Set 3: Multiple-Choice Questions on Differentiation

In each of Questions 1–5 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = (4x + 1)(1 - x)^3$

(A) $-12(1 - x)^2$ **(B)** $(1 - x)^2(1 + 8x)$ **(C)** $(1 - x)^2(1 - 16x)$

(D) $3(1 - x)^2(4x + 1)$ **(E)** $(1 - x)^2(16x + 7)$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ **(B)** $\frac{6x-5}{(3x+1)^2}$ **(C)** $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ **(E)** $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ **(B)** $-\frac{1}{\sqrt{3-2x}}$ **(C)** $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ **(E)** $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ **(B)** $-30(5x+1)^{-4}$ **(C)** $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ **(E)** $\frac{30}{(5x+1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ **(B)** $3x^{-1/3} - 2x^{-1/2}$ **(C)** $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ **(E)** $2x^{-1/3} - 2x^{-1/2}$

In questions 6–13, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

6. If $A = f + 2g$, then $A'(3) =$

(A) -2 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 10

7. If $B = f \bullet g$, then $B'(2) =$

(A) -20 (B) -7 (C) -6 (D) -1 (E) 13

8. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{3}$ **(C)** $-\frac{1}{9}$ **(D)** $\frac{1}{9}$ **(E)** $\frac{1}{3}$

9. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2\sqrt{10}}$ **(C)** 2 **(D)** $\frac{2}{\sqrt{10}}$ **(E)** $4\sqrt{10}$

10. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ **(B)** $-\frac{1}{4}$ **(C)** $\frac{13}{25}$ **(D)** $\frac{13}{16}$ **(E)** $\frac{22}{25}$

11. If $M(x) = f(g(x))$, then $M'(1) =$

(A) -12 **(B)** -6 **(C)** 4 **(D)** 6 **(E)** 12

12. If $P(x) = f(x^3)$, then $P'(1) =$

(A) 2 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 54

13. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) -2 **(B)** $-\frac{1}{25}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$ **(E)** 2

In Questions 14–21 find y'.

14. $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

(A) $x + \dfrac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\dfrac{4x-1}{4x\sqrt{x}}$

(D) $\dfrac{1}{\sqrt{x}} + \dfrac{1}{4x\sqrt{x}}$ **(E)** $\dfrac{4}{\sqrt{x}} + \dfrac{1}{x\sqrt{x}}$

15. $y = \sqrt{x^2 + 2x - 1}$

(A) $\dfrac{x+1}{y}$ **(B)** $4y(x + 1)$ **(C)** $\dfrac{1}{2\sqrt{x^2 + 2x - 1}}$

(D) $-\dfrac{x+1}{(x^2 + 2x - 1)^{3/2}}$ **(E)** none of these

16. $y = \dfrac{x}{\sqrt{1-x^2}}$

(A) $\dfrac{1-2x^2}{(1-x^2)^{3/2}}$ **(B)** $\dfrac{1}{1-x^2}$ **(C)** $\dfrac{1}{\sqrt{1-x^2}}$

(D) $\dfrac{1-2x^2}{(1-x^2)^{1/2}}$ **(E)** none of these

17. $y = \ln \dfrac{e^x}{e^x - 1}$

(A) $x - \dfrac{e^x}{e^x - 1}$ **(B)** $\dfrac{1}{e^x - 1}$ **(C)** $-\dfrac{1}{e^x - 1}$

(D) 0 **(E)** $\dfrac{e^x - 2}{e^x - 1}$

18. $y = \tan^{-1} \dfrac{x}{2}$

(A) $\dfrac{4}{4+x^2}$ **(B)** $\dfrac{1}{2\sqrt{4-x^2}}$ **(C)** $\dfrac{2}{\sqrt{4-x^2}}$

(D) $\dfrac{1}{2+x^2}$ **(E)** $\dfrac{2}{x^2+4}$

19. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$

(D) $\dfrac{1}{\sec x + \tan x}$ **(E)** $-\dfrac{1}{\sec x + \tan x}$

20. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$

(D) $\dfrac{4}{(e^x + e^{-x})^2}$ **(E)** $\dfrac{1}{e^{2x} + e^{-2x}}$

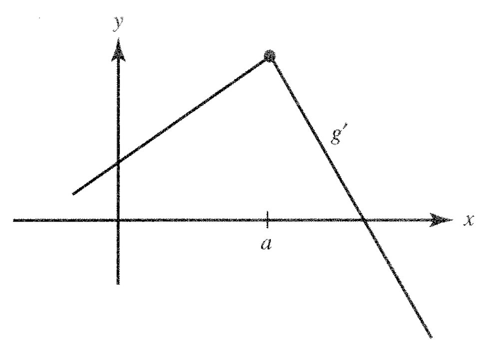
21. $y = \ln(x\sqrt{x^2+1})$

(A) $1 + \dfrac{x}{x^2+1}$ **(B)** $\dfrac{1}{x\sqrt{x^2+1}}$ **(C)** $\dfrac{2x^2+1}{x\sqrt{x^2+1}}$

(D) $\dfrac{2x^2+1}{x(x^2+1)}$ **(E)** none of these

22. The graph of g' is shown here. Which of the following statements are true of g at $x = a$?

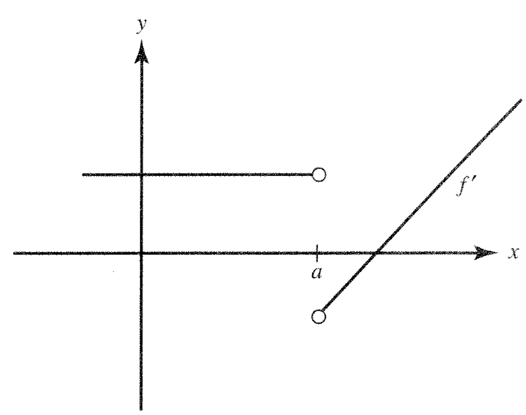
I. g is continuous
II. g is differentiable
III. g is increasing



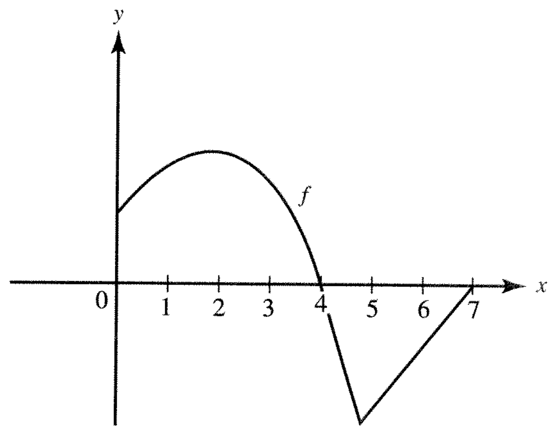
(A) I only **(B)** III only **(C)** I and III only
(D) II and III only **(E)** I, II, and III

