Unit I

Definitions of Various Terms. In this section we will define and explain the various terms which are used in the definition of probability.

Trial and Event. Consider an experiment which, though repeated under essentially identical conditions, does not give unique results but may result in any one of the several possible outcomes. The experiment is known as a *trial* and the outcomes are known as *events* or *cases*. For example :

(i) Throwing of a die is a trial and getting 1(or 2 or 3, ...or 6) is an event.

(ii) Tossing of a coin is a trial and getting head (H) or tail (T) is an event.

(iii) Drawing two cards from a pack of well-shuffled cards is a trial and getting a king and a queen are events.

Random

20 — 4

Non Random

10 — Batsman

2

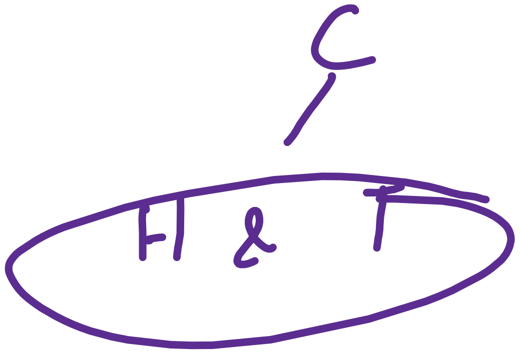
Exhaustiye Events. The total number of possible outcomes in any trial is known as exhaustive events or exhaustive cases. For example :

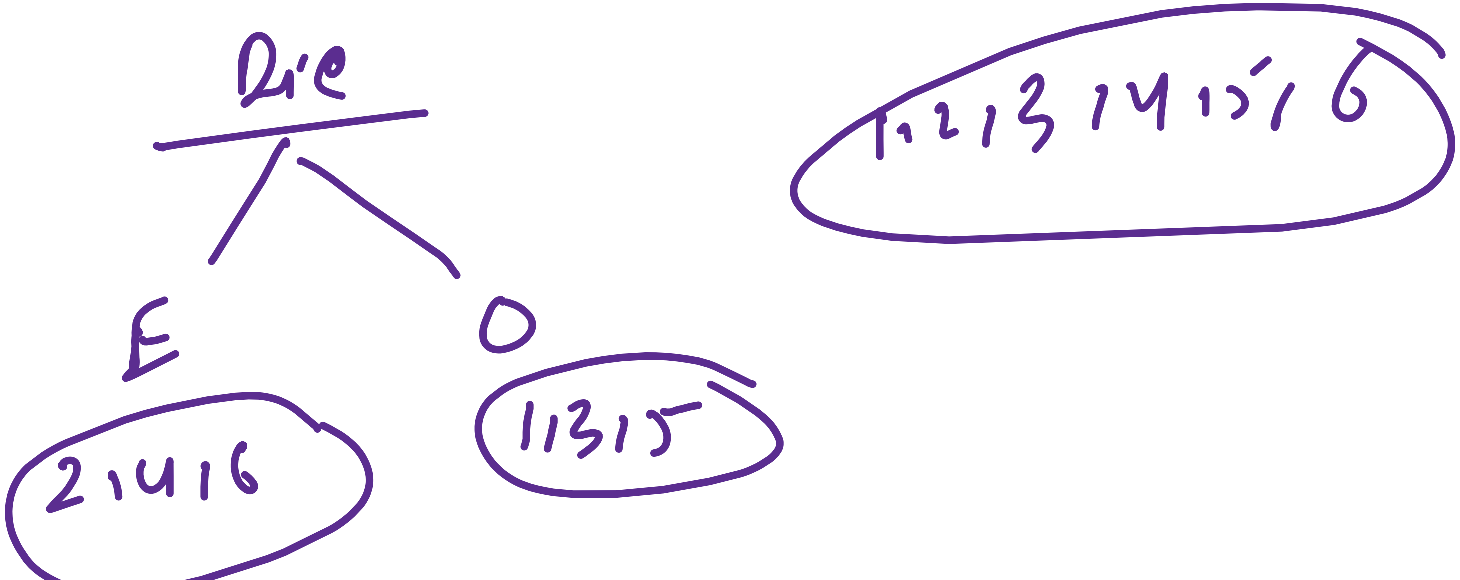
(i) In tossing of a coin there are two exhaustive cases, *viz.*, head and tail (the possibility of the coin standing on an edge being ignored).

(ii) In throwing of a die, there are six exhaustive cases since any one of the 6 faces 1, 2, ...,6 may come uppermost.

(iii) In drawing two cards from a pack of cards the exhaustive number of cases is ${}^{52}C_2$, since 2 cards can be drawn out of 52 cards in ${}^{52}C_2$ ways.

(iv) In throwing of two dice, the exhaustive number of cases is $6^2 = 36$, since any of the 6 numbers 1 to 6 on the first die can be associated with any of the six numbers on the other die.





