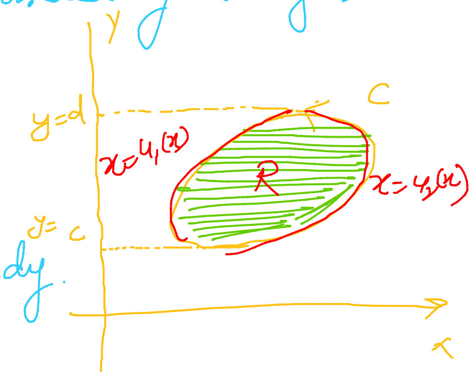
L43 24 April 2021

24 April 2021 11:13

Green's Theorem

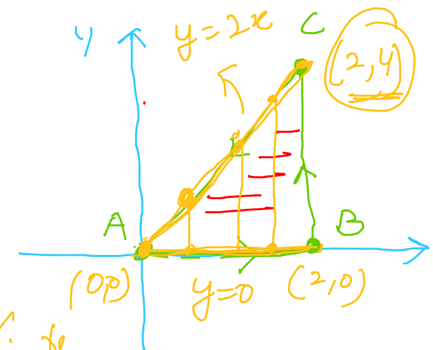
Let C be a piecewise smooth simple closed curve bounded by a region R.

If f, g, $\frac{\partial f}{\partial y}$ and $\frac{\partial g}{\partial x}$ are continuous on R then.



$$\oint_C f(x,y)\,dx + g(x,y)\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy.$$

Use Green Theorem to Evaluate $\oint_C (x+y)\,dx + x^2\,dy$. C is the triangle with vertices at $(0,0)$, $(2,0)$ and $(2,4)$ taken in that order.



$$\oint_C (x+y)\,dx + x^2\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$$

$f = x+y \qquad \frac{\partial f}{\partial y} = 1$

$g = x^2 \qquad \frac{\partial g}{\partial x} = 2x$

$$= \iint_R (2x-1)\,dx\,dy$$

$$= \int_0^2 \int_0^{2x} (2x-1)\,dy\,dx$$

Limits

$$\boxed{\begin{aligned} 0 \le x \le 2 \\ 0 \le y \le 2x \end{aligned}}$$

$(0,0)$ —— $(2,4)$

$$y-0 = \frac{4-0}{2-0}(x-0)$$

$$\boxed{y=2x}$$

$$= \int_0^2 \left[(2x-1)(y)_0^{2x}\right] dx$$

$$= \int_0^2 (2x-1)(2x)\,dx$$

$$= \int_0^2 (4x^2-2x)\,dx = \left[4\cdot\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}}\right]_0^2$$

$$= \frac{4\times 8}{3} - 4 = \frac{32-12}{3} = \frac{20}{3}$$

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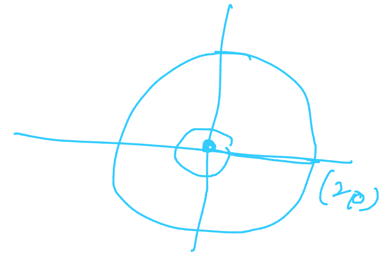
$\oint_C x^3 dy - y^3 dx$, C is the circle $\left.\begin{array}{l} x = 2\cos\theta \\ y = 2\sin\theta \end{array}\right\}$ $0 \le \theta \le 2\pi$

$\oint_C x^3 dy - y^3 dx = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx dy$ $\qquad x^2 + y^2 = 4$

$f = -y^3 \quad \frac{\partial f}{\partial y} = -3y^2$

$g = x^3 \quad \frac{\partial g}{\partial x} = 3x^2$

$= \iint_R (3x^2 + 3y^2) dx dy$



$= 3 \iint_R (x^2 + y^2) dx dy$

$= 3 \iint_R 4 \, dx dy = 12 \iint_R dx dy$ $\qquad x^2 + y^2 = 4$

$\iint_R dx dy$

$= \int_0^{2\pi} \int_0^2 r \, dr \, d\theta$

$= 12(\pi (2)^2)$

$= 48\pi$

$x = r\cos\theta$
$y = r\sin\theta$

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56. By using Green's theorem $\oint (\overset{f}{x^2 + y^2}) dx + (\overset{g}{y + 2x}) dy$ over curve C is same as

a. $\iint (2 - 2y) dxdy$ over the region R

b. $\iint (2 + 2y) dxdy$ over the region R

$\int f dx + g dy = \iint \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx dy = \iint (2 - 2y) \, dx dy$

c. $\iint (2 - y) dxdy$ over the region R

d. $\iint (2 + y) dxdy$ over the region R