OBJECTIVE TYPE QUESTIONS

A. Fill up the blanks

1. The formula for the Fourier coefficients a_n, b_n for $f(x)$ in $(-\pi, \pi)$ are ______________.
2. If $f(x)$ is an even function in $(-\pi, \pi)$, then the Fourier coefficients are $a_n =$ __________, $b_n =$ __________.
3. If $f(x) = x^2 + x$ is expressed as a Fourier series in $(-2, 2)$, then $f(2) =$ ____________.
4. If the Fourier series for the function $f(x)=\begin{cases}0, & 0<x<\pi \\ \sin x, & \pi<x<2\pi\end{cases}$ is

   $$f(x)=-\frac{1}{\pi}+\frac{2}{\pi}\left[\frac{\cos 2x}{1\cdot 3}+\frac{\cos 4x}{3\cdot 5}+\frac{\cos 6x}{5\cdot 7}+\cdots\right]+\frac{\sin x}{2}$$, then $\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}-\cdots=$ ______________.
5. The half-range sine series for $f(x) = x$ in $(0, \pi)$ is __________.
6. The Dirichlet's conditions for $f(x)$ is $c < x < c + 2\pi$ to have a Fourier series expansion are ___________.
7. The value of $f(2)$ in the half-range cosine series for $f(x) = x^2$ in $(0, 2)$ is ___________.
8. The root mean square value of $f(x) = x^2$ in $(0, 6)$ is ___________.
9. The half-range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$ is $x(\pi-x)=\frac{8}{\pi}\left[\frac{\sin x}{1^3}+\frac{\sin 3x}{3^3}+\frac{\sin 5x}{5^3}+\cdots\right]$ then the value of $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots=$ ________
10. The half-range cosine series for $f(x) = (x - 1)^2$ in $(0, 1)$ is $f(x)=\frac{1}{3}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi x$, then the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}$ is ___________.
11. The Fourier series for $f(x) = x$ in $(0, 2\pi)$ is $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$, then the value of $\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ is ___________.
12. If the half-range cosine series of $f(x)=\begin{cases}\pi x, & 0\le x\le 1 \\ \pi(2-x), & 1<x\le 2\end{cases}$ is

    $$f(x)=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos \pi x}{1^2}+\frac{\cos 3\pi x}{3^2}+\frac{\cos 5\pi x}{5^2}+\cdots\right]$$ then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=$ ____________.
13. If the Fourier series of $f(x) = x(2\pi - x)$ in $(0, 2\pi)$ is $x(2\pi-x)=\frac{2\pi^2}{3}-4\sum_{n=1}^{\infty}\frac{\cos nx}{x^2}$, then the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=$ ____________.

14. If $f(x)$ is discontinuous at $x = a$, then the sum of the Fourier series of $f(x)$ when $x = a$ is ________.
15. The Parseval's identity for the half-range cosine expansion of $f(x)$ in $(0, 1)$ is ________.

