L17 24 Feb 2021

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$(D^2+3D+2)y = e^{2x}\sin x$

P.I $\quad \frac{1}{D^2+3D+2}\cdot e^{2x}\sin x = e^{2x}\cdot\frac{1}{(D+2)^2+3(D+2)+2}\cdot\sin x$

$D^2 \to -1$

$= e^{2x}\cdot\frac{1}{D^2+4+4D+3D+6+2}\cdot\sin x$

$= e^{2x}\cdot\frac{1}{D^2+7D+12}\cdot\sin x = e^{2x}\cdot\frac{1}{-1+7D+12}\cdot\sin x$

$= e^{2x}\,\frac{7D-11}{(7D+11)(7D-11)}\,\sin x$

$y_c = c_1e^{-x}+c_2e^{-2x}$

$= e^{2x}\cdot\frac{(7D-11)}{49D^2-121}\,\sin x$

$D^2 \to -1$

$121 + 49 = 170$

$= \frac{e^{2x}}{-170}\left[7D-1\right]\sin x$

$y_p = -\frac{e^{2x}}{170}\left[7(\cos x)-\sin x\right]$ Ans

$\sin ax$ or $\cos ax$

$D^2 \to -a^2$

Case No:- 5 $\quad s(x) = x.V$ where V is a function of x $\quad \sin x \quad a=1$, $D^2 \to -1$

$\frac{1}{f(D)}x.V = x.\frac{1}{f(D)}.V + \frac{d}{dD}\left(\frac{1}{f(D)}\right)\cdot V$

Case I
1. $s(x) = e^{ax}$
2. $s(x) = \sin ax$ or $\cos ax$
3. $s(x) = x^m$
4. $s(x) = e^{ax}\cdot f(x)$

$(D^2-1)y = x\sin x$

A.E $\quad D^2=1$, $D=\pm1$ $\quad y_c = c_1e^{-x}+c_2e^{x}$ &

A.E $D=1$ $D=\pm 1$ $y_C = c_1 e + c_2 e$ ②

(4) $g(x) = e \cdot f(x)$

$D^2 \to -a^2$

P.I

$$\frac{1}{D^2-1} \cdot x \sin x = x \cdot \frac{1}{D^2-1} \cdot \sin x + \frac{d}{dD}\left(\frac{1}{D^2-1}\right) \cdot \sin x$$

$\frac{1}{x}$

$$= x \cdot \frac{1}{-1-1} \sin x + \left(\frac{-2D}{(D^2-1)^2}\right) \sin x$$

$$= \frac{x}{-2} \sin x - 2\left[\frac{D}{(-1-1)^2} \sin x\right]$$

$$y_p = \frac{-x}{2} \sin x - \frac{2}{4}\left[\cos x\right] \quad ②$$

Euler formula

$$e^{ix} = \cos x + i \sin x$$

$(D^2-1)y = x \sin x$

$$\frac{1}{D^2-1} x \sin x = \text{Imaginary part of}\left(\frac{1}{D^2-1} \cdot x e^{ix}\right)$$

$$\boxed{\frac{1}{D^2-1} \cdot x \cos x}$$

$$= \frac{1}{D^2-1} \cdot x e^{ix} = e^{ix} \cdot \frac{1}{(D+i)^2-1} \cdot x$$

$$= e^{ix} \cdot \frac{1}{D^2-1+2Di-1} \cdot x$$

$$= \frac{e^{ix}}{-2} \frac{1}{\left[1-\left(\frac{D^2+2Di}{2}\right)\right]} \cdot x$$

$D(x) = 1$

$D^2(x) = 0$

$$= \frac{e^{ix}}{-2}\left[1-\left(\frac{D^2+2Di}{2}\right)\right]^{-1} \cdot x$$

$$= \frac{e^{ix}}{-2}\left[1+\left(\frac{D^2+2Di}{2}\right)\right] x$$

$$= \frac{-e^{ix}}{2}\left[1 + Di\right]x$$

$$= \frac{-e^{ix}}{2}\left\{x + i\right\} = \frac{-1}{2}(\cos x + i\sin x)(x + i)$$

