

Case No:-3 :-  $\frac{1}{f(D)} \cdot x^m = (f(D))^{-1} \cdot x^m$

$\downarrow$  binomial

Find the general solution of:  $y'' + 25y = 4x^2$

C.F.

$$D^2 + 25 = 0$$

$$D^2 = -25$$

$$D = \pm i5$$

$$y_C = C_1 \cos 5x + C_2 \sin 5x.$$

$$\left| \begin{array}{l} \frac{1}{D^2 + 25} \cdot 4x^2 = \frac{1}{25(1 + \frac{D^2}{25})} \cdot 4x^2 \\ = \frac{4}{25} \left(1 + \frac{D^2}{25}\right)^{-1} \cdot x^2 \end{array} \right.$$

$$\left\{ \begin{array}{l} (1+x)^{-1} = 1-x+x^2-x^3+x^4-\dots \\ (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\dots \\ (1-x)^{-1} = 1+x+x^2+x^3+\dots \end{array} \right.$$

$$\frac{4}{25} \left(1 + \frac{D^2}{25}\right) \cdot x^2 = \frac{4}{25} \left(1 - \frac{D^2}{25} + \frac{D^4}{(25)^2} - \dots\right) x^2$$

$$= \frac{4}{25} \left(x^2 - \frac{D^2(x^2)}{25} + \frac{D^4(x^2)}{(25)^2} - \dots\right)$$

$$\begin{aligned} D(x^2) &= 2x \\ D^2(x^2) &= 2 \\ D^3(x^2) &= 0 \end{aligned}$$

$$= \frac{4}{25} \left(x^2 - \frac{2}{25} + 0 - \dots\right) = \frac{4}{25} \left(25x^2 - 2\right) x^2$$

Find the general solution of:  $y'' - 6y' + 9y = 4x^2 - 1$

C.F.

A.E.

$$D^2 - 6D + 9 = 0$$

$$(D-3)^2 = 0$$

$$D = 3, 3$$

$$y_C = (C_1 + C_2 x) e^{3x}$$

$$\begin{aligned}
 \text{P.I} \quad & \frac{1}{D^2 - 6D + 9} \cdot (4x^2 - 1) = \frac{1}{9(1 + \frac{D^2 - 6D}{9})} (4x^2 - 1) \\
 & = \frac{1}{9} \left( 1 + \frac{D^2 - 6D}{9} \right)^{-1} (4x^2 - 1) \quad \checkmark \\
 & = \frac{1}{9} \left[ 1 - \left( \frac{D^2 - 6D}{9} \right) + \left( \frac{D^2 - 6D}{9} \right)^2 - \dots \right] (4x^2 - 1) \quad \checkmark \\
 & = \frac{1}{9} \left[ 1 - \left( \frac{D^2 - 6D}{9} \right) + \frac{1}{81} (36D^2) \right] (4x^2 - 1) \\
 & = \frac{1}{9} \left[ (4x^2 - 1) - \frac{1}{9} [8 - 6(8x)] + \frac{36}{81} (8) \right]
 \end{aligned}$$

$$\begin{aligned}
 D(4x^2 - 1) &= 8x \\
 D^2(4x^2 - 1) &= 8 \\
 D^3(4x^2 - 1) &= 0 \\
 \hline
 (D^2 - 6D)^2 &= 8 \\
 -\cancel{8} + 36D - 12\cancel{D}^2 &
 \end{aligned}$$

$$\frac{4x^2 - 1}{9} - \frac{8}{9} + \frac{48}{9}x + \frac{36}{81} \times 8 = 0$$

$$\# (D^2 - 5D + 6)y = x + e^{mx}, \quad m \neq \underline{2, 3}$$

$$\begin{aligned}
 \text{GF} \quad & A \cdot E \quad D^2 - 5D + 6 = 0 \\
 & D^2 - 2D - 3D + 6 = 0 \\
 & D(D-2) - 3(D-2) = 0 \\
 & (D-3)(D-2) = 0 \\
 & D = 2, 3 \quad \checkmark
 \end{aligned}$$

