1. A body of weight 300 N is lying on a rough horizontal plane having coefficient of friction 0.3. Find the magnitude of the force which can move the body, while acting at an angle of 25° with the horizontal. **(P=87.1 N)**

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.

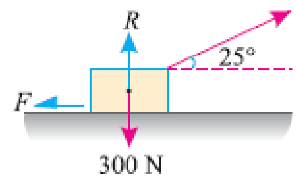


Fig. 8.2.

Resolving the forces horizontally,

$$F = P\cos\alpha = P\cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P\sin\alpha = 300 - P\sin 25°$$
$$= 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

or $$90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$\therefore$ $$P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \textbf{Ans.}$$

2. A body resting on a rough horizontal plane required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of the friction. **(W=991.2 N and coefficient of friction=0.173)**

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°

Let W = Weight of the body

R = Normal reaction, and

μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (*a*).

Resolving the forces horizontally,

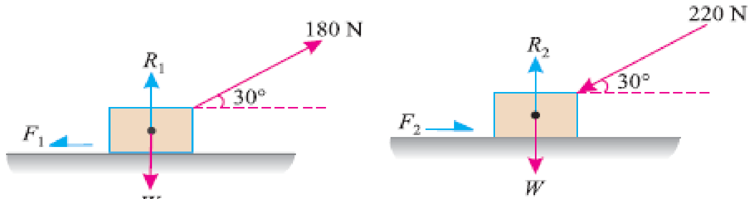
$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9 \text{ N}$$

and now resolving the forces vertically,

$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90 \text{ N}$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \quad \ldots(i)$$



(*a*) Pull of 180 N

(*b*) Pull of 220 N

Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (*b*).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2),

$$190.5 = \mu.R_2 = \mu (W + 110) \quad \ldots(ii)$$

Dividing equation (*i*) by (*ii*)

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu (W - 90)}{\mu (W + 110)} = \frac{W - 90}{W + 110}$$

$$155.9\, W + 17\,149 = 190.5\, W - 17\,145$$

$$34.6\, W = 34\,294$$

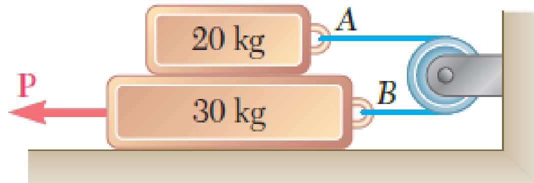
or $$W = \frac{34\,294}{34.6} = 991.2 \text{ N} \quad \textbf{Ans.}$$

Now substituting the value of W in equation (*i*),

$$155.9 = \mu\,(991.2 - 90) = 901.2\,\mu$$

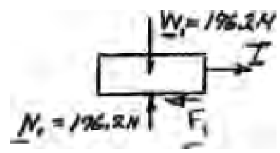
$\therefore$ $$\mu = \frac{155.9}{901.2} = 0.173 \quad \textbf{Ans.}$$

3. The Coefficients of friction are μ_S=0.4 and μ_K=0.3 between all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached as shown (b) is removed. (**P=353.2 N and P=196.2 N)**



SOLUTION

(*a*) Free body: 20-kg block

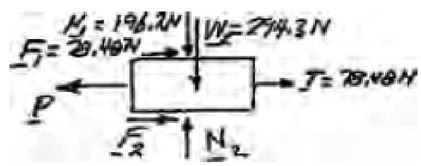


$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$

Free body: 30-kg block



$$W_2 = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$N_2 = 196.2\text{ N} + 294.3\text{ N} = 490.5\text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

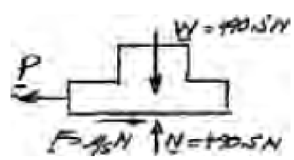
$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

$$P = 78.48\text{ N} + 196.2\text{ N} + 78.48\text{ N} = 353.2\text{ N}$$

$\mathbf{P} = 353\text{ N} \leftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together



