L32 03 April 2021

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Find the parametric form of the curve of intersection of the plane $y=x$ and the surface $z=\sqrt{16-x^2-y^2}$

Let $x=t$

$y=x=t$

$z=\sqrt{16-x^2-y^2}$

$z=\sqrt{16-t^2-t^2}$

$z=\sqrt{16-2t^2}$

$y^2=4ax$

$x=at^2$

$y=2at$

$\vec{r}(t)=x\hat{i}+y\hat{j}+z\hat{k}$

$$\boxed{\vec{r}(t)=t\hat{i}+t\hat{j}+\sqrt{16-2t^2}\,\hat{k}}$$

$0\le t\le 2\sqrt{2}$

& $16-2t^2\ge 0$

$16\ge 2t^2$

$2\sqrt{2}\ge t$

(2)

$x=y$, $y=z$

$x=t$, $y=t$, $z=t$

$\vec{r}(t)=t\hat{i}+t\hat{j}+t\hat{k}$

(3)

$x+y+z=3$, $y-z=0$

$$\boxed{y=z=t}$$

$x+t+t=3$

$$\boxed{x=3-2t}$$

$r(t)=x\hat{i}+y\hat{j}+z\hat{k}=(3-2t)\hat{i}+t\hat{j}+t\hat{k}$

(4)

$y^2+z^2=9$, $x=9-y^2$, $y=3\sin t$

$x=9-(3\sin t)^2$

$x=9-9\sin^2 t=9(1-\sin^2 t)$

$$\boxed{x=9\cos^2 t}$$

$$\boxed{x = 9\cos^2 t}$$

$$y^2 + z^2 = 9 \Rightarrow z^2 = 9 - y^2 = 9 - 9\sin^2 t$$

$$z^2 = 9(1-\sin^2 t)$$

$$z^2 = 9\cos^2 t$$

$$\boxed{z = \pm 3\cos t}$$

$$\vec{r}(t) = 9\cos^2 t\,\hat{i} + 3\sin t\,\hat{j} \pm (3\cos t)\,\hat{k}$$

Limit, Continuity and Differentiability of (Vector function) :-

Vector function

$\vec{v}(t)$

$$\lim_{x \to a} f(x) = l$$

$$\boxed{x \to a}$$

Rat

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$$\lim_{t \to a} |\vec{v}(t) - \vec{l}| = 0$$

$$\boxed{\lim_{t \to a} \vec{v}(t) = \vec{l}}$$

$$\lim_{x \to a} |f(x) - l| = 0$$

$$\vec{v}(t) = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$

$$\vec{l} = l_1\hat{i} + l_2\hat{j} + l_3\hat{k}$$

$$\lim_{t \to a} v_1(t) = l_1, \quad \lim_{t \to a} v_2(t) = l_2, \quad \lim_{t \to a} v_3(t) = l_3$$

Continuity

Lim exists

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

$$\boxed{\lim_{t \to a} \vec{v}(t) = \vec{v}(a)}$$

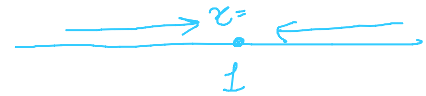
$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)}$$

$x = a$

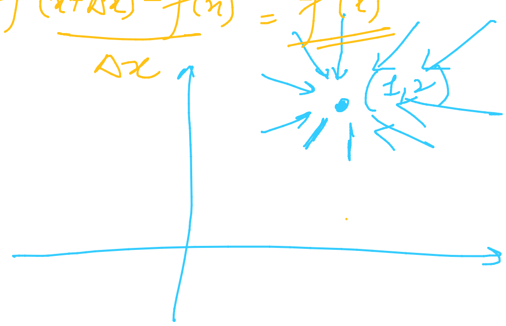
Differentiability

$$\lim_{\Delta t\to 0} \frac{\vec{v}(t+\Delta t)-\vec{v}(t)}{\Delta t} \text{ exists}$$

$$= \frac{d\vec{v}}{dt} = \vec{v}\,'(t)$$



$$\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = f'(x)$$



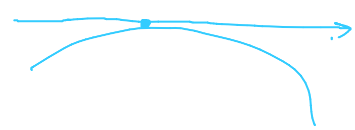
$$\vec{v}(t) = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$

$$\boxed{\vec{v}\,'(t) = v_1'(t)\,\hat{i} + v_2'(t)\,\hat{j} + v_3'(t)\,\hat{k}}$$

$$\left.\frac{dy}{dx}\right|_{x=a}$$

$$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\frac{d\vec{r}}{dt} = \vec{r}\,'(t) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$$



→ tangent line to the curve at the point $P(x,y,z)$

Find the tangent vector to the curve

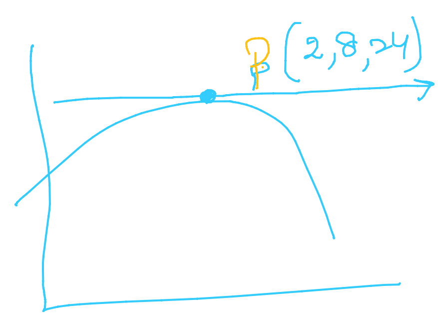
$x=t,\ y=2t^2,\ z=3t^3,\quad t=2.$

$$\vec{r}(t) = t\hat{i} + 2t^2\hat{j} + 3t^3\hat{k}$$

The tangent vector

$$\vec{r}\,'(t) = \frac{d\vec{r}}{dt} = \hat{i} + 4t\,\hat{j} + (9t^2)\,\hat{k}$$

$$\vec{r}\,'(2) = \left.\frac{d\vec{r}}{dt}\right|_{t=2} = \hat{i} + 8\hat{j} + 36\hat{k}$$



Parametric form

tangent line

$$\frac{x-2}{\ } = \frac{y-8}{\ } = \frac{z-24}{\ } = t$$

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n} = t$$

$$\frac{x-2}{1} = \frac{y-8}{8} = \frac{z-24}{36} = t$$

$$x = 2+t, \quad y = 8+8t, \quad z = 24+36t \qquad \vec{r} = \vec{a} + \vec{b}t$$

$$\vec{r} = (2+t)\hat{i} + (8+8t)\hat{j} + (24+36t)\hat{k}$$