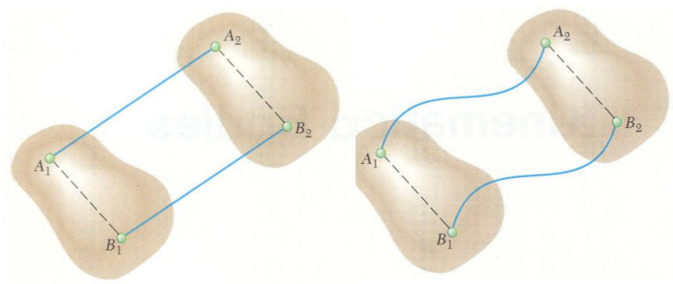
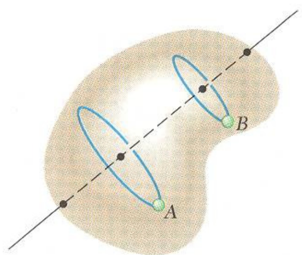
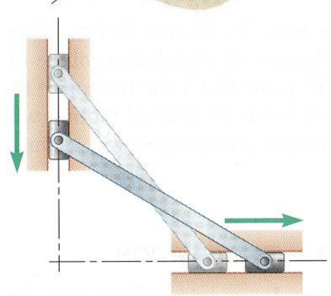
Engineering Mechanics: Dynamics

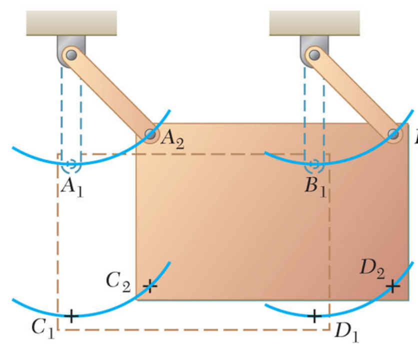
Introduction



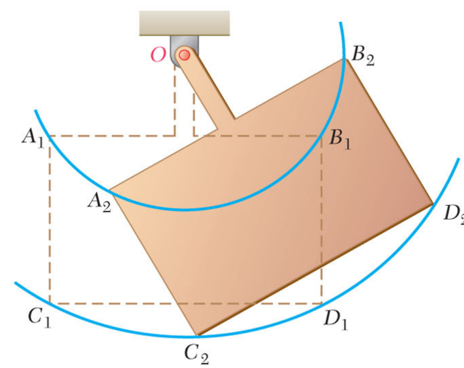




- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.
- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation - Fig (a)
  - rotation about a fixed axis - Fig (b)
  - general plane motion

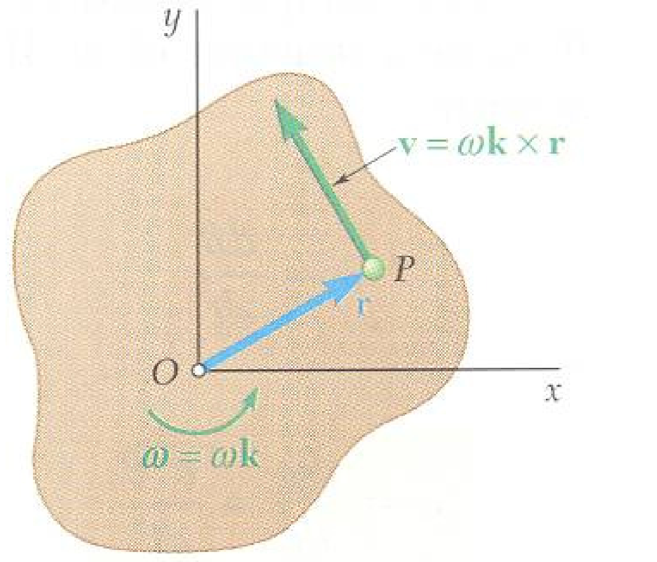


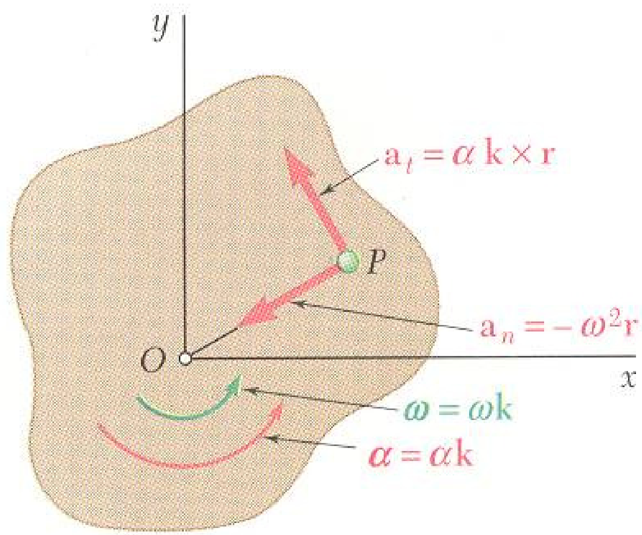
(a) Curvilinear translation



(b) Rotation

Rotation About a Fixed Axis.





- Consider the motion of a rigid body in a plane perpendicular to the axis of rotation.

