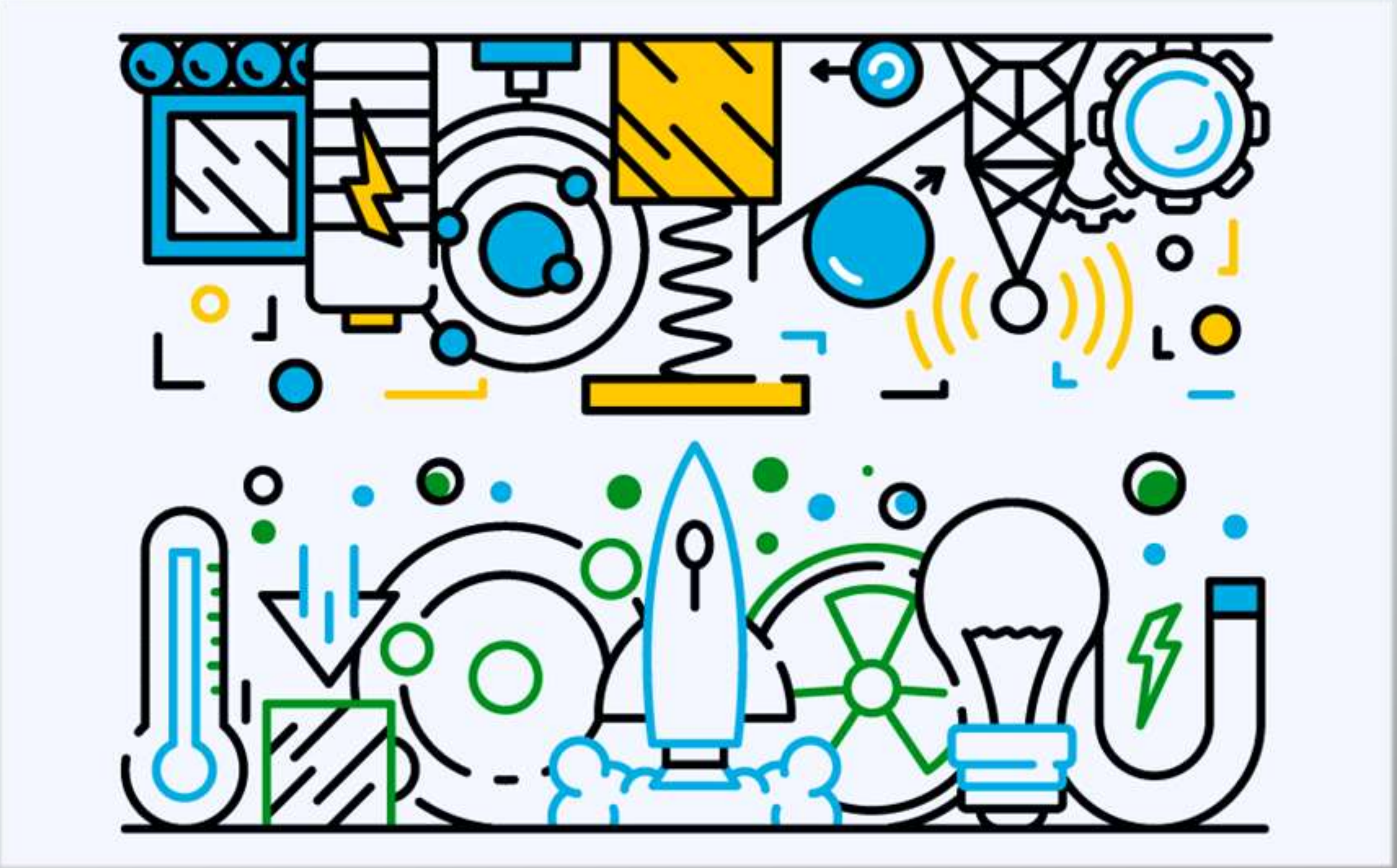
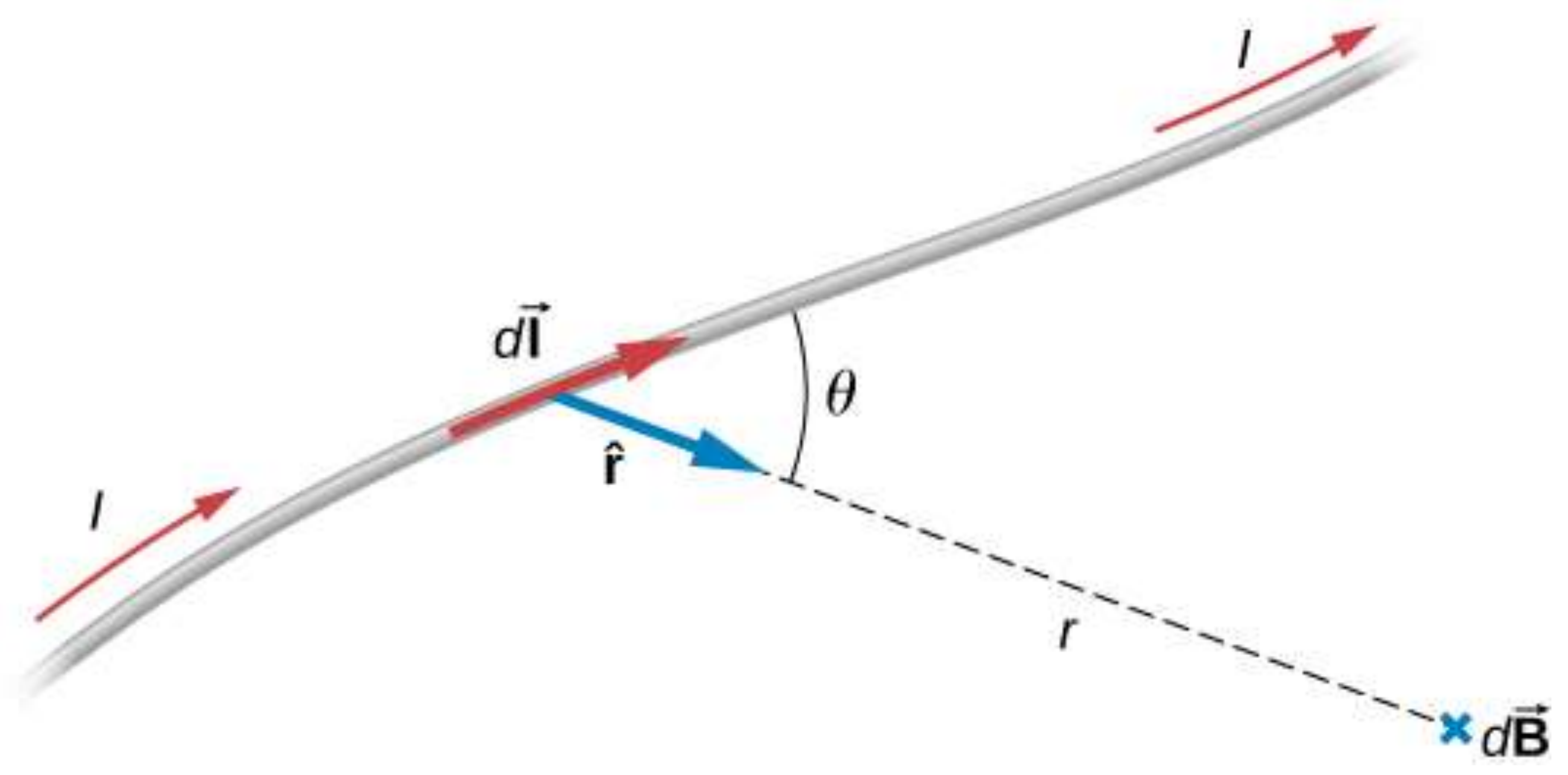
Engineering Physics (FIC 102)

UNIT-V



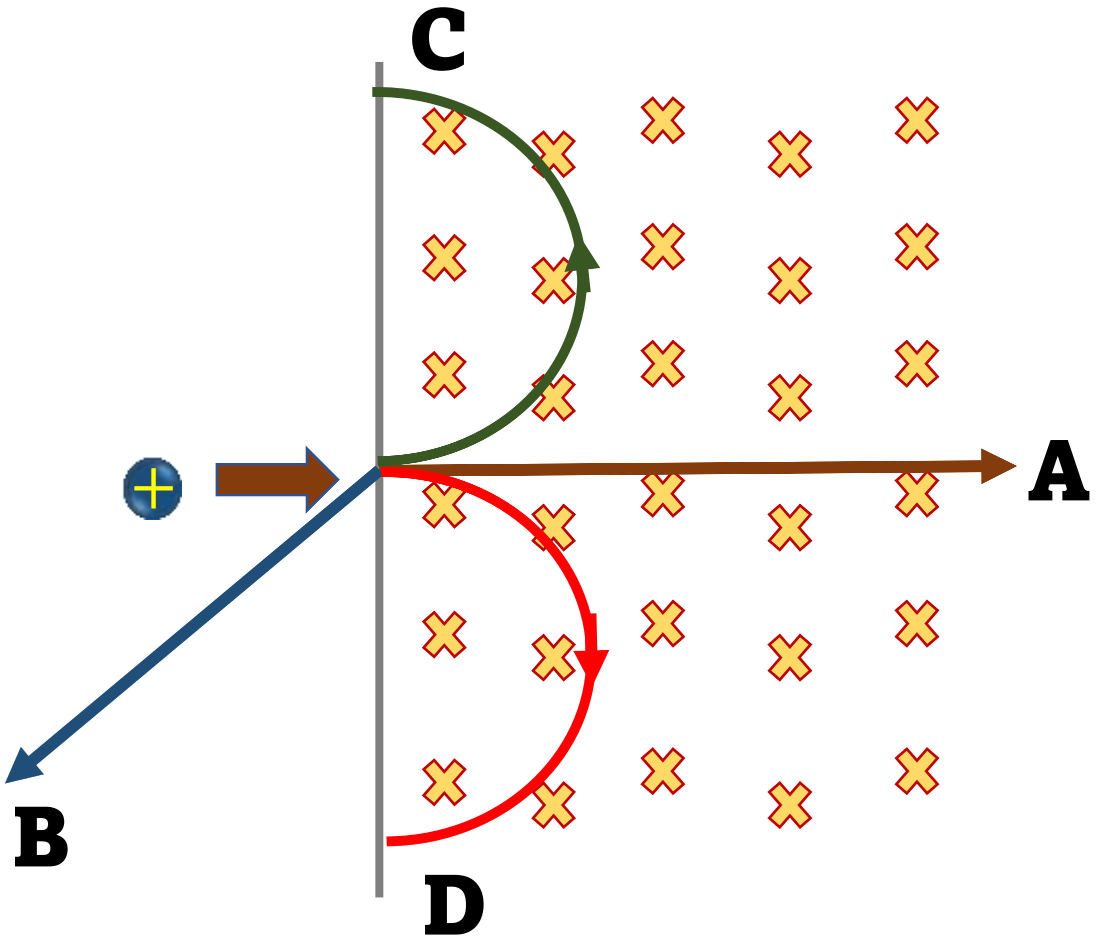
CONTENT



Biot Savart Law

LECTURE-01

CONCEPT QUESTION



A proton moving horizontally enters a uniform magnetic field perpendicular to the proton's velocity, as shown in Figure. Describe the subsequent motion of the proton.

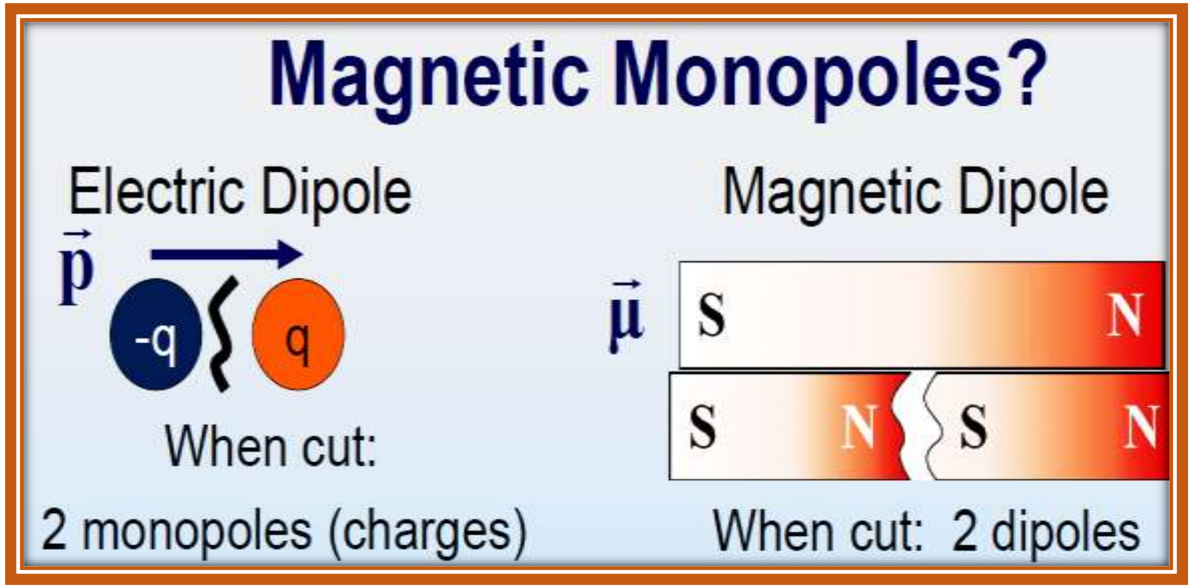
A. Path A

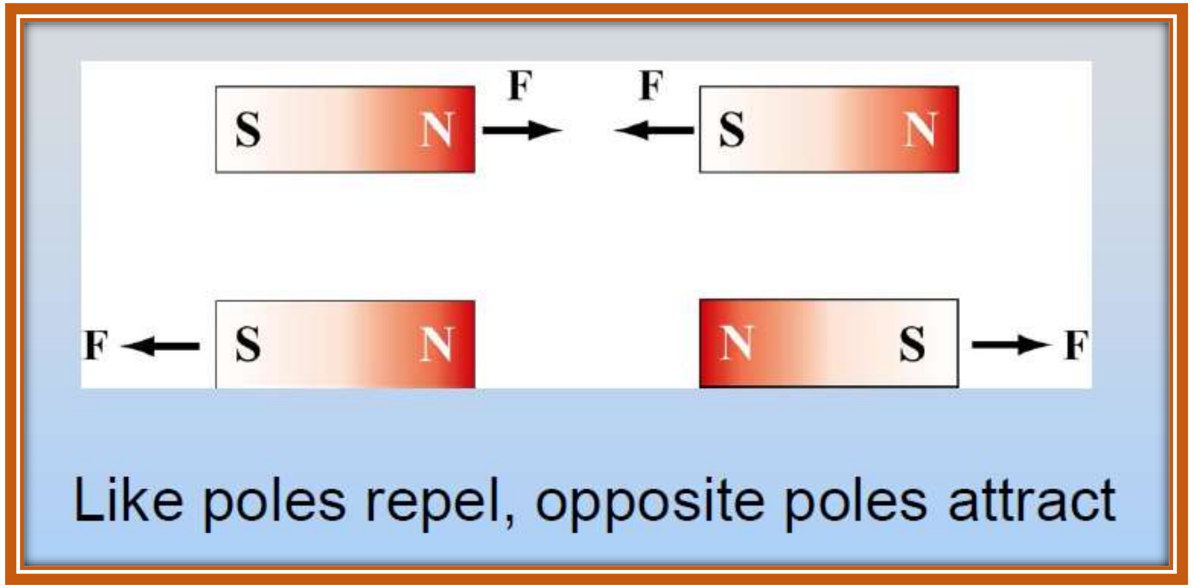
B. Path B

C. Path C

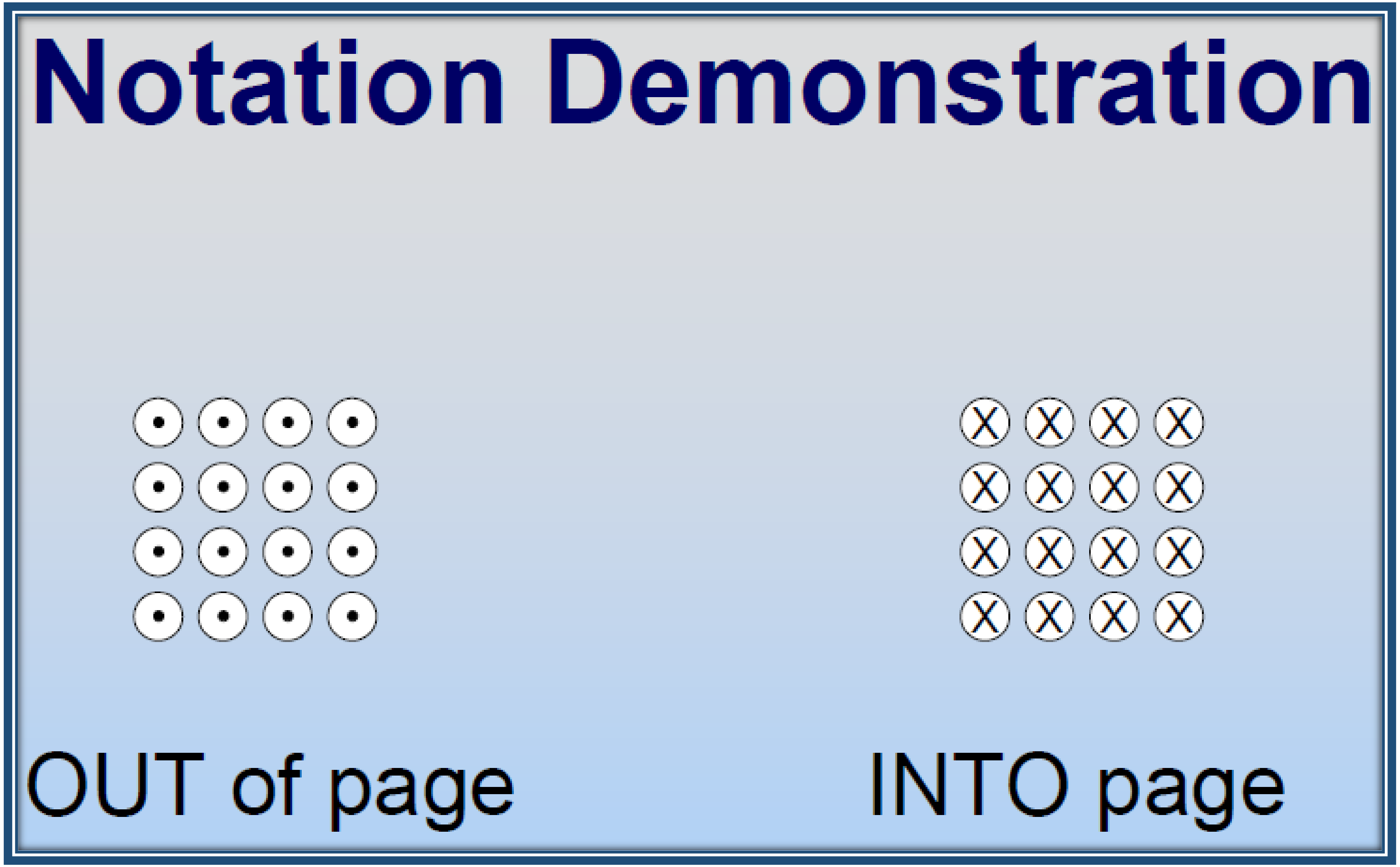
D. Path D

Magnet


Magnetic Monopoles?
Electric Dipole
Magnetic Dipole
$\vec{p}$
-q
q
$\vec{\mu}$
S
N
S
N
S
N
When cut:
2 monopoles (charges)
When cut: 2 dipoles