Imaginary part $y_p = \frac{-1}{2}\left[\cos x + x\sin x\right]$ Ans

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P.I

$(D^2 - 3D + 2)y = 2e^{2x}$

$f(x) = e^{ax}$

$D \to a$

$$\frac{1}{D^2-3D+2} \cdot 2e^{2x} = 2 \cdot \frac{1}{4-3(2)+2} \cdot e^{2x}$$

$$= 2 \cdot \frac{1}{\cancel{6}-\cancel{6}} \cdot e^{2x} \to \text{case of failure}$$

$$= 2x \cdot \frac{1}{2D-3} \cdot e^{2x} = 2x \cdot \frac{1}{4-3} \cdot e^{2x}$$

$$= 2xe^{2x} \text{ Ans}$$

$(D^2 - 3D + 2)y = \sin 3x$

$D^2 \to -a^2$

$$\frac{1}{D^2-3D+2} \cdot \sin 3x = \frac{1}{-9-3D+2}\sin 3x$$

$$= \frac{1}{-7-3D}\sin 3x = -\left(\frac{7-3D}{49-9D^2}\right)\sin 3x$$

$$\frac{-1}{7+3D} \cdot \frac{7-3D}{7-3D}$$

$$\frac{7-3D}{(7)^2-(3D)^2}$$

$$= -\left[\frac{7-3D}{49-9(-9)}\right]\sin 3x$$

$$= -\left[\frac{7-3D}{}\sin 3x\right]$$

49
81

$$= -\left[\frac{7-3D}{130}\sin 3x\right]$$

$$\begin{array}{r} 49 \\ 81 \\ \hline 130 \end{array}$$

$$= \frac{-1}{130}\left[7\sin 3x - 3\cos 3x \cdot 3\right]$$

$$-\frac{7}{130}\sin 3x + \frac{9}{130}\cos 3x$$

Case 3

$$\frac{1}{D^2+2D-3}\cdot(x^2+x+1)$$

$$= \frac{1}{-3\left[1-\left(\frac{D^2+2D}{3}\right)\right]}\cdot x^2+x+1$$

$$D(x^2+x+1) = 2x+1$$

$$D^2(x^2+x+1) = 2$$

$$D^3(x^2+x+1) = 0$$

$$(1-x)^{-1} = 1+x+x^2+x^3$$

$$= \frac{-1}{3}\left[1-\left(\frac{D^2+2D}{3}\right)\right]^{-1}(x^2+x+1)$$

$$= \frac{-1}{3}\left[1+\left(\frac{D^2+2D}{3}\right)+\left(\frac{D^2+2D}{3}\right)^2\right](x^2+x+1)$$

$$(D^2+2D)^2 = \cancel{D^4}+4D^2+\cancel{4D^3}$$

$$= \frac{-1}{3}\left[1+\left(\frac{D^2+2D}{3}\right)+\left(\frac{4D^2}{9}\right)\right]\cdot(x^2+x+1)$$

$$y_p = \frac{-1}{3}\left[(x^2+x+1)+\frac{1}{3}\left[2+2(2x+1)\right]+\frac{4}{9}(2)\right]$$

$(D^3-D^2-6D)y = 1+x^2$

$$\frac{1}{D^3-D^2-6D}\cdot(x^2+1) = \frac{-1}{6D\left[1-\left(\frac{D^3-D^2}{6D}\right)\right]}\cdot(x^2+1)$$

$$= \frac{-1}{6D}\left[1 - \left(\frac{D^3 - D^2}{6D}\right)\right]^{-1}(x^2+1)$$

$$= \frac{-1}{6D}\left\{1 - \left(\frac{D^2 - D}{6}\right)\right]^{-1}(x^2+1)$$

$$= \frac{-1}{6D}\left[1 + \left(\frac{D^2 - D}{6}\right) + \left(\frac{D^2 - D}{6}\right)^2\right] \cdot (x^2+1)$$

$$= \frac{-1}{6D}\left[1 + \left(\frac{D^2 - D}{6}\right) + \left(\frac{D^2}{36}\right)\right] \cdot (x^2+1)$$

$$= \frac{-1}{6D}\left\{(x^2+1) + \frac{1}{6}\left[2 - 2x\right] + \frac{1}{36}(2)\right]$$

$$= \frac{-1}{6}\frac{1}{D}\left\{x^2 - \frac{x}{3} + \left(\frac{25}{18}\right)\right]$$

$$= \frac{-1}{6}\left\{\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^2}{2} + \left(\frac{25}{18}\right)x\right]$$

$$\boxed{\begin{aligned} &D(x^2+1) = 2x \\ &D^2(x^2+1) = 2 \\ &D^3(\ ) = 0 \end{aligned}}$$

$$(D^2 - D)^2 = \cancel{D^4} + D^2 - \cancel{2D^3}$$

$$1 + \frac{1}{3} + \frac{1}{18}$$