- Velocity of any point *P* of the slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega\vec{k} \times \vec{r}$$
$$v = r\omega$$

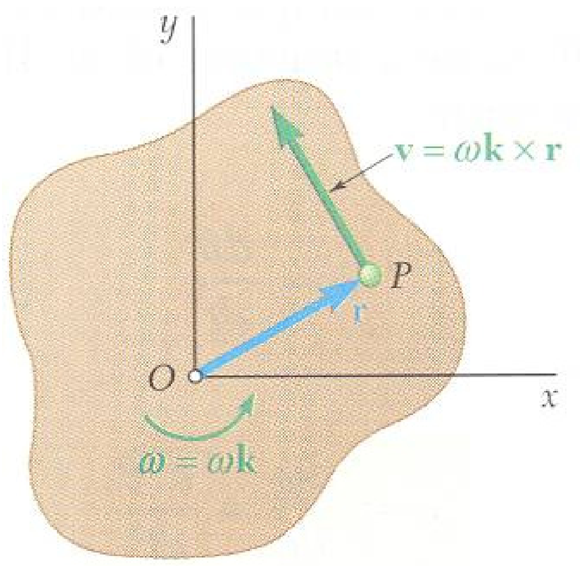
- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$= \alpha\vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha\vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \dfrac{d\theta}{dt}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation,* $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

- *Uniformly Accelerated Rotation,* $\alpha =$ constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

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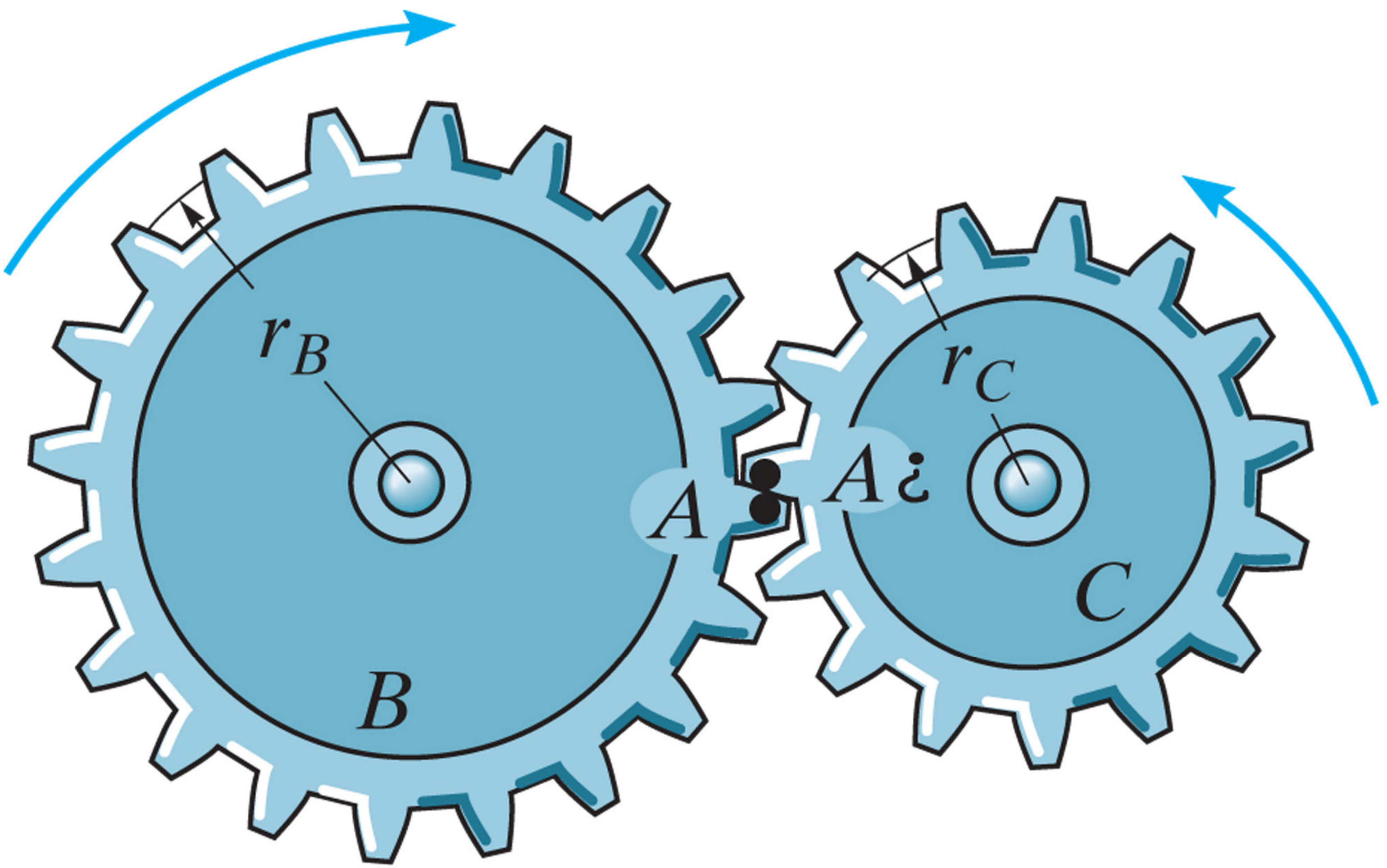
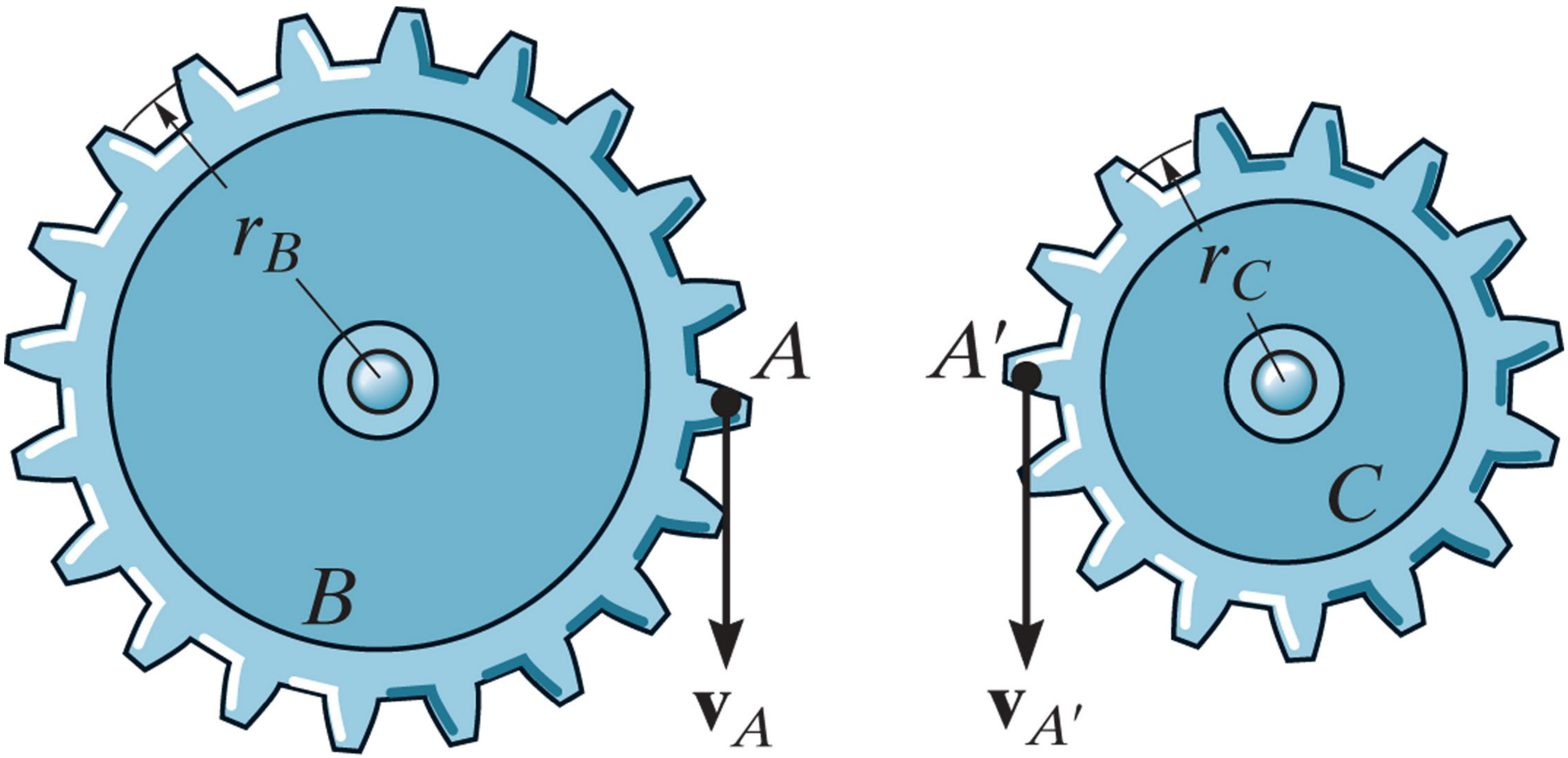
Two Rotating Bodies in Contact