B. Choose the correct answer

1. The value of the constant term in the Fourier series expansion of $\cos^2 x$ in $(-\pi, \pi)$ is
   (a) 1 (b) $\frac{1}{2}$ (c) $\frac{\pi}{2}$ (d) π
2. The value of b_n in the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ is
   (a) 0 (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{\pi^2}{4}$
3. The value of a_n in the Fourier series of $f(x) = x - x^3$ in $(-\pi, \pi)$ is
   (a) $\frac{\pi}{2}(2-\pi^2)$ (b) $\frac{\pi}{4}(2-\pi^2)$ (c) 0 (d) None of these
4. The Fourier of $f(x)=\begin{cases}\sin x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi,\end{cases}$ of period 2π is
   $f(x)=\frac{1}{\pi}+\frac{1}{2}\sin x-\frac{2}{\pi}\left[\frac{\cos 2x}{1\cdot 3}+\frac{\cos 4x}{3\cdot 5}+\frac{\cos 6x}{3\cdot 7}+\ldots\right]$, then the value of $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots$ is
   (a) 1 (b) π (c) $\frac{1}{2}$ (d) $\frac{\pi}{2}$
5. The Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ is $\frac{\pi^2}{3}+\sum_{n=1}^{\infty}(-1)^n\left[\frac{4}{n^2}\cos nx-\frac{2}{n}\sin nx\right]$, then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is
   (a) $\frac{\pi-2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$
6. If $f(x) = 2x$ in $(0, 4)$, then the value of a_2 in the Fourier series expansion of period 4 is
   (a) 4 (b) 2 (c) 0 (d) 3
7. The root mean square value of $f(x) = 1 - x$ in zz$0 < x < 1$ is
   (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1
8. If the Fourier series for $f(x)$ in $(0, 2\pi)$ is $f(x) = \sum_{n=1}^{\infty}\frac{\sin nx}{n}$, then the root mean value is
   (a) $\frac{\pi}{2\sqrt{3}}$ (b) $\frac{\pi}{\sqrt{3}}$ (c) $\frac{\pi}{3\sqrt{2}}$ (d) $\frac{\pi^2}{\sqrt{3}}$
9. The Fourier coefficient b_n for $x\sin x$ in $[-\pi, \pi]$ is
   (a) $\frac{1}{2}$ (b) 0 (c) $\frac{\pi}{\sqrt{3}}$ (d) $\frac{\pi}{3}$
10. The Fourier series for $f(x)=\begin{cases}-k, & -\pi < x < 0 \\ k, & 0 < x < \pi\end{cases}$ is $f(x)=\frac{4k}{\pi}\left[\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\ldots\right]$,
    then the value of $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi^2}{4}$
11. The half-range cosine series for $f(x) = x$ in $(0, \pi)$ is $x = \frac{\pi}{2}-\frac{4}{\pi}\sum_{n \text{ is odd}}\frac{\cos nx}{n^2}$, then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is
    (a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{12}$ (d) $\frac{\pi}{4}$

12. The half-range cosine series for $f(x)=x(\pi-x)$ in $0<x<\pi$ is $x(\pi-x)=\frac{\pi^2}{6}-\left[\frac{\cos 2x}{1^2}+\frac{\cos 4x}{2^2}+\frac{\cos 6x}{3^2}+\ldots\right]$,

then the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}=$ (a) $\frac{\pi^4}{8}$ (b) $\frac{\pi^4}{96}$ (c) $\frac{\pi^4}{90}$ (d) $\frac{\pi^2}{90}$

13. If the Fourier series of $f(x)=x(2l-x)$ is $(0, 2l)$ of period $2l$ is $f(x)=\frac{2}{3}l^2-\frac{4}{\pi^2}l^2\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{l}\right)$,

then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$ is (a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{12}$ (d) $\frac{\pi^2}{4}$

14. If $x=\frac{l}{2}-\frac{4l}{\pi^2}\left(\cos\frac{\pi x}{l}+\frac{1}{3^2}\cos\frac{3\pi x}{l}+\frac{1}{5^2}\cos\frac{5\pi x}{l}+\cdots\right)$ in $0<x<l, f(x+2l)=f(x)$,

then the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$ is (a) $\frac{\pi^2}{32}$ (b) $\frac{\pi^4}{96}$ (c) $\frac{\pi^4}{90}$ (d) None of these

15. If the half-range cosine series for $f(x)=(x-1)^2, 0<x<1$, is $f(x)=\frac{1}{3}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi x$, then the value of

$\sum_{n=1}^{\infty}\frac{1}{n^4}$ is (a) $\frac{\pi^4}{90}$ (b) $\frac{\pi^4}{96}$ (c) $\frac{\pi^2}{16}$ (d) None of these

ANSWERS

A. Fill up the blanks

1. $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx,\ n=0,1,2,3,\ldots,$ $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx,\ n=1,2,3,\ldots$

2. $a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nx\,dx,\ n=0,1,2,\ldots$ and $b_n=0$, 3, 4

3. 4

4. $\frac{\pi-2}{4}$

5. $2\left[\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-\cdots\right]$

6. Refer definition 17.3, page 17.2.

7. 4

8. $\frac{l^2}{\sqrt{5}}$

9. $\frac{\pi^3}{32}$

10. $\frac{\pi^4}{90}$

11. $\frac{8\pi^2}{3}$

12. $\frac{\pi^2}{8}$

13. $\frac{\pi^2}{6}$

14. $\frac{1}{2}[f(a-)+f(a+)]$

15. $\int_0^1[f(x)^2]dx=\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}a_n^2$

B. Choose the correct answer

1. (b) 2. (a) 3. (c) 4. (c) 5. (b) 6. (c) 7. (b) 8. (a) 9. (b) 10. (c)
11. (b) 12. (c) 13. (c) 14. (b) 15. (a)