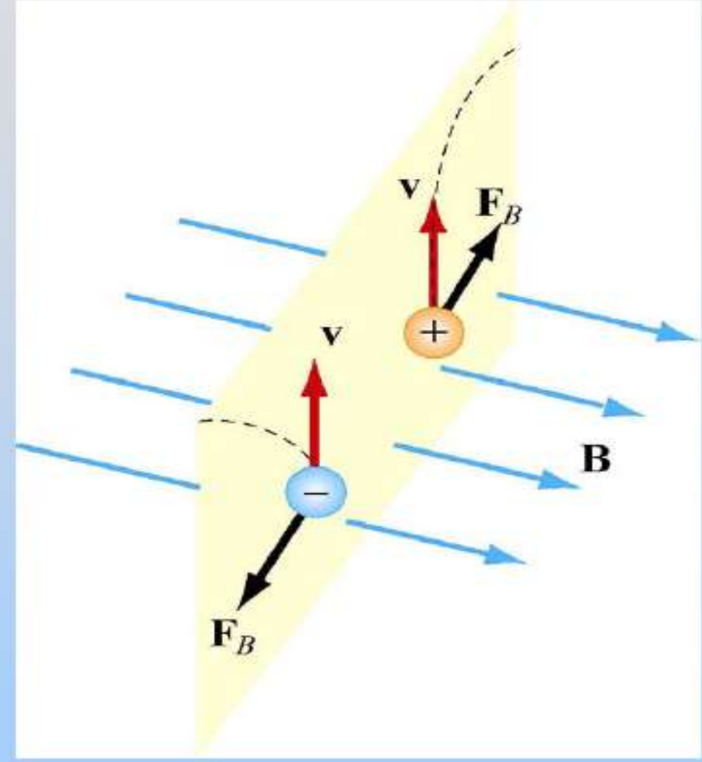

S
N
F
F
S
N
F
S
N
N
S
F
Like poles repel, opposite poles attract

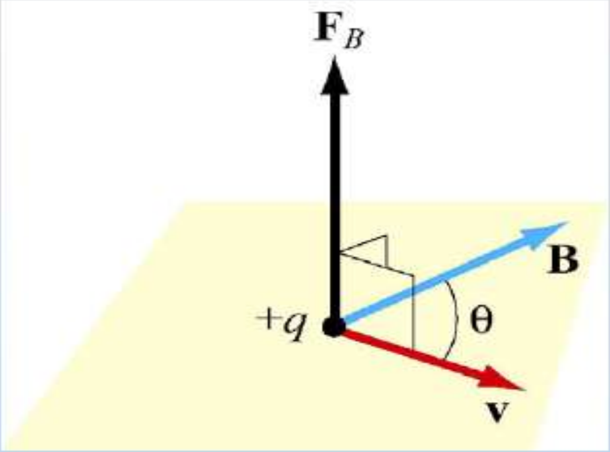
Magnetic field



Magnetism

Moving Charges Feel Magnetic Force





$$\boxed{\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}}$$

Magnetic force perpendicular both to:
Velocity **v** of charge and magnetic field **B**

Force on Moving Charge

Magnetic Field B: Units

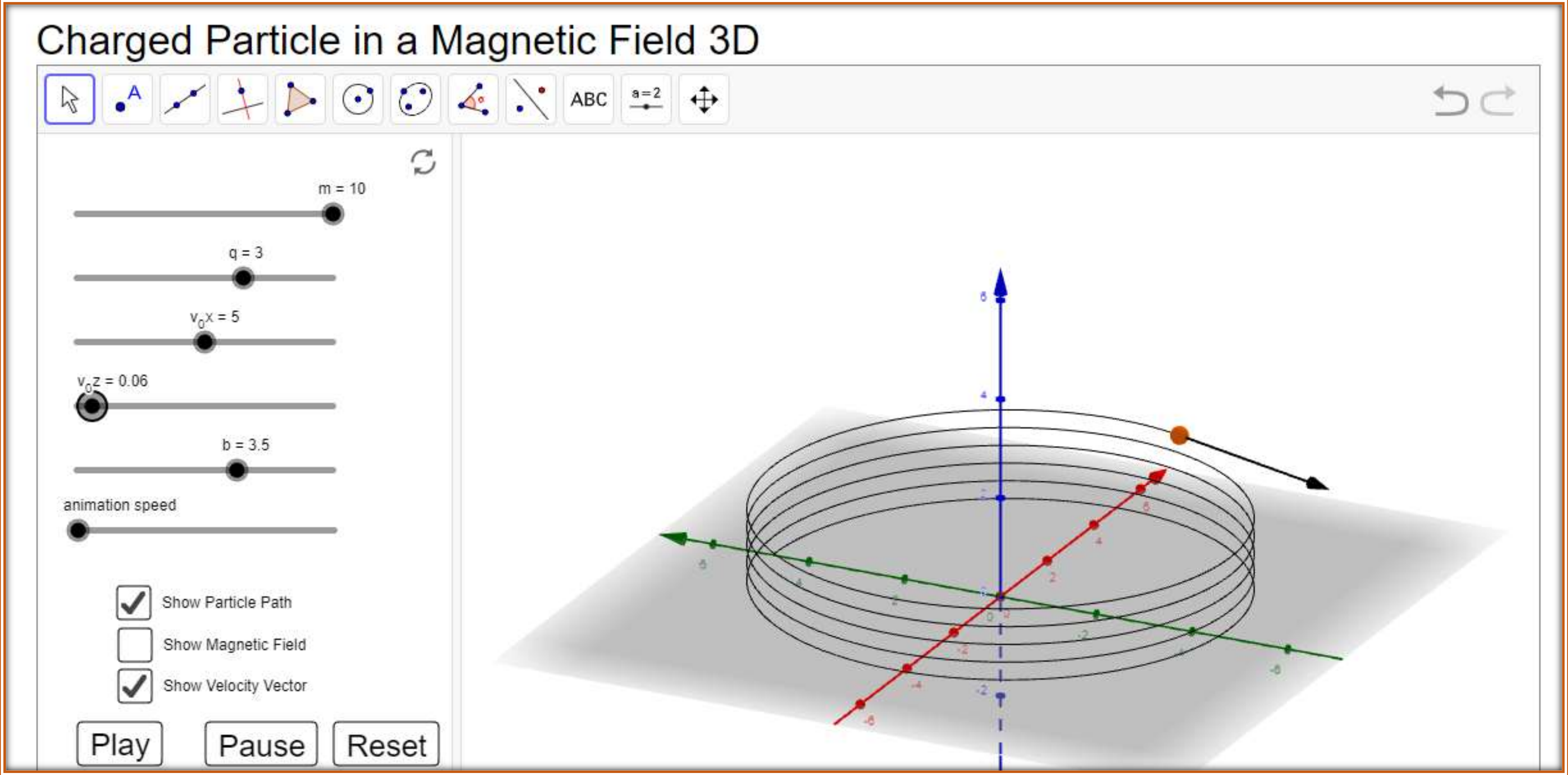
Since $\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$\text{B Units} = \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})} = 1\frac{\text{N}}{\text{C}\cdot\text{m/s}} = 1\frac{\text{N}}{\text{A}\cdot\text{m}}$$

This is called 1 Tesla (T)

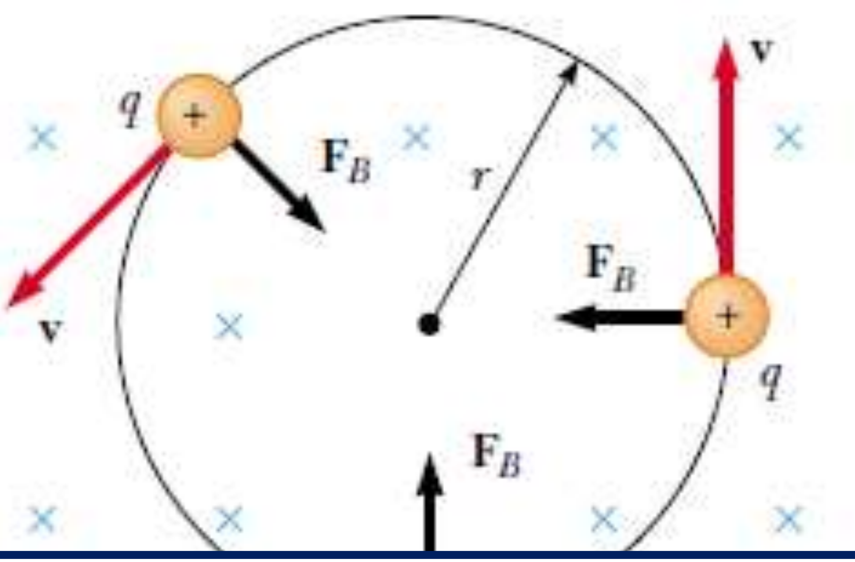
$$1\ \text{T} = 10^4\ \text{Gauss (G)}$$

INTERACTIVE PRESENTATION



Charge Particle in Magnetic Field

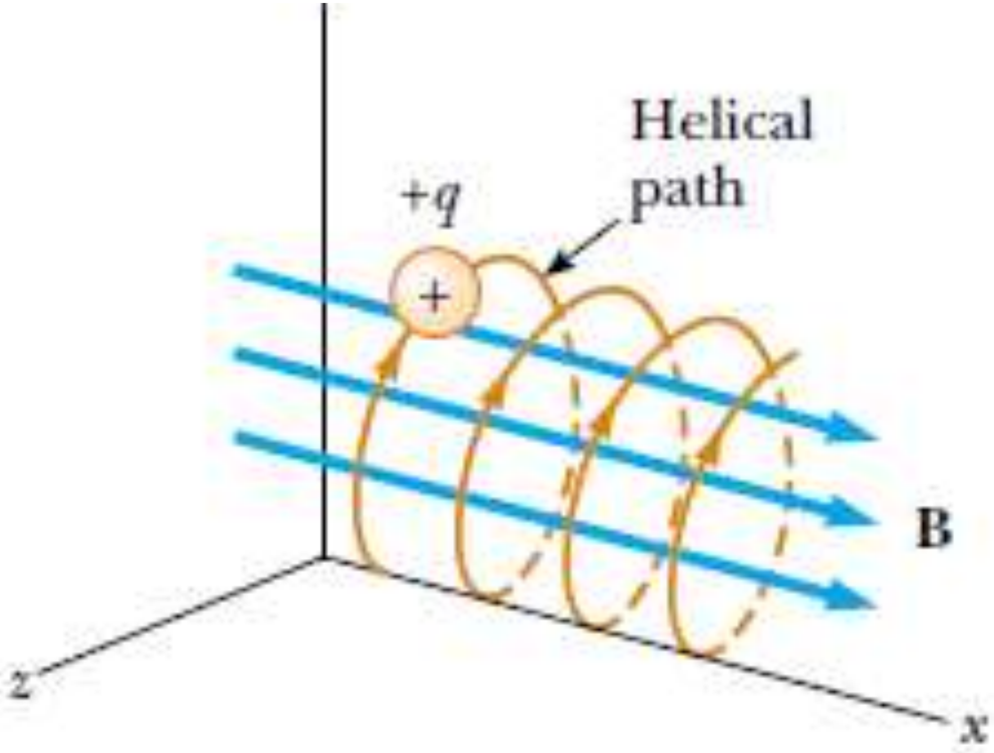
$\vec{v}$ *exactly perp to* $\vec{B}$



$$\text{Radius of rotation} \Rightarrow \ r = \frac{mv}{qB}$$

$$\text{Cyclotron frequency} \ \Rightarrow \omega = \frac{qB}{m}$$

$\vec{v}$ *not exactly perp. to* $\vec{B}$



Hellical motion

Lorentz Force

Putting it Together: Lorentz Force

Charges Feel…

$$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

Electric Fields

$$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Magnetic Fields

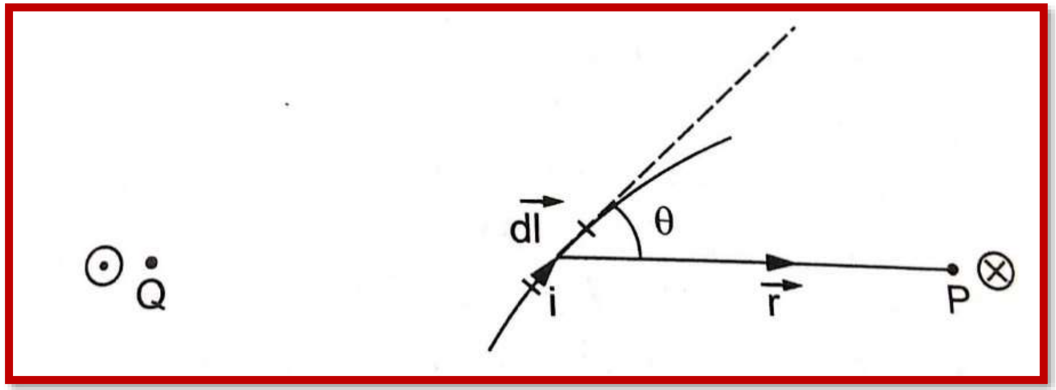
$$\boxed{\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)}$$

This is the final word on the force on a charge

Biot-Savart's law

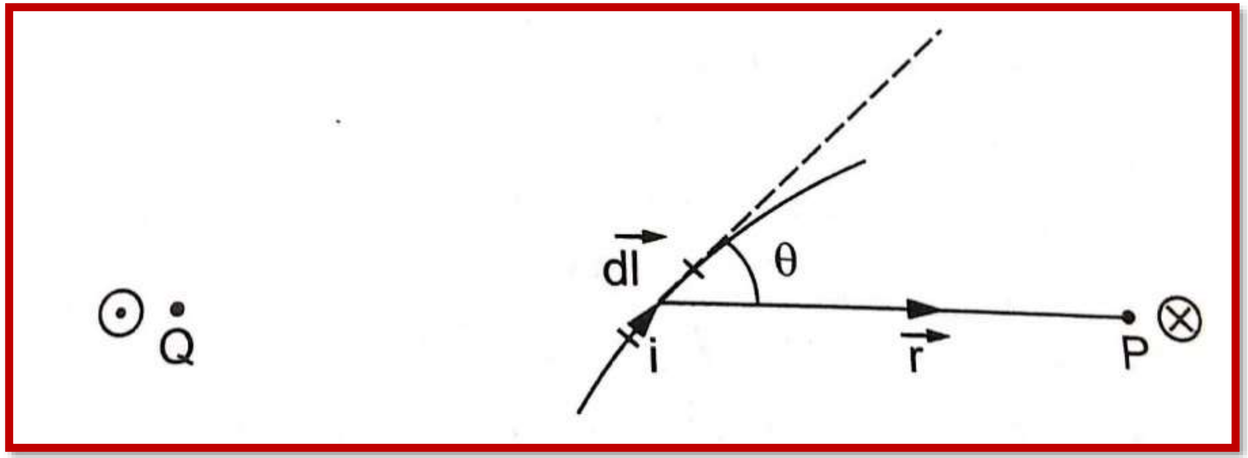
- Biot-Savart Law describes the magnetic field created by a current-carrying wire.
- The magnetic field ($\overrightarrow{dB}$) at a point P due to a current element ($I\,\overrightarrow{dl}$) *is directly proportional to the*
- length of the element dl, $dB \propto dl$
- current I, $dB \propto I$
- sine of the angle ($sin\theta$), $dB \propto \sin(\theta)$ and
- *is inversely proportional to* the square of the distance of the given point from the current element, i.e. r $dB \propto \frac{1}{r^2}$

$$dB \propto \frac{I\,dl\,sin\theta}{r^2}$$



Biot-Savart's law

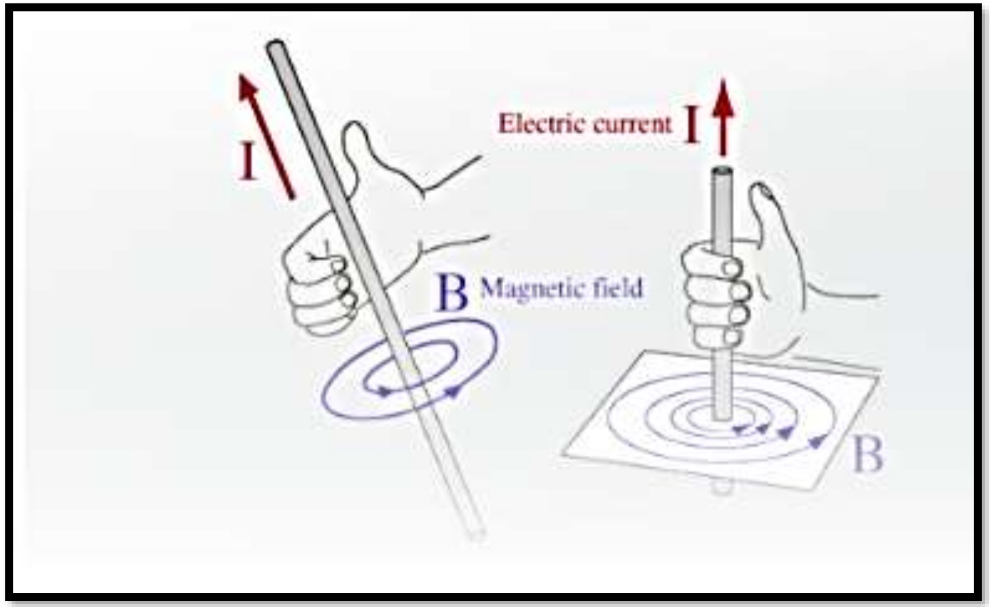
$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,sin\theta}{r^2}$$

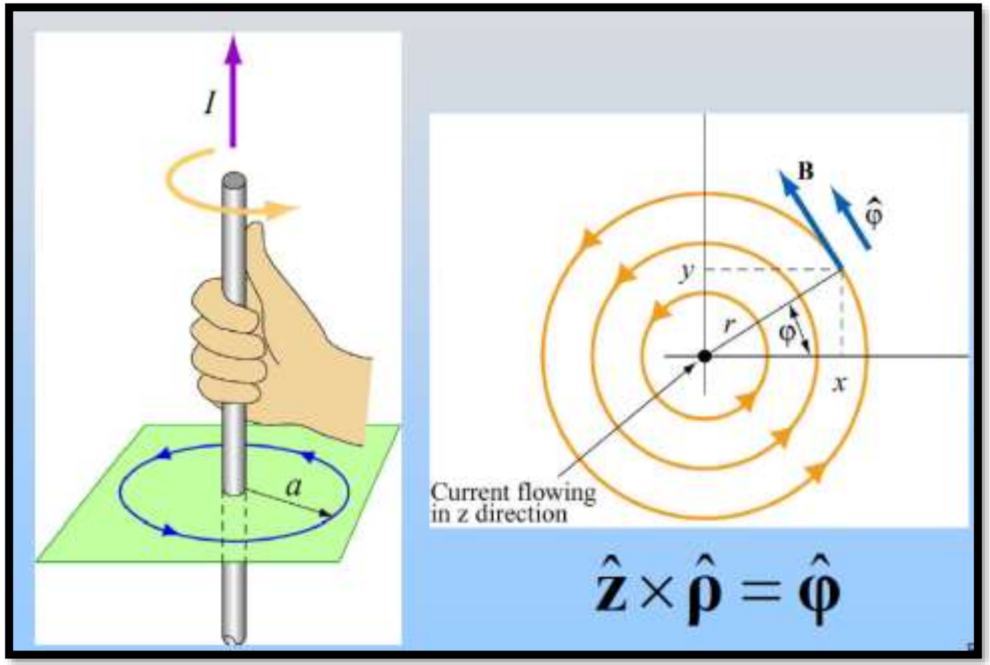


Here μ_0 is called the permeability of vacuum. *It's value is* $4\pi \times 10^{-7}\ T\ m\ A^{-1}$

- ✓ In vector form, $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2}$
- ✓ The direction of field is perpendicular to the plane containing the current element and the point P

The right hand rule

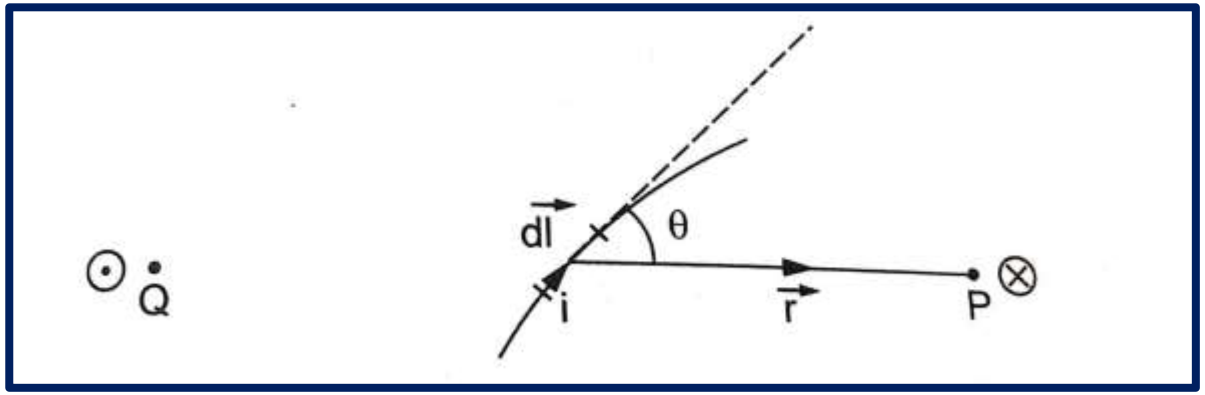




- The magnetic field created by a current-carrying wire takes the form of concentric circles. But we have to be able to figure out if those circles point clockwise or counter-clockwise. To do that we use a right-hand rule.
- In general, the direction going into the plane is denoted by an encircled cross and the direction coming out of the plane by an encircled dot.