23. A function f has the derivative shown. Which of the following statements is false?

(A) f is continuous at $x = a$
(B) $f(a) = 0$
(C) f has a vertical asymptote at $x = a$
(D) f has a jump discontinuity at $x = a$
(E) f has a removable discontinuity at $x = a$



24. The function f whose graph is shown has $f' = 0$ at $x =$



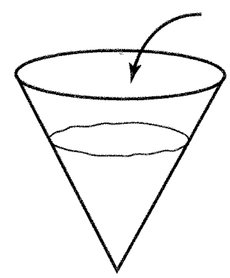
(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7

25. A differentiable function f has values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

(A) 8 **(B)** 12 **(C)** 18 **(D)** 40 **(E)** 80

26. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is



(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing

In Questions 27–33, find $\frac{dy}{dx}$.

27. $y = x^2 \sin \frac{1}{x}$ $\quad (x \neq 0)$

(A) $2x \sin\frac{1}{x} - x^2 \cos\frac{1}{x}$ **(B)** $-\frac{2}{x} \cos\frac{1}{x}$ **(C)** $2x \cos\frac{1}{x}$

(D) $2x \sin\frac{1}{x} - \cos\frac{1}{x}$ **(E)** $-\cos\frac{1}{x}$

28. $y = \frac{1}{2 \sin 2x}$

(A) $-\csc 2x \cot 2x$ **(B)** $\frac{1}{4 \cos 2x}$ **(C)** $-4 \csc 2x \cot 2x$

(D) $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ **(E)** $-\csc^2 2x$

29. $y = e^{-x} \cos 2x$

(A) $-e^{-x}(\cos 2x + 2 \sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x} \sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x} \sin 2x$

30. $y = \sec^2 \sqrt{x}$

(A) $\dfrac{\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$ (B) $\dfrac{\tan\sqrt{x}}{\sqrt{x}}$ (C) $2\sec\sqrt{x}\tan^2\sqrt{x}$

(D) $\dfrac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$ (E) $2\sec^2\sqrt{x}\tan\sqrt{x}$

31. $y = x \ln^3 x$

(A) $\dfrac{3\ln^2 x}{x}$ (B) $3\ln^2 x$ (C) $3x\ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ (E) none of these

32. $y = \frac{1+x^2}{1-x^2}$

(A) $-\dfrac{4x}{(1-x^2)^2}$ (B) $\dfrac{4x}{(1-x^2)^2}$ (C) $\dfrac{-4x^3}{(1-x^2)^2}$

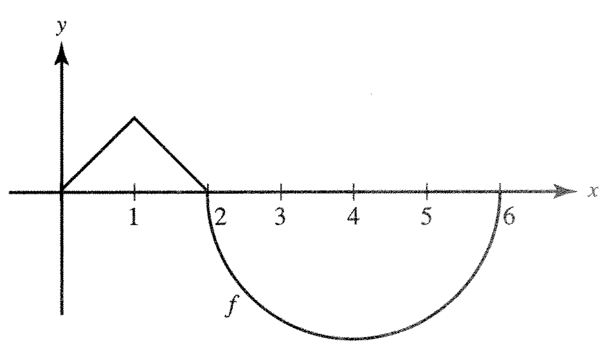
(D) $\dfrac{2x}{1-x^2}$ (E) $\dfrac{4}{1-x^2}$

33. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\dfrac{1}{2\sqrt{1-x^2}}$ (B) $\dfrac{2}{\sqrt{1-x^2}}$ (C) $\dfrac{1+x}{\sqrt{1-x^2}}$

(D) $\dfrac{x^2}{\sqrt{1-x^2}}$ (E) $\dfrac{1}{\sqrt{1+x}}$

Use the graph to answer Questions 34–36. It consists of two line segments and a semicircle.



34. $f'(x) = 0$ for $x =$

(A) 1 only
(B) 2 only
(C) 4 only
(D) 1 and 4
(E) 2 and 6

35. $f'(x)$ does not exist for $x =$

(A) 1 only **(B)** 2 only **(C)** 1 and 2
(D) 2 and 6 **(E)** 1, 2, and 6

36. $f'(5) =$

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** 2 **(E)** $\sqrt{3}$

37. At how many points on the interval $[-5,5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

38. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

(A) -0.10 **(B)** -0.20 **(C)** -5 **(D)** -10 **(E)** -25

In each of Questions 39–42 a pair of equations is given which represents a curve parametrically. Choose the alternative that is the derivative $\frac{dy}{dx}$.

***39.** $x = t - \sin t$ and $y = 1 - \cos t$

(A) $\frac{\sin t}{1-\cos t}$ **(B)** $\frac{1-\cos t}{\sin t}$ **(C)** $\frac{\sin t}{\cos t - 1}$
(D) $\frac{1-x}{y}$ **(E)** $\frac{1-\cos t}{t-\sin t}$

***40.** $x = \cos^3 \theta$ and $y = \sin^3 \theta$

(A) $\tan^3 \theta$ **(B)** $-\cot \theta$ **(C)** $\cot \theta$ **(D)** $-\tan \theta$ **(E)** $-\tan^2 \theta$

***41.** $x = 1 - e^{-t}$ and $y = t + e^{-t}$

(A) $\frac{e^{-t}}{1-e^{-t}}$ **(B)** $e^{-t} - 1$ **(C)** $e^{t} + 1$ **(D)** $e^{t} - e^{-2t}$ **(E)** $e^{t} - 1$

***42.** $x = \frac{1}{1-t}$ and $y = 1 - \ln(1-t)$ $(t < 1)$

(A) $\frac{1}{1-t}$ **(B)** $t - 1$ **(C)** $\frac{1}{x}$ **(D)** $\frac{(1-t)^2}{t}$ **(E)** $1 + \ln x$

*An asterisk denotes a topic covered only in Calculus BC.

In each of Questions 43–46, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$.

