Example 4·6. *A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, two officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner:*

(i) There must be one from each category.

(ii) It should have at least one from the purchase department.

(iii) The chartered accountant must be in the committee.

Sol

Production	3
Purchase	4
Sales	2
CA	1
	10

$TC = {}^{10}C_4$

(i) $FC = {}^{3}C_1 \times {}^{4}C_1 \times {}^{2}C_1 \times {}^{1}C_1$

$$P = \frac{3 \times 4 \times 2 \times 1}{{}^{10}C_4}$$

Sol (11)

Purchased (4)	other (6)
1	3
2	2
3	1
4	0

$^4C_1 \times {}^6C_3 +$
$^4C_2 \times {}^6C_2 +$
$^4C_3 \times {}^6C_1 +$
$^4C_4 \times {}^6C_0$

$TC = {}^{10}C_4$

$$P = \frac{^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^4C_4 \times {}^6C_0}{^{10}C_4}$$

(iii) CA — — — other (9)

Sol

$FC = {}^{9}C_{3}$ $\quad TC = {}^{10}C_{4}$

$$P = \frac{{}^{9}C_{3}}{{}^{10}C_{4}} = \frac{\frac{9\times 8\times 7}{3!}}{\frac{10\times 9\times 8\times 7}{4!}}$$

$$\boxed{P = \frac{2}{5}}$$

$$= \frac{\cancel{9}\times\cancel{8}\times\cancel{7}}{3!}\times\frac{4!}{10\times\cancel{9}\times\cancel{8}\times\cancel{7}} = \frac{4\times\cancel{3!}}{\cancel{3!}\times 10} = \frac{4}{10} = \frac{2}{5}$$

7. *(a)* If the letters of the word RANDOM be arranged at random, what is the chance that there are exactly two letters between A and O.

(b) Find the probability that in a random arrangement of the leters of the word 'UNIVERSITY', the two I's do not come together.

Sol (a) R~~A~~ND~~O~~M

_ _ _ _ _ _

$FC' = \underline{3} \times \underline{2} \times {}^4P_4$

$= 3 \times 2 \times 4!$

$P = \dfrac{3 \times 2 \times 4!}{6 \times 5 \times 4!} = \dfrac{1}{5}$

${}^6P_6 = 6! = TC = 6 \times 5 \times 4!$

O

A

A + + O + +

_ A _ _ O _

_ _ A _ _ O

Q

$TC = \dfrac{10!}{2!}$

UNIVERSITY

I - 2

UNIIVERSTY

Two I's are Together

UNIVRSITY

When Two I's are not together

UNIVRISTY

UNI VIRSTY

FC = TC - when Two I come together

IIUNVERSTY $\rightarrow$ $9!$

IIV
FIV

UIINUVERSITY
UNIIVERSITY

$$fc = \frac{10!}{2!} - 9!$$

$$P = \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = 1 - \frac{9!}{\frac{10!}{2!}} = 1 - \frac{9! \times 2!}{10!}$$

$$P = 1 - \frac{\cancel{9!} \times 2}{10 \times \cancel{9!}} = 1 - \frac{2}{10} = 1 - \frac{1}{5} = \frac{4}{5}$$

Three digit numbers are formed by using the digits 1, 2, 3, ~~4~~ and 5 without repeating any digit. What is the probability that a chosen number is an even number?

(a) 2/5 (b) 3/5 (c) 3/16 (d) 3/4

1, 2, 3, 4, 5

Sol

_ _ _

2/4

_ _ 4

$TC = {}^5P_3 = 5 \times 4 \times 3$

${}^4P_2 \times {}^2P_1$

$FC = {}^4P_2 \times {}^2P_1 = 4 \times 3 \times 2$

$P = \frac{4 \times 3 \times 2}{5 \times 4 \times 3} = \frac{2}{5}$

(c) In random arrangements of the letters of the word 'ENGINEERING', what is the probability that vowels always occur together?