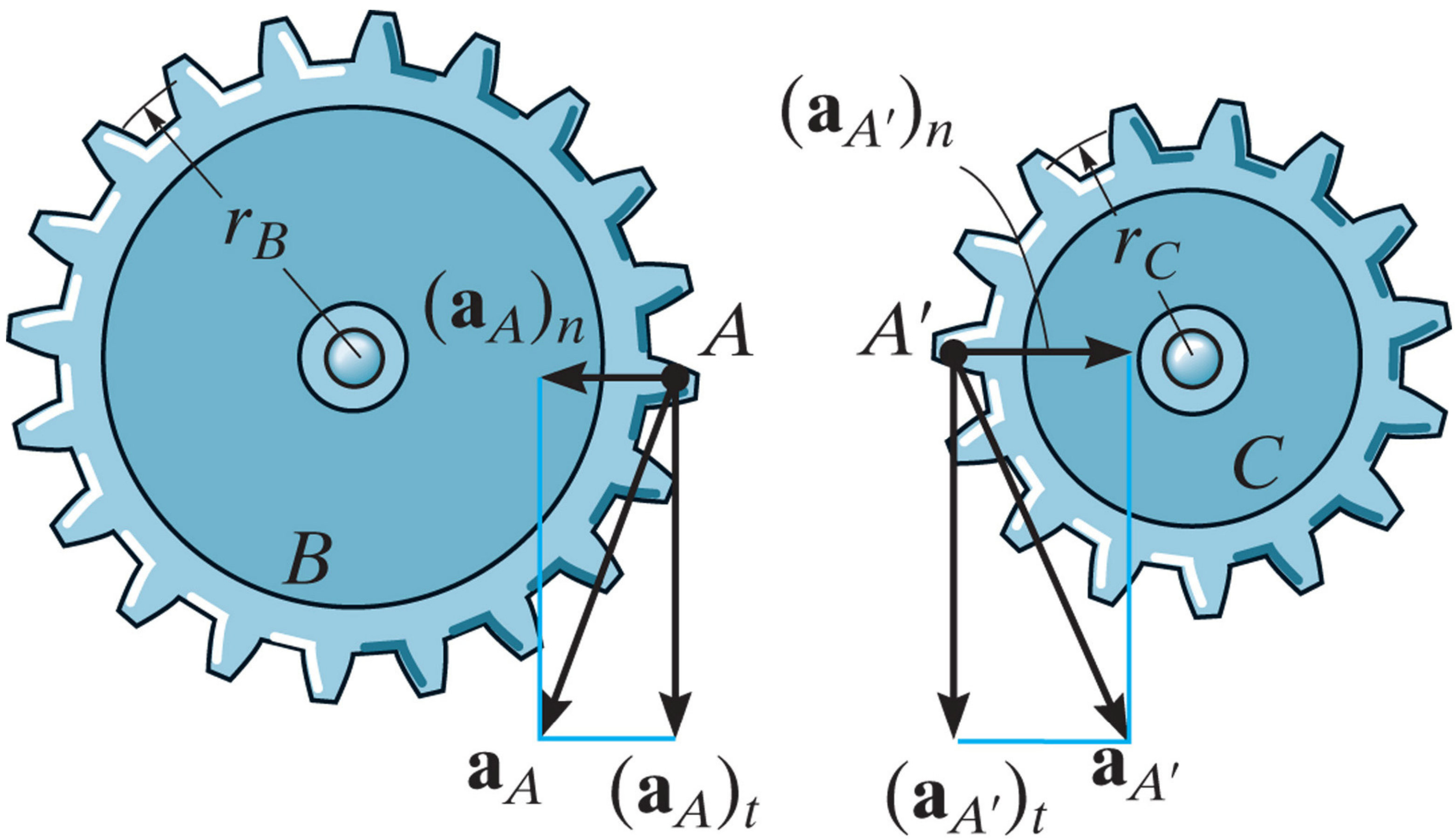
(a)