43. $x^3 - xy + y^3 = 1$

(A) $\dfrac{3x^2}{x-3y^2}$ (B) $\dfrac{3x^2-1}{1-3y^2}$ (C) $\dfrac{y-3x^2}{3y^2-x}$

(D) $\dfrac{3x^2+3y^2-y}{x}$ (E) $\dfrac{3x^2+3y^2}{x}$

44. $x + \cos(x + y) = 0$

(A) $\csc(x + y) - 1$ (B) $\csc(x + y)$ (C) $\dfrac{x}{\sin(x+y)}$

(D) $\dfrac{1}{\sqrt{1-x^2}}$ (E) $\dfrac{1-\sin x}{\sin y}$

45. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ (B) $-\cot y$ (C) $\dfrac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ (E) $\dfrac{2-\cos x}{\sin y}$

46. $3x^2 - 2xy + 5y^2 = 1$

(A) $\dfrac{3x+y}{x-5y}$ (B) $\dfrac{y-3x}{5y-x}$ (C) $3x + 5y$

(D) $\dfrac{3x+4y}{x}$ (E) none of these

Individual instructions are given in full for each of the remaining questions of this set.

***47.** If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then when $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) 1 (B) -1 (C) 0 (D) 3 (E) $\dfrac{1}{2}$

48. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ (B) $\{0, -1, -2\}$ (C) $\{-1, +2\}$
(D) $\{0\}$ (E) $\{0, 1, 2\}$

49. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

(A) -32 (B) -16 (C) -4 (D) -2 (E) $-\dfrac{1}{2}$

*An asterisk denotes a topic covered only in Calculus BC.

50. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\frac{1}{3}$ **(B)** -1 **(C)** -3 **(D)** 1 **(E)** none of these

51. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

(A) 0 **(B)** $\frac{1}{5}$ **(C)** -5 **(D)** $-\frac{1}{5}$ **(E)** nonexistent

52. If $y = a \sin ct + b \cos ct$, where a, b, and c are constants, then $\frac{d^2y}{dt^2}$ is

(A) $ac^2(\sin t + \cos t)$ **(B)** $-c^2y$ **(C)** $-ay$
(D) $-y$ **(E)** $a^2c^2 \sin ct - b^2c^2 \cos ct$

53. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \circ g)'(3)$ equals

(A) 0 **(B)** 1 **(C)** 6 **(D)** 9 **(E)** none of these

54. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 **(B)** 1.0 **(C)** 5.0 **(D)** 8.0 **(E)** 32.0

55. If $f(x) = \frac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) all reals
(B) all reals except $x = 1$ and $x = -1$
(C) all reals except $x = -1$
(D) all reals except $x = \frac{1}{3}$ and $x = -1$
(E) all reals except $x = 1$

56. If $y = e^x(x - 1)$ then $y''(0)$ equals

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** none of these

57. If $y = \sqrt{x^2 + 1}$, then the derivative of y^2 with respect to x^2 is

(A) 1 **(B)** $\frac{x^2+1}{2x}$ **(C)** $\frac{x}{2(x^2+1)}$ **(D)** $\frac{2}{x}$ **(E)** $\frac{x^2}{x^2+1}$

58. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\frac{-\sqrt{x}}{(x^2+1)^2}$ **(B)** $-(x + 1)^{-2}$ **(C)** $\frac{-2x}{(x^2+1)^2}$

(D) $\frac{1}{(x+1)^2}$ **(E)** $\frac{1}{2\sqrt{x}(x+1)}$

***59.** If $x = e^{\theta} \cos \theta$ and $y = e^{\theta} \sin \theta$, then, when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx}$ is

(A) 1 **(B)** 0 **(C)** $e^{\pi/2}$ **(D)** nonexistent **(E)** -1

***60.** If $x = \cos t$ and $y = \cos 2t$, then $\frac{d^2y}{dx^2}$ $(\sin t \neq 0)$ is

(A) $4 \cos t$ **(B)** 4 **(C)** $\frac{4y}{x}$ **(D)** -4 **(E)** $-4 \cot t$

61. If $y = x^2 + x$, then the derivative of y with respect to $\frac{1}{1-x}$ is

(A) $(2x + 1)(x - 1)^2$ **(B)** $\frac{2x+1}{(1-x)^2}$ **(C)** $2x + 1$

(D) $\frac{3-x}{(1-x)^3}$ **(E)** none of these

62. $\lim_{h \to 0} \frac{(1+h)^6 - 1}{h}$ is

(A) 0 **(B)** 1 **(C)** 6 **(D)** ∞ **(E)** nonexistent

63. $\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 **(B)** $\frac{1}{12}$ **(C)** 1 **(D)** 192 **(E)** ∞

64. $\lim_{h \to 0} \frac{\ln(e+h) - 1}{h}$ is

(A) 0 **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** e **(E)** nonexistent

65. $\lim_{x \to 0} \frac{\cos x - 1}{x}$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** ∞ **(E)** none of these

66. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the mean-value theorem because

(A) $f(0)$ is not defined **(B)** $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist **(D)** $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

67. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

*An asterisk denotes a topic covered only in Calculus BC.

68. If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leqslant x \leqslant \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line?

(A) 1 **(B)** -1 **(C)** $\sqrt{2}$ **(D)** 0 **(E)** nowhere

69. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

(A) -9 **(B)** $-\frac{1}{9}$ **(C)** $\frac{1}{9}$ **(D)** 3 **(E)** 9

70. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1)$ equals

(A) -1 **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{3}$ **(D)** 1 **(E)** 3

71. Suppose $y = f(x)$ and $x = f^{-1}(y)$ are mutually inverse functions. If $f(1) = 4$ and $\frac{dy}{dx} = -3$ at $x = 1$, then $\frac{dx}{dy}$ at $y = 4$ equals

(A) $-\frac{1}{3}$ **(B)** $-\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** 3 **(E)** 4

72. Let $y = f(x)$ and $x = h(y)$ be mutually inverse functions. If $f'(2) = 5$, then what is the value of $\frac{dx}{dy}$ at $y = 2$?

(A) -5 **(B)** $-\frac{1}{5}$ **(C)** $\frac{1}{5}$

(D) 5 **(E)** It cannot be determined from the information given.