Sol

$$TC = \frac{11!}{3! \times 3! \times 2! \times 2!}$$

$$FC = \frac{7!}{3!\,2!} \times \frac{5!}{3!\,2!}$$

$\boxed{E\,E\,E\,I\,I}$ N N N G G R

$\boxed{E\,I\,I\,E\,E\,I}$

E N G I N E E R I N G

✓ E – 3

N – 3

G – 2

✓ I – 2

R – 1

$\overline{11}$

$$P = \frac{FC}{TC} = \frac{\dfrac{7! \times 5!}{3!\,2!\;\;3!\,2!}}{\dfrac{11!}{3!\;3!\,2!\;2!}}$$

$$P = \frac{\cancel{7!} \times \cancel{5} \times 4 \times \cancel{3} \times \cancel{2} \times 1}{11 \times \cancel{10} \times \underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}} \times \cancel{7!}} = \frac{1}{66}$$

$$\boxed{P = \frac{1}{66}}$$

Two fair dice are rolled once. What is the probability that the sum of the results is at least 8?

(a) 1/9 (b) 5/12 (c) 7/12 (d) 17/36

Sol

$Tc = 6 \times 6 = 36$

$fc = 5+4+3+2+1 = 15$

$P = \frac{\cancel{15}\ 5}{\cancel{36}\ 12} = \frac{5}{12}$

$\boxed{P = \frac{5}{12}}$

Sum	No
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

A positive integer is chosen at random from 1 to 100. What is the probability that the integer chosen is either a multiple of 5 or a multiple of 8?

(a) 8/25 (b) 6/11 (c) 3/10 (d) 1/50

Sol TC = 100

$5 \rightarrow \frac{100}{5} = 20$, 5, 10, 15, 20, 25 .... 100 20

$8 \rightarrow \frac{100}{8} = 12.5$, 8, 16, 24, 32, ... 96 12

12.5

5 and 8 LCM 5 and 8 = 40 40, 80 2

$$FC = 20 + 12 - 2$$

$$FC = 30$$

$$TC = 100$$

$$P = \frac{30}{100} = 0.3$$

$$= 3/10$$

4·4·1. Sets and Elements of Sets. A set is a well defined collection or aggregate of all possible objects having given properties and specified according to a well defined rule. The objects comprising a set are called elements, members or points of the set. Sets are often denoted by capital letters, *viz.*, A, B, C, etc. If x is an element of the set A, we write symbolically $x \in A$ (x belongs to A). If x is not a member of the set A, we write $x \notin A$ (x does not belong to A). Sets are often described by describing the properties possessed by their members. Thus the set A of all non-negative rational numbers with square less than 2 will be written as $A = \{x : x \text{ rational}, x \geq 0, x^2 < 2\}$.

If every element of the set A belongs to the set B, *i.e.*, if $x \in A \Rightarrow x \in B$, then we say that A is a subset of B and write symbolically $A \subseteq B$ (A is contained in B) or $B \supseteq A$ (B contains A). Two sets A and B are said to be *equal* or *identical* if $A \subseteq B$ and $B \subseteq A$ and we write $A = B$ or $B = A$.

A *null* or an *empty* set is one which does not contain any element at all and is denoted by ϕ.

Remarks. 1. Every set is a subset of itself.

2. An empty set is subset of every set.

3. A set containing only one element is conceptually distinct from the element itself, but will be represented by the same symbol for the sake of convenience.

4. As will be the case in all our applications of set theory, especially to probability theory, we shall have a fixed set S (say) given in advance, and we shall be concerned only with subsets of this given set. The underlying set S may vary from one application to another, and it will be referred to as *universal set* of each particular discourse.

Operation on Sets

The union of two given sets A and B, denoted by $A \cup B$, is defined as a set consisting of all those points which belong to either A or B or both. Thus symbolically,

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}.$$

Similarly

$$\bigcup_{i=1}^{n} A_i = \{ x : x \in A_i \text{ for at least one } i = 1, 2, \ldots, n \}$$

The *intersection* of two sets A and B, denoted by $A \cap B$, is defined as a set consisting of all those elements which belong to both A and B. Thus

$$A \cap B = \{ x : x \in A \text{ and } x \in B \}.$$

Similarly

$$\bigcap_{i=1}^{n} A_i = \{ x : x \in A_i \text{ for all } i = 1, 2, ..., n \}$$

For example, if $A = \{1, 2, 5, 8, 10\}$ and $B = \{2, 4, 8, 12\}$, then

$$A \cup B = \{1, 2, 4, 5, 8, 10, 12\} \text{ and } A \cap B = \{2, 8\}.$$