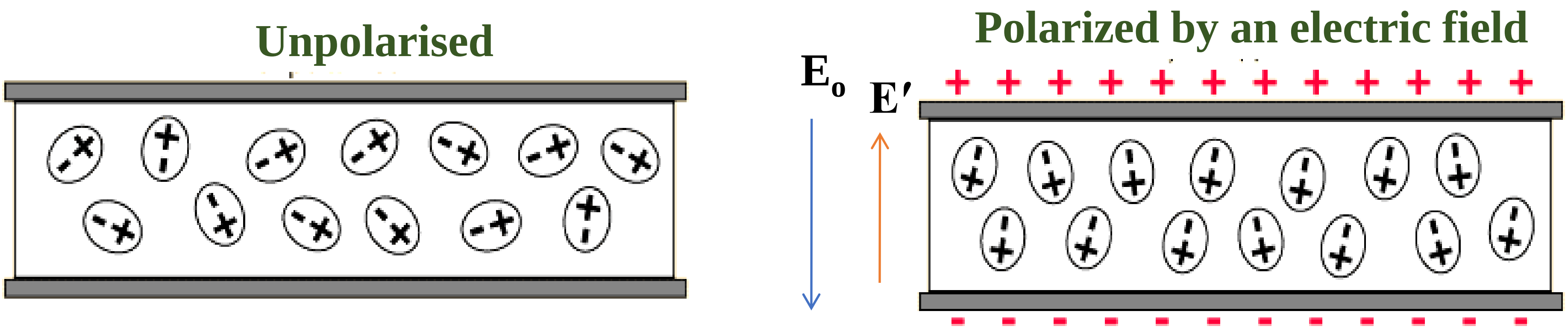
Dielectrics

- Dielectrics are the materials effectively with no charge carriers.
- They are mostly insulators.
- In dielectrics, the electrons are tightly bound the nucleus. So, there are no free electrons.
- They are characterized by high specific resistance (10^{10} to 10^{20} ohm-m).
- They exhibit negative temperature coefficient of resistance and large insulation resistance. The resistance of dielectric materials decreases with increase in temperature.
- The band-gap is very large.
- It is used for the charge storage in capacitors
- The capacitance of a parallel plate capacitor increases by introducing a slab of dielectric medium between the parallel plate capacitor.

Dielectric polarization

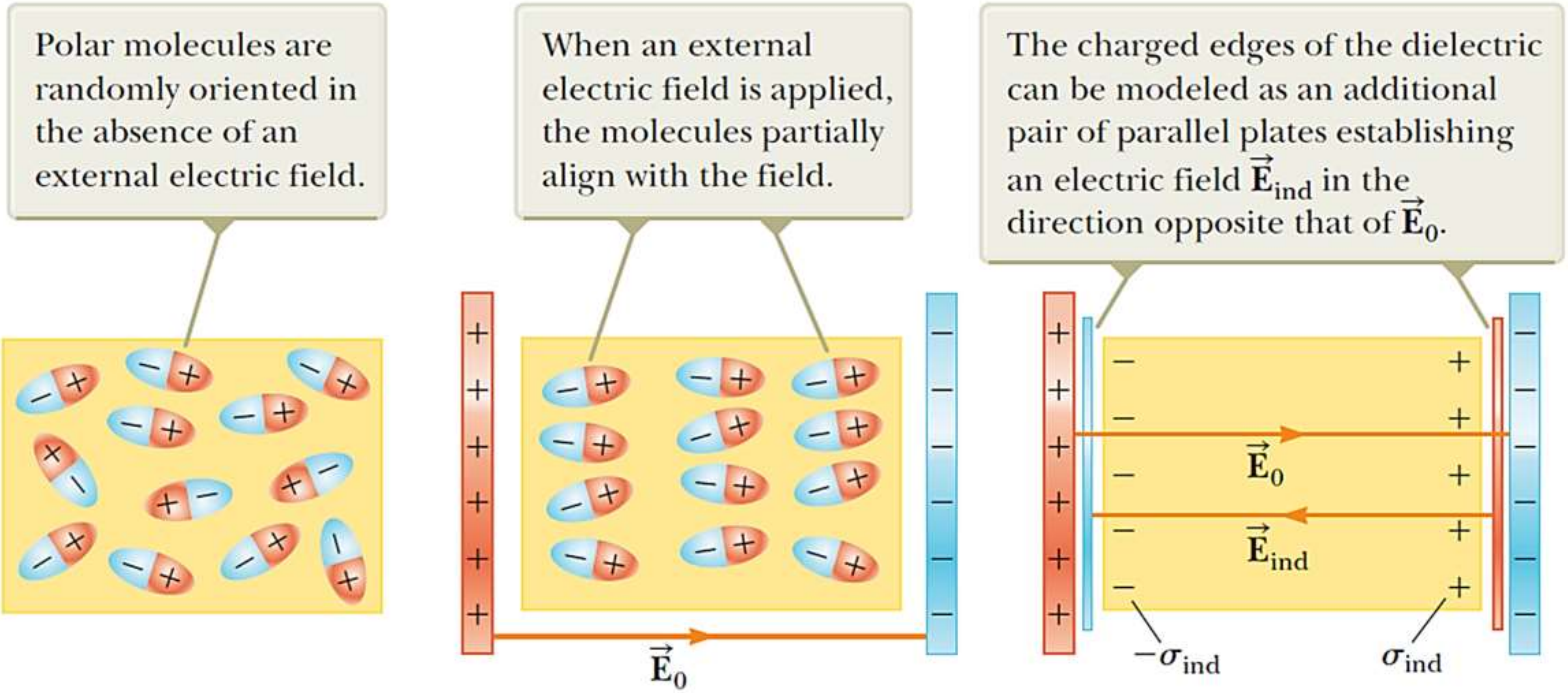
•Consider an electrically neutral slab of an isotropic dielectric inserted between the plates of a charged parallel plate capacitor.