Two Rotating Bodies in Contact-Same Velocities & Tangential Acceleration



(b)

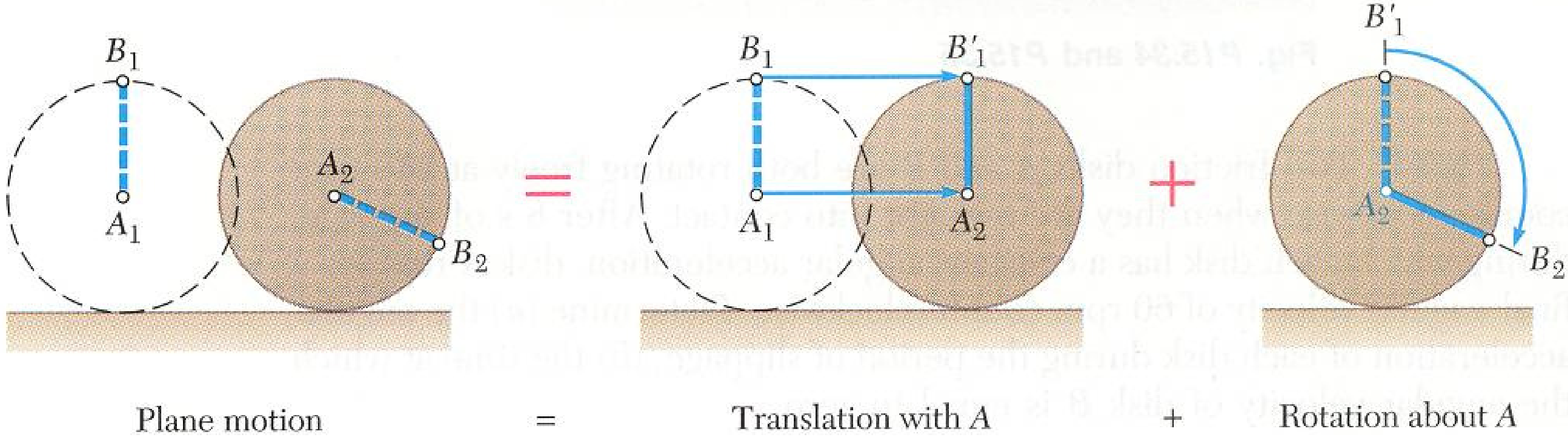
Two Rotating Bodies in Contact – Different Normal Accelerations



