L5 30 Jan 2021

30 January 2021 11:04

$(2y^2xe^y + y + 1)y\,dx$ $\qquad f(xy)$

$(2y^3xe^y + y^2 + y)dx + (y^3x^2e^y - xy - 2x)dy = 0$

$M\,dx + N\,dy = 0$

$M = 2y^3xe^y + y^2 + y$

$\frac{\partial M}{\partial y} = 2xe^y(3y^2) + 2xy^3e^y + 2y + 1$

$= 6xy^2e^y + 2xy^3e^y + 2y + 1$

$N = y^3e^yx^2 - xy - 2x$

$\frac{\partial N}{\partial x} = y^3e^y(2x) - y - 2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ the eqn is not exact.

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 6xy^2e^y + \cancel{2xy^3e^y} + 2y + 1 - \cancel{2xy^3e^y} + y + 2$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{6xy^2e^y + 3y + 3}{M} = \frac{3\cancel{(2xy^2e^y + y + 1)}}{y\cancel{(2xy^2e^y + y + 1)}}$

$= \frac{3}{y} = g(y)$

$I.F = e^{\int -\frac{3}{y}dy} = e^{-3\int \frac{1}{y}dy} = e^{-3\log y} = \cancel{e}^{\log y^{-3}} = \frac{1}{y^3}$

$\frac{(2y^3xe^y + y^2 + y)}{y^3}dx + \frac{(y^3x^2e^y - xy - 2x)}{y^3}dy = 0$

$\int M\,dx + \int (\text{Terms of } N \text{ free from } x)\,dx = c$
y const.

Sol $\int_{y \text{ const}} \left(2xe^y + \frac{1}{y} + \frac{1}{y^2}\right)dx + \int (0) = \cancel{f_0}\; C$

$\frac{\cancel{2}x^2}{\cancel{2}}e^y + \frac{1}{y}x + \frac{1}{y^2}x = C$ Ans

Q8. Which of the following is the solution of $(x^3 + xy^2 - xa^2)dx + (x^2y - y^3 - b^2y)dy = 0$

(a) $\cancel{x^3} - 3axy + y^3 = c$ **(b)** $x^4 + 2x^2y^2 - 2a^2x^2 - 2b^2y^2 - y^4 = c$

(c) $\cancel{x^3} - 6x^2y - 6xy^2 + y^3 = c$ **(d)** $\cancel{x^3} + 3x^2y^2 + y^4 = c$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 2xy$

$\frac{x^4}{4} + \frac{y^2x^2}{2} - \frac{a^2x^2}{2} - \frac{y^4}{4} - \frac{b^2y^2}{2} = C$

$$\boxed{x^4 + 2x^2y^2 - 2a^2x^2 - y^4 - 2b^2y^2 = c}$$ Q3

Q9. Which of the following is the solution of $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$

(a) $x^3 - 3axy + y^3 = c$ (b) $x^4 + 2x^2y^2 - 2a^2x^2 - 2b^2y^2 - y^4 = c$

(c) $x^3 - 6x^2y - 6xy^2 + y^3 = c$ (d) $x^3 + 3x^2y^2 + y^4 = c$

$$\frac{\partial M}{\partial y} = 12xy \qquad \frac{\partial N}{\partial x} = 12xy$$

Sol

$$\frac{3x^3}{3} + y^2 \cdot \frac{6x^3}{x} \cdot 3 + \frac{4y^4}{y} = c$$

$$x^3 + 3x^2y^2 + y^4 = c$$

Q2. What is the relationship between a and b so that the $(x + x^8 + ay^2)\,dx + (y^8 - y + bxy)dy = 0$ is exact

(a) $b = 2a$ (b) $a = b$ (c) $a \neq b$ (d) $a = 1, b = 3$

$$M = x + x^8 + ay^2 \qquad \frac{\partial M}{\partial y} = 2ay \qquad \frac{\partial N}{\partial x} = by$$

$$2ay = by$$

$$\boxed{2a = b}$$

Find I.F

$$\frac{ydx - xdy}{x^2} + \frac{e^{1/x}\,dx}{x^2} = 0$$

$$\frac{1}{x} = t \qquad -\frac{1}{x^2}dx = dt$$

$$-d\left(\frac{y}{x}\right) + \left(-\int e^t\,dt\right)$$

$$\int -d\left(\frac{y}{x}\right) - e^t = c$$

$$\boxed{-\frac{y}{x} - e^{1/x} = c}$$

Q5. Solution of $xdy + ydx = 0$ is represented by

(a) $xy = c$ (b) $\frac{x}{y} = c$ (c) $x + y = c$ (d) $x - y = c$

$$xdy + ydx = 0$$

$$\int d(xy) = 0$$

$\downarrow$

$\int d(xy) = 0$

$xdy + ydx = 0$

$xy = c$

5

For the eqn of the type

$$x^a y^b (my\,dx + nx\,dy) + x^{a'} y^{b'} (m'y\,dx + n'x\,dy) = 0$$

I.F is $x^h y^k$ where $\frac{a+h+1}{m} = \frac{b+k+1}{n}$, $\frac{a'+h+1}{m'} = \frac{b'+k+1}{n'}$

$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

$\Rightarrow xy(ydx + xdy) + x^2y^2(2ydx - xdy) = 0$

$a = 1 \quad b = 1 \quad m = 1, \; n = 1 \qquad a' = 2, \; b' = 2, \; m' = 2, \; n' = -1$

$\frac{a+h+1}{m} = \frac{b+k+1}{n} \Rightarrow \frac{1+h+1}{1} = \frac{1+k+1}{1} \Rightarrow \boxed{h = k}$

$\frac{a'+h+1}{m'} = \frac{b'+k+1}{n'} \Rightarrow \frac{2+h+1}{2} = \frac{2+k+1}{-1} \Rightarrow -2-h-1 = 4+2k+2$

$\Rightarrow 6+2k+h+3 = 0$

$\boxed{h+2k+9=0}$

$k+2k+9=0$

$3k+9=0$

$k = -3 \qquad h = -3$

$$I.F = x^h y^k = \frac{1}{x^3y^3}$$