52

♥ Heart 13

♦ Diamond 13

♠ Spade 13

♣ club 13

26 – Red

26 – Black

13 → K, Q, J → face cards, Court cards.

13 → 2 – 10 → Number card.

13 → A → Ace

$6 \times 6 = \underline{36}$

(1,1)	1 2	1 3	1 4	1 5	1 6
(2,1)	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	(4,6)
5 1	5 2	5 3	5 4	5 5	(5,6)
6 1	6 2	6 3	6 4	6 5	(6,6)

Sum :	2	3	4	5	6	7	8	9	10	11	12
No	1	2	3	$\underline{4}$	5	6	5	4	3	2	1

$/36$

Favourable Events or Cases. The number of cases favourable to an event in a trial is the number of outcomes which entail the happening of the event. For example,

(i) In drawing a card from a pack of cards the number of cases favourable to drawing of an ace is 4, for drawing a spade is 13 and for drawing a red card is 26.

(ii) In throwing of two dice, the number of cases favourable to getting the sum 5 is : (1,4) (4,1) (2,3) (3,2), *i.e.*, 4.

B – 26

Q

K-S — B

K-C — A

K-D — R

K-H

(a)

4 — K — 52

Exhaustive

52 — Red Card — 26

B

$4 \cup 26$

$A \cup B$ — 28

(B) Not Exhaustive

52

Red — 26

Black — 26

$R \cup B$ — 52

Exhaustive

Exhaustive Die

E O

2, 4, 6   1, 3, 5

E ∪ O → {1, 2, 3, 4, 5, 6}

Die {1, 2, 3, 4, 5, 6}

E

(2, 4, 6)

B → No. divisible by 3

(3, 6)

E ∪ B → 2, 3, 4, 6

E & B are Not Exhaustive

Mutually exclusive events. Events are said to be *mutually exclusive* or *incompatible* if the happening of any one of them precludes the happening of all the others, *i.e.*, if no two or more of them can happen simultaneously in the same trial. For example :

(*i*) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others, in the same trial, is ruled out.

(*ii*) Similarly in tossing a coin the events head and tail are mutually exclusive.

Mutually Ex

52

K

K-H
K-D
K-S
K-C

Q

Q-H
Q-D
Q-S
Q-C

Not Mutually Ex.

52

K

K-H
K-D
K-S
K-C

Red Card

H + D

K-H

K-D

Die

A - E

2, 4, 6

B - Number divisible by 3

3, 6

(a) Not Mutually Exclusive

(b) Mutually Exclusive.