***73.** $\lim\limits_{x\to\infty} \frac{e^x}{x^{50}}$ equals

(A) 0 **(B)** 1 **(C)** $\frac{1}{50!}$ **(D)** ∞ **(E)** none of these

74. Suppose $\lim\limits_{x\to 0} \frac{g(x)-g(0)}{x} = 1$. It follows necessarily that

(A) g is not defined at $x = 0$
(B) g is not continuous at $x = 0$
(C) The limit of $g(x)$ as x approaches 0 equals 1
(D) $g'(0) = 1$
(E) $g'(1) = 0$

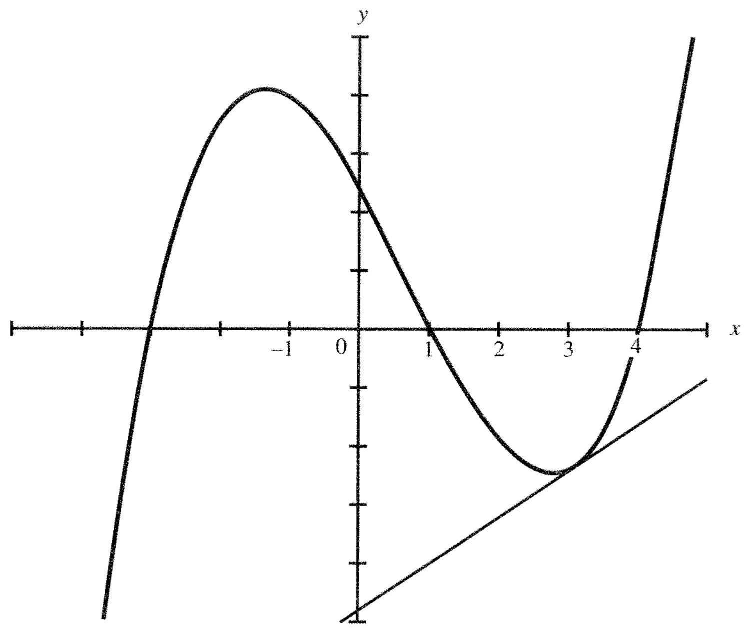
75. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\sec(xy)$ **(B)** $\frac{\sec(xy)}{x}$ **(C)** $\frac{\sec(xy) - y}{x}$

(D) $-\frac{1 + \sec(xy)}{x}$ **(E)** $\sec(xy) - 1$

*An asterisk denotes a topic covered only in Calculus BC.

Use this graph of $y = f(x)$ for Questions 76 and 77.



76. $f'(3)$ is most closely approximated by

(A) 0.3 **(B)** 0.8 **(C)** 1.5 **(D)** 1.8 **(E)** 2

77. The rate of change of $f(x)$ is least at $x \simeq$

(A) –3 **(B)** –1.3 **(C)** 0 **(D)** 0.7 **(E)** 2.7

Use the following definition of the *symmetric difference quotient for* $f'(x_0)$ for Questions 78–81: for small values of h,

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

78. $f'(x_0)$ equals the exact value of the derivative at $x = x_0$

(A) only when f is linear
(B) whenever f is quadratic
(C) if and only if $f'(x_0)$ exists
(D) whenever $|h| < 0.001$.
(E) none of these

79. To how many places is the symmetric difference accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.08$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

80. To how many places is $f'(x_0)$ accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.001$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

81. The value of $f'(0)$ obtained using the symmetric difference quotient with $f(x) = |x|$ and $h = 0.001$ is

(A) -1 **(B)** 0 **(C)** ± 1 **(D)** 1 **(E)** indeterminate

82. If $\frac{d}{dx}f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx}f(h(x))$ equals

(A) $g(\sin x)$ **(B)** $\cos x \cdot g(x)$ **(C)** $g'(x)$
(D) $\cos x \cdot g\,(\sin x)$ **(E)** $\sin x \cdot g(\sin x)$

83. $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x}$ is

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 0 **(E)** ∞

84. $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 4x}$ is

(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{4}$ **(D)** 0 **(E)** nonexistent

85. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x}$ is

(A) nonexistent **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

86. $\lim\limits_{x \to 0} \dfrac{\tan \pi x}{x}$ is

(A) $\frac{1}{\pi}$ **(B)** 0 **(C)** 1 **(D)** π **(E)** ∞

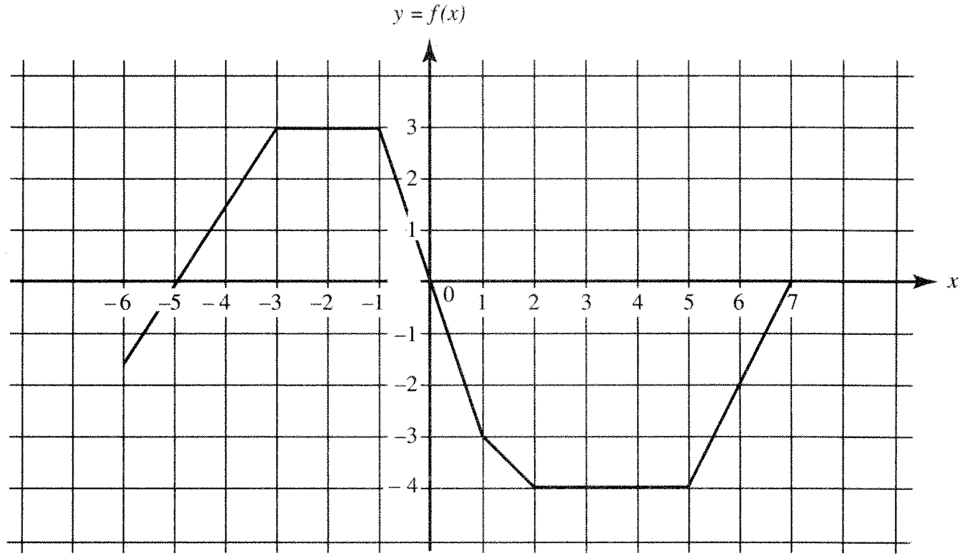
87. $\lim\limits_{x \to \infty} x^2 \sin \dfrac{1}{x}$

(A) is 1 **(B)** is 0 **(C)** is ∞
(D) oscillates between -1 and 1 **(E)** is none of these

88. Let $f(x) = 3^x - x^3$. The tangent to the curve is parallel to the secant through (0,1) and (3,0) for x equal

(A) only to 0.984 **(B)** only to 1.244 **(C)** only to 2.727
(D) to 0.984 and 2.804 **(E)** to 1.244 and 2.727

Questions 89 through 96 are based on the following graph of $f(x)$, sketched on $-6 \leqslant x \leqslant 7$. Assume the horizontal and vertical grid lines are equally spaced at unit intervals.



89. On the interval $1 < x < 2$, $f(x)$ equals

(A) $-x - 2$ **(B)** $-x - 3$ **(C)** $-x - 4$ **(D)** $-x + 2$ **(E)** $x - 2$

90. Over which of the following intervals does $f'(x)$ equal zero?

I. $(-6,-3)$ II. $(-3,-1)$ III. $(2,5)$

(A) I only **(B)** II only **(C)** I and II only
(D) I and III only **(E)** II and III only

91. How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

92. For $-6 < x -3$, $f'(x)$ equals

(A) $-\frac{3}{2}$ **(B)** -1 **(C)** 1 **(D)** $\frac{3}{2}$ **(E)** 2

93. Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of 6 horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1$, $f'(x) = -3$.