Biot-Savart's law

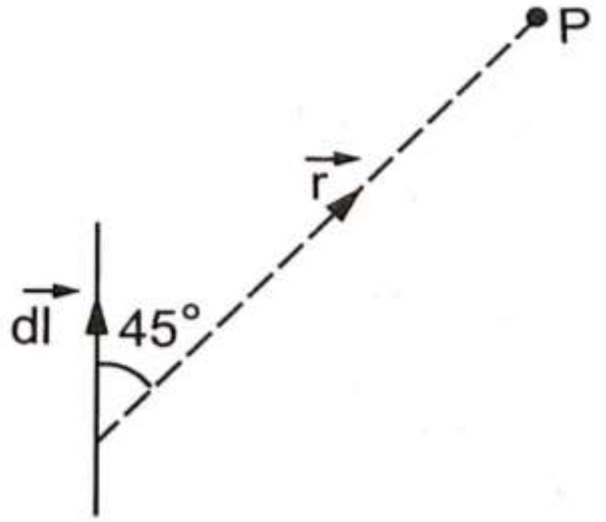
- ✓ In the figure the magnetic field at the point ***P*** goes into the plane of the diagram and that at ***Q*** comes out of this plane.
- ✓ We can use this law to find the total magnetic field at any point in space due to the current in a complete circuit



$$\vec{B} = \int_{wire} \overrightarrow{dB} = \frac{\mu_0}{4\pi}\int_{wire} \frac{I\,\overrightarrow{dl} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int_{wire} \frac{I\,\overrightarrow{dl} \times \hat{r}}{r^2}$$

POLL QUESTION

A wire placed along a north-south direction carries a current of ***10A*** from south to north. *Find the magnetic field* due to ***1 cm*** piece of wire at a point ***200 cm*** north east from the piece.



A. *2.2 nT*

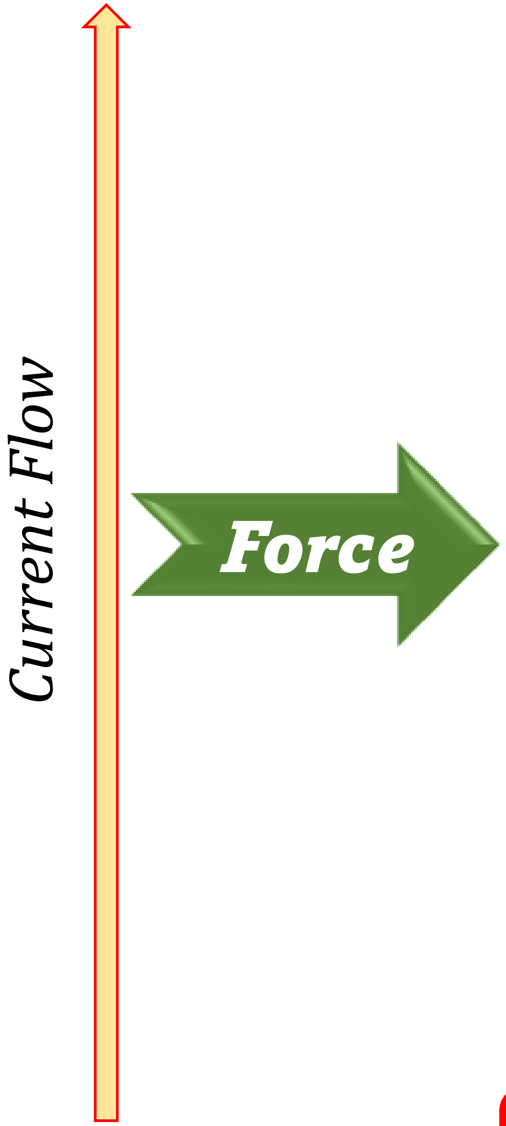
B. *3.4 nT*

C. *1.8 nT*

D. *4.6 nT*

$$dB = \frac{\mu_0}{4\pi}\frac{I\ dl\ sin\theta}{r^2} = 10^{-7} \times 10\ \frac{(10^{-2})sin45°}{(2)^2} = 1.8 \times 10^{-9}\ T$$

CONCEPT QUESTION



A wire carries current in the plane of this paper toward the top of the page. The wire experiences a magnetic force toward the right edge of the page. The direction of the magnetic field causing this force is

(a) Upwards in the page,

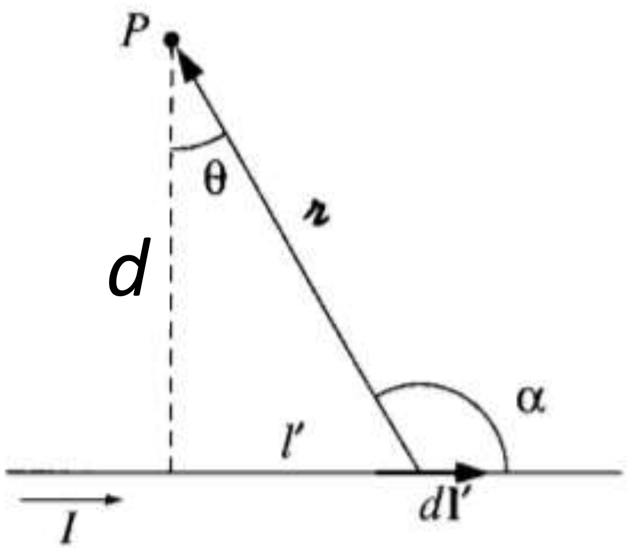
(b) Downwards in the page,

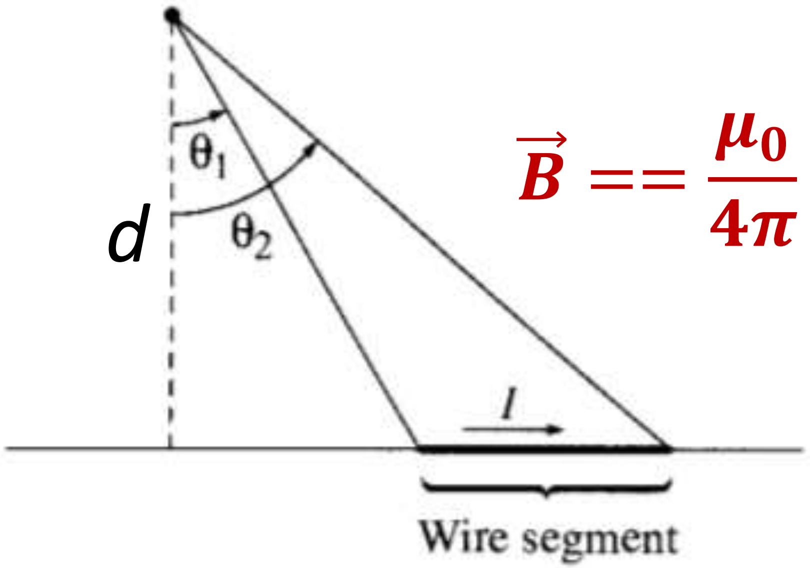
(c) Into the page,

(d) Out of the page.

Application of Biot Savart – Finite Wire

Find the magnetic field at a distance ***s*** from a long straight wire carrying current ***I***.





$$\vec{B} == \frac{\mu_0}{4\pi}\int_{wire} \frac{I\,\overrightarrow{dl'} \times \hat{r}}{r^2}$$

Solution:

- Consider a straight wire carrying current ***I*.** Let P be a point at distance ***s*** from it. The point O is the foot of the perpendicular from P to the wire.
- Let us consider an element ***dl′*** of the wire at a distance ***l′*** from O. The vector joining element ***dl′*** with the point P is $\vec{r}$.
- Let α be the angle between $\overrightarrow{dl'}$ and $\vec{r}$. The magnetic field at P due to the element is

In the diagram $(dl' \times \hat{r})$ has the magnitude of $dl' sin\alpha = dl' \sin(90+\theta) = dl' cos\theta$

From the figure, $tan\theta = \frac{l'}{d}$ or $l' = d\ tan\theta$

$$dl' = d\ sec^2\theta\ d\theta = \frac{d}{cos^2\theta} d\theta$$

and $\mathrm{d} = r\ cos\theta, so\ \frac{1}{r^2} = \frac{cos^2\theta}{d^2}$

If θ_1 and θ_2 are the values of θ corresponding to the lower and upper end respectively.

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{wire} \frac{I\ \overrightarrow{dl'} \times \hat{r}}{r^2} =$$

$$\frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{cos^2\theta}{d^2}\right)\left(\frac{d}{cos^2\theta}\right) cos\theta\ \mathrm{d}\theta = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2} cos\theta\ \mathrm{d}\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2} cos\theta\ \mathrm{d}\theta = \frac{\mu_0 I}{4\pi z}(sin\theta_2 - \sin\theta_1)$$

$$\boldsymbol{\vec{B} = \frac{\mu_0 I}{4\pi d}(sin\theta_2 - sin\,\theta_1)}$$

Field due to a long, straight wire

(infinite wire)

In this case, $\theta_1 = -\frac{\pi}{2}$ *and* $\theta_2 = \frac{\pi}{2}$

$$\vec{B} = \frac{\mu_0 I}{4\pi d}(sin\theta_2 - sin\,\theta_1) = \frac{\mu_0 I}{2\pi d}$$

$$\boldsymbol{\vec{B} = \frac{\mu_0 I}{2\pi d}\hat{\varphi}}$$

Magnetic Force Current Carrying Wires

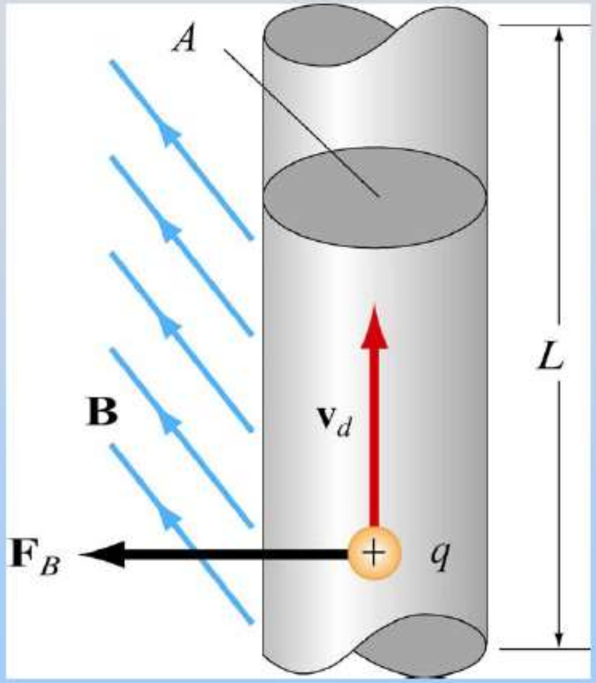
Magnetic Force on Current-Carrying Wire

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

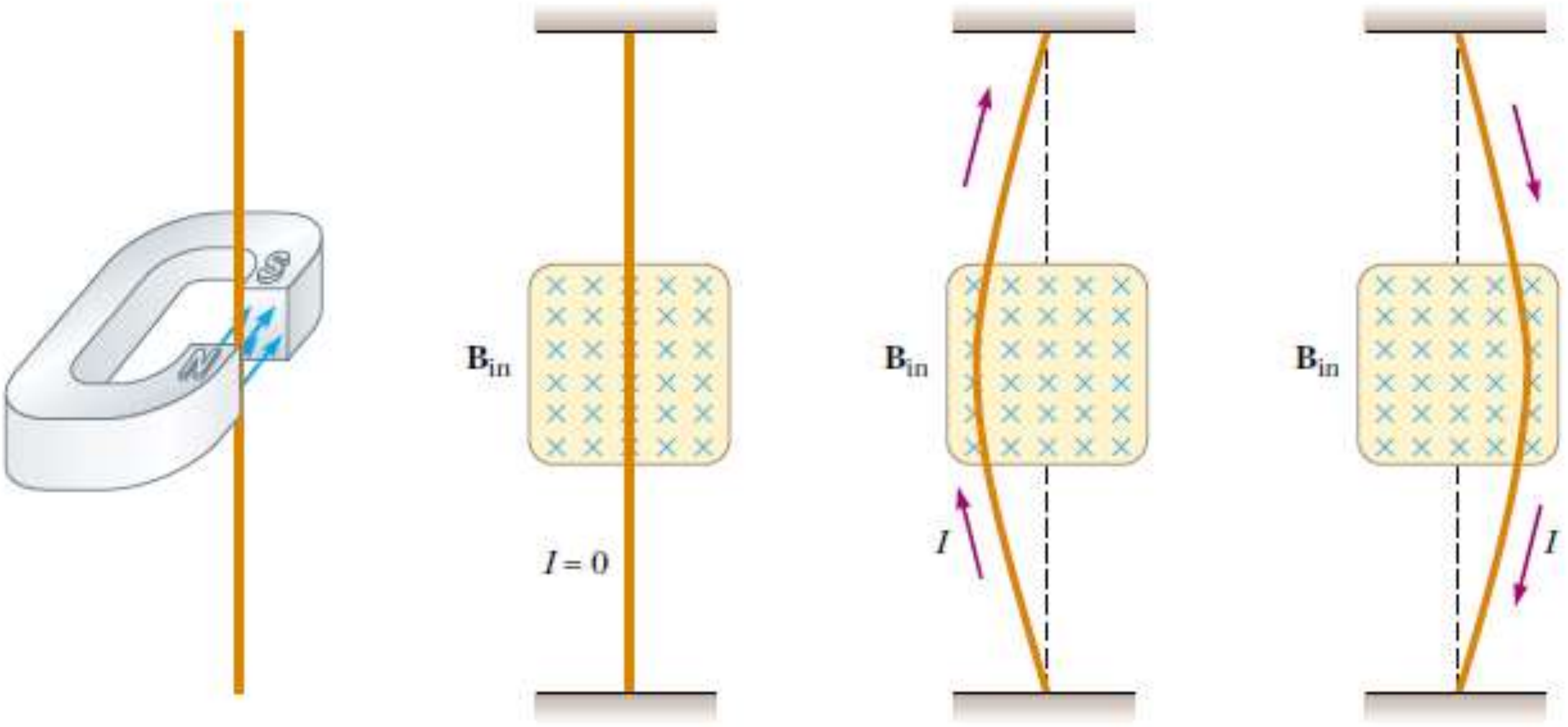
$$= (\text{charge})\frac{\text{m}}{\text{s}} \times \vec{\mathbf{B}}$$

$$= \frac{\text{charge}}{\text{s}}\text{m} \times \vec{\mathbf{B}}$$

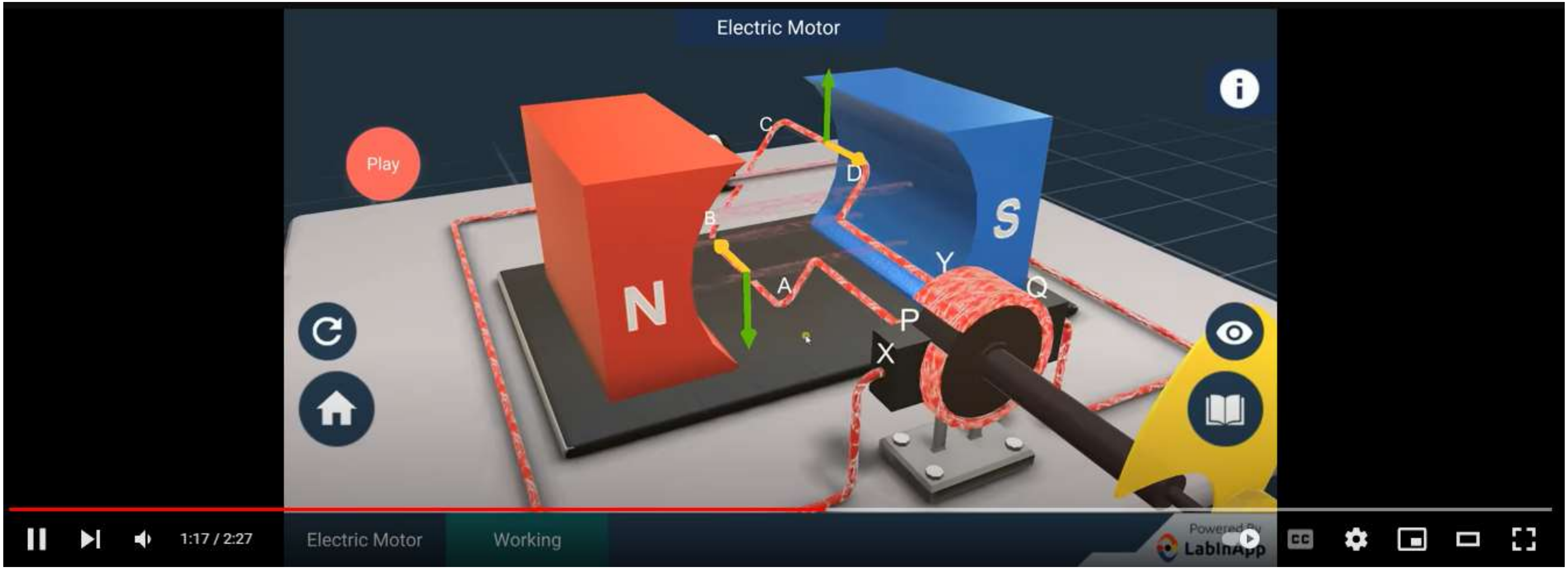
$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)}$$



Magnetic Force on Current Carrying Wires



INTERACTIVE PRESENTATION



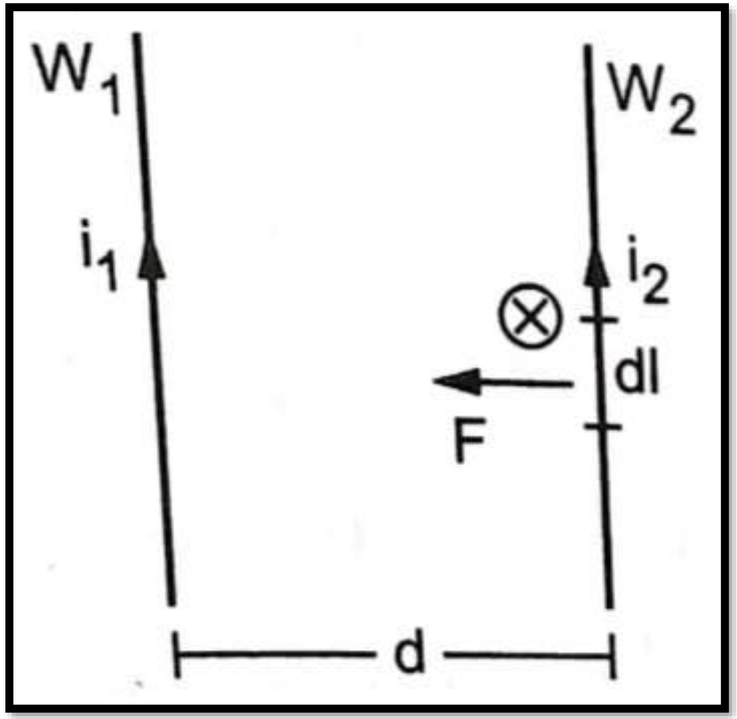
Current Carrying Wires

Consider two long wires $\mathbf{W_1}$ and $\mathbf{W_2}$ kept parallel to each other and carrying current $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ respectively ***in the same direction.***

The separation between the wires is d

Consider a small element ***dl*** of the wire $\mathbf{W_2}$**.** The magnetic field at ***dl*** due to the wire $\mathbf{W_1}$ is $\mathrm{B} = \boldsymbol{\frac{\mu_0 I_1}{2\pi d}}$

We know that the magnetic force due to current $\boldsymbol{I}$ is $\boldsymbol{dF_{mag} = I \cdot (\overrightarrow{dl} \times \vec{B})}$



Thus the magnetic force at the element $\boldsymbol{I_2 dl}$ is $\mathrm{d}F = I_2 \left(\frac{\mu_0 I_1}{2\pi d}\right) dl$

Thus the force per unit length of the wire W_2 due to the wire W_1

$$\boxed{\boldsymbol{f = \frac{dF}{dl} = \left(\frac{\mu_0 I_1 I_2}{2\pi d}\right)}}$$

Definition of Ampere

If $I_1 = I_2 = 1A$, $d = 1m$, so $f = I_2\left(\frac{\mu_0 I_1}{2\pi d}\right) = 2 \times 10^{-7} N/\mathrm{m}$

If two parallel, long wires kept 1m apart in vacuum, carry equal currents in the same direction and there is a force of $2 \times 10^{-7} N/\mathrm{m}$, the current in each wire is said to be 1 ampere.

SOLVED EXAMPLE: *Two long straight wires each carrying an electric current of* ***5.0A*** *are kept parallel to each other at a separation of* ***2.5cm****. Find the magnitude of the magnetic force experienced by* ***10 cm*** *of a wire.*

Solution:

$$Bf = \frac{dF}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(2 \times 10^{-7})(5)(5)}{2.5 \times 10^{-2}} = 20 \times 10^{-5} T$$

The force experienced by 10cm of this wire due to the other is

$$F = f dl = (10 \times 10^{-2})(20 \times 10^{-5}) = 2 \times 10^{-5} N$$

POLL QUESTION

A circular coil of radius **1.5cm** carries a current of **1.5 A**. if the coil has **25** turns, find the magnetic field at the centre.

