L41 21 April 2021

21 April 2021 12:13

① $\int_C f(x,y,z)\,ds$

② $\int_C \vec{u}\cdot d\vec{r}$

Evaluate ① $\boxed{\int_C f(x,y)\,dx}$ and ② $\int_C f(x,y)\,dy$

$f(x,y) = xy$ $\quad x = 3\cos t, \; y = 3\sin t$

$0 \le t \le \pi/4$

$\frac{dx}{dt} = -3\sin t$

$\int_C f(x,y)\,dx = \int_0^{\pi/4} f(t)\frac{dx}{dt}\,dt = \int_0^{\pi/4} (3\cos t)(3\sin t)\cdot(-3\sin t)\,dt$

$= -27\int_0^{\pi/4} \sin^2 t\cos t\,dt$

$\sin t = u$
$\cos t\,dt = du$
$-27\int u^2\,du$

$\int f^n(x) f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1}$

$= -27\left|\frac{\sin^3 t}{3}\right|_0^{\pi/4} = -\frac{27}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3\right]$

Evaluate

$\int_C f(x,y,z)\,dy$

$\frac{dy}{dt} = 1$

$f(x,y,z) = x+y+z$, $\quad x = t, \; y = t, \; z = t^2 \qquad 1 \le t \le 2$

$\int_C f(x,y,z)\,dy = \int_1^2 f(t)\cdot\frac{dy}{dt}\cdot dt = \int_1^2 (t+t+t^2)\cdot 1\,dt$

$= \int_1^2 (2t+t^2)\,dt = \left|\frac{\cancel{2}t^2}{\cancel{2}} + \frac{t^3}{3}\right|_1^2$

$= \left(4+\frac{8}{3}\right) - \left(1+\frac{1}{3}\right) = 3 + \frac{7}{3} = \frac{16}{3}$

$\int_c ds = \cdots\ldots\ldots?$, where C is the curve $x = 3\cos t, y = 3\sin t, 0 \le t \le \pi$

(a) 3π (b) π (c) 2π (d) $3\pi/2$

$x = 3\cos t$ $\quad 3\pi$

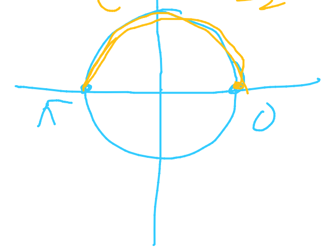
$$\int_C ds = \int_0^{\pi} \frac{ds}{dt}\, dt = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{(-3\sin t)^2 + (3\cos t)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{9}\, dt = 3\,[t]_0^{\pi} = 3\pi$$

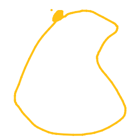
$x = 3\cos t$, $y = 3\sin t$ $\quad 3\pi$

$\frac{2\pi r}{2} = \frac{2\pi(3)}{2} = \frac{6\pi}{2}$



If V represent the force field then the work done by V along any simple closed path is ...?

(a) 0 (b) V (c) $2V$ (d) none of these



\# Evaluate the Line Integral $\int f\,dx + g\,dy + h\,dz$

$f = 2x+y,\quad g = y^2,\quad h = (x+z)\qquad C:\ y^2 = 2x,\ z = x,\quad 0 \le x \le 2$

$y = t$

$x = \frac{t^2}{2} \Rightarrow dx = \frac{2t}{2}\,dt$ $\qquad y^2 = 2x,\quad y = \pm\sqrt{2x}$

$z = \frac{t^2}{2}$

$$\int \left[\underbrace{\left(2\cdot\frac{t^2}{2} + t\right)}_{f(x)} \underbrace{t}_{dx} + t^2 + \left(\frac{t^2}{2} + \frac{t^2}{2}\right) t \right] dt$$

$$\int_{-2}^{2} \left(t^3 + t^2 + t^2 + t^3\right) dt = 2\left[\int_{-2}^{2} (t^3 + t^2)\, dt\right]$$

$0 \le x \le 2$

$0 \le \frac{t^2}{2} \le 2$

$0 \le t^2 \le 4$

$$= 2\cdot\left[\frac{t^4}{4} + \frac{t^3}{3}\right]_{-2}^{2}$$

$$= 2\left[\frac{8}{3} + \frac{8}{3}\right] = 2\cdot\frac{2\times 8}{3} = \frac{32}{3}$$

$\int_c r.dr = \cdots \ldots \ldots ?$, where C is the circle $x^2 + y^2 = a^2, z = 0$ and $r = xi + yj + zk$

(a) 0 (b) 1 (c) -1 (d) 3

?

$x = a\cos t$
$y = a\sin t$

$$\int x\,dx + y\,dy + \cancel{z\,dz}$$

$$= \int x\,dx + y\,dy$$

$$= \int a\cos t(-a\sin t) + (a\sin t)(a\cos t)\,dt = 0$$