•Dielectric materials have no free charge carriers. Hence current does not flow in the dielectric material.



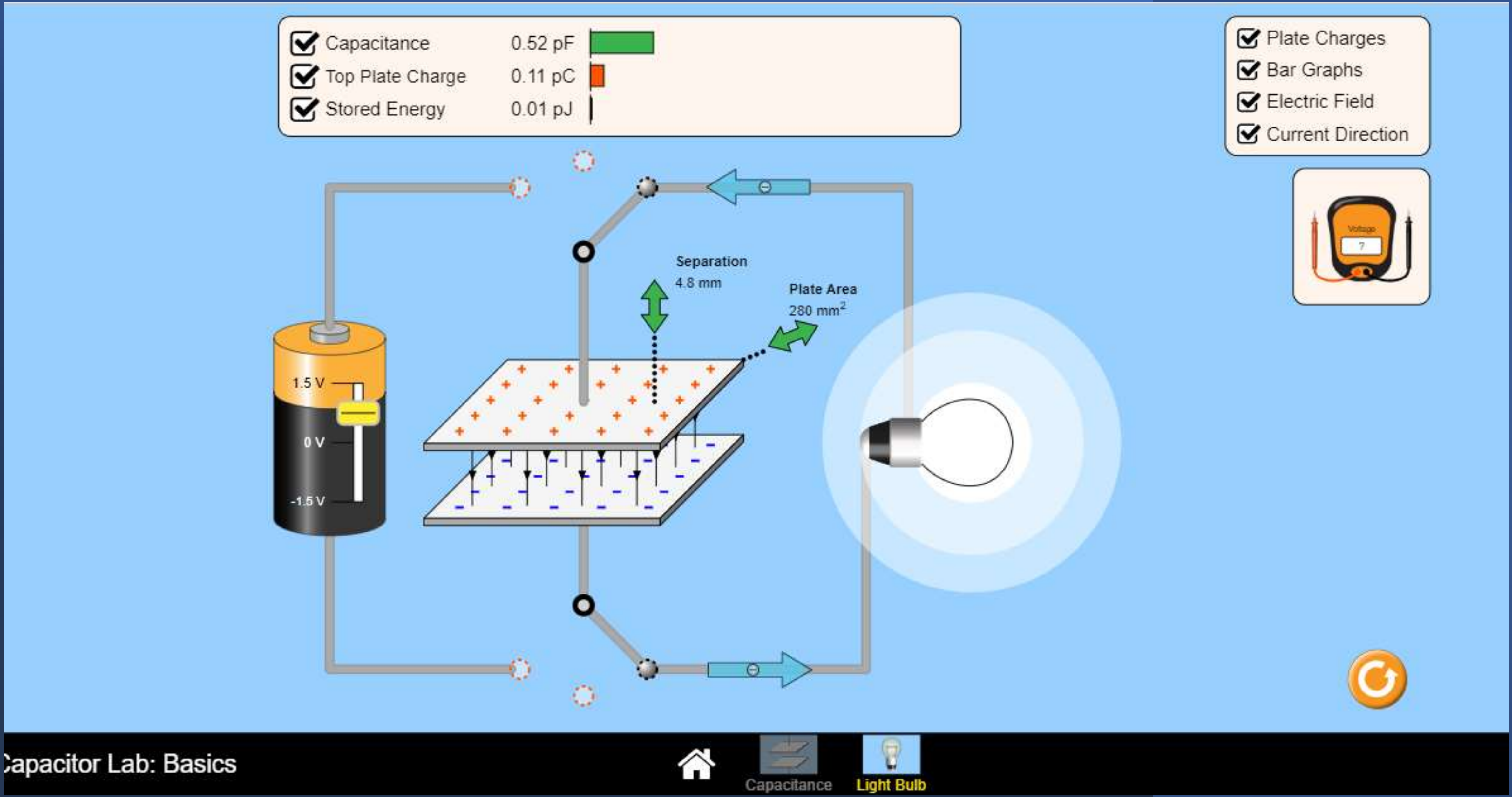
•However, the electric field can act on the bound charges in the dielectric. These bound charges are not free to migrate through the dielectric. The action of the field E_o on the bound charges consist in displacing the bound charges relative to one other.