***94.** The graph in the xy-plane represented by $x = 3 + 2 \sin t$ and $y = 2 \cos t - 1$, for $-\pi \leq t \leq \pi$, is

(A) a semicircle **(B)** a circle **(C)** an ellipse
(D) half of an ellipse **(E)** a hyperbola

95. $\lim\limits_{x \to 0} \dfrac{\sec x - \cos x}{x^2}$ equals

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** none of these

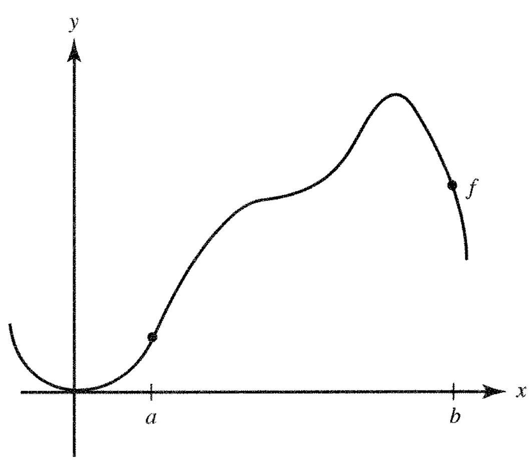
96. The table gives the values of a function f that is differentiable on the interval [0,1]:

x	0.10	0.20	0.30	0.40	0.50	0.60
$f(x)$	0.171	0.288	0.357	0.384	0.375	0.336

The best approximation of $f'(0.10)$ according to this table is

(A) 0.12 **(B)** 1.08 **(C)** 1.17 **(D)** 1.77 **(E)** 2.88

97. At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

*An asterisk denotes a topic covered only in Calculus BC.

Answers for Set 3: Differentiation

1. C	21. D	41. E	61. A	81. B
2. A	22. E	42. C	62. C	82. D
3. B	23. C	43. C	63. B	83. B
4. B	24. A	44. A	64. B	84. C
5. E	25. D	45. D	65. B	85. E
6. B	26. E	46. B	66. E	86. D
7. B	27. D	47. E	67. B	87. C
8. E	28. A	48. E	68. A	88. E
9. D	29. A	49. E	69. B	89. A
10. C	30. D	50. A	70. C	90. E
11. A	31. E	51. D	71. A	91. E
12. B	32. B	52. B	72. E	92. D
13. D	33. C	53. C	73. D	93. B
14. D	34. C	54. D	74. D	94. B
15. A	35. E	55. E	75. C	95. C
16. E	36. B	56. D	76. B	96. C
17. C	37. D	57. A	77. D	97. D
18. E	38. C	58. B	78. B	
19. A	39. A	59. E	79. B	
20. D	40. D	60. B	80. E	

Many of the explanations provided include intermediate steps that would normally be reached on the way to a final algebraically simplified result. You may not need to reach the final answer.

NOTE: the formulas or rules in parentheses referred to in the explanations are given on pages 46 and 47.

1. C. By the product rule, (5),

$$\begin{aligned} y' &= (4x+1)[3(1-x)^2(-1)] + (1-x)^3 \cdot 4 \\ &= (1-x)^2(-12x-3+4-4x) \\ &= (1-x)^2(1-16x) \end{aligned}$$

2. A. By the quotient rule, (6).

$$y' = \frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2} = \frac{-7}{(3x+1)^2}.$$

3. B. Since y = $(3-2x)^{1/2}$, by the power rule, (3).

$$y' = \frac{1}{2}(3-3x)^{-1/2} \cdot (-2) = -\frac{1}{\sqrt{3-2x}}.$$

4. B. Since $y = 2(5x+1)^{-3}, y' = -6(5x+1)^{-4}(5)$.

5. E. $y' = 3\left(\frac{2}{3}\right)x^{-1/3} - 4\left(\frac{1}{2}\right)x^{-1/2}.$

6. B. $(f + 2g)'(3) = f'(3) + 2g'(3) = 4 + 2(-1)$

7. B. $(f \cdot g)'(2) = f(2) \cdot g'(2) + g(2) \cdot f'(2) = 5(-2) + 1(3)$

8. E. $\left(\frac{1}{g}\right)'(1) = -1 \cdot \frac{1}{[g(1)]^2} \cdot g'(1) = -1 \cdot \frac{1}{3^2}(-3)$

9. D. $(\sqrt{f})'(3) = \frac{1}{2}[f(3)]^{-1/2} \cdot f'(3) = \frac{1}{2}(10^{-1/2}) \cdot 4$

10. C. $\left(\frac{f}{g}\right)'(0) = \frac{g(0) \cdot f'(0) - f(0) \cdot g'(0)}{[g(0)]^2} = \frac{5(1) - 2(-4)}{5^2}$

11. A. $M'(1) = f'(g(1)) \cdot g'(1) = f'(3)g'(1) = 4(-3)$

12. B. $[f(x^3)]' = f'(x^3) \cdot 3x^2$, so $P'(1) = f'(1^3) \cdot 3 \cdot 1^2 = 2 \cdot 3$

13. D. $f(S(x)) = x$ implies that $f'(S(x) \cdot S'(x) = 1$. So,

$$S'(3) = \frac{1}{f'(S(3))} = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)}$$

14. D. Rewrite: $y = 2x^{1/2} - \frac{1}{2}x^{1/2}$; so $y' = x^{-1/2} + \frac{1}{4}x^{-3/2}$.

15. A. Rewrite: $y = (x^2 + 2x - 1)^{1/2}$. (Use rule (3).)

16. E. Use the quotient rule:

$$y = \frac{\sqrt{1-x^2} \cdot 1 - x \cdot \frac{1 \cdot -2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{1}{(1-x^2)^{3/2}}$$

17. C. Since

$$y = \ln e^x - \ln(e^x - 1),$$

$$y = x - \ln(e^x - 1),$$

$$y' = 1 - \frac{e^x}{e^x - 1} = \frac{e^x - 1 - e^x}{e^x - 1} = -\frac{1}{e^x - 1}.$$

18. E. Use formula (18): $y' = \dfrac{\frac{1}{2}}{1 + \frac{x^2}{4}}$.

19. A. Use formulas (13), (11), and (9):

$$y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}.$$

20. D. By the quotient rule,

$$y' = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}$$

21. D. Since $y = \ln x + \frac{1}{2}\ln(x^2 + 1)$,

$$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2 + 1} = \frac{2x^2 + 1}{x(x^2 + 1)}.$$

22. E. Since $g'(a)$ exists, g is differentiable and thus continuous; $g'(a) > 0$.

23. C. Near a vertical asymptote the slopes must approach $\pm\infty$.

24. A. There is only one horizontal tangent.

25. D. Using the symmetric difference quotient,

$$f'(1.5) = \frac{f(1.6) - f(1.4)}{1.6 - 1.4} = \frac{8}{0.2}$$

26. E. Since the water level rises more slowly as the cone fills, the answer is (C) or (E). Furthermore, at every instant the portion of the cone containing water is similar to the entire cone; the volume is proportional to the cube of the depth of the water. The rate of change of depth (the derivative) is therefore not linear.