A. 5.89 mT

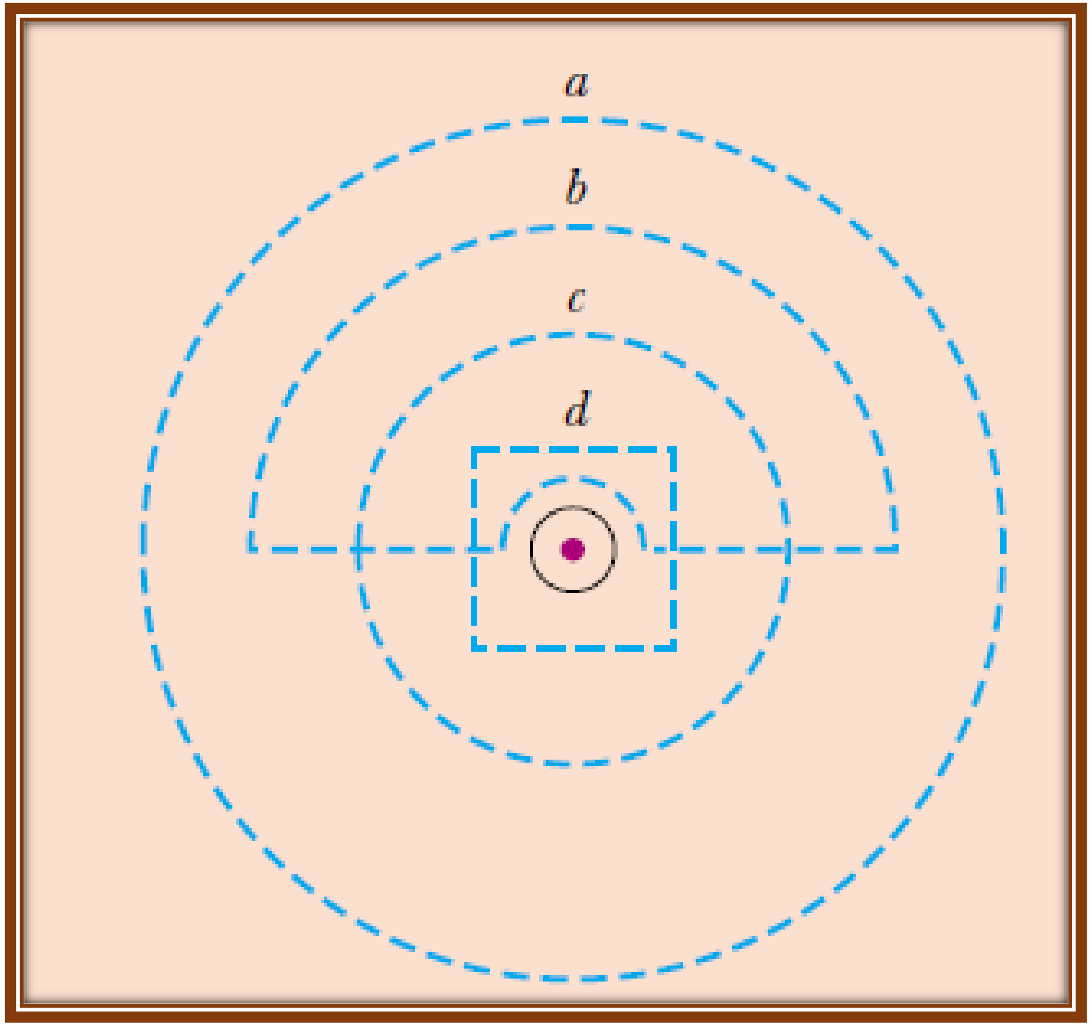
B. 3.45 mT

C. 2.69 mT

D. $\boxed{\text{1.57 mT}}$

$$B = \frac{\mu_0 NI}{2a} = \frac{(4\pi \times 10^{-7})(1.5) \times 25}{2 \times 1.5 \times 10^{-2}} = 1.57 \times 10^{-3} T$$

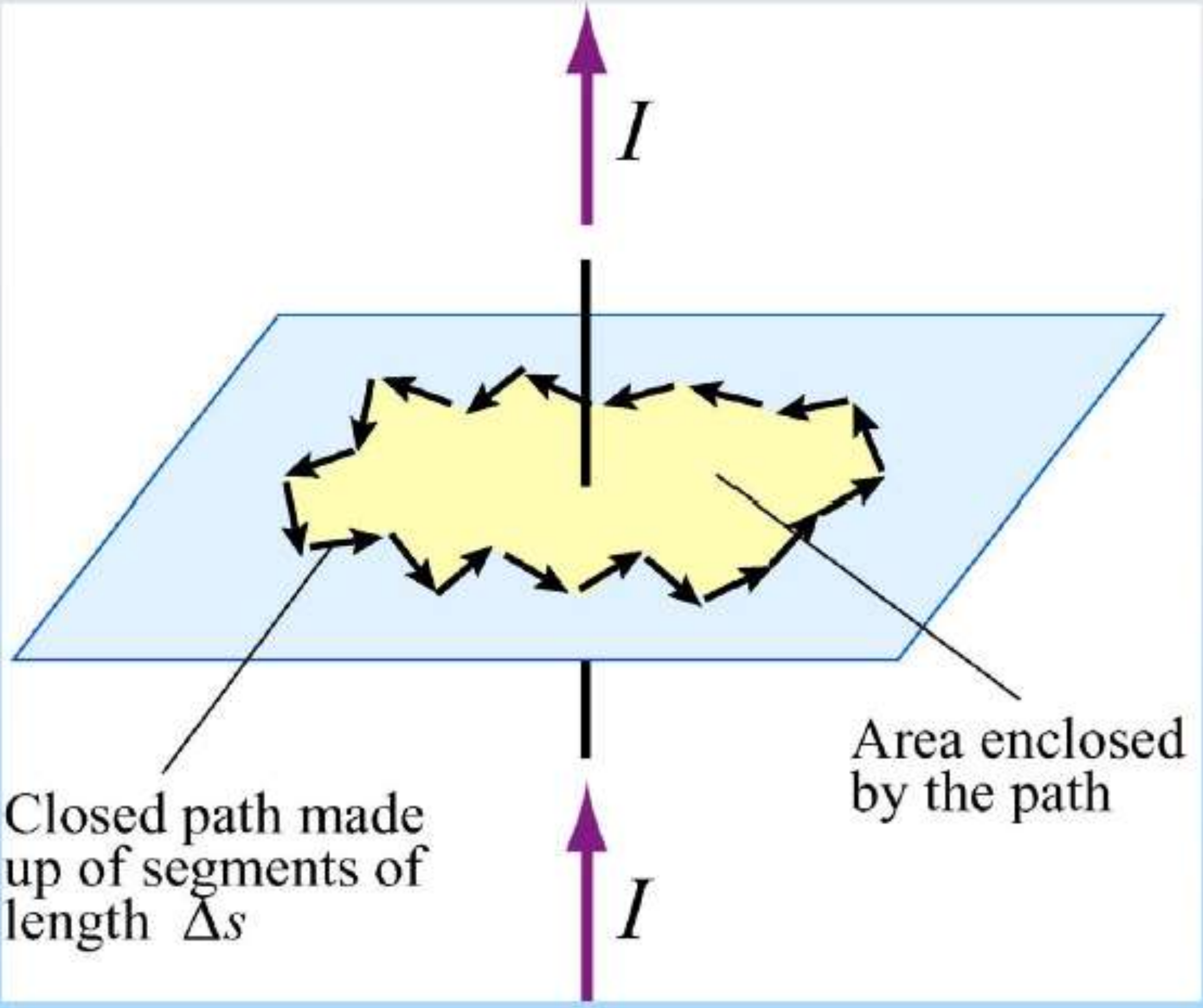
CONCEPT QUESTION



Rank the magnitudes of line integrals of magnetic field ($\oint \vec{B}.\overrightarrow{dl}$) for the closed paths in adjacent Figure, from least to greatest.

$$Path\ a \equiv Path\ c \equiv Path\ d$$
$$> Path\ b\ (Zero!)$$

Ampere's Law : the idea

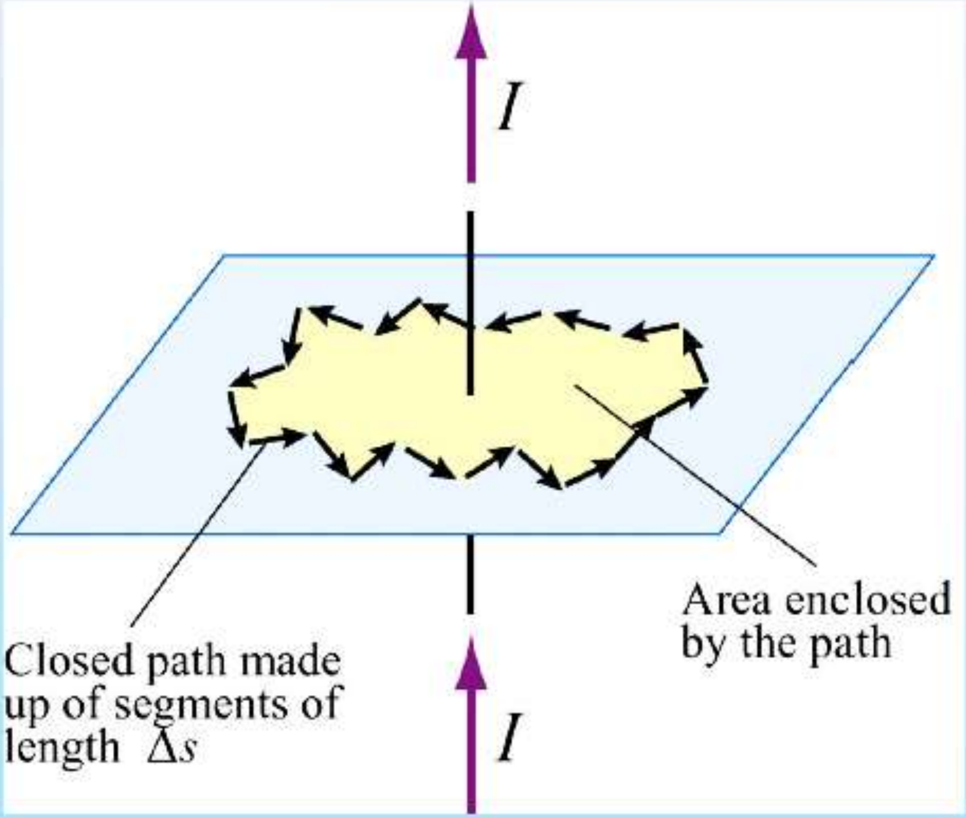


In order to have a B field around a loop, there must be current punching through the loop

Ampere's Law : the equation

Mathematically,

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0\, I_{enc}$$



The line integral is around any closed contour bounding an open surface *S*.

I_{enc} is current through S:

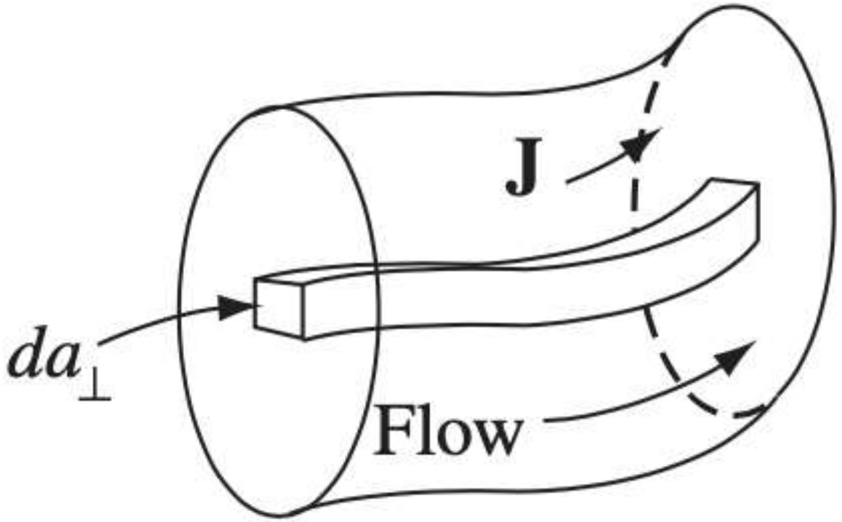
$$I_{enc} = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

Volume & Surface current

$$\vec{J} = \frac{dI}{da_{Perp.}} \Rightarrow [\vec{J}] = {}^{A}/_{m^2}$$

$$\vec{J} = \rho\vec{v}$$

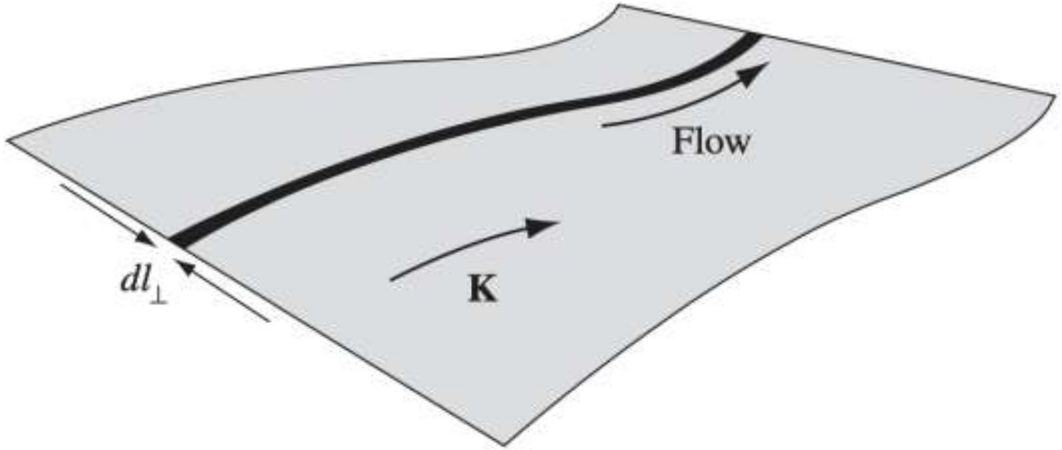
$$dq = \rho dv$$



$$\vec{K} = \frac{dI}{dl_{Perp.}} \Rightarrow [\vec{K}] = {}^{A}/_{m}$$

$$\vec{K} = \sigma\vec{v}$$

$$dq = \sigma da$$



Ampere's Law : Application Protocol

A useful law that relates the net magnetic field along a closed loop to the electric current passing through the loop.

Mathematically,

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \, I_{enc}$$

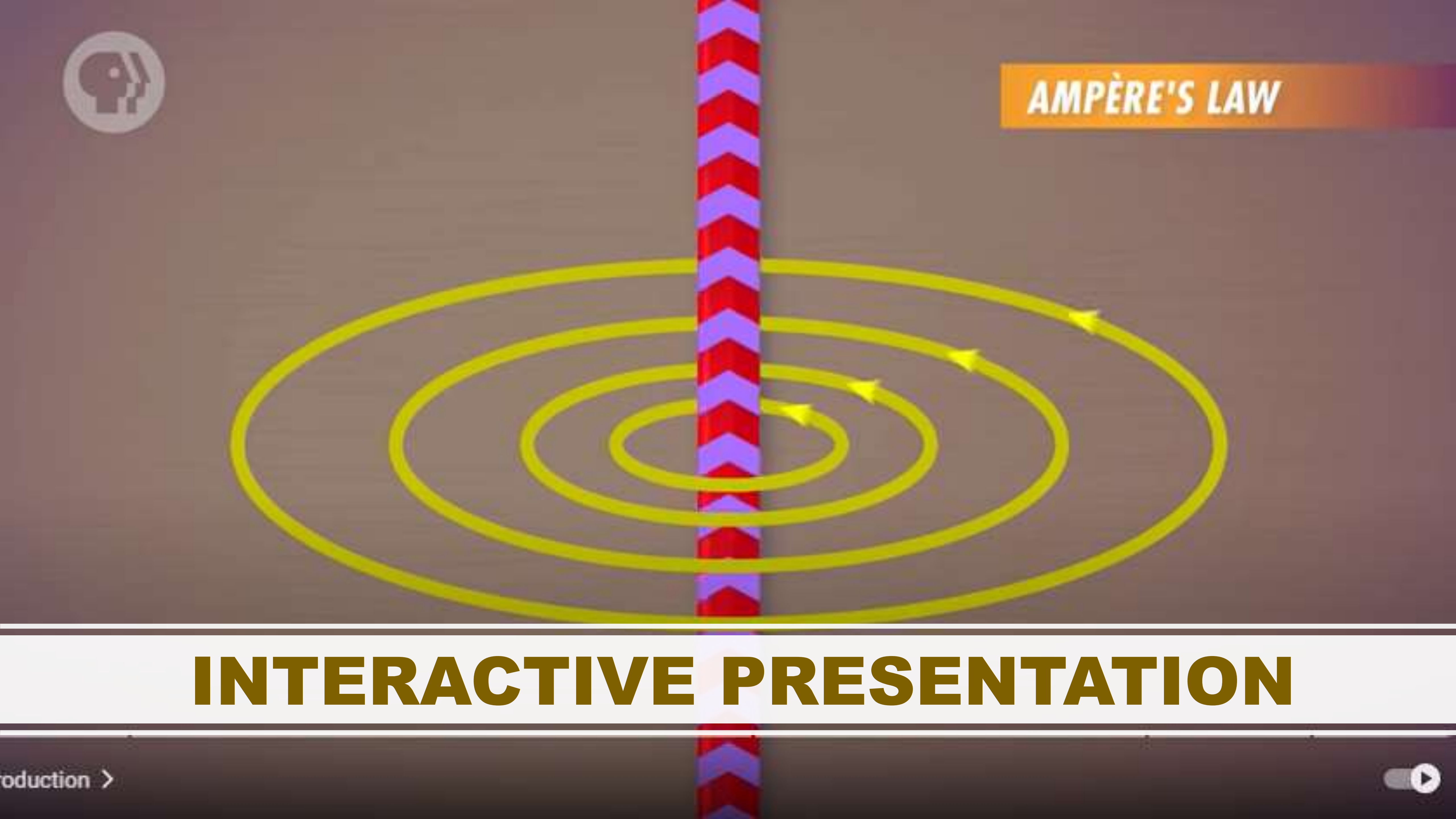
To apply Ampere's law to calculate the magnetic field, we can use the following procedure:

(1) Draw an Amperian loop using symmetry arguments.

(2) Find the current enclosed by the Amperian loop.

(3) Calculate the line integral $\oint \vec{B}.\overrightarrow{dl}$ around the closed loop.

(4) Equate $\oint \vec{B}.\overrightarrow{dl}$ with $\mu_0 \mathrm{I}_{enc}$ and solve for B



INTERACTIVE PRESENTATION

SOLVED EXAMPLE

Using Ampere's law, find the magnetic field at a distance ***s*** *from a long straight wire carrying current* ***I****.*

Solution:

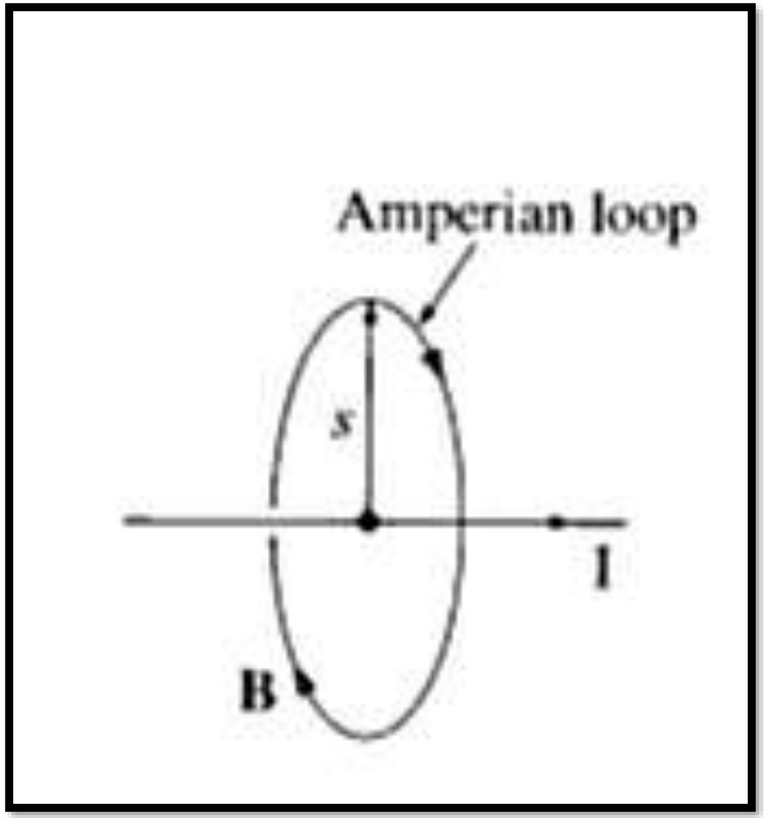
The direction of ***B*** is circling around the wire as indicated by the right-hand rule.

B is constant around the loop of radius ***s***.