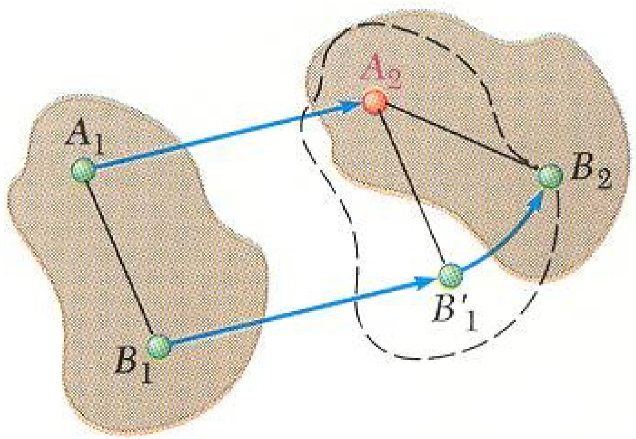
(c)

Engineering Mechanics: Dynamics

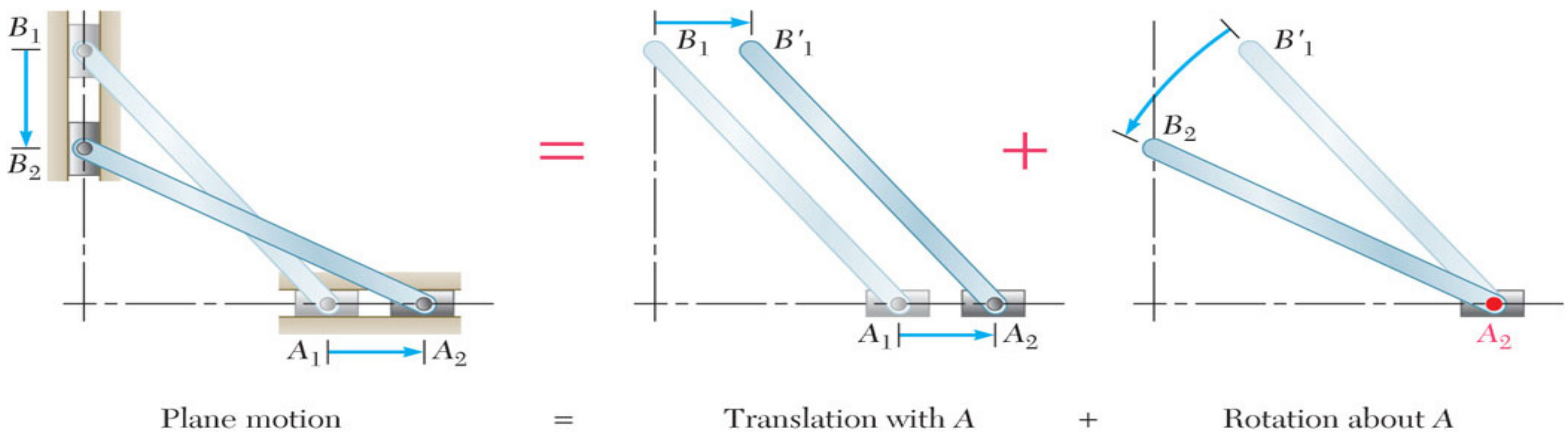
General Plane Motion



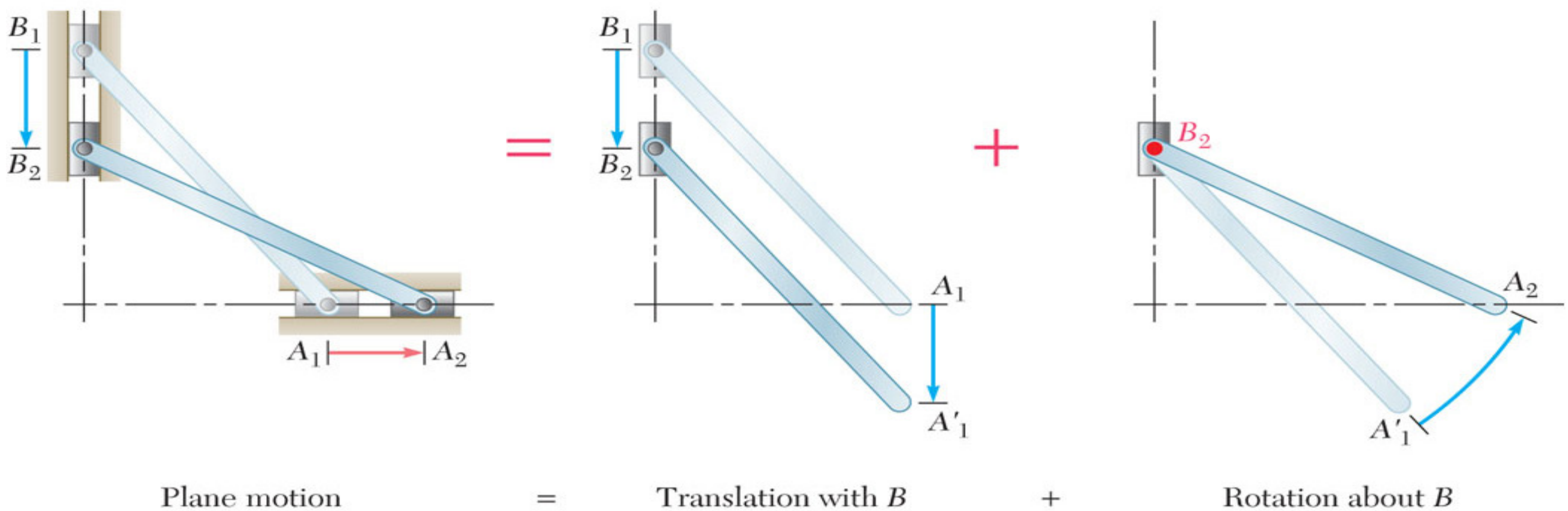
- *General plane motion* is neither a translation nor a rotation.
- General plane motion can be considered as the *sum* of a translation and rotation.
- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2