•The negative charges (electrons) are displaced in a direction opposite to the field, while the positive charges are displaced in the same direction as that of the applied field.

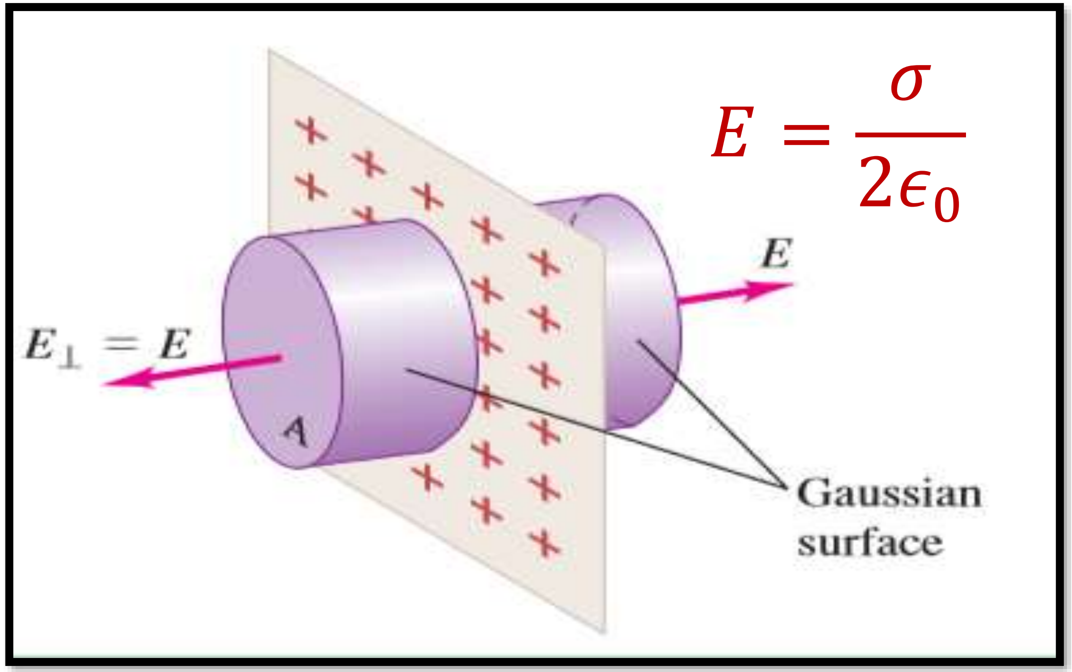


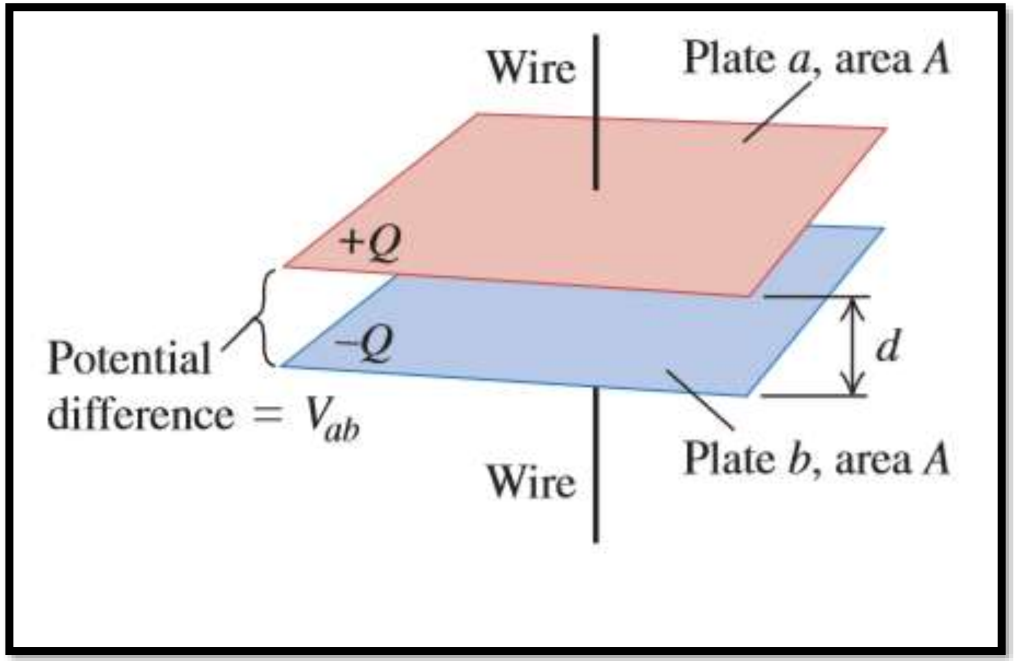
Polar molecules are randomly oriented in the absence of an external electric field.
When an external electric field is applied, the molecules partially align with the field.
$\vec{\mathbf{E}}_0$
The charged edges of the dielectric can be modeled as an additional pair of parallel plates establishing an electric field $\vec{\mathbf{E}}_{ind}$ in the direction opposite that of $\vec{\mathbf{E}}_0$.
$\vec{\mathbf{E}}_0$
$\vec{\mathbf{E}}_{ind}$
$-\sigma_{ind}$
σ_{ind}