From Ampere' s law

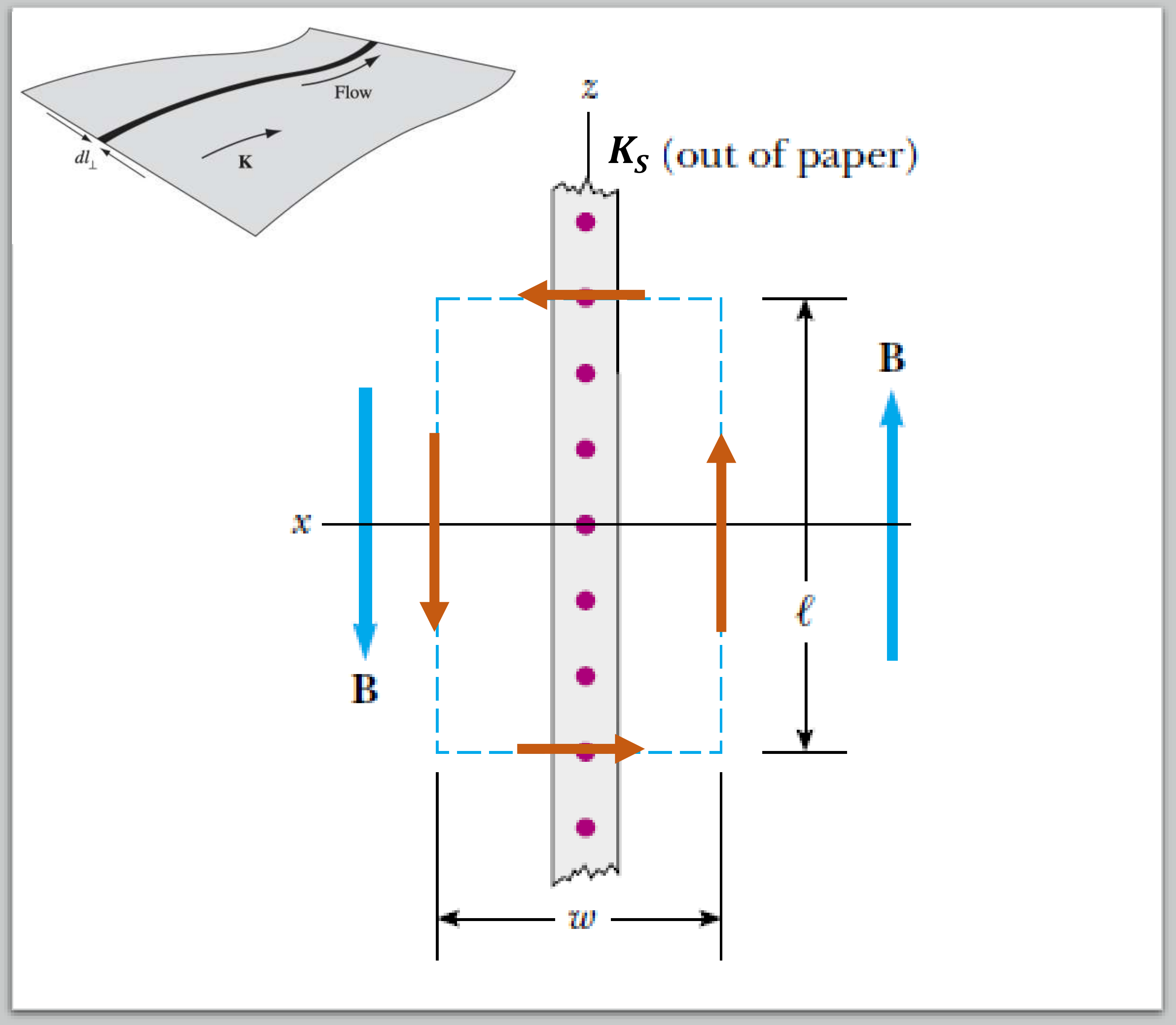
$$\oint \vec{B}.\overrightarrow{dl} = B \oint \overrightarrow{dl} = B\ 2\pi\ s = \mu_0\ I_{enc} = \mu_0 I$$

$$\Rightarrow \boldsymbol{B} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi s}}$$



SOLVED EXAMPLE

A thin, infinitely large sheet lying in the *yz* plane carries a current of surface current density K_S. The current is in the *y* direction, and K_S represents the surface current density measured along the *z* axis. Find the magnetic field near the sheet.

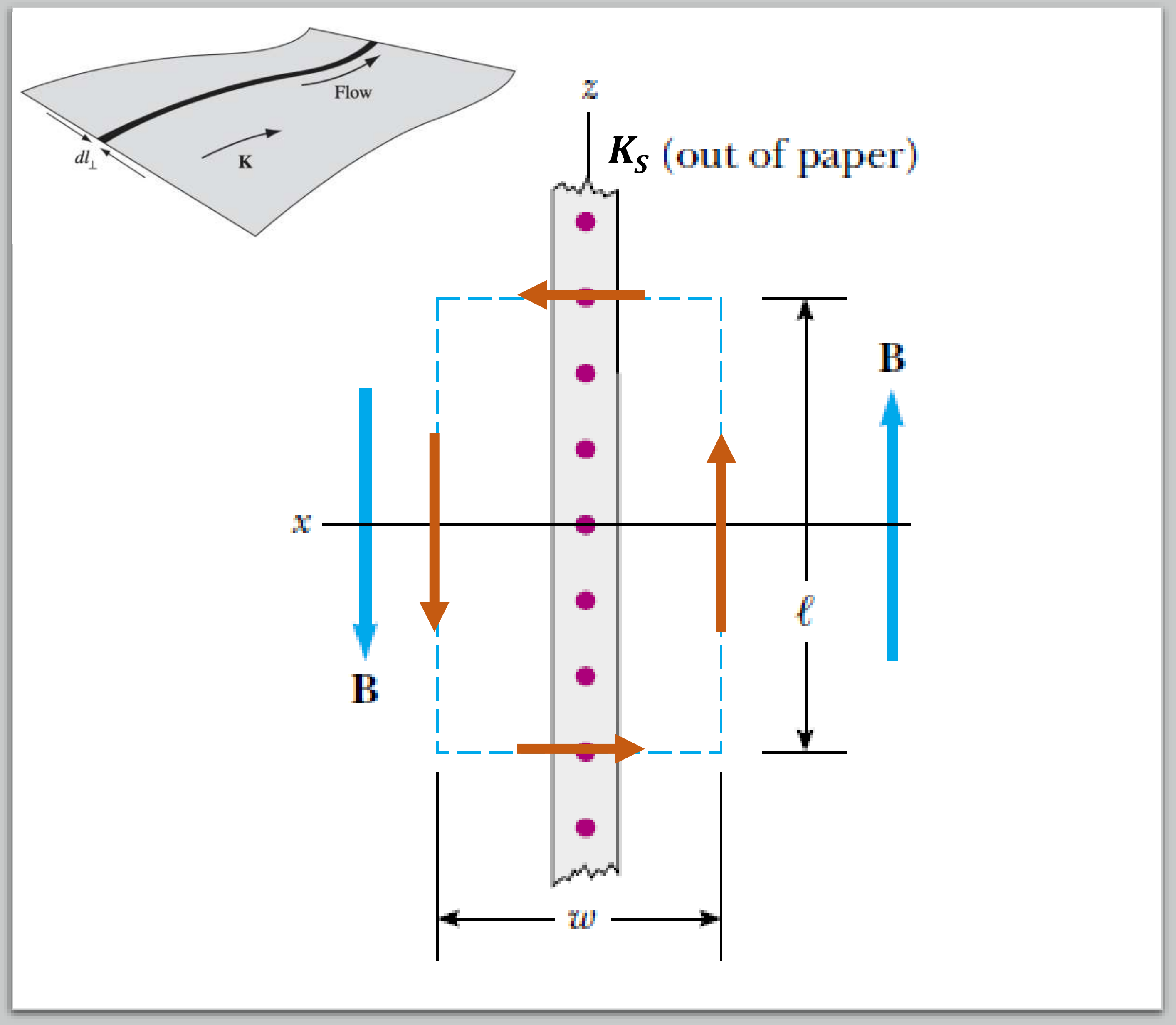


SOLVED EXAMPLE

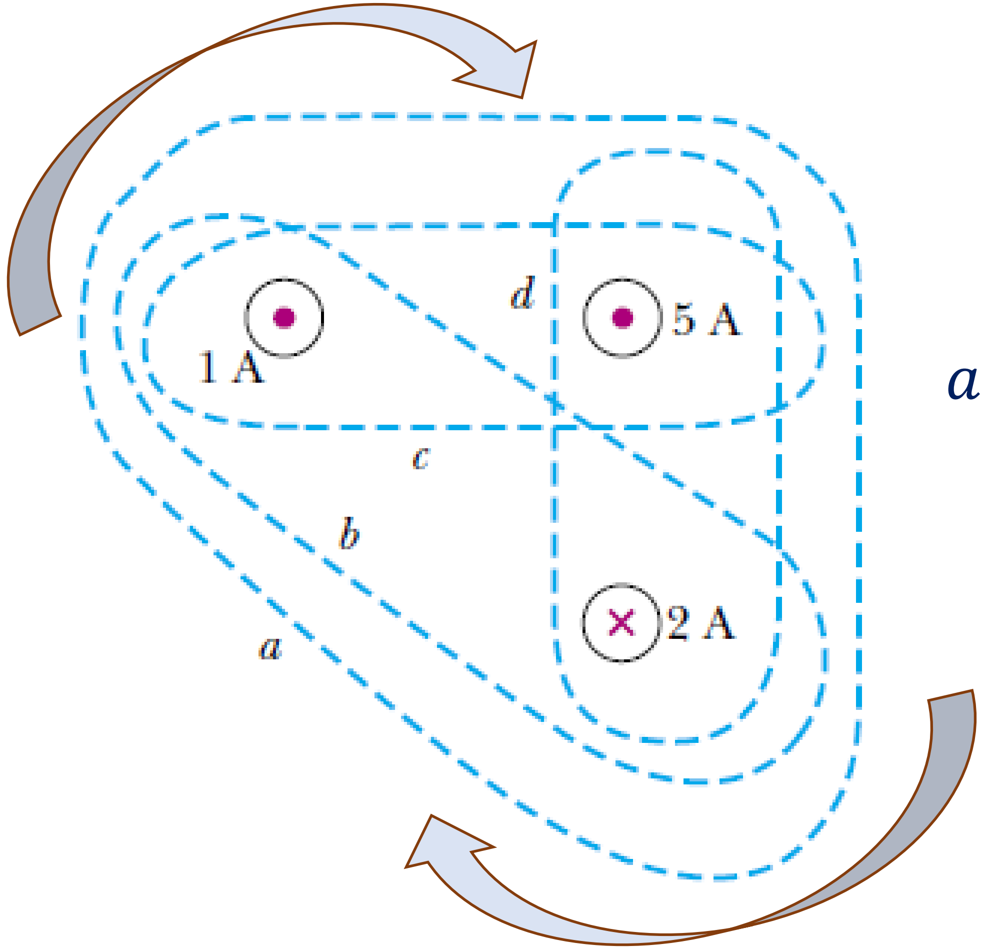
$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I$$

$$2Bl = \mu_0 K_S l \Rightarrow B = \frac{\mu_0 K_S}{2}$$

$$Remember \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$



POLL QUESTION



Calculate the ***clockwise line integral*** *of magnetic field* $\left[\oint \vec{B}.\overrightarrow{dl}\right]$ *along the closed paths* a, b, c, d *shown in the figure*:

$$Path\ a = +4\mu_0$$

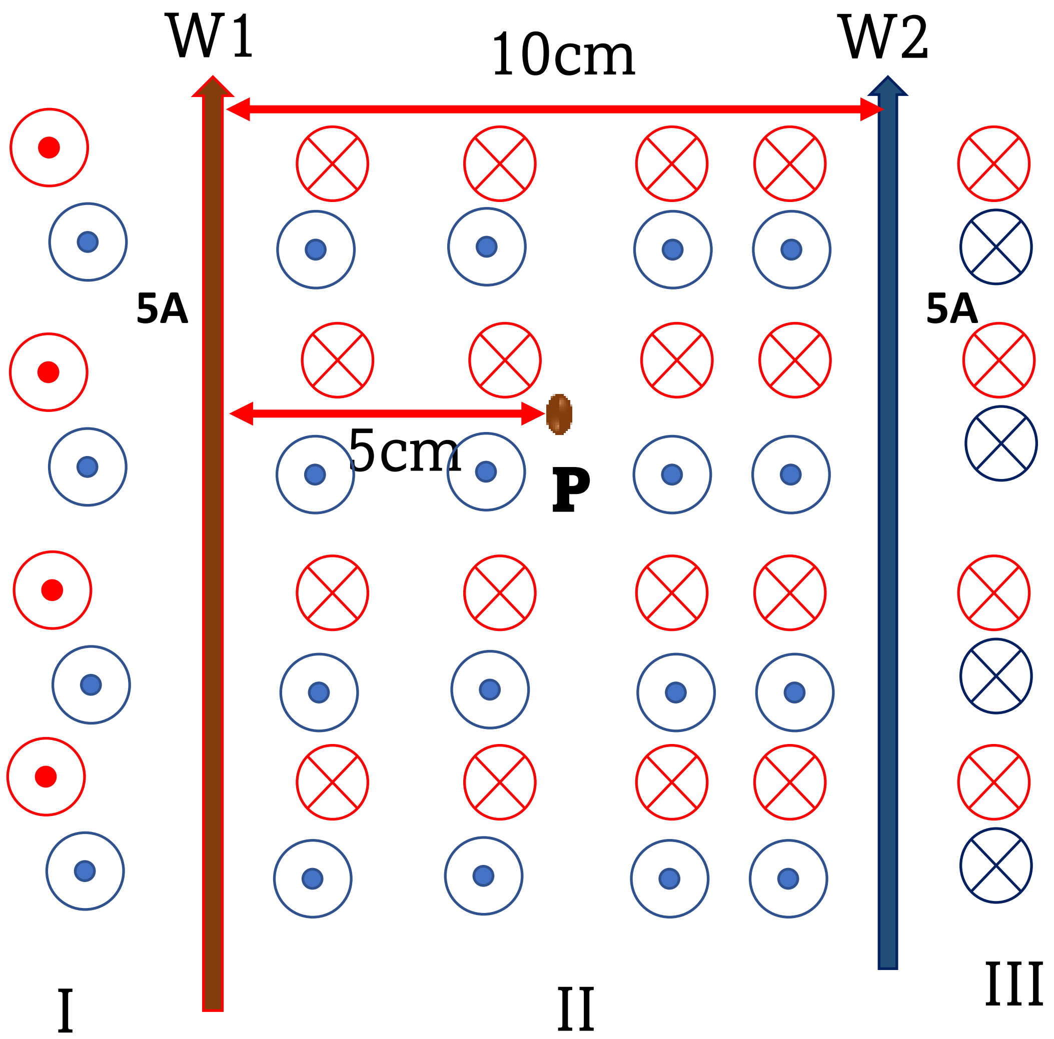
$$Path\ b = -1\mu_0$$

$$Path\ c = +6\mu_0$$

$$Path\ d = +3\mu_0$$

CONCEPT QUESTION

The adjacent figure shows two long, straight wires carrying electric currents in the same directions, each carrying 5A current. The separation between the wires is 10.0 cm. Find the magnetic field at a point P midway between the wires.



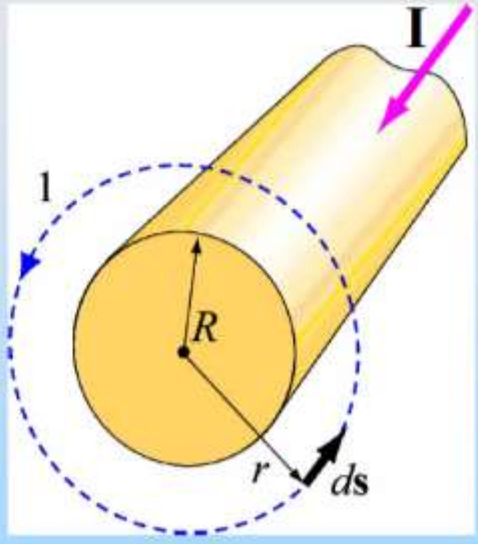
A. $80\mu T$

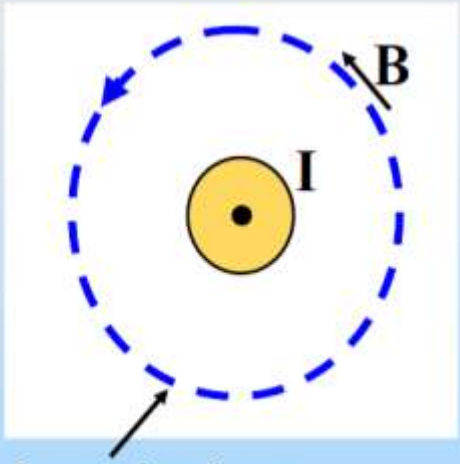
B. $120\mu T$

C. $160\mu T$

$\boxed{D.\ 0}$

Magnetic Field due to Infinite Wire

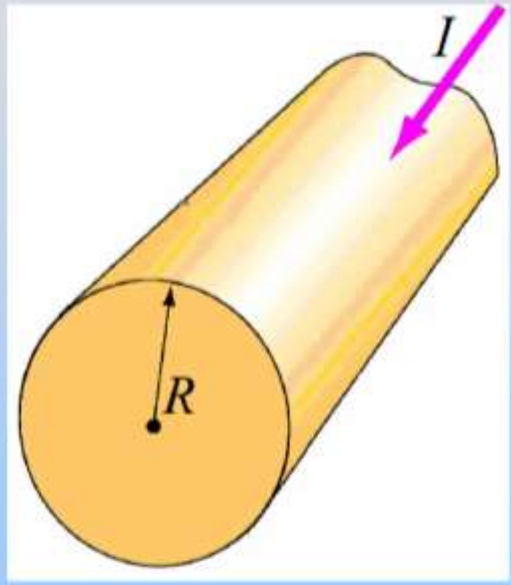




<u>Amperian Loop</u>:

B is Constant & Parallel

I Penetrates



A cylindrical conductor has radius R and a uniform current density with total current I

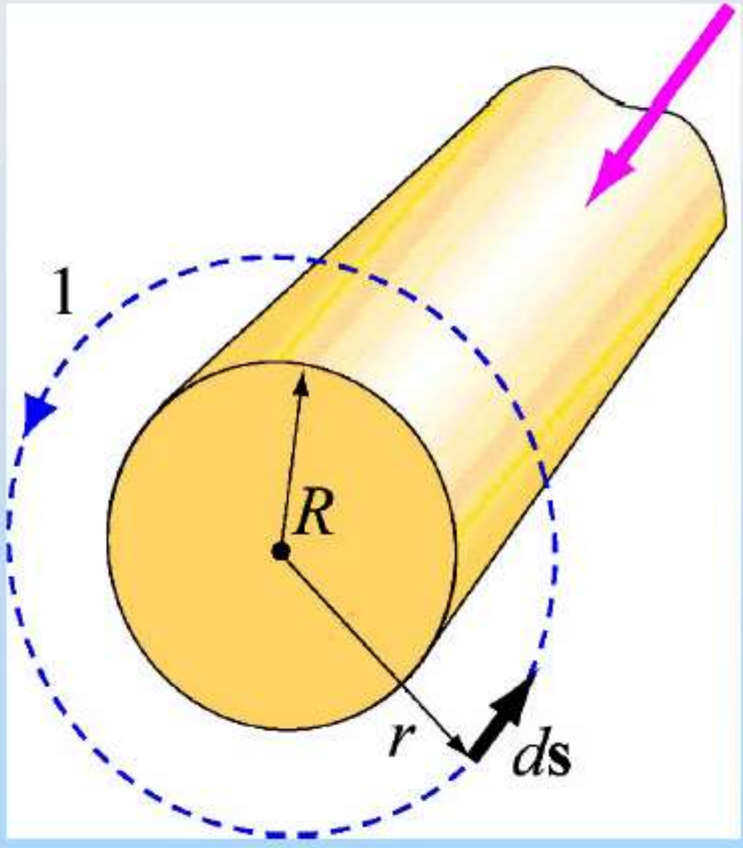
Find B everywhere

Two regions:

(1) outside wire ($r \geq R$)

(2) inside wire ($r < R$)