Die

E

2, 4, 6

O

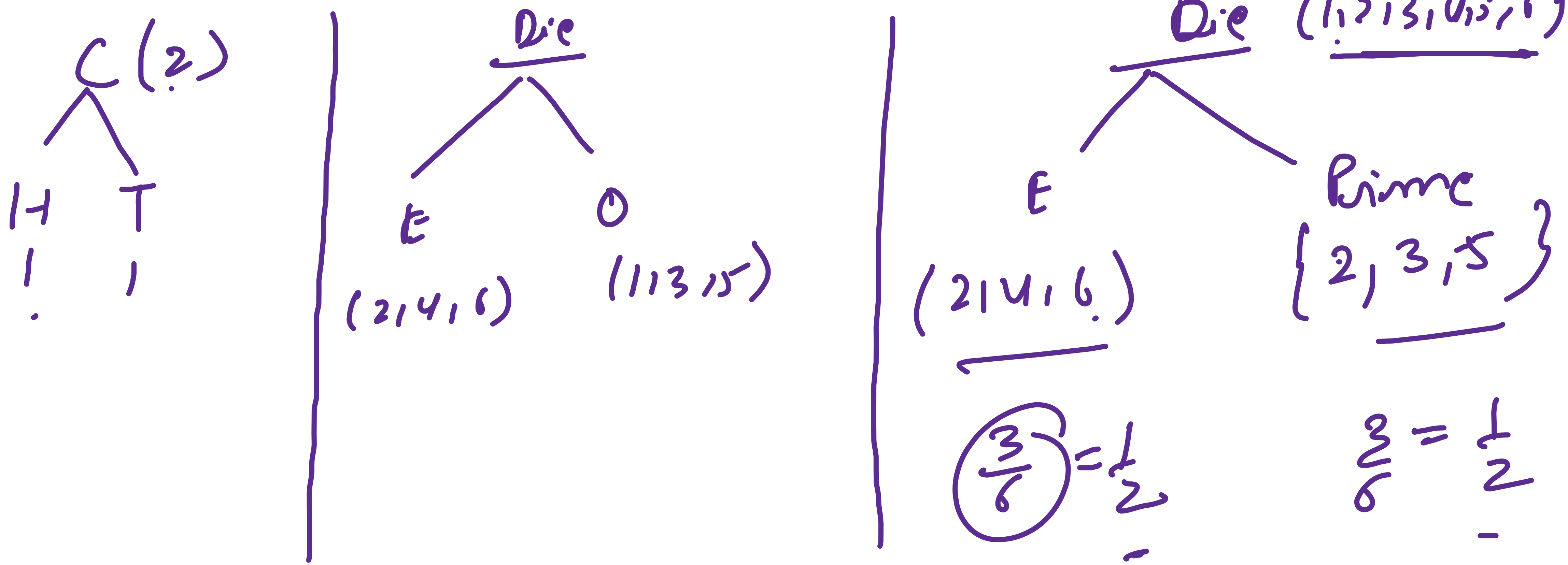
1, 3, 5

Mutually Ex.

Equally likely events. Outcomes of a trial are set to be equally likely if taking into consideration all the relevant evidences, there is no reason to expect one in preference to the others. For example

(i) In tossing an unbiased or uniform coin, head or tail are equally likely events.

(ii) In throwing an unbiased die, all the six faces are equally likely to come.

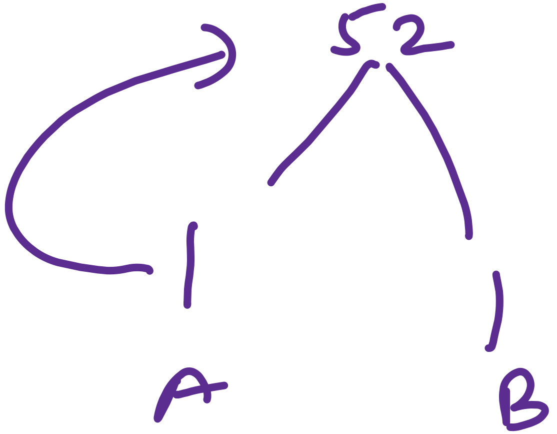


Independent events. Several events are said to be independent if the happening (or non-happening) of an event is not affected by the supplementary knowledge concerning the occurrence of any number of the remaining events. For example

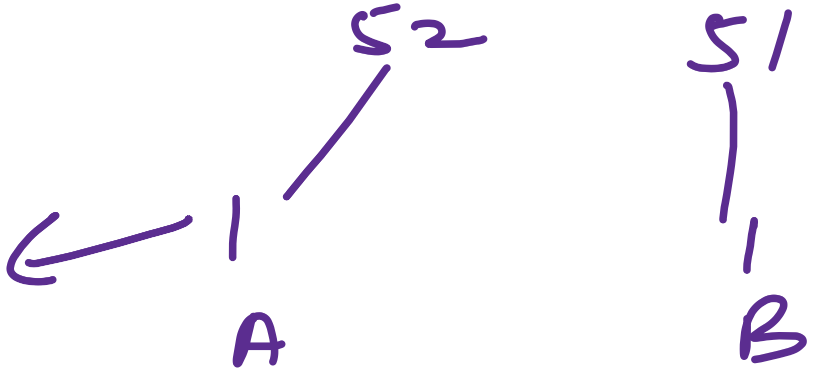
(i) In tossing an unbiased coin the event of getting a head in the first toss is independent of getting a head in the second, third and subsequent throws.

(ii) If we draw a card from a pack of well-shuffled cards and replace it before drawing the second card, the result of the second draw is independent of the first draw. But, however, if the first card drawn is not replaced then the second draw is dependent on the first draw.

Independent E
(with replacement)



Dependent
(without replacement)



Mathematical or Classical or 'a priori' Probabability

Definition. If a trial results in n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event E, then the probability 'p' of happening of E is given by

$$\boxed{p = P(E) = \frac{\text{Favourable number of cases}}{\text{Exhaustive number of cases}} = \frac{m}{n}}$$

Remark

$$0 \le m \le n$$

$$\frac{0}{n} \le \frac{m}{n} \le \frac{n}{n}$$

$$\boxed{0 \le P(E) \le 1}$$

$P(E) = 1$

E is a sure event
certain

$P(E) = 0$

E is impossible event

the odds in favour of E $= \dfrac{\text{favourable cases}}{\text{unfavourable cases}} = \dfrac{n_1}{n_2}$

the odds against E $= \dfrac{\text{unfavourable cases}}{\text{favourable cases}} = \dfrac{n_2}{n_1}$

$n_1 \rightarrow$ favourable cases

$n_2 \rightarrow$ unfavourable cases

$Tc = n_1 + n_2$

$$\boxed{P(E) = \frac{n_1}{n_1 + n_2}}$$

Ex $E \to$ number divisible by 3 on

Throwing a die $\{1, 2, 3, 4, 5, 6\}$

$E \to 3, 6$ $n_1 = 2$ favourable cases

$n_2 = 4$ $1, 2, 4, 5$ unfavourable cases.