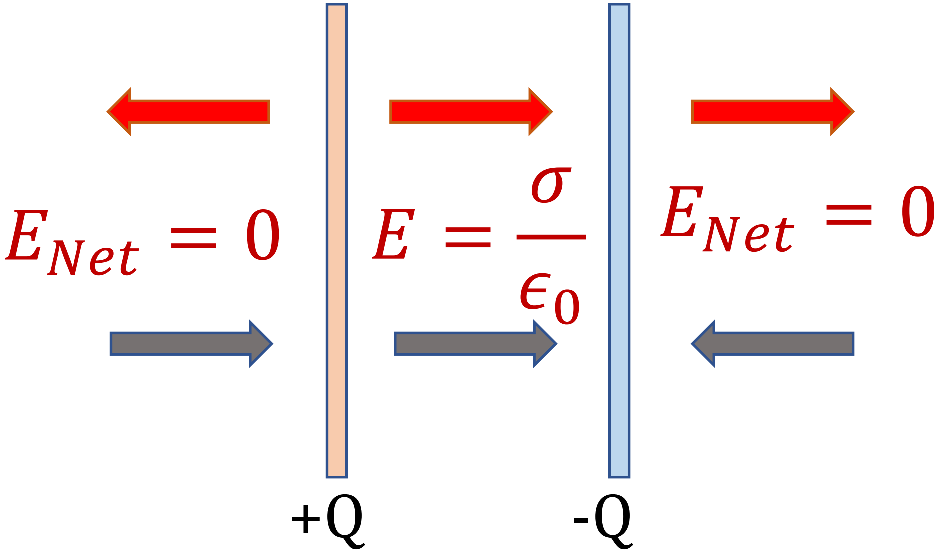
INTERACTIVE PRESENTATION



PARALELL PLATE CAPACITOR







$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \Rightarrow V = E \times d = \frac{Qd}{\epsilon_0 A}$$

$$\boldsymbol{C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}}$$

POLL QUESTION

The plates of a parallel-plate capacitor in vacuum are *5.00 mm* apart and in *2.00 m*^{2} *area*. A *10.0-kV* potential difference is applied across the capacitor. Compute the capacitance:

A. *4.56 nF*

B. *3.54 nF*

C. *5.67 pF*

D. *7.89 pF*

POLL QUESTION

The plates of a parallel-plate capacitor in vacuum are *5.00 mm* apart and in *2.00 m² area*. A *10.0-kV* potential difference is applied across the capacitor. Compute (b) the charge on each plate:

A. 12.4 μC

B. 27.6 μC

C. 35.4 μC

D. 45.4 μC

POLL QUESTION

The plates of a parallel-plate capacitor in vacuum are *5.00 mm* apart and in *2.00 m² area*. A *10.0-kV* potential difference is applied across the capacitor. Compute (c) the magnitude of the electric field between the plates.