General Plane Motion

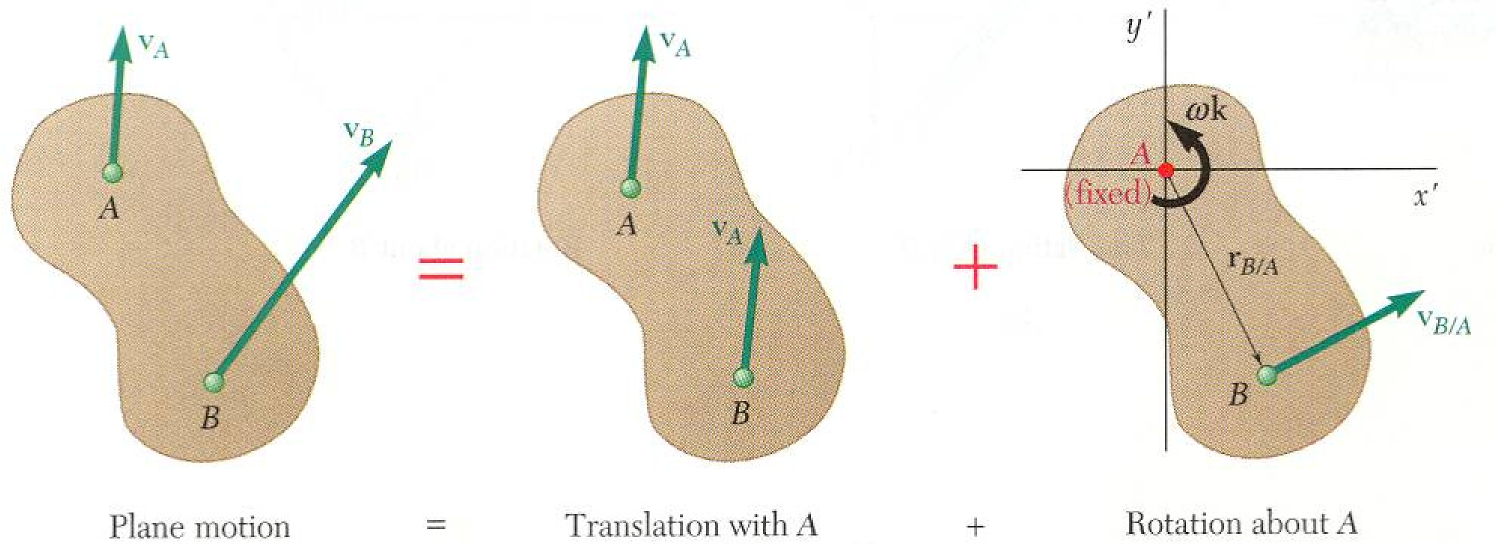


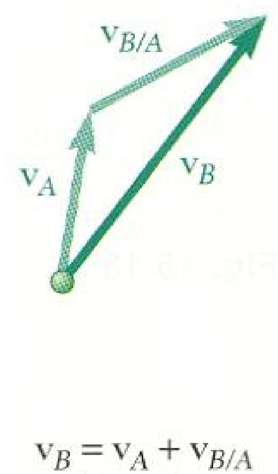
(a)



(b)

Absolute and Relative Velocity in Plane Motion





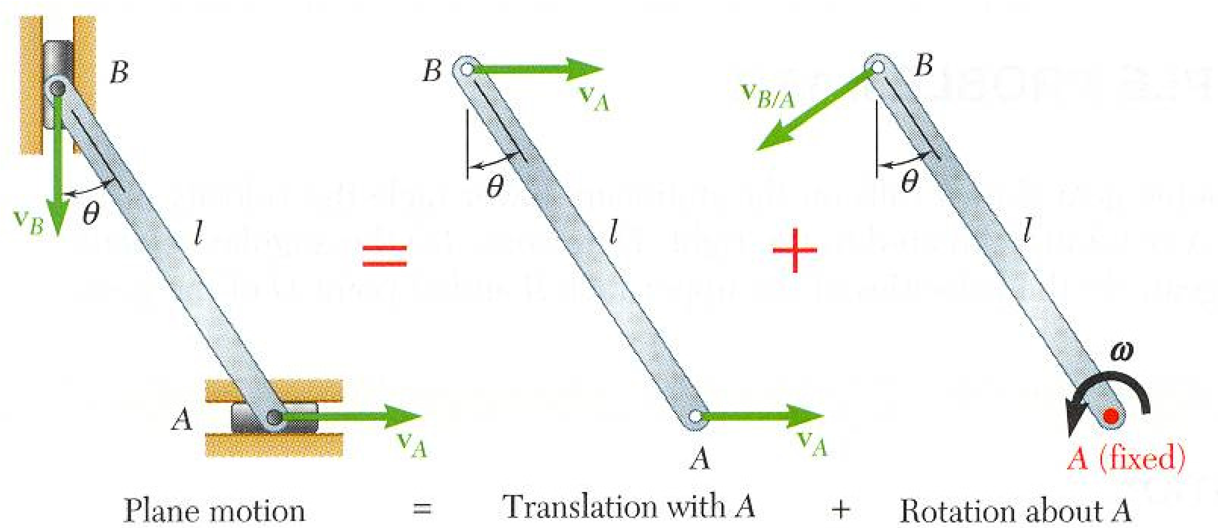
- Any plane motion can be replaced by a translation of an arbitrary reference point A and a simultaneous rotation about A.