Magnetic Field due to Infinite Wire



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry → Amperian Circle

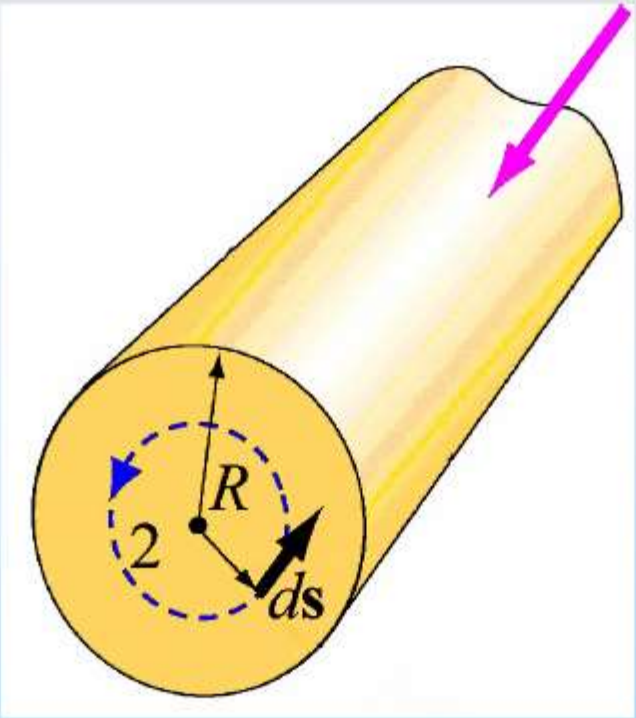
B-field counterclockwise

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B(2\pi r)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \text{counterclockwise}$$

Magnetic Field due to Infinite Wire



Region 2: Inside wire (r < R)

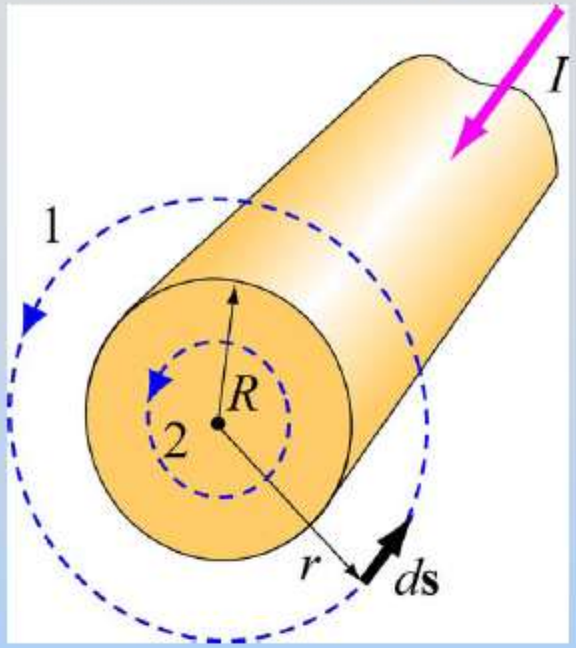
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B(2\pi r)$$

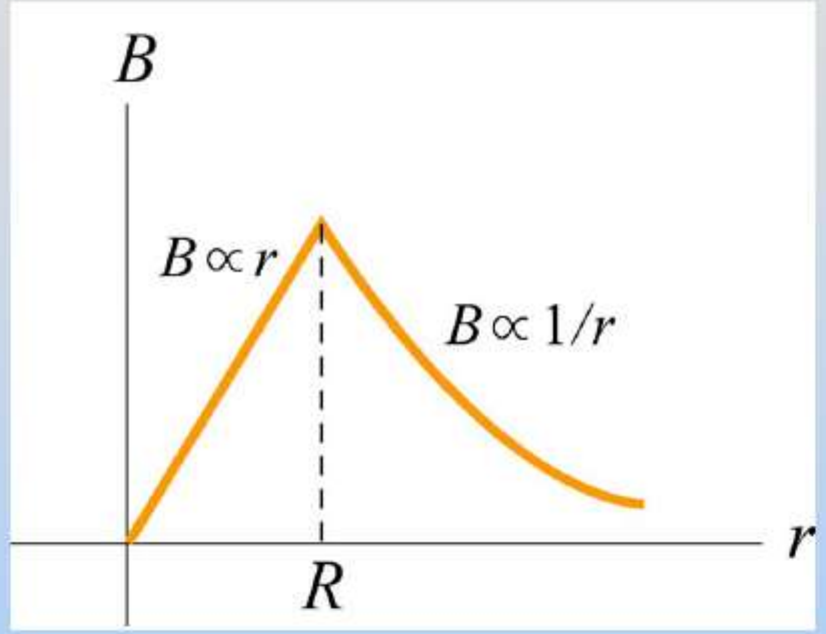
$$= \mu_0 I_{enc} = \mu_0 I \left(\frac{\pi r^2}{\pi R^2} \right)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \text{ counterclockwise}$$

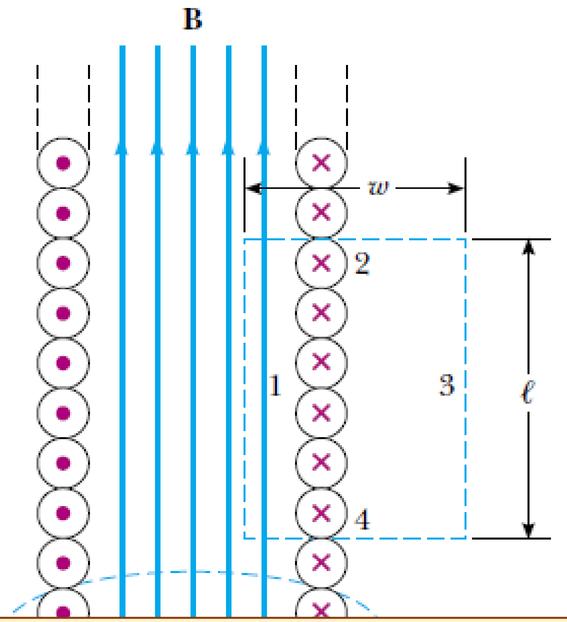
Could also say: $J = \dfrac{I}{A} = \dfrac{I}{\pi R^2}$; $I_{enc} = J A_{enc} = \dfrac{I}{\pi R^2}(\pi r^2)$

Magnetic Field due to Infinite Wire

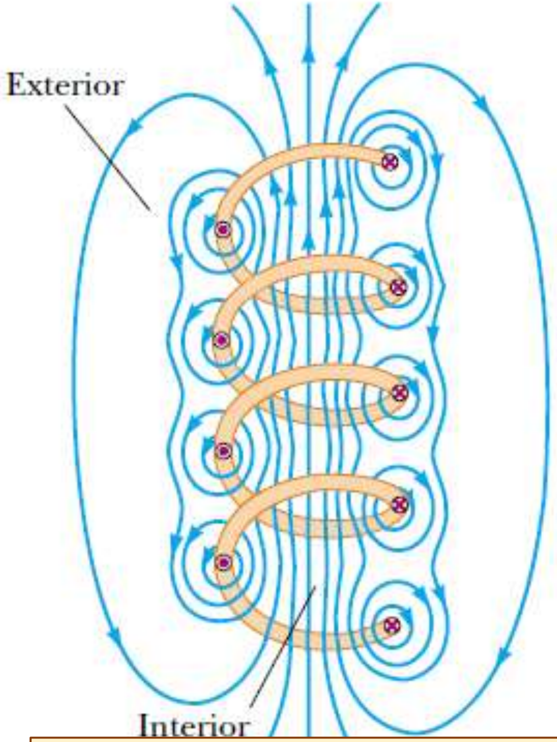




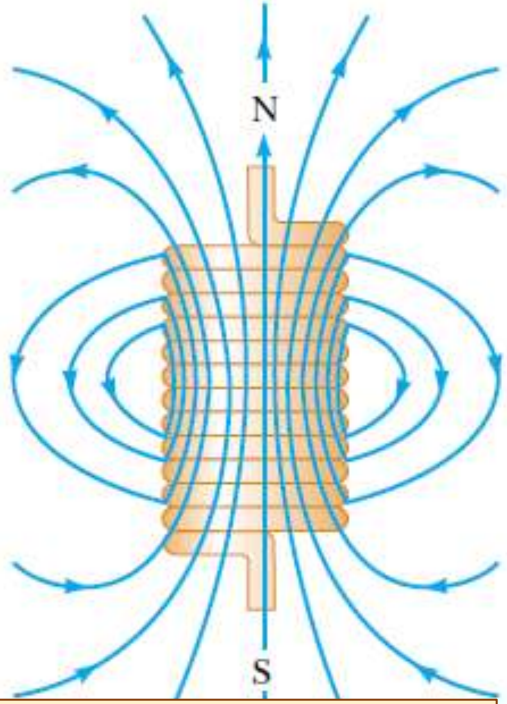
$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$



✓ *Tightly wound*
✓ *Infinite length*



Loosely wound

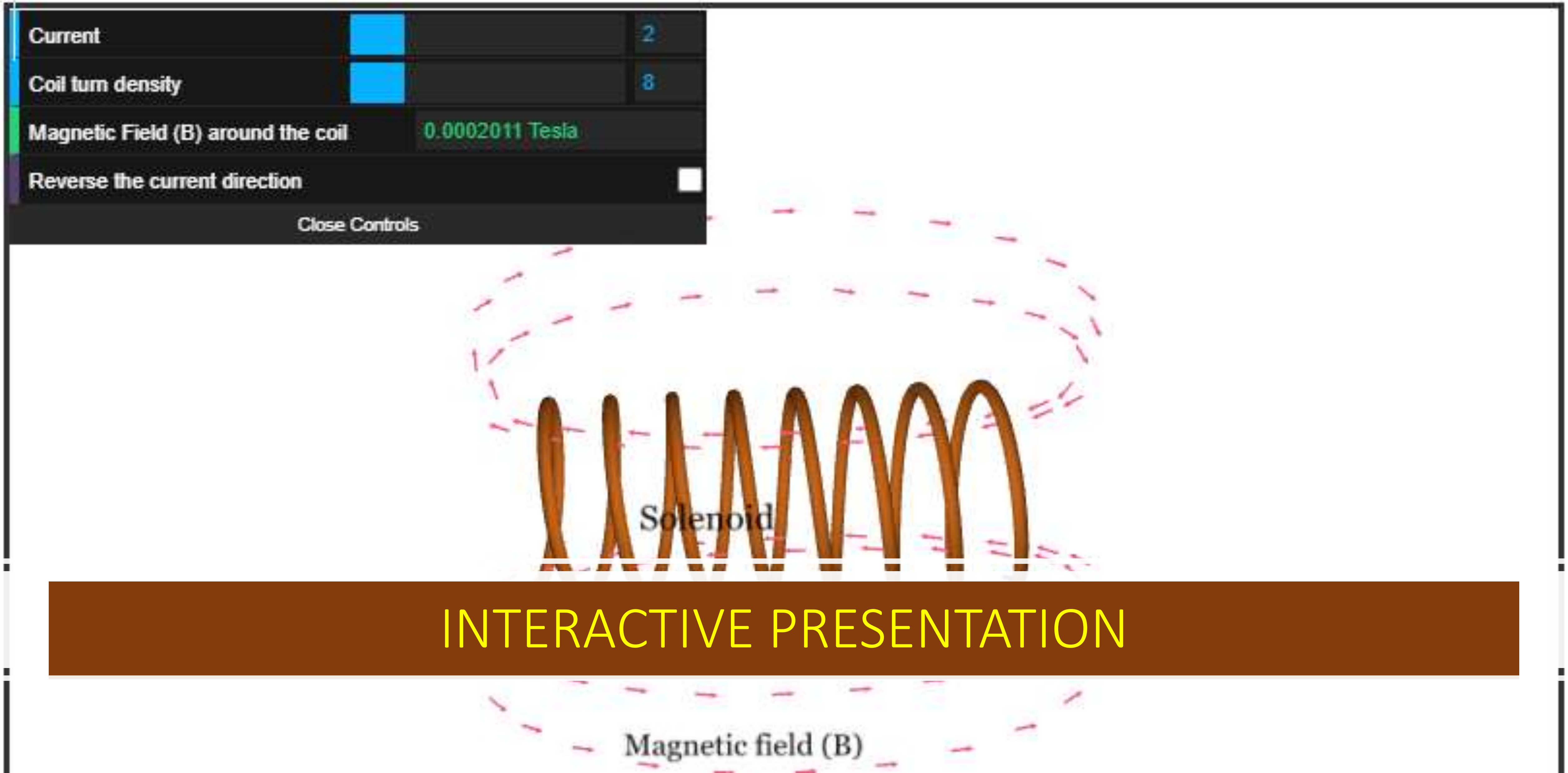


Tightly wound

Magnetic Field due to Solenoid

A solenoid is a long coil of wire tightly/loosely wound in the helical form.

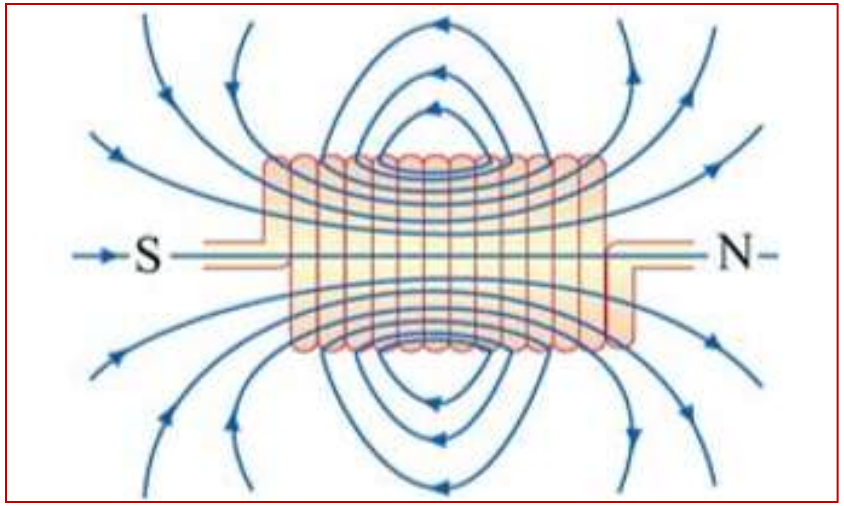
Magnetic field formed around a current carrying Solenoid

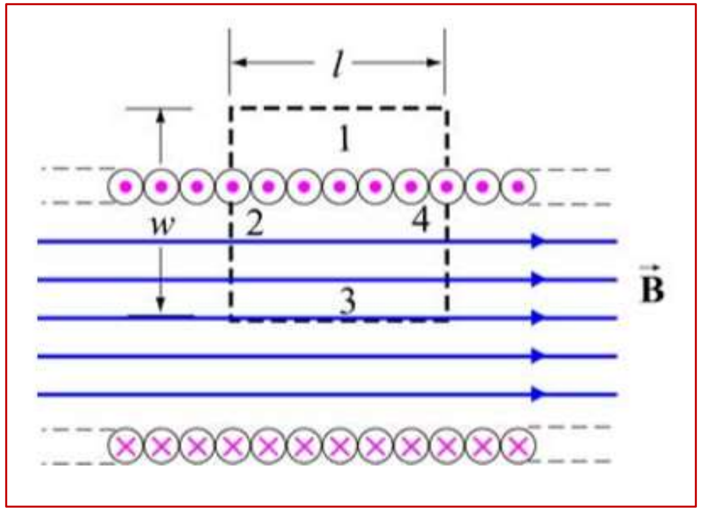


INTERACTIVE PRESENTATION

Magnetic Field due to Infinite Solenoid

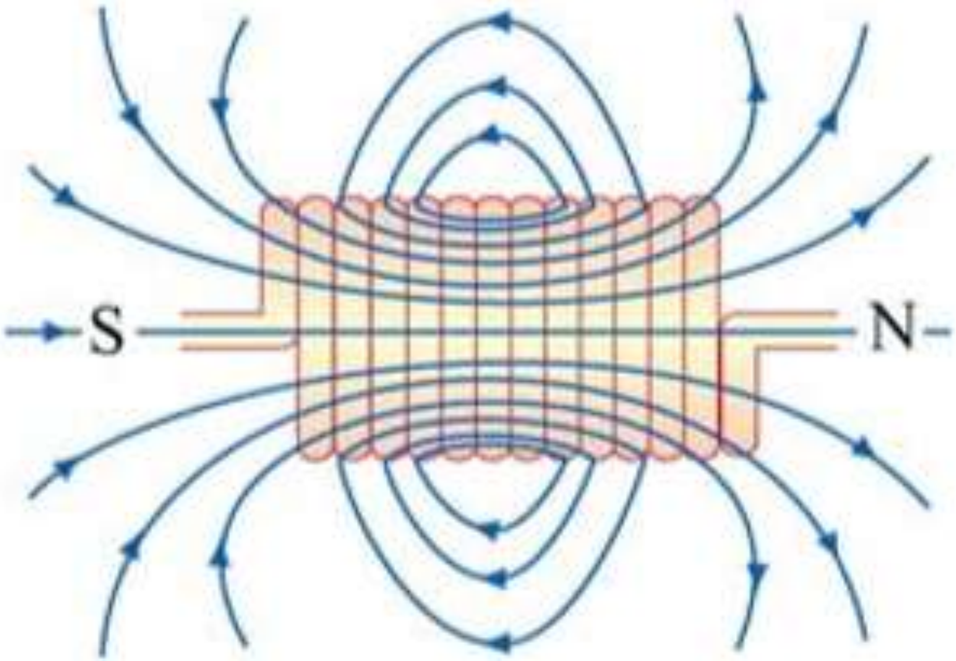
Find the magnetic field of a very long solenoid consisting of n closely wound turns per unit length on a cylinder of radius R, each carrying a steady current I.

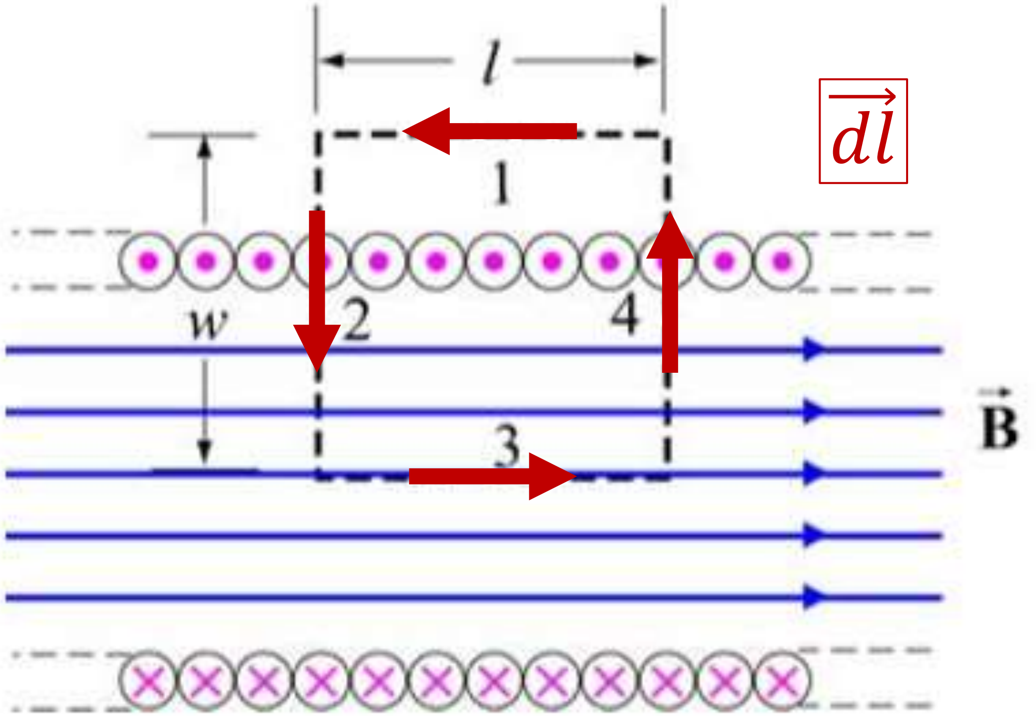




- ✓ If the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter.
- ✓ For an "ideal" solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.
- ✓ We can use Ampere's law to calculate the magnetic field strength inside an ideal solenoid.