A. $2 \times 10^6 \mathrm{N/C}$

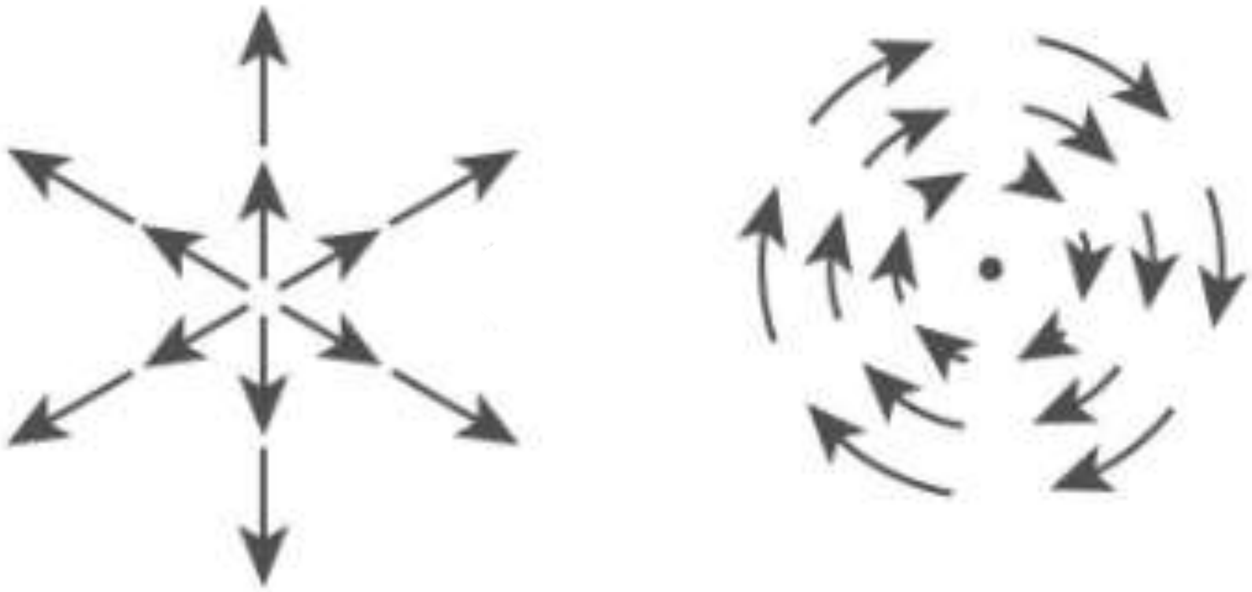
B. $4 \times 10^6 \mathrm{N/C}$

C. $6 \times 10^6 \mathrm{N/C}$

D. $8 \times 10^6 \mathrm{N/C}$

OPTIONAL SLIDES
on
Mathematical Foundations

Mathematical Foundations



Partial differentiation

Let *f(x, y)* be a function of two independent variables *x and y*.

The total differential *df* of the function *f(x, y)* is defined as

$$df = \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy$$

For the partial derivative $\left(\frac{\partial f}{\partial x}\right)$, y is held constant and x is considered as a variable. Similarly for the partial derivative $\left(\frac{\partial f}{\partial y}\right)$, x is held constant and y is considered as a variable

CONCEPT QUESTION

$$Evaluate\ \frac{\partial f}{\partial x}\ when\ f(x,y,z) = x^2 + y^3 + z^4$$

A. $2x + y^3 + z^4$

B. $2x$

C. $2x + 3y^2 + 4z^3$

D. 0

Gradient, divergence and curl

Del operator:

The del operator is defined through the spatial derivative with respect to space coordinates.

In Cartesian coordinates, $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$; *Here* $\hat{x}, \hat{y}$ *and* $\hat{z}$ *are the unit vectors* along X, Y and Z axes respectively.

Gradient

✓Let Φ (x, y, z) be a scalar function. The gradient of a scalar function Φ is

✓$Grad\ \varphi = \vec{\nabla}\varphi = \hat{x}\frac{\partial \varphi}{\partial x} + \hat{y}\frac{\partial \varphi}{\partial y} + \hat{z}\frac{\partial \varphi}{\partial z}$; where $\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}$ are the partial derivative of the scalar function Φ *with respect to x, y, and z respectively.*

✓ The gradient of any scalar function is a vector

✓ The gradient $\vec{\nabla}\varphi$ points in the direction of maximum increase of the function φ

✓ The magnitude of the gradient of φ gives the rate of increase along the maximal direction.

Divergence

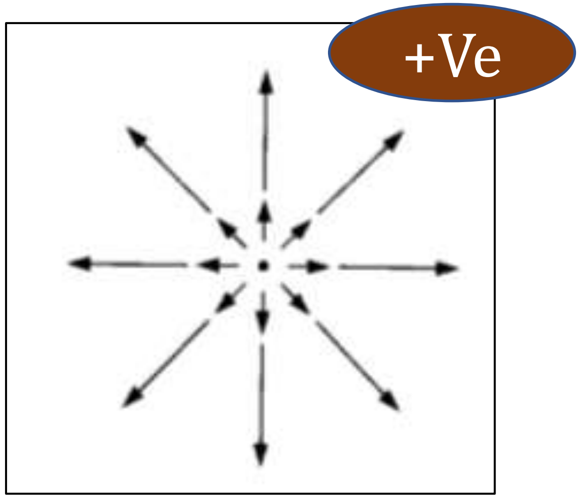
- Mathematically the divergence of a vector function is defined as the dot product of the del operator and the given vector function