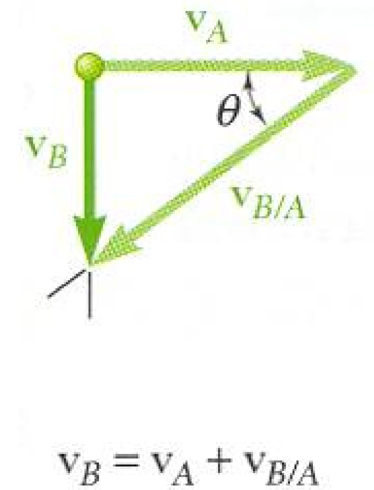
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{AB} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{AB}$$

Absolute and Relative Velocity in Plane Motion-Example

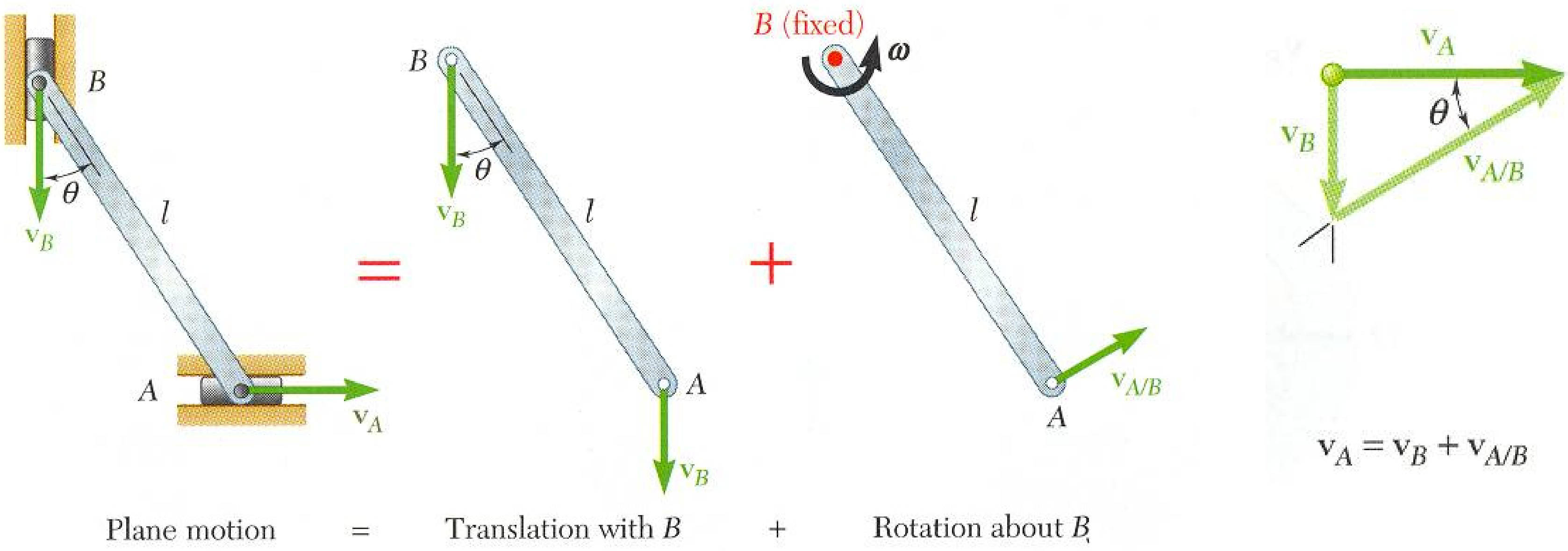




- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.

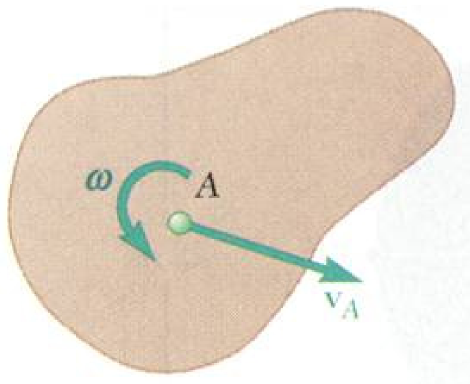
•The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

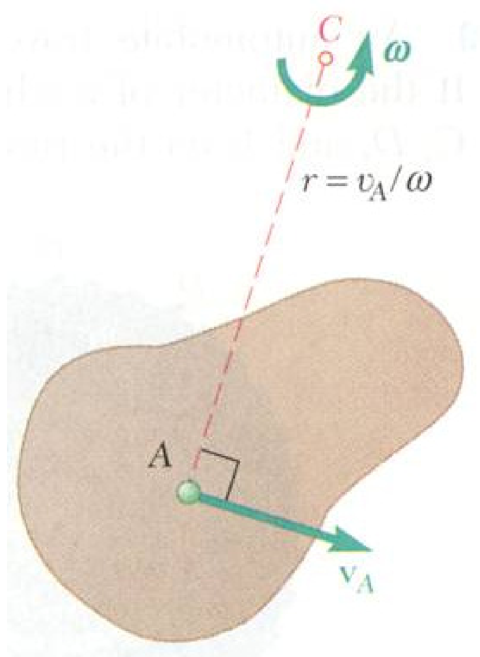
Absolute and Relative Velocity in Plane Motion



- Selecting point B as the reference point and solving for the velocity v_A of end A and the angular velocity ω leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity ω of the rod in its rotation about B is the same as its rotation about A. Angular velocity is not dependent on the choice of reference point.