Magnetic Field due to Infinite Solenoid





$\boxed{\overrightarrow{dl}}$

$$\oint \vec{B}.\overrightarrow{dl} = \int_1 \cancel{\vec{B}.\overrightarrow{dl}} + \int_2 \cancel{\vec{B}.\overrightarrow{dl}} + \int_3 \vec{B}.\overrightarrow{dl} + \int_4 \cancel{\vec{B}.\overrightarrow{dl}}$$

$\vec{B} \perp \overrightarrow{dl}$

$\vec{B} = 0$

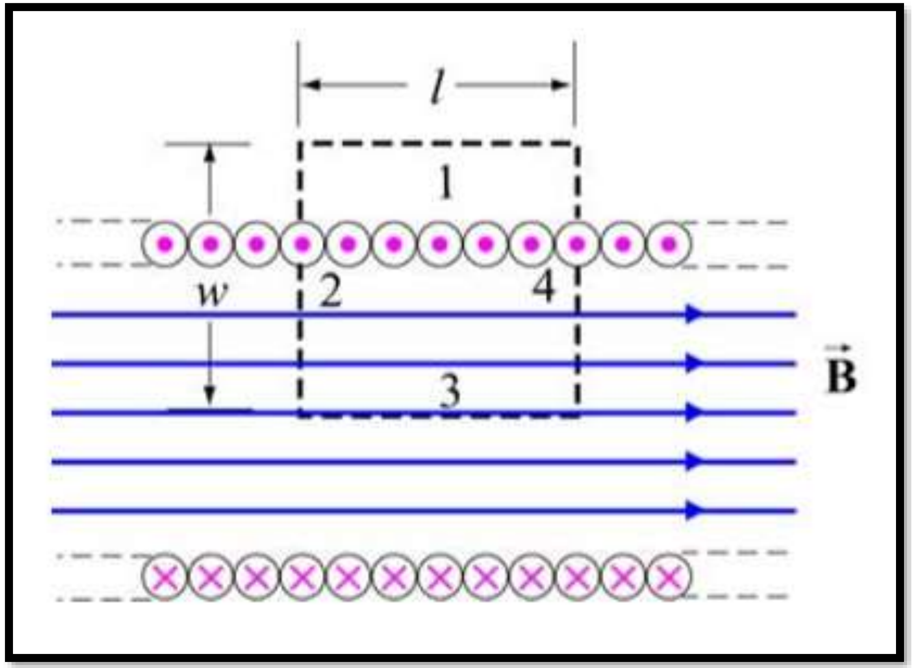
Magnetic Field due to Infinite Solenoid

$$\oint \vec{B}.\overrightarrow{dl} = \int_1 \vec{B}.\overrightarrow{dl} + \int_2 \vec{B}.\overrightarrow{dl} + \int_3 \vec{B}.\overrightarrow{dl} + \int_4 \vec{B}.\overrightarrow{dl}$$

$$= 0 + 0 + Bl + 0 = Bl$$

On the other hand, the total current enclosed by the Amperian loop is $I_{enc} = NI$, where N is the total number of turns. Applying Ampere's law yields $\oint \vec{B}.\overrightarrow{dl} = Bl = \mu_0 NI$

Or $B = \mu_0 \left(\frac{N}{l}\right) I = \mu_0 nI$ *or* $\boldsymbol{B = \mu_0 nI}$ **,** where n represents the number of turns per unit length



POLL QUESTION

A long solenoid is formed by winding *20 turns per cm*. *What current is necessary* to produce a magnetic field of *20 mT* inside the solenoid?

A. 2A

B. 4A

C. 8A

D. 16A

Solution:

The magnetic field inside the solenoid is $\vec{B} = \mu_0 n I$

Or $20 \times 10^{-3}\ T = (4\pi \times 10^{-7}\ T\ m\ A^{-1})(20 \times 10^{2} m^{-1})\mathrm{I}$

Or $I = 8\ A$

POLL QUESTION

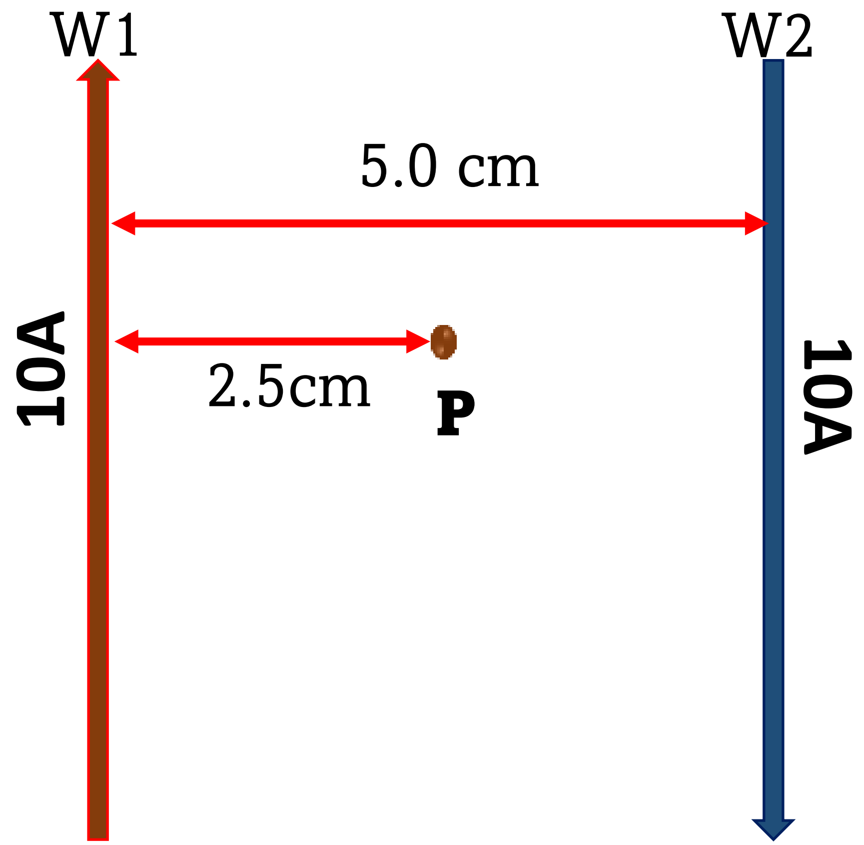
The following figure shows two long, straight wires carrying electric currents in opposite directions. The separation between the wires is 5.0 cm. Find the magnetic field at a point P midway between the wires.

A. $80\mu T$

B. $120\mu T$

C. $160\mu T$

D. 0

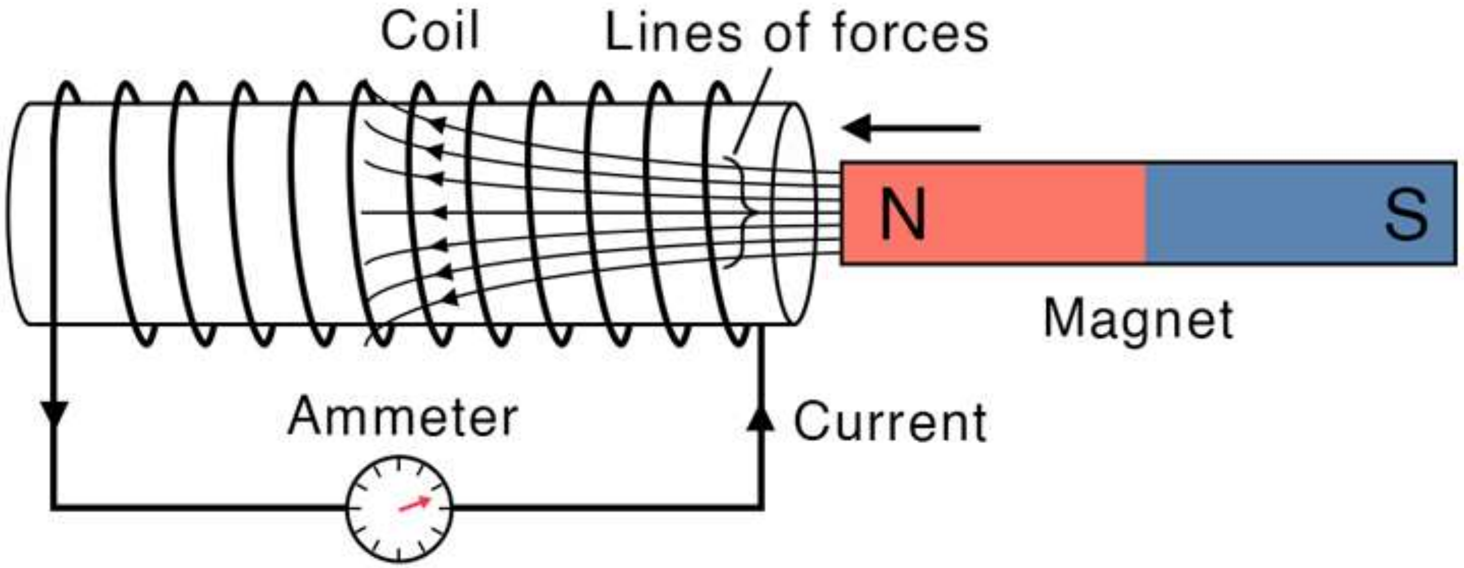


Solution:

$$B = \frac{\mu_0 I}{2\pi d} = \frac{(2\times10^{-7})(10)}{(2.5\times10^{-2})} = 80 \times 10^{-6}\ T$$

The net field $2 \times 80 \times 10^{-6}\ T = 160 \times 10^{-6}\ T$

LECTURE - 05



Faraday's & Lenz's Law

What is EMF?

$$\varepsilon = \int \vec{E} \cdot \vec{dl}$$

Looks like potential. It's a "driving force" for current

ES force at the source (f_s)

Chemical force in the battery

Mechanical pressure converting into electrical impulse in piezoelectric crystal

in a thermocouple it’s a temperature gradient

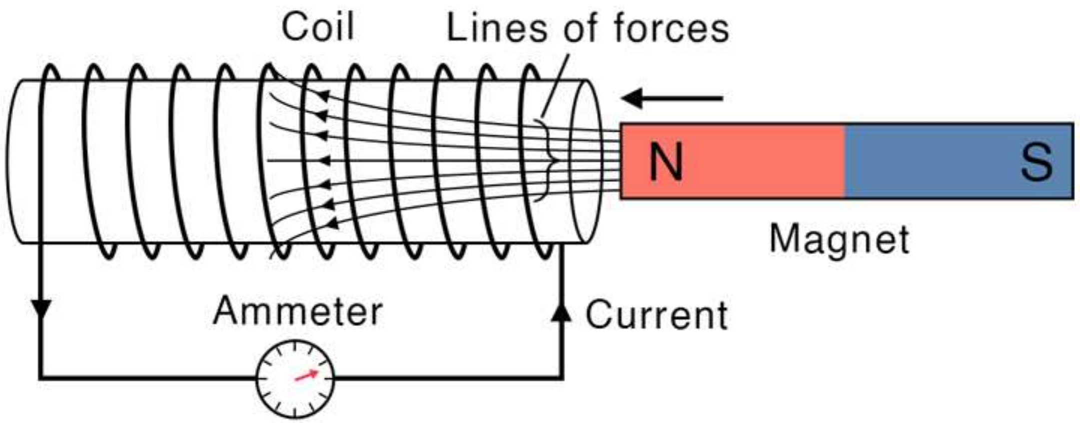
Light in a Photo-electric Cell

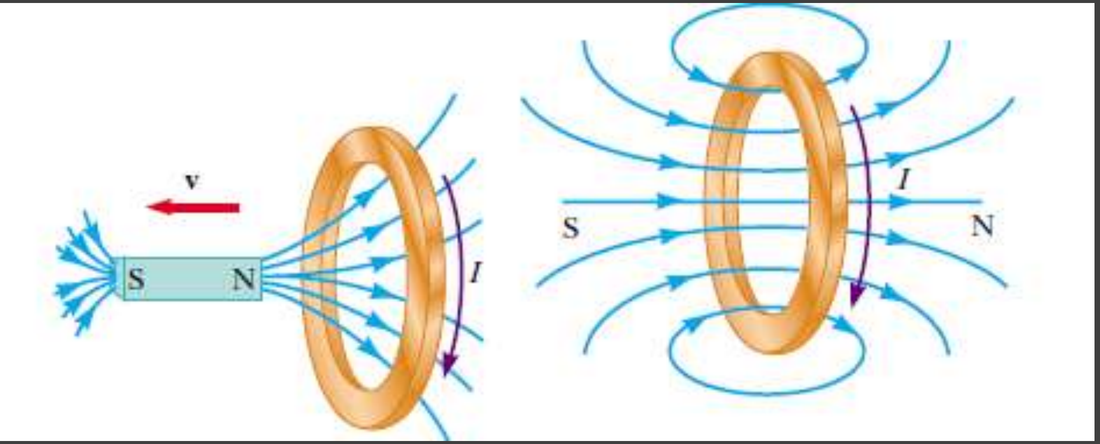
in a Van de Graaff generator, the electrons are literally loaded onto a conveyer belt and swept along

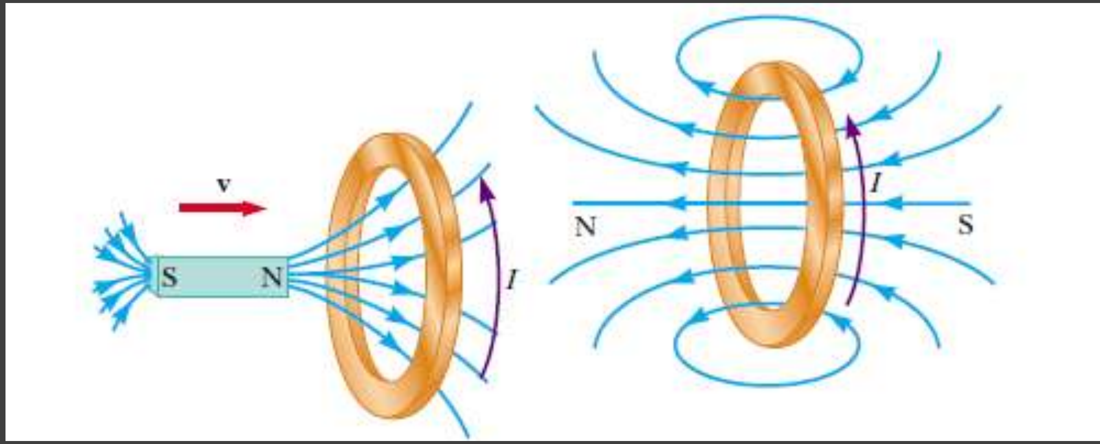
Faraday's Law of Induction

$$\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$$

A changing magnetic flux *induces* an EMF





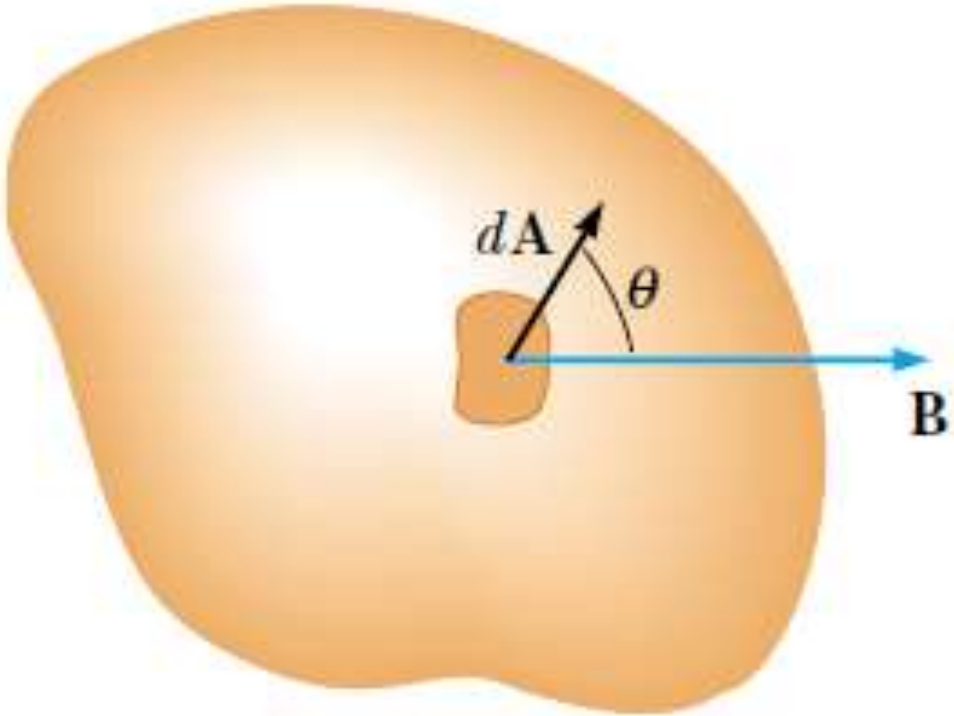


Lenz's Law

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

Magnetic Flux

- $\Phi_B = \int \vec{B} \cdot \overrightarrow{ds}$
- $Uniform\ Field \Rightarrow \Phi_B = BACos\theta$
- $Unit \Rightarrow T.m^2 \equiv Weber\ (Wb)$



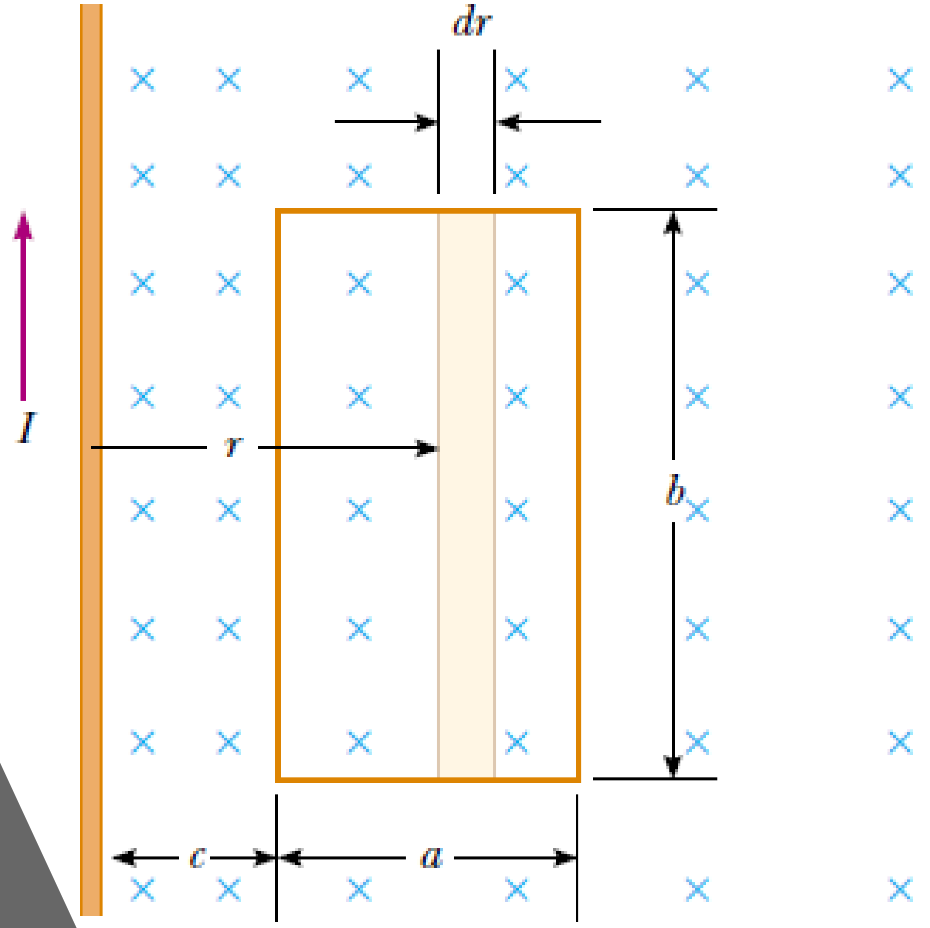
Find the total magnetic flux through the loop due to the current in the wire.