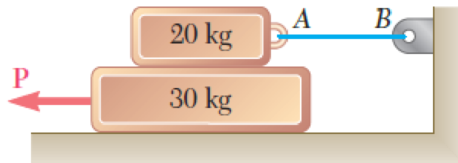
$$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$$

$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

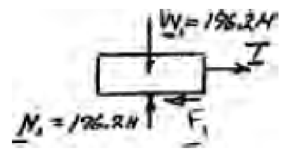
$\mathbf{P} = 196.2\text{ N} \leftarrow$ ◀

4. The Coefficients of friction are μ_S=0.4 and μ_K=0.3 between all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached as shown (b) is removed **(P=275 N and P=196.2 N)**



SOLUTION

(*a*) Free body: 20-kg block

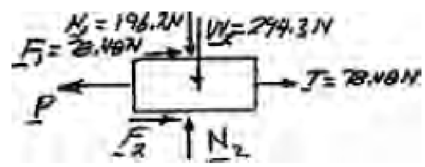


$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

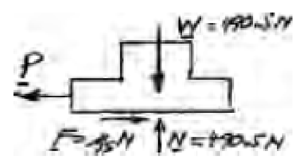
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$

$\mathbf{P} = 275 \text{ N} \leftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together



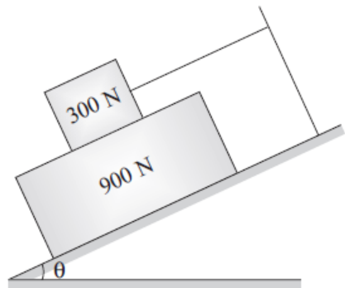
$$W = (50 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 490.5 \text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \leftarrow$ ◀

5. What should be the value of θ which will make the motion of 900 N block down the block to impend? The coefficient of friction for all contact surfaces is 1/3. **(θ=29.1°)**



Solution: 900 N block is on the verge of moving downward. Hence frictional forces F_1 and F_2 [Ref. Fig. 5.6(*b*)] act up the plane on 900 N block. Free body diagrams of the blocks are as shown in Fig. 5.6(*b*).

Consider the equilibrium of 300 N block.

Σ Forces normal to plane = 0 →

$$N_1 - 300 \cos \theta = 0 \quad \text{or} \quad N_1 = 300 \cos \theta \qquad \text{...}(i)$$

From law of friction,

$$F_1 = \frac{1}{3} N_1 = 100 \cos \theta \qquad \text{...}(ii)$$

For 900 N block:

Σ Forces normal to plane = 0 →

$$N_2 - N_1 - 900 \cos \theta = 0$$

$$N_2 = N_1 + 900 \cos \theta$$

$$= 300 \cos \theta + 900 \cos \theta$$

$$= 1200 \cos \theta.$$

From law of friction,

$$F_2 = \mu_2 N_2 = \frac{1}{3} \times 1200 \cos \theta = 400 \cos \theta.$$

Σ Forces parallel to the plane = 0 →

$$F_1 + F_2 - 900 \sin \theta = 0$$

$$100 \cos \theta + 400 \cos \theta = 900 \sin \theta$$

$$\tan \theta = \frac{500}{900}$$

$$\theta = \mathbf{29.05°}$$

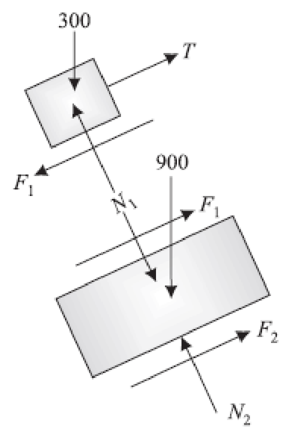
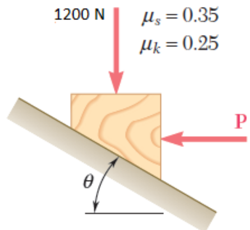