27. D. $y' = x^2 \cos\frac{1}{x}\left(-\frac{1}{x^2}\right) + \sin\frac{1}{x}(2x).$

28. A. Since $y = \frac{1}{2}\csc 2x$, $y' = \frac{1}{2}(-\csc 2x \cot 2x \cdot 2)$.

29. A. $y' = e^{-x}(-2\sin 2x) + \cos 2x(-e^{-x}).$

30. D. Use formulas (3) and (11):

$$y' = 2\sec\sqrt{x} \cdot \sec\sqrt{x}\tan\sqrt{x} \cdot \left(\frac{1}{2\sqrt{x}}\right).$$

31. E. $y' = \frac{x(3\ln^2 x)}{x} + \ln^3 x$. The correct answer is $3\ln^2 x + \ln^3 x$.

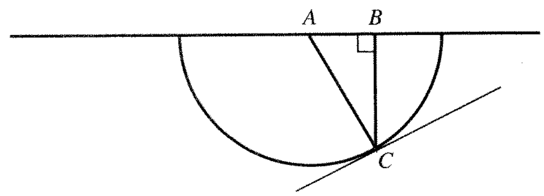
32. B. $y' = \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}.$

33. C. $y' = \frac{1}{\sqrt{1-x^2}} - \frac{1 \cdot (-2x)}{2\sqrt{1-x^2}}.$

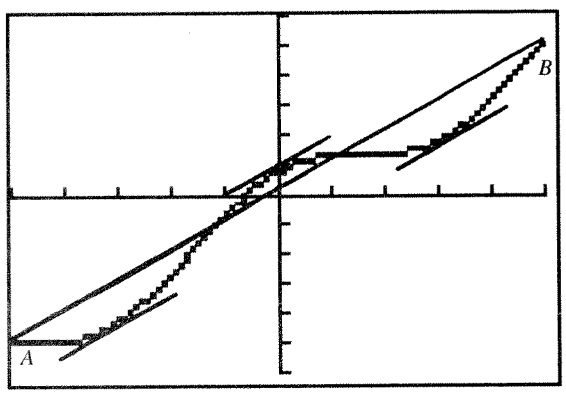
34. C. The only horizontal tangent is at $x = 4$. Note that $f'(1)$ does not exist.

35. E. The graph has corners at $x = 1$ and $x = 2$; the tangent line is vertical at $x = 6$.

36. B. Consider triangle ABC: $AB = 1$; radius $AC = 2$; thus, $BC = \sqrt{3}$ and AC has $m = -\sqrt{3}$. The tangent line is perpendicular to the radius.



37. D. The graph of $y = x + \cos x$ is shown in window $[-5,5] \times [-6,6]$. The average rate of change is represented by the slope of secant segment $\overline{AB}$. There appear to be 3 points at which tangent lines are parallel to $\overline{AB}$.



38. C. $f'(2) \approx \dfrac{f(2)-f(1.98)}{2-1.98} = \dfrac{4.00-4.10}{0.02}$

39. A. $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\sin t}{1-\cos t}$.

40. D. $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{3\sin^2\theta\cos\theta}{-3\cos^2\theta\sin\theta}$.

41. E. $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{1-e^{-t}}{e^{-t}} = e^t - 1$.

42. C. Since $\dfrac{dy}{dt} = \dfrac{1}{1-t}$ and $\dfrac{dx}{dt} = \dfrac{1}{(1-t)^2}$, then

$$\frac{dy}{dx} = 1 - t = \frac{1}{x}.$$

43. C. Let y' be $\dfrac{dy}{dx}$; then $3x^2 - (xy' + y) + 3y^2y' = 0$; $y'(3y^2 - x) = y - 3x^2$.

44. A. $1 - \sin(x + y)(1 + y') = 0;\ \dfrac{1-\sin(x+y)}{\sin(x+y)} = y'$.

45. D. $\cos x + \sin y \cdot y' = 0;\ y' = -\dfrac{\cos x}{\sin y}$.

46. B. $6x - 2(xy' + y) + 10yy' = 0;\ y'(10y - 2x) = 2y - 6x$.

47. E. $\dfrac{dy}{dx} = \dfrac{4t^3 - 6t^2}{2t} = 2t^2 - 3t(t \neq 0);\ \dfrac{d^2y}{dx^2} = \dfrac{4t-3}{2t}$. Replace t by 1.

48. E. $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x - 1)(x - 2)$.

49. E. $f'(x) = 8x^{-1/2};\ f''(x) = -4x^{-3/2} = -\dfrac{4}{x^{3/2}};\ f''(4) = -\dfrac{4}{8}$.

50. A. $f(x) = 3 \ln x;\ f'(x) = \dfrac{3}{x};\ f''(x) = \dfrac{-3}{x^2}$. Replace x by 3.

51. D. $2x + 2yy' = 0;\ y' = -\dfrac{x}{y};\ \ y'' = -\dfrac{y - xy'}{y^2}$. At (0,5), $y'' = -\dfrac{5-0}{25}$.

52. B. $y' = ac \cos ct - bc \sin ct;$
$y'' = -ac^2 \sin ct - bc^2 \cos ct$.

53. C. $(f \circ g)'$ at $x = 3$ equals $f'(g(3)) \cdot g'(3)$ equals $\cos u$ (at $u = 0$) times $2x$ (at $x = 3$) $= 1 \cdot 6 = 6$.

54. D. $f'(1) \simeq \dfrac{5^{1.002} - 5^1}{0.002} = \dfrac{5.016 - 5}{0.002}$.

55. E. Here $f'(x)$ equals $\dfrac{-x^2 - x - 1}{(x - 1)^3}$.

56. D. $y' = e^x \cdot 1 + e^x(x - 1) = xe^x$
$y'' = xe^x + e^x$ and $y''(0) = 0 \cdot 1 + 1 = 1$.

57. A. $\dfrac{dy^2}{dx^2} = \dfrac{\frac{dy^2}{dx}}{\frac{dx^2}{dx}}$. Since $y^2 = x^2 + 1$, $\dfrac{dy^2}{dx^2} = \dfrac{2x}{2x}$.