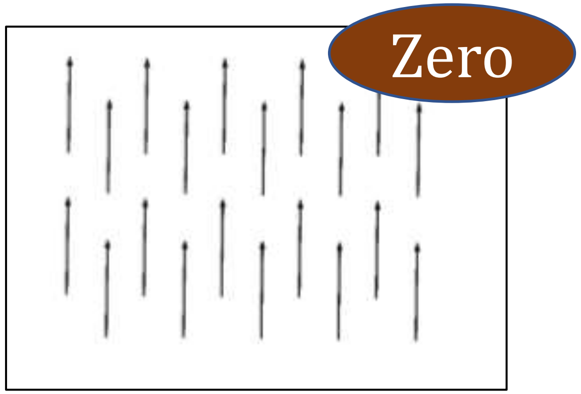
- $\vec{\nabla}.\vec{A} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

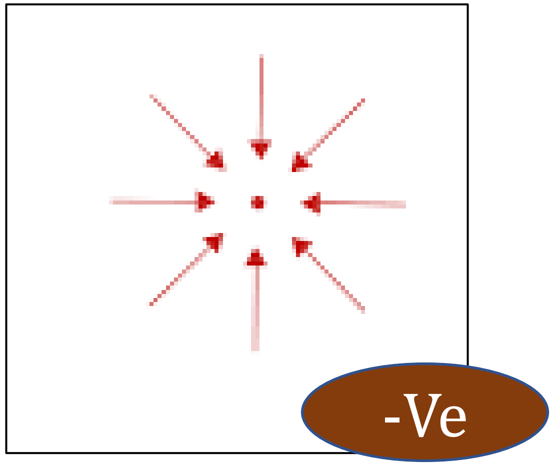
- This is a scalar quantity termed as divergence. The divergence of a vector field at any point is the amount of flux per unit volume *diverging from that point*.

- A point of positive divergence is a source, and a point of negative divergence is a sink or drain.

- *A vector field with zero divergence is said to be solenoidal or incompressible.*

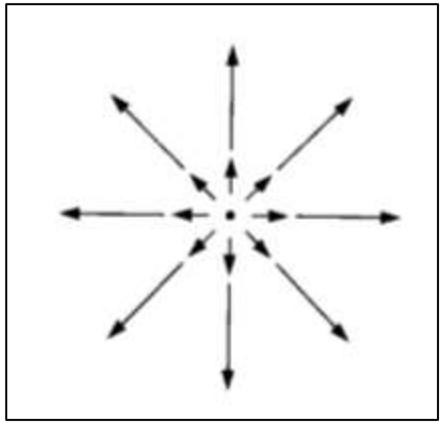






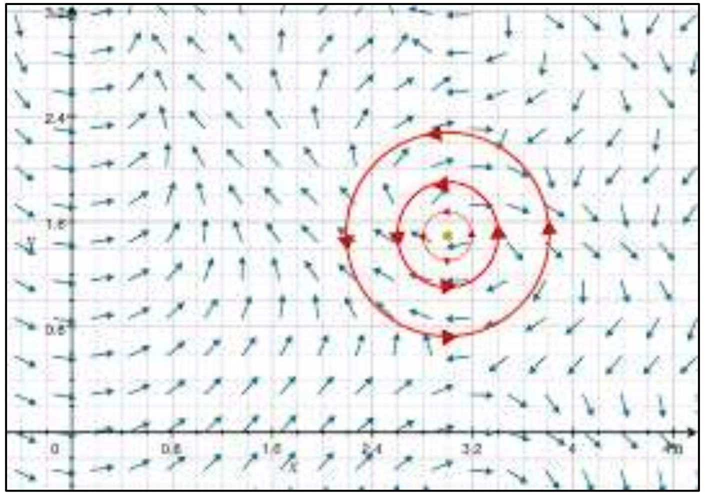
Curl

✓Mathematically the curl of a vector function is defined as the cross product of the del operator and the given vector function



