Poisson Dist

$n \to \infty$
$p \to 0$

$np = \lambda$ constant

rare events

$$\frac{100}{100000} = 0.01$$

Poisson dist is discrete probability dist of a discrete random variable x. is suitable for rare events for which probability of occurrence is very small $p \to 0$, and n number of trials are very large.

Probability mass function for random variable x is given by

$$f(x,\lambda) = P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,1,2,\ldots$$

$$= 0, \quad \text{otherwise}$$

(I)

$$\sum_{x=0}^{\infty} p(x) = \sum_{x=0}^{\infty} f(x, \lambda) = 1$$

(II)

$$p(x) \geq 0$$

λ is called the parameter of poisson dist, so poisson dist is uniparametric dist.

λ = mean of dist

λ = variance

so poisson dist

mean = Variance = λ

Ex ① Number of misprints.

② Number of road accident

③ Number of defective in production centre.

Remark ①

Poisson dist can be approximated by Binomial Dist

when $n \to \infty$, $p \to 0$

$np = \lambda$ (constant)

λ = mean.

E ① Find mean of poisson dist.

Sol $f(x, \lambda) = p(x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}, \quad x = 0, 1, 1, \ldots\ldots$

$$E(x) = \bar{x} = \sum_{x=0}^{\infty} x \, p(x) = \sum_{x=0}^{\infty} x \, f(x)$$

$$= \sum_{x=0}^{\infty} x \, \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$\{ x! = x(x-1)! \}$

$$E(x) = \bar{x} = \sum_{x=0}^{\infty} x \frac{e^{-\lambda} \lambda^{x-1+1}}{x(x-1)!}$$

$$= e^{-\lambda} \lambda^{1} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$E(x) = \lambda e^{-\lambda} \left(\frac{\lambda^{0}}{0!} + \frac{\lambda^{1}}{1!} + \frac{\lambda^{2}}{2!} \cdots \right)$$

$$E(x) = \lambda e^{-\lambda} e^{\lambda} = \lambda e^{-\lambda+\lambda} = \lambda e^{0}$$

$$\boxed{E(x) = \lambda}$$

$$V(x) = E(x^2) - \left(E(x)\right)^2$$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda} \lambda^x}{x!} - \lambda^2$$

$$\boxed{V(x) = \lambda}$$

Ex ① The average number of phone calls per minute coming into a switch board between 2 and 4 pm is 2.5. Determine the probability that during one particular minute there will be (a) 0 (b) 1 (c) 2 (d) atmost 4 (e) atleast 6 phone calls.

Sol $\lambda = 2.5$ $p(x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$

$$p(x=0) = \frac{e^{-2.5}(2.5)^{0}}{0!}$$

$$p(x=1) = \frac{e^{-2.5}(2.5)^{1}}{1!}$$

$$p(x=2) = \frac{e^{-2.5}(2.5)^{2}}{2!}$$

$$P(x \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$$

$$= \frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!}$$

$$+ \frac{e^{-2.5}(2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!} +$$

$$\frac{e^{-2.5}(2.5)^4}{4!}$$

$$P(X \geq 6) = P(6) + P(7) + P(8) \ldots\ldots$$

$$= 1 - \left[P(0) + P(1) + P(2) + P(3) + P(4) + P(5) \right]$$

$$= 1 - \left[\frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!} + \frac{e^{-2.5}(2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!} + \frac{e^{-2.5}(2.5)^4}{4!} + \frac{e^{-2.5}(2.5)^5}{5!} \right]$$

Ex 2 Suppose that on the average one person in 1000 makes a numerical mistake in preparing ITR. If 10000 forms are selected randomly and examined find the probability that 6, 7 or 8 of the forms will be in error.

Sol $\frac{1}{1000}$ $n = 10000$

$$\lambda = np = 10000 \times \frac{1}{1000} = 10$$

$$P(6, 7 \text{ or } 8) = P(6) + P(7) + P(8)$$

$$= \frac{e^{-10}\, 10^{6}}{6!} + \frac{e^{-10}\, 10^{7}}{7!} + \frac{e^{-10}\, 10^{8}}{8!}$$

Ex 3 Suppose 300 misprints are distributed randomly throughout a ~~book~~ of 500 pages. Find the probability that a given page contain (i) exactly Two misprints

(ii) Two or more misprints

$$\lambda = \frac{300}{500} = \frac{3}{5}$$

mis = 300

page = 500

$$\lambda = 0.6$$

$$p(x=2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-0.6}(0.6)^2}{2!}$$

$$p(x \geq 2) = p(2) + p(3) + \cdots \; p(300)$$

$$= 1 - \left[\boxed{p(0)} + p(1)\right]$$

$$P(x \geq 2) = 1 - \left[\frac{e^{-0.6} (0.6)^0}{0!} + \frac{e^{-0.6} (0.6)^1}{1!} \right]$$

Ex 4 In a factory producing blades the probability of any blade being defective is $0.002 = p$

If blades are supplied in packet of 10, determine the number of packets containing (a) no defective (b) 1 defective

(c) Two defectives in an consignment of 10000 packets $\lambda = np = 0.02$

Sol $p = 0.002$ $n = 10$ $N = 10000$

$$E(0) = N\,P(0) = 10000 \frac{e^{-0.02}(0.02)^0}{0!}$$

$$= 9801.98 \simeq 9802$$

$$E(1) = N\,p(1)$$

$$= 10000 \; \frac{e^{-0.02}\,(0.02)^1}{1!}$$

$$= 196.04 \cong 196$$

$$E(2) = N\,p(2) = 10000 \; \frac{e^{-0.02}\,(0.02)^2}{2!}$$

$$= 1.96 \cong 2$$