ANS:

$$B = \frac{\mu_0 I}{2\pi r};\ ds = b.dr$$

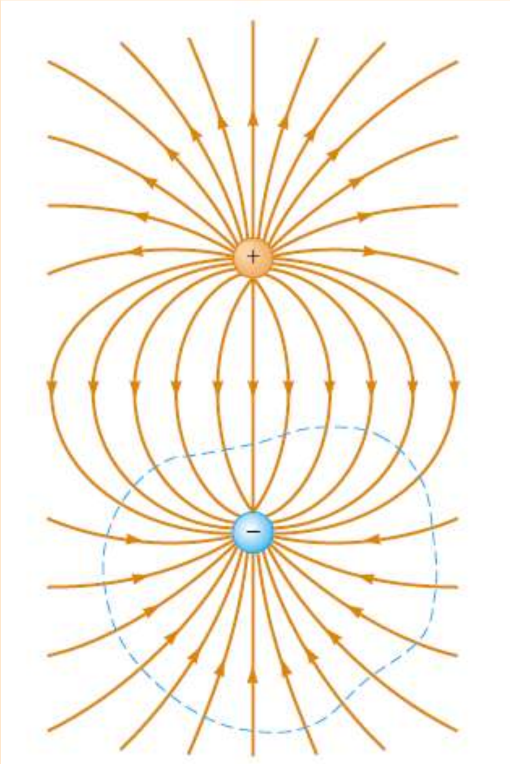
$$\Phi_B = \int \vec{B} \cdot \overrightarrow{ds} = \frac{\mu_0 I b}{2\pi} \int_c^{c+a} \frac{dr}{r}$$

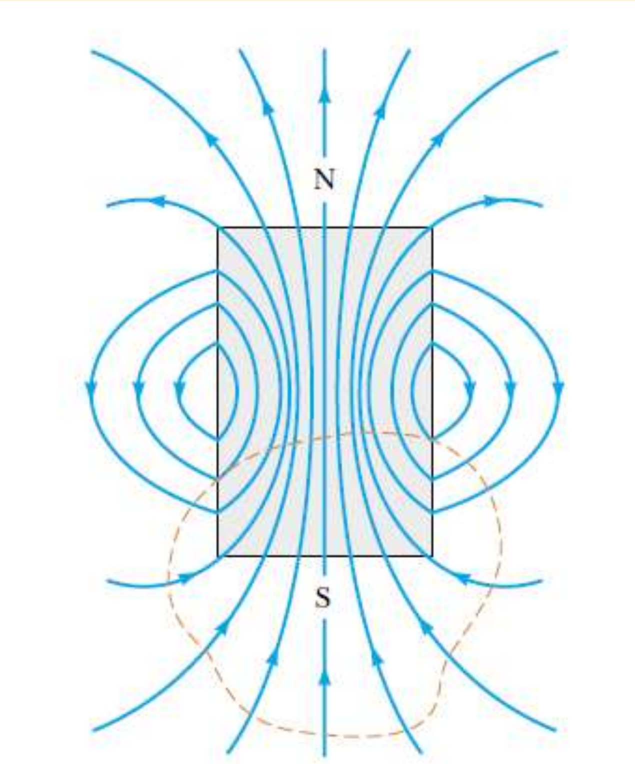
$$\Phi_B = \int \vec{B} \cdot \overrightarrow{ds} = \frac{\mu_0 I b}{2\pi} \ln \frac{c+a}{a}$$



SOLVED EXAMPLE

Gauss Law of Magnetism
No Magnetic Monopole



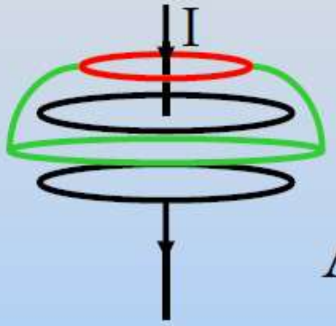


- Gauss Theorem
- $\int_v (\vec{\nabla}.\vec{F})\,dv = \oint_s \vec{F}.\overrightarrow{ds}$
- $\oint \vec{B}\cdot\overrightarrow{ds} = 0$
- $\Rightarrow \int(\vec{\nabla}.\vec{B})dv = 0$
- $\Rightarrow (\vec{\nabla}.\vec{B}) = 0$
- Notice the difference with ES field

Displacement current and Maxwell-Ampere's Law

Ampere's Law: Capacitor

Consider a charging capacitor:



Use Ampere's Law to calculate the magnetic field just above the top plate

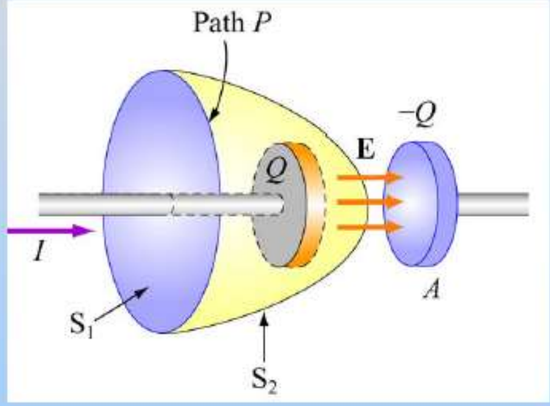
$$\text{Ampere's law: } \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

1) Red Amperian Area, $I_{enc} = I$
2) Green Amperian Area, $I = 0$

What's Going On?

Displacement Current

We don't have current between the capacitor plates but we do have a changing E field. Can we "make" a current out of that?



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$\boxed{\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d}$$

Displacement current and Maxwell-Ampere's Law

Maxwell-Ampere's Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

$$= \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

CONCEPT QUESTION

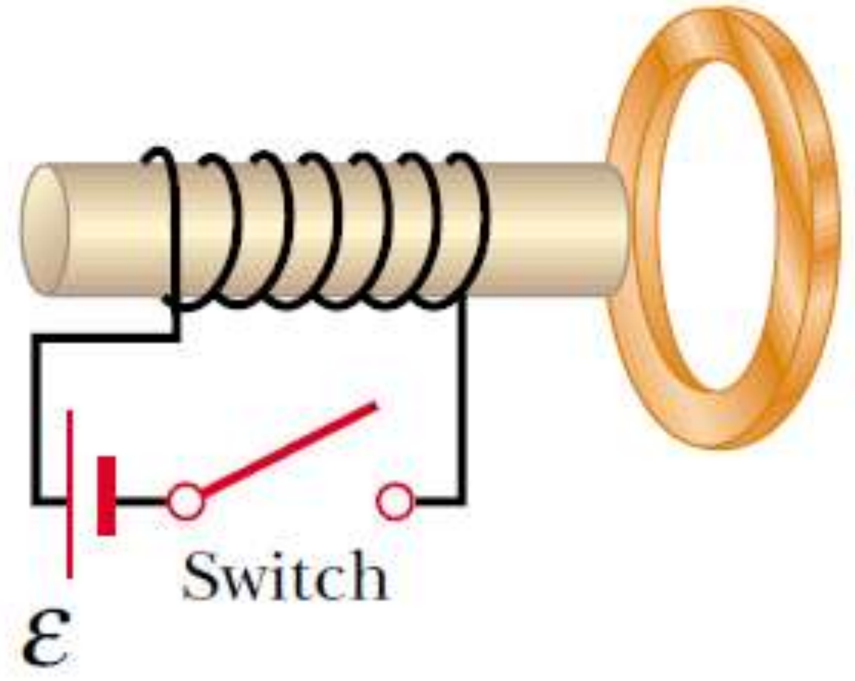
A metal ring is placed near a solenoid, as shown in Figure. Find the direction of the induced current in the ring

(A) at the instant, the switch in the circuit containing the solenoid is thrown closed,

(B) after the switch has been closed for several seconds, and

(C) at the instant, the switch is thrown open.

(D) After the switch is thrown open for several seconds



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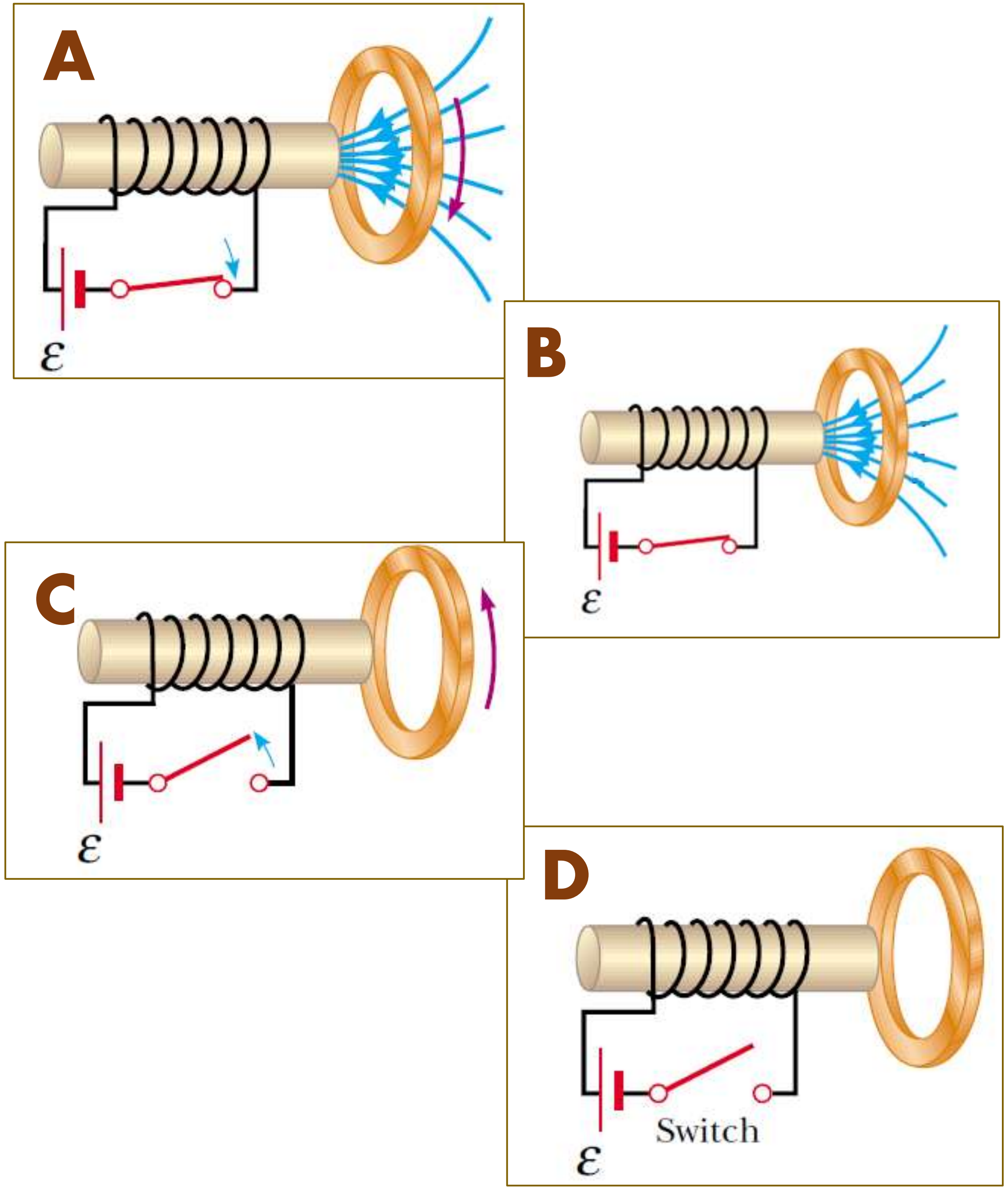

Electromagnetism before Maxwell

❖Maxwell in 1862 formulated the basic laws of electricity and magnetism in the form of four fundamental equations. These equations explain the experimental observations of Gauss law, Faraday's law and Ampere's law.

❖The integral form of Maxwell's relations are

1. $\oint_s \vec{E}.\overrightarrow{ds} = \frac{1}{\epsilon_0}\int_v \rho \, dv$ (**Gauss law of Electrostatic**)

2. $\oint_s \vec{B}.\overrightarrow{ds} = 0$ (**Gauss law of Magnetostatics**)

3. $\oint_l \vec{E}.\overrightarrow{dl} = -\int_s \frac{\partial\vec{B}}{\partial t}.\overrightarrow{ds}$ (**Faraday's law of electromagnetic induction**)

4. $\oint_l \vec{B}.\overrightarrow{dl} = \mu_0 \int_s \vec{J}.\overrightarrow{ds}$ (**Ampere's law**)

DIFFERENTIAL FORMS USING VECTOR CALCULUS

(i) $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$ (Gauss's law),

(ii) $\nabla \cdot \mathbf{B} = 0$ (Gauss Law of Magnetostatics)

(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday's law),

(iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (Ampère's law).

INTERACTIVE PRESENTATION

Displacement current :

Maxwell's Contribution

Maxwell's relations

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3. $\oint_l \vec{E}.\overrightarrow{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$ (**Faraday's law of electromagnetic induction**)

4. $\oint_l \vec{B}.\overrightarrow{dl} = \mu_0 \int_s \vec{J}.\overrightarrow{ds} + \mu_0\epsilon_0 \int_s \frac{\partial \vec{E}}{\partial t}.\overrightarrow{ds}$ (**Ampere's law modified by Maxwell**)

(i) $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$ (Gauss's law),

(ii) $\nabla \cdot \mathbf{B} = 0$ (Gauss Law of Magnetostatics)

(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday's law),

(iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ (Ampère's law with Maxwell's correction).

Maxwell's Equations in free space

❖ In 1862 Maxwell formulated the basic laws of electricity and magnetism in the form of four fundamental equations. These equations explain the experimental observations of Gauss law, Faraday's law and Ampere's law.

Gauss' law for $\vec{E}$	$\vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\epsilon_0}$
Gauss' law for $\vec{B}$	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's law of induction	$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$
Ampere's law with Maxwell's correction	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

In free space: $\rho = 0$ and $J = 0$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Additional information: Changing electric flux produces magnetic field

Differential Form With Proper Representation

$$\left.\begin{array}{ll}\text{(i)}\ \nabla\cdot\mathbf{E}=\dfrac{1}{\epsilon_0}\rho, & \text{(iii)}\ \nabla\times\mathbf{E}+\dfrac{\partial\mathbf{B}}{\partial t}=\mathbf{0}, \\[2ex] \text{(ii)}\ \nabla\cdot\mathbf{B}=0, & \text{(iv)}\ \nabla\times\mathbf{B}-\mu_0\epsilon_0\dfrac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J},\end{array}\right\}$$

all electromagnetic fields are ultimately attributable to charges and currents

Maxwell's equations tell you how charges produce fields

MAXWELL'S EQUATION IN FREE SPACE

In free space: ρ = 0 and J = 0

Feel the beautiful symmetry

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$$

PHYSICAL SIGNIFICANCE OF MW EQUATION

Maxwell's equation combined with Lorentz's force law constitutes the entire Classical Electrodynamics.

Magnetic monopoles DO NOT exist in this universe.

ONLY moving charge particle create magnetic field.

Electric monopoles DO exist and that can create electro-static fields.

The left-hand side of all the four relations are identical but the right-hand side are not identical. This indicates that electrical and magnetic properties are not symmetric

PHYSICAL SIGNIFICANCE OF MW EQUATION

The left-hand side of all the four relations are identical but the right-hand side are not identical.

This indicates that electrical and magnetic properties are not symmetric

The time variation of electric field produces magnetic field.

Similarly, time variation of magnetic field produces electric field.

POLL QUESTION

Which is the only correct mathematical relation according to Maxwell′s Equation:

$$\text{A)}\vec{\nabla}\cdot\vec{\mathrm{E}} = -\frac{\partial\vec{\mathrm{E}}}{\partial \mathrm{t}},$$

$$\text{B)}\vec{\nabla}\cdot\vec{\mathrm{B}} = -\frac{\partial\vec{\mathrm{E}}}{\partial \mathrm{t}},$$

$$\boxed{\text{C)}\vec{\nabla}\cdot\vec{\mathrm{B}} = 0,}$$

$$\text{D)}\vec{\nabla}\times\vec{\mathrm{B}} = 0$$

OPTIONAL

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \, I_{enc}$$

Differential form of Ampere's Law

If J is the current density in the medium, $I_{enc} = \int \vec{J}.\overrightarrow{ds}$

Thus $\oint \vec{B}.\overrightarrow{dl} = \mu_0 \int_s \vec{J}.\overrightarrow{ds}$

Applying Stoke's theorem on LHS $\oint \vec{B}.\overrightarrow{dl} \Rightarrow \int_s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$

So $\int_s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds} = \mu_0 \int_s \vec{J}.\overrightarrow{ds}$

$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \Rightarrow \text{differential form of Ampere's law}}$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Differential form of Faraday's Law

$$\boldsymbol{\Phi_B = \int \vec{B} \cdot \overrightarrow{ds} \;\&\; \varepsilon = \int \vec{E} \cdot \overrightarrow{dl}} \Rightarrow \oint_l \vec{E}.\overrightarrow{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

Applying Stoke's theorem on LHS $\oint \vec{E}.\overrightarrow{dl} \Rightarrow \int_s (\vec{\nabla} \times \vec{E}).\overrightarrow{ds}$

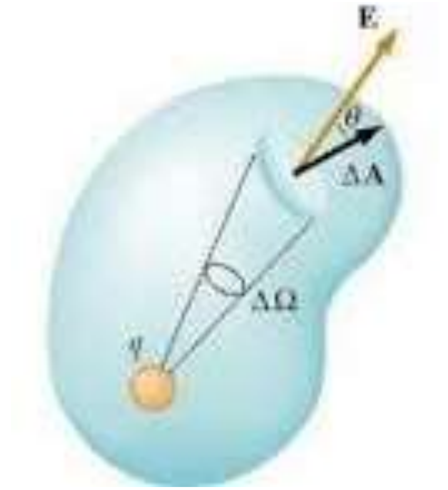
So $\int_s (\vec{\nabla} \times \vec{E}).\overrightarrow{ds} = -\int_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow$ differential form of Ampere's law

Maxwell's 1st relation

- ❖ It is based on the Gauss's law of electrostatics.
- ❖ *The total electric flux passing through any closed surface is equal to the $1/\varepsilon_0$ times the total charge enclosed by that surface.*

$$\oint_s \vec{E}.\overrightarrow{dS} = \frac{Q_{enc}}{\epsilon_0}$$



Rewriting Q in terms of the charge density, ρ $\quad Q_{enc} = \int_v \rho \, dv$

Thus $$\oint_s \vec{E}.\overrightarrow{ds} = \frac{1}{\epsilon_0}\int_v \rho \, dv$$ *Maxwell's 1st relation in integral form*

By applying Gauss divergence theorem $$\oint_s \vec{E}.\overrightarrow{dS} = \int_v (\vec{\nabla}.\vec{E})dv$$

So Gauss law becomes $$\int_v (\vec{\nabla}.\vec{E})dv = \int_v \frac{\rho}{\epsilon_0} dv$$

Since this holds for any volume, the integrands must be equal,

$$\boxed{\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0}}$$

Maxwell's 1st relation in differential form

Maxwell's 2nd relation:

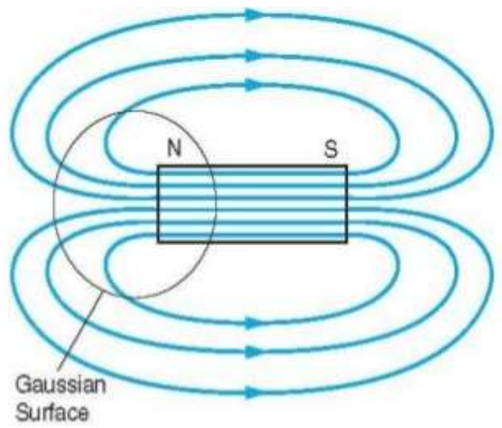
- It is based on the Gauss's law of magnetostatics
- *The total magnetic flux emerging through any closed surface is zero*

✓There are no magnetic monopoles.

✓The number of magnetic field lines emerging from any volume bounded a closed surface is equal to the number of lines entering the volume.

$$\varphi_m = \oint_s \vec{B}.\overrightarrow{ds} = 0$$

Maxwell's 2nd relation in integral form



By applying Gauss divergence theorem

$$\oint_s \vec{B}.\overrightarrow{dS} = \int_v (\vec{\nabla}.\vec{B})dv = 0$$

Hence

$$\boxed{\vec{\nabla}.\vec{B} = 0}$$

Maxwell's 2nd relation in differential form

Maxwell's 3rd relation

- It is based on the Faraday's law of electromagnetic induction
- *Whenever there is a change in the magnetic flux linked with a circuit, an emf is induced. The magnitude of the induced emf is directly proportional to the negative rate of variation of the magnetic flux linked with the circuit*

$$e = -\frac{\partial \varphi_m}{\partial t}$$

We know the magnetic flux through the entire circuit is $\varphi_m = \int_s \vec{B}.\overrightarrow{ds}$

The emf in a circuit can be represented as the line integral of the electric field around the closed path $e = \oint_l \vec{E}.\overrightarrow{dl}$

Hence $\oint_l \vec{E}.\overrightarrow{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$

Maxwell's 3rd relation in integral form

By using Stoke's theorem

$$\oint_l \vec{E}.\overrightarrow{dl} = -\int_s (\vec{\nabla} \times \vec{E}).\overrightarrow{ds}$$

Since this holds for any surface, the integrands must be equal, $\Rightarrow$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Maxwell's 3rd relation in differential form

Maxwell's 4th relation:

1. It is based on the Ampere's law modified by Maxwell.
2. *The line integral of the magnetic field B about any closed path is equal to the μ_0 times of the net current I_{enc} flowing through the area bounded by the curve*

Mathematically, $$\oint \vec{B}.\overrightarrow{dl} = \mu_0\, I_{enc}$$

If J is the current density in the medium, $I_{enc} = \int \vec{J}.\overrightarrow{ds}$

Thus $\oint \vec{B}.\overrightarrow{dl} = \mu_0 \int_s \vec{J}.\overrightarrow{ds}$ … … … … … … … … … … … … … … (1)

Applying Stoke's theorem $\oint \vec{B}.\overrightarrow{dl} = \int_s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$

So $\int_s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds} = \mu_0 \int_s \vec{J}.\overrightarrow{ds}$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ ……………….(2)

Taking the divergence on both sides, $\vec{\nabla}.\vec{J} = 0,$ *Since the divergence of curl of a vector is zero*

However, the equation of continuity for time varying fields is given by

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0, \textit{where } \rho \textit{ is the charge density}$$

This shows that Ampere's law is valid only for static fields and not for time varying fields

- Maxwell postulated that like changing magnetic field induces an electric field, a changing electric field also induces a magnetic field. $$\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- The magnetomotive force around a closed path is equal to the sum of the conduction current and the displacement current (rate of change of electric field).

- $\oint \vec{B}.\vec{dl} = \mu_0 \int_s (\vec{J} + \vec{J}_d).\vec{ds} = \mu_0 \int_s \vec{J}.\vec{ds} + \mu_0 \epsilon_0 \int_s \frac{\partial \vec{E}}{\partial t}.\vec{ds}$ *Maxwell's 4th relation in differential form*

- From equation 2, we can write

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \vec{J}_d) = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}$$

Maxwell's 4th relation in differential form