58. B. Note that $f(g(x)) = \dfrac{1}{x+1}$.

59. E. When simplified, $\dfrac{dy}{dx} = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$.

60. B. Since (if $\sin t \neq 0$)

$$\frac{dy}{dt} = -2\sin 2t = -4\sin t \cos t \text{ and } \frac{dx}{dt} = -\sin t,$$

then $\dfrac{dy}{dx} = 4\cos t$. Then,

$$\frac{d^2y}{dx^2} = -\frac{4\sin t}{-\sin t}.$$

61. A. $$\frac{dy}{d\left(\frac{1}{1-x}\right)} = \frac{\frac{dy}{dx}}{\frac{d\left(\frac{1}{1-x}\right)}{dx}} = \frac{2x+1}{\frac{1}{(1-x)^2}}.$$

NOTE: Since each of the limits in Questions 62 through 65 yields an indeterminate form of the type $\frac{0}{0}$, we can apply L'Hôpital's rule in each case, getting identical answers.

62. C. The limit given is the derivative of $f(x) = x^6$ at $x = 1$.

63. B. The given limit is the definition for $f'(8)$, where $f(x) = \sqrt[3]{x}$;
$$f'(x) = \frac{1}{3x^{2/3}}.$$

64. B. This is $f'(e)$, where $f(x) = \ln x$.

65. B. The limit is the derivative of $f(x) = \cos x$ at $x = 0$; $f'(x) = -\sin x$.

66. E. Since $f'(x) = \frac{2}{3x^{1/3}}$, $f'(0)$ is not defined; $f'(x)$ must be defined on $(-8,8)$.

67. B. Sketch the graph of $f(x) = 1 - |x|$; note that $f(-1) = f(1) = 0$ and that f is continuous on $[-1,1]$. Only (B) holds.

68. A. Note that $f(0) = f(\sqrt{3}) = 0$ and that $f'(x)$ exists on the given interval. By the MVT, there is a number c in the interval such that $f'(c) = 0$. If $c = 1$, then $6c^2 - 6 = 0$. (-1 is not in the interval.)

69. B. Since the inverse, h, of $f(x) = \frac{1}{x}$ is $h(x) = \frac{1}{x}$, $h'(x) = -\frac{1}{x^2}$. Replace x by 3.

70. C. Since $f'(x) = 6x^2 - 3$, therefore $h'(x) = \frac{1}{6x^2-3}$; also, $f(x)$, or $2x^3 - 3x$, equals -1, by observation, for $x = 1$. So $h'(-1)$ or $\frac{1}{6x^2-3}$ (when $x = 1$) equals $\frac{1}{6-3} = \frac{1}{3}$.

71. A. Use $\left(\frac{dx}{dy}\right)_{y=y_0} = \frac{1}{\left(\frac{dy}{dx}\right)_{x=x_0}}$, where $f(x_0) = y_0$.
$$\left(\frac{dx}{dy}\right)_{y=4} = \frac{1}{\left(\frac{dy}{dx}\right)_{x=1}} = \frac{1}{-3}.$$

72. E. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$. However, to evaluate $\frac{dx}{dy}$ for a particular value of y, we need to evaluate $\frac{1}{\frac{dy}{dx}}$ at the corresponding value of x. This question does not say what value of x yields $y = 2$.

73. D. After 50(!) applications of L'Hôpital's rule we get $\lim_{x\to\infty} \frac{e^x}{50!}$, which "equals" ∞. A perfunctory examination of the limit, however, shows immediately that the answer is ∞. In fact, $\lim_{x\to\infty} \frac{e^x}{x^n}$ for any positive integer n, no matter how large, is ∞.

74. D. The given limit is the derivative of $g(x)$ at $x = 0$.

75. C. $\cos(xy)(xy' + y) = 1;\ x\cos(xy)y' = 1 - y\cos(xy);$

$$y' = \frac{1 - y\cos(xy)}{x\cos(xy)}.$$

76. B. The tangent line appears to contain (3,–2.6) and (4,–1.8).

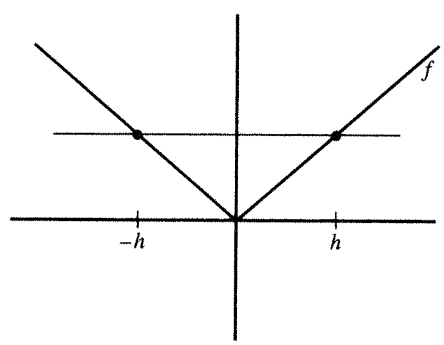
77. D. $f'(x)$ is least at the point of inflection of the curve, at about 0.7.

78. B. Note that $\frac{(x+h)^2 - (x-h)^2}{2h} = 2x$, the derivative of $f(x) = x^2$. You should be able to confirm that the symmetric difference quotient works for the general quadratic $g(x) = ax^2 + bx + c$.

79. B. By calculator, $f'(0) = 1.386294805$ and $\frac{4^{0.08} - 4^{-0.08}}{0.16} = 1.3891\ldots.$

80. E. Now $\frac{4^{0.001} - 4^{-0.001}}{0.002} = 1.386294805.$

81. B. Note that any line determined by two points equidistant from the origin will necessarily be horizontal.



82. D. Note that $\frac{d}{dx} f(h(x)) = f'(h(x)) \cdot h'(x) = g(h(x)) \cdot h'(x) = g(\sin x) \cdot \cos x.$

NOTE: In Questions 83 through 87 the limits are all indeterminate forms of the type $\frac{0}{0}$. We have therefore applied L'Hôpital's rule in each one. The indeterminacy can also be resolved by introducing $\frac{\sin a}{a}$ which approaches 1 as a approaches 0. The latter technique is presented in square brackets.

83. B. $\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0} \frac{2\cos 2x}{1} = \frac{2 \cdot 1}{1} = 2.$

[Using $\sin 2x = 2 \sin x \cos x$ yields $\lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)\cos x = 2 \cdot 1 \cdot 1 = 2.$]

84. C. $\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3\cos 3x}{4\cos 4x} = \frac{3 \cdot 1}{4 \cdot 1} = \frac{3}{4}.$

[We rewrite $\frac{\sin 3x}{\sin 4x}$ as $\frac{\sin 3x}{3x} \cdot \frac{4x}{\sin 4x} \cdot \frac{3}{4}$. As $x \to 0$, so do $3x$ and $4x$; the fraction approaches $1 \cdot 1 \cdot \frac{3}{4}$.]