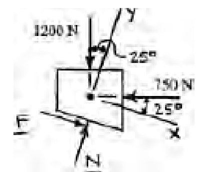
Fig. 5.6(b)

6. Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when θ=25and P=750N. **(in equilibrium & F=172.6 N)**



SOLUTION

<u>Assume equilibrium</u>:



$\overset{+}{\searrow} \Sigma F_x = 0$: $F + (1200\text{ N})\sin 25° - (750\text{ N})\cos 25° = 0$

$F = +172.589$ N $\qquad$ $\mathbf{F} = 172.589$ N ↘

$+\nearrow \Sigma F_y = 0$: $N - (1200\text{ N})\cos 25° - (750\text{ N})\sin 25° = 0$

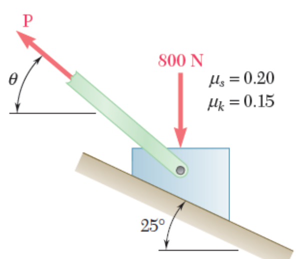
$N = +1404.53$ N $\qquad$ $\mathbf{N} = 1404.53$ N ↗

Maximum friction force: $F_m = \mu_s N = 0.35(1404.53\text{ N}) = 491.587$ N

<u>Block is in equilibrium</u> ◀

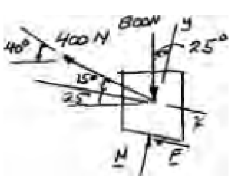
Since $F < F_m$, $\qquad$ $\mathbf{F} = 172.6$ N ↘ ◀

7. Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when θ= 40 and P=400 N **(in equilibrium & F=48.3 N)**



SOLUTION

<u>Assume equilibrium:</u>



$+\nearrow \Sigma F_y = 0$: $N - (800\text{ N})\cos 25° + (400\text{ N})\sin 15° = 0$

$N = +621.5$ N $\qquad$ $\mathbf{N} = 621.5$ N ↑

$\overset{+}{\searrow} \Sigma F_x = 0$: $-F + (800\text{ N})\sin 25° - (400\text{ N})\cos 15° = 0$

$F = +48.28$ N $\qquad$ $\mathbf{F} = 48.28$ N ↖

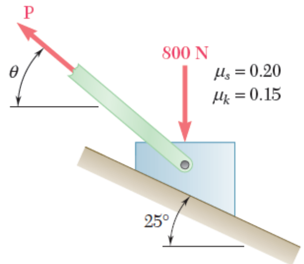
Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.20(621.5\text{ N})$$
$$= 124.3\text{ N}$$

<u>Block is in equilibrium</u> ◀

Since $F < F_m$, $\qquad$ $\mathbf{F} = 48.3$ N ↖ ◀

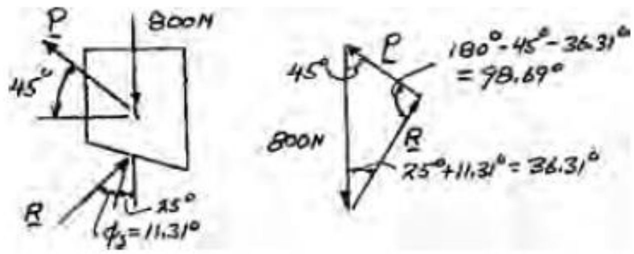
8. Knowing that θ=45, determine the range of values of P for which equilibrium is maintained **(222N≤P≤479N)**



To start block up the incline:

$$\mu_s = 0.20$$

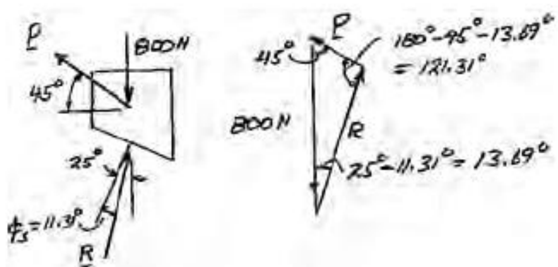
$$\phi_s = \tan^{-1} 0.20 = 11.31°$$



Force triangle:

$$\frac{P}{\sin 36.31°} = \frac{800\text{ N}}{\sin 98.69°} \qquad P = 479.2\text{ N} \triangleleft$$