$$D^2 \Rightarrow -9^2$$

$$\text{P.I} \quad \frac{1}{D^2 - 5D + 6} \cdot (x + e^{mx}) = \frac{1}{D^2 - 5D + 6} \cdot x + \frac{1}{D^2 - 5D + 6} \cdot e^{mx}$$

$$\Rightarrow \frac{1}{6 \left[ 1 + \left( \frac{D^2 - 5D}{6} \right) \right]} \cdot x + \frac{1}{m^2 - 5m + 6} \cdot e^{mx} \quad \underline{m \neq 2, 3}$$

$$D \Rightarrow m$$

$$= \frac{1}{6} \left[ 1 + \left( \frac{D^2 - 5D}{6} \right) \right]^{-1} \cdot x + \frac{1}{2x^2 - 5x + 6} \cdot e^{mx}$$

$$D(x) = 1$$

$$= \frac{1}{6} \left[ 1 - \left( \frac{D^2 - 5D}{6} \right) \right] \cdot x + \frac{1}{2x^2 - 5x + 6} \cdot e^{mx}$$

$$D^2(x) = 0$$

$$= \frac{1}{6} \left[ x + \frac{5}{6} \right] + \frac{1}{2x^2 - 5x + 6} \cdot e^{mx}$$

**Q34.** The general solution of the equation  $y'' + 16y = x^2$  is

- (a) ~~A cos x + B sin x + 4x<sup>2</sup>~~
- (b) A cos 4x + B sin 4x + 4x<sup>2</sup> -  $\frac{1}{2}$
- (c) A cos 4x + B sin 4x + 4x
- (d) None of these

$$(D^2 + 16)y = x^2$$

$$D^2 + 16 = 0$$

$$D^2 = -16$$

$$D = \pm 4i$$

$$\frac{1}{D^2 + 16} \cdot x^2 = \frac{1}{16} \left( 1 + \frac{D^2}{16} \right)^{-1} x^2$$

$$y_C = C_1 \cos 4x + C_2 \sin 4x$$

$$\frac{1}{16} \left[ \left( 1 + \frac{D^2}{16} \right)^{-1} x^2 \right] = \frac{1}{16} \left[ 1 - \frac{D^2}{16} \right] x^2$$

$$D(x^2) = 2x$$

$$D^2(x^2) = 2$$

$$D^3(x^2) = 0$$

$$\frac{1}{16} \left( x^2 - \frac{1}{16} \cdot \frac{x^2}{8} \right) = \frac{1}{16} \left( x^2 - \frac{1}{8} \right)$$

# Case 4  $\therefore s(x) = \frac{e^{ax} \cdot g(x)}{f(D)}$

$$\frac{1}{f(D)} \cdot e^{ax} \cdot g(x) = e^{ax} \cdot \frac{1}{f(D+a)} \cdot g(x)$$

$$f(x) = x^m$$

$$f(m) = 0$$

P.I. Find the general solution of:  $y'' - 4y' + 5y = 24e^{2x} \sin x$

P.I.  $\frac{1}{D^2 - 4D + 5} \cdot 24 \cdot \underline{\underline{e^{2x} \sin x}} = 24 \cdot \frac{1}{D^2 - 4D + 5} \cdot e^{2x} \sin x$

$$= 24 \cdot e^{2x} \cdot \frac{1}{(D+2)^2 - 4(D+2) + 5} \cdot \sin x$$

$$= 24 \cdot e^{2x} \cdot \frac{1}{D^2 + 4 + 4D - 4D - 8 + 5} \cdot \sin x$$

$$= 24 \cdot e^{2x} \cdot \frac{1}{D^2 + 1} \sin x = 24 \cdot e^{2x} \cdot \frac{1}{-1+1} \sin x$$

↓ Case of failure

$$= 24 \cdot e^{2x} x \frac{1}{2D} \sin x$$

$$= \frac{12}{2} \cdot e^{2x} x \frac{1}{D} \sin x$$

$$= \underline{12 \cdot e^{2x} x (-\cos x)} = -12 \cdot e^{2x} \cos x$$

D

$$\frac{-1}{D} = \frac{1}{D}$$

Find the general solution of:  $y'' - y' - 6y = xe^{-2x}$