Positive divergence but zero curl

$$✓\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

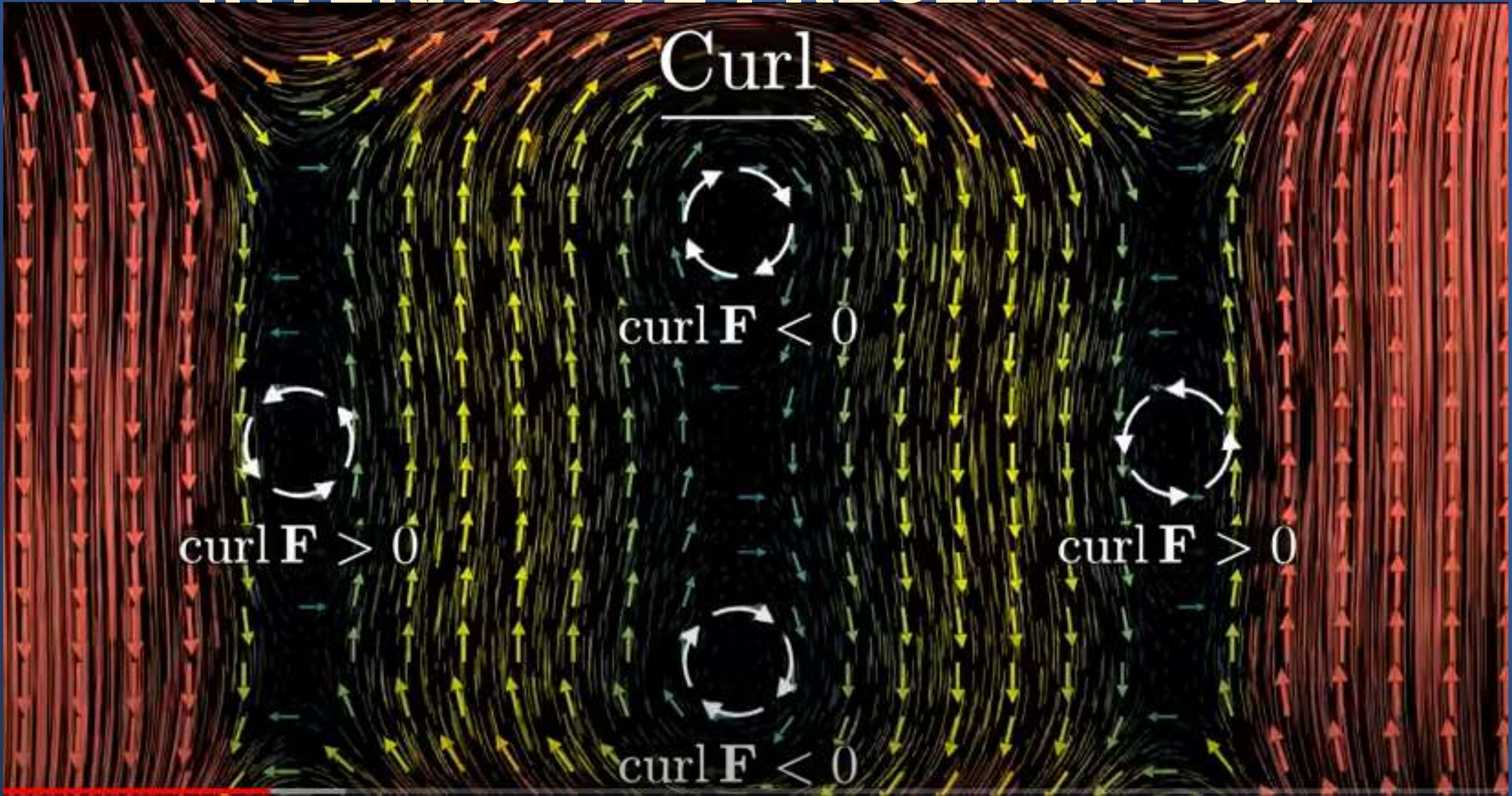


Positive curl but zero divergence

✓The curl of a vector is the measure of how much the vector $\vec{F}$ swirls around the point in question.

✓ A vector field with zero curl is said to be irrotational.

INTERACTIVE PRESENTATION



Divergence : ~2:15 to 3:35

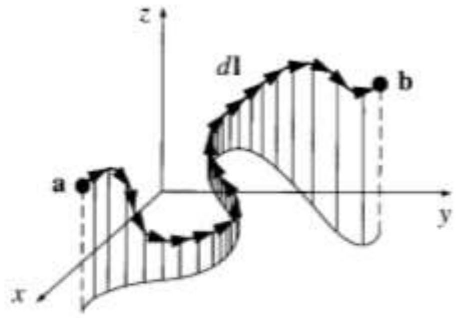
Curl : ~4:33 to 5:20

Line, Surface and Volume integral

Line integral:

- A line integral is an expression of the form $\int_a^b \vec{v}.\overrightarrow{dl}$, where $\vec{v}$ is a vector function and

  $\overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ is the infinitesimal displacement vector.



- Here the integral is to be carried out along the path from point "*a*" to a point "*b*" instead of integrating over an integral *[a, b]*.