Instantaneous Center of Rotation in Plane Motion





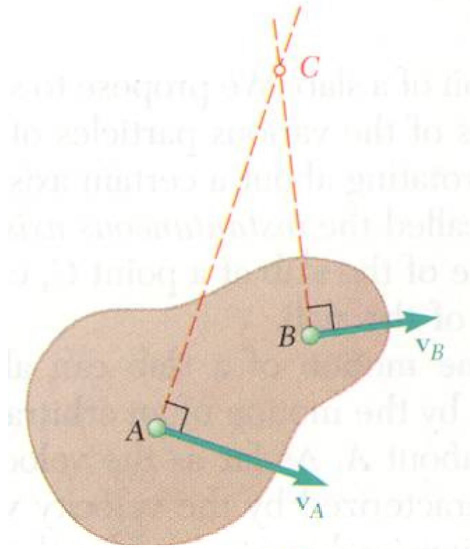
- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.

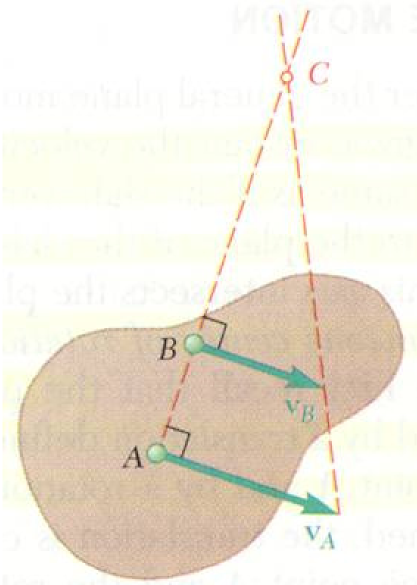
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.

- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

Instantaneous Center of Rotation in Plane Motion





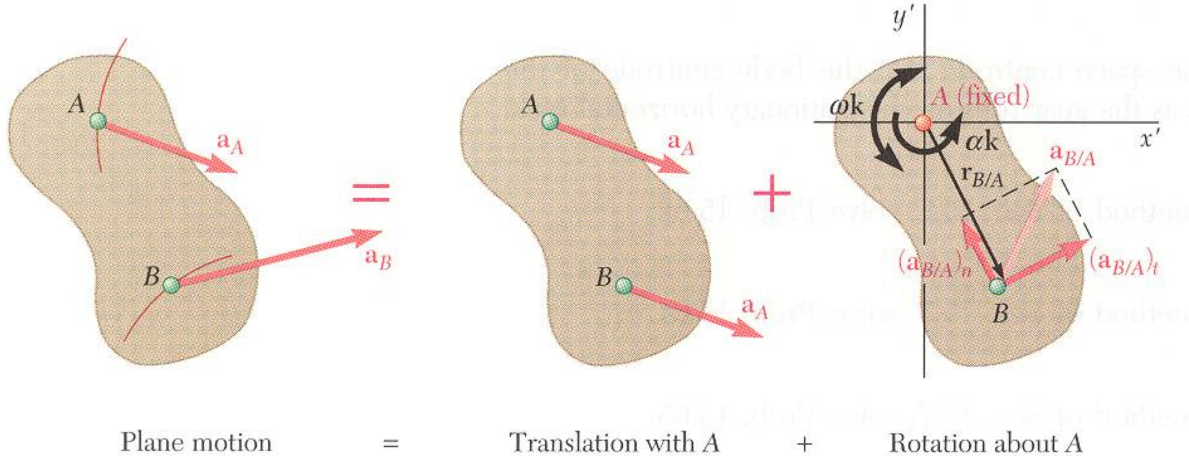
- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .

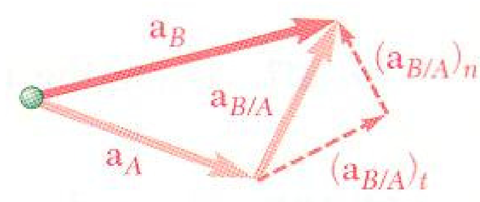
- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.

- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.

- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

Absolute and Relative Acceleration in Plane Motion





- Absolute acceleration of a particle of the slab,

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about A includes tangential and normal components,

$$\left(\vec{a}_{B/A}\right)_t = \alpha\vec{k}\times\vec{r}_{AB} \qquad \left(a_{B/A}\right)_t = r\alpha$$

$$\left(\vec{a}_{B/A}\right)_n = -\omega^2\vec{r}_{AB} \qquad \left(a_{B/A}\right)_n = r\omega^2$$