odd in favour E : $\frac{2}{4} = \frac{1}{2}$ $1 : 2$

odd against E $\frac{4}{2} = \frac{2}{1}$ $2 : 1$

$P(E) = \frac{1}{1+2} = \frac{1}{3}$

$PE = \frac{2}{6}$

$= \frac{1}{3}$

Remarks. 1. Probability 'p' of the happening of an event is also known as the probability of success and the probability 'q' of the non- happening of the event as the probability of failure.

sure

2. If $P(E) = 1$, E is called *a certain event* and if $P(E) = 0$, E is called an *impossible* event.

3. **Limitations of Classical Definition.** This definition of Classical Probability breaks down in the following cases :

(i) If the various outcomes of the trial are not equally likely or equally probable. For example, the probability that a candidate will pass in a certain test is not 50% since the two possible outcomes, *viz.*, sucess and failure (excluding the possibility of a compartment) are not equally likely.

(ii) If the exhaustive number of cases in a trial is infinite.

Statistical or Empirical Probability

Definition (Von Mises). *If a trial is repeated a number of times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event happens to the number of trials, as the number of trials become indefinitely large, is called the probability of happening of the event. (It is assumed that the limit is finite and unique).*

Symbolically, if in n trials an event E happens m times, then the probability 'p' of the happening of E is given by

$$\boxed{p = P(E) = \lim_{n \to \infty} \frac{m}{n}}$$

$n = 1000$

$m = 510$ (H)

$P = \frac{510}{1000}$

$P = 0.510 \cong 0.5$

Q1

Marks obtained	No of Student
0 - 10	10
10 - 20	12
20 - 30	15
30 - 40	18
40 - 50	5
	60

$P(20 \le x \le 30) = \frac{15}{60}$

$= \frac{1}{4}$

$P(x > 40) = \frac{5}{60}$

$= \frac{1}{12}$

Q2 What is the chance that a leap year selected at random will contain 53 Sundays?

Sol 366

(a) $\frac{1}{7}$ (B) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

$$7\overline{)366}\ (52$$
$$35$$
$$16$$
$$14$$
$$2$$

Sat	Sunday
Sund	Mond
Mond	Tue
Tue	wed
wed	Thrusday
Thrusday	Fri
Fri	Saturday

$TC = 7$

$FC = 2$

$$\boxed{P = \frac{2}{7}}$$

Q3 A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue?

Sol

R	W	B	Total
3	6	7	16

$$TC = {}^{16}C_2 = \frac{16 \times 15}{2!} = \frac{\cancel{16}^{8} \times 15}{\cancel{2}} = 8 \times 15$$

$$FC = {}^{6}C_1 \times {}^{7}C_1 = 6 \times 7$$

$$P = \frac{\cancel{6}^{3} \times 7}{\cancel{8}^{4} \times \cancel{15}^{5}} = \frac{7}{20}$$

$$\boxed{P = \frac{7}{20}}$$

A book contains 100 pages. A page is chosen at random. What is the chance that the sum of the digits on the page is equal to 8?

(a) 0.08 (b) 0.09 (c) 0.90 (d) 0.10

1 ——— 100

8, 17, 26, 35, 44, 53, 62, 71, 80,

TC = 100

FC = 9

$P = \frac{9}{100} = 0.09$

Example (a) *Two cards are drawn at random from a well-shuffled pack of 52 cards. Show that the chance of drawing two aces is 1/221.*

(b) *From a pack of 52 cards, three are drawn at random. Find the chance that they are a king, a queen and a knave.* — Jack

(c) *Four cards are drawn from a pack of cards. Find the probability that*

(i) *all are diamond,* (ii) *there is one card of each suit, and* (iii) *there are two spades and two hearts.*

(a) $TC = {}^{52}C_2$

$FC = {}^{4}C_2$

$P = \frac{{}^{4}C_2}{{}^{52}C_2}$

$= \frac{\frac{4 \times 3}{2!}}{\frac{52 \times 51}{2!}} = \frac{\cancel{4} \times \cancel{3}}{\cancel{52}_{13} \times \cancel{51}_{17}}$

$\boxed{P = \frac{1}{221}}$

(b) $TC = {}^{52}C_3$

$FC = {}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1 = 4 \times 4 \times 4$

$$\boxed{P = \frac{4 \times 4 \times 4}{{}^{52}C_3}}$$

(c) $TC = {}^{52}C_4$

(i) $FC = {}^{13}C_4$ $\qquad P = \dfrac{{}^{13}C_4}{{}^{52}C_4}$

(ii) $FC = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$$P = \frac{13 \times 13 \times 13 \times 13}{{}^{52}C_4}$$

(iii) $FC = {}^{13}C_2 \times {}^{13}C_2$

$$P = \frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$$