85. E. $\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{\sin x}{1} = 0.$

[We can replace $1 - \cos x$ by $2 \sin^2 \frac{x}{2}$, getting $\lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x} = \lim_{x\to 0} \frac{\sin^2 \frac{x}{2}}{\frac{x}{2}} =$ $\lim_{x\to 0} \sin \frac{x}{2} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right) = 0 \cdot 1.$]

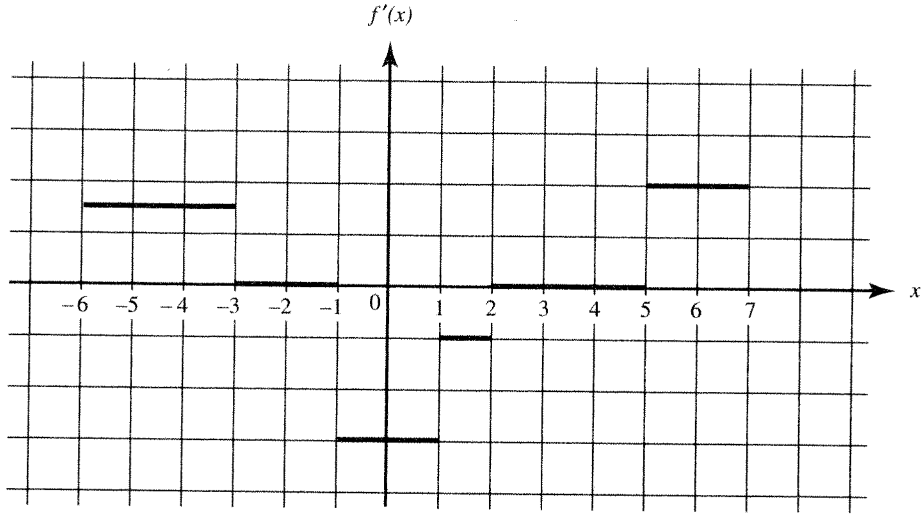
86. D. $\lim_{x\to 0} \frac{\tan \pi x}{x} = \lim_{x\to 0} \frac{(\sec^2 \pi x)\cdot\pi}{1} = 1 \cdot \pi = \pi.$

[$\frac{\tan \pi x}{x} = \frac{\sin \pi x}{x \cos \pi x} = \pi \cdot \frac{\sin \pi x}{\pi x} \cdot \frac{1}{\cos x}$; as x (or πx) approaches 0, the original fraction approaches $\pi \cdot 1 \cdot \frac{1}{1} = \pi$.]

87. C. The limit is easiest to obtain here if we rewrite: $\lim_{x\to\infty} x^2 \sin \frac{1}{x} = \lim_{x\to\infty} x \frac{\sin(1/x)}{(1/x)} = \infty \cdot 1 = \infty.$

88. E. Since $f(x) = 3^x - x^3$, $f'(x) = 3^x \ln 3 - 3x^2$. Further, f is continuous on [0,3] and f' is differentiable on (0,3). So the MVT applies. We therefore seek c such that $f'(c) = \frac{f(3) - f(0)}{3} = -\frac{1}{3}$. Using a graphing calculator, we key in Y_1 = `3^x ln 3 - 3X^2 +` $\frac{1}{3}$. We can either graph Y_1 and use the [root] option (twice; where Y_1 has x-intercepts) or use the [solve] option (estimating solutions at $\simeq 1$ and $\simeq 2.5$). The graphics calculator shows that c may be either 1.244 or 2.727. These are the x-coordinates of points on the graph of $f(x)$ at which the tangents are parallel to the secant through points (0,1) and (3,0) on the curve.

89. A. The line segment passes through (1,–3) and (2,–4).



Use the graph of $f'(x)$ for Questions 90 through 94.

90. E. $f'(x) = 0$ when the slope of $f(x)$ is 0; that is, when the graph of f is a horizontal segment.

91. E. The graph of $f'(x)$ jumps at each corner of the graph of $f(x)$, namely, at x equal to –3, –1, 1, 2, and 5.

92. D. On the interval (–6,–3), $f(x) = \frac{3}{2}(x + 5)$.

93. B. Verify that all choices but (B) are true. The graph of $f'(x)$ has five (not four) jump discontinuities.

94. B. Since $x - 3 = 2 \sin t$ and $y + 1 = 2 \cos t$, we have

$$(x - 3)^2 + (y + 1)^2 = 4$$

This is the equation of a circle with center at (3,–1) and radius 2. In the domain given, $-\pi \leqslant t \leqslant \pi$, the entire circle is traced by a particle moving counterclockwise, starting from and returning to (3, –3).

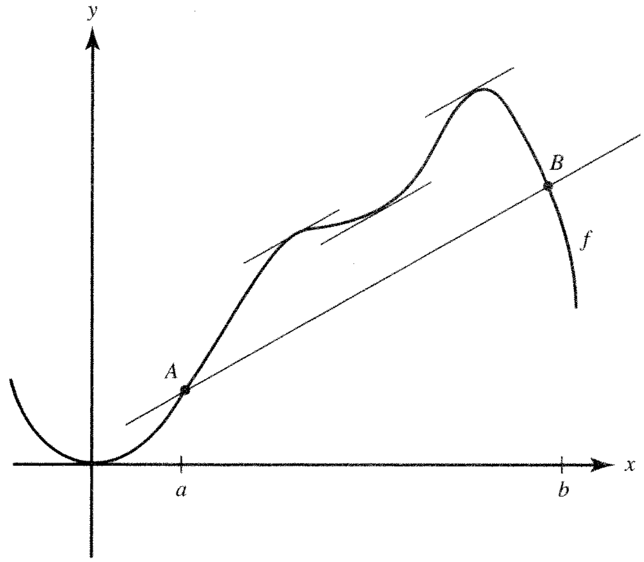
95. C. Using L'Hôpital's rule,

$$\lim_{x\to 0} \frac{\sec x - \cos x}{x^2} = \lim_{x\to 0} \frac{\sec x \tan x + \sin x}{2x}$$

$$= \lim_{x\to 0} \frac{\sec^3 x + \sec x \tan^2 x + \cos x}{2} = \frac{1 + 1 \cdot 0 + 1}{2} = 1.$$

96. C. The best approximation to $f'(0.10)$ is $\frac{f(0.20) - f(0.10)}{0.20 - 0.10}$.

97. D.



The average rate of change is represented by the slope of secant segment $\overline{AB}$. There appear to be 3 points at which the tangent lines are parallel to $\overline{AB}$.