To prevent block from moving down:

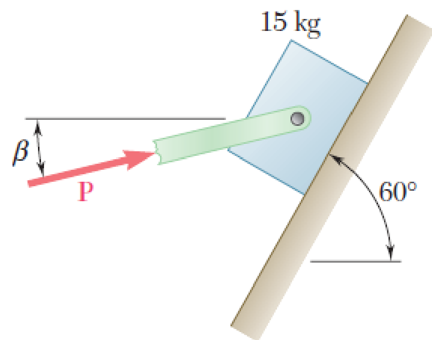


Force triangle:

$$\frac{P}{\sin 13.69°} = \frac{800\text{ N}}{\sin 121.31°} \qquad P = 221.61\text{ N} \triangleleft$$

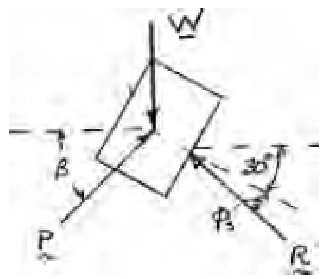
Equilibrium is maintained for $222\text{ N} \le P \le 479\text{ N}$ ◀

9. Knowing that the coefficient of friction between the 15-kg block and the incline is μs = 0.25, determine (a) the smallest value of P required to maintain the block in equilibrium, (b) the corresponding value of β. **(P=108.8 N, β=46°)**



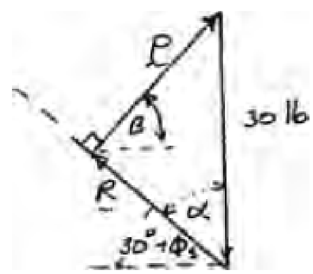
SOLUTION

FBD block (Impending motion downward):



$$W = (15\text{ kg})(9.81\text{ m/s}^2) = 147.150\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



(a) *Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$

So $\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$

and $P = (147.150\text{ N})\sin\alpha = (147.150\text{ N})\sin(45.964°)$

$P = 108.8\text{ N}$ ◀

(b) $\beta = 46.0°$ ◀

10. A block of weight W_1=100N rests on an inclined plane and another weight W_2 is attached to the first weight through a string as shown in fig below. If the coefficient of friction between the block and plane is 0.3, determine the maximum and minimum values of W_2 so that equilibrium can exist **(24≤W≤76)**

(*i*) When weight W_1 = 100 N is about to slide down the plane

(*ii*) When weight W_1 = 100 N is about to slide up the plane

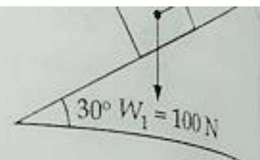


Fig. 9.37

Case (*i*): Refer Fig. 9.38 (*a*) for the free body diagram for impending motion down the plane.