- If the path in question forms a closed loop (i.e. if *b*=*a*). The line integral is given by $\oint \vec{v}.\overrightarrow{dl}$

- Ex: Work done by a force, $\vec{F}, w = \int \vec{F}.\overrightarrow{dl}$

- Generally, the value of line integral depends on the path taken from "*a*" to "*b*". However there is a special class of vector functions for which the line integral is independent of path known as *conservative systems*.

Surface integral:

- A surface integral is an expression of the form $\int_s \vec{v}.\overrightarrow{ds}$, where $\vec{v}$ is a vector function and $\overrightarrow{ds}$ is the infinitesimal patch of area with direction perpendicular to the surface.

- For a closed surface, the surface integral can be written as $\oint \vec{v}.\overrightarrow{ds}$

- Ex: If $\vec{v}$ describes the flow of fluid (mass per unit area per unit time), then $\int \vec{v}.\overrightarrow{ds}$ represents the total mass of the fluid per unit time passing through the surface (flux).

Volume integral:

- A volume integral is an expression of the form $\int_v T d\tau$, where T is a scalar function and $d\tau = dx\ dy\ dz$ is the infinitesimal volume element.
- Occasionally we shall also encounter the volume integral of vector functions
- If T is the density of a substance, then its volume integral give the total mass.

Gauss theorem

- Gives us relation between surface and volume integrals
- The surface integral of the normal component of a vector *F* over a closed surface *s* is equal to the volume integral of the divergence of the vector *F* over the volume *v* enclosed by surface *s*.

$$\int_v (\vec{\nabla}.\vec{F})\, dv = \oint_s \vec{F}.\overrightarrow{ds}$$

Stoke's theorem

- Gives us relation between line and surface integrals
- The line integral of the normal component of a vector *F* around a closed curve is equal to the surface integral of the curl of the vector *F* takes over the surface *s* surrounded by the closed curve.

$$\int_s (\vec{\nabla} \times \vec{F}).\overrightarrow{ds} = \oint_l \vec{F}\, \overrightarrow{dl}$$

SOLVED EXAMPLE

Find the gradient of a) $r = \sqrt{x^2 + y^2 + z^2}$, and b) $r^2 = x^2 + y^2 + z^2$

Solution:

a) $\vec{\nabla} r = \hat{x}\frac{\partial r}{\partial x} + \hat{y}\frac{\partial r}{\partial y} + \hat{z}\frac{\partial r}{\partial z} = \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{1}{2}\frac{2y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{1}{2}\frac{2z}{\sqrt{x^2+y^2+z^2}}\hat{z}$

$$\vec{\nabla} r = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}} = \frac{\vec{r}}{r} = \hat{r}$$

b) $\vec{\nabla} r^2 = \hat{x}\frac{\partial(x^2+y^2+z^2)}{\partial x} + \hat{y}\frac{\partial(x^2+y^2+z^2)}{\partial y} + \hat{z}\frac{\partial(x^2+y^2+z^2)}{\partial z} = 2x\,\hat{x} + 2y\hat{y} + 2z\hat{z} = 2\vec{r}$

SOLVED EXAMPLE

Calculate the divergence of the following vector functions

$(a)\vec{V} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

$(b)\vec{V} = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

Solution:

a. $\vec{\nabla}.\vec{V} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).(x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z})$

$\vec{\nabla}.\vec{V} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(3xz^2) + \frac{\partial}{\partial z}(-2xz) = 2\mathrm{x}$

b. $\vec{\nabla}.\vec{V} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).(xy\hat{x} + 2yz\hat{y} + 3zx\hat{z})$

$\vec{\nabla}.\vec{V} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(3zx) = \mathrm{y} + 2\mathrm{z} + 3\mathrm{x}$

SOLVED EXAMPLE

Calculate the curl of the following vector functions

$(a)\vec{V} = -y\hat{x} + x\hat{y}$

$(b)\vec{V} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

Solution:

$$a. \vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix} = 2\hat{z}$$

$$\text{b. } \vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = -6xz\hat{x} + 2z\hat{y} + 3z^2\hat{z}$$

POLL QUESTION

Calculate the divergence of $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

A. 0

B. 1

C. 2

$\boxed{\text{D. 3}}$

$$\oint_{s} \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$$

Differential form of Gauss Law

By applying Gauss divergence theorem

$$\oint_{s} \vec{E}.\overrightarrow{dS} = \int_{v} (\vec{\nabla}.\vec{E})d\tau$$

Rewriting Q in terms of the charge density, ρ

$$Q_{enc} = \int_v \rho \, d\tau$$

So Gauss law becomes

$$\int_v (\vec{\nabla}.\vec{E})d\tau = \int_v \frac{\rho}{\epsilon_0} \, d\tau$$

Since this holds for any volume, the integrands must be equal,

$$\boxed{\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0} \; (Gauss\ law\ in\ differential\ form)}$$