Under equilibrium conditions,

$$\Sigma F \text{ (along the plane)} = 0$$

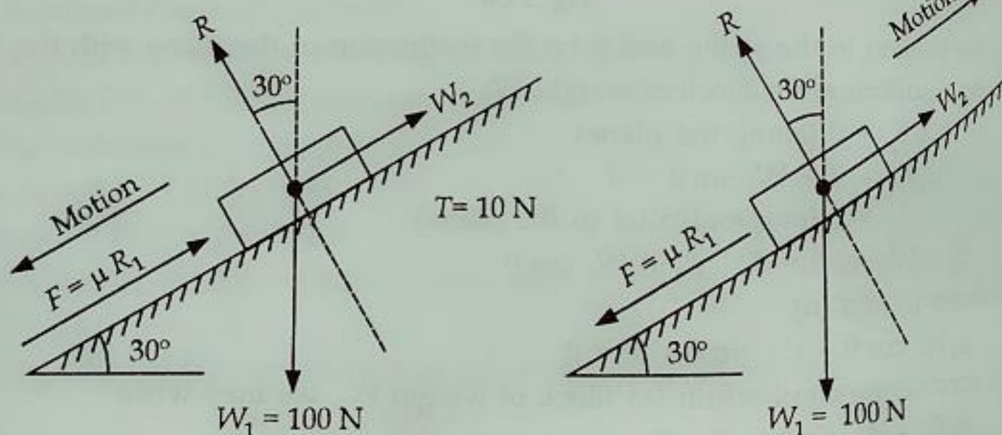
$$W_2 + \mu R - W_1 \sin 30^\circ = 0$$

$$W_2 = W_1 \sin 30^\circ - \mu R = 100 \sin 30^\circ - 0.3 R = 50 - 0.3 R$$

$$\Sigma F \text{ (perpendicular to the plane)} = 0$$

$$R = 100 \cos 30^\circ = 86.6 \text{ N}$$

$$\therefore \quad W_2 = 50 - 86.6 \times 0.3 = 24.02 \text{ N} \simeq \textbf{24 N}$$



(*a*) (*b*)

Case (*ii*): Refer Fig. 9.38 (*b*) for the free body diagram for impending motion up the plane.

Under equilibrium conditons,

$$\Sigma F \text{ (along the plane)} = 0$$

$$W_2 - W_1 \sin 30^\circ - \mu R = 0$$

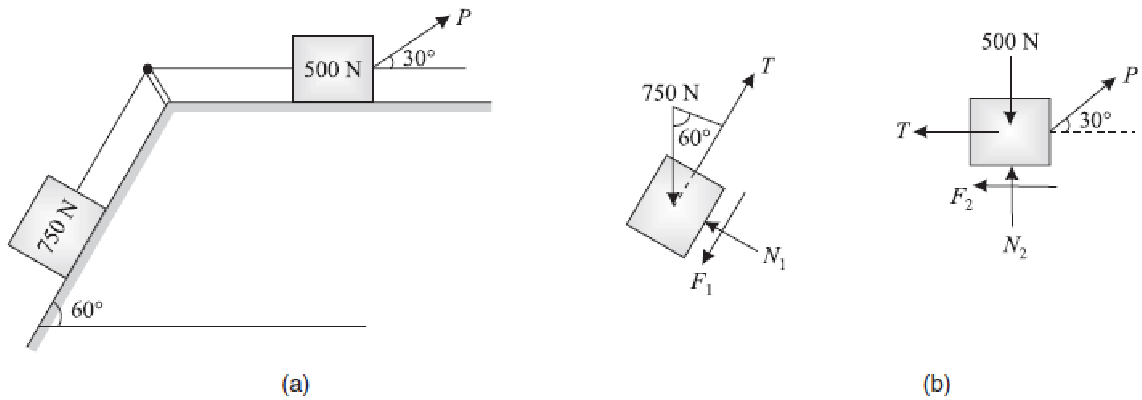
$$W_2 = \mu R + W_1 \sin 30^\circ = 0.3 R + 100 \sin 30^\circ = 0.3 R + 50$$

$$\Sigma F \text{ (perpendicular to plane)} = 0$$

$$R = 100 \cos 30^\circ = 86.6$$

$$\therefore \quad W_2 = 0.3 \times 86.6 + 50 = 75.98 \text{ N} \simeq \textbf{76 N}$$

11. What is the value of P in the system shown in figure to cause the motion to impend? Assume the pulley is smooth and coefficient of friction between the other contact surfaces is 0.2. **(P=853.5 N)**



Solution: Free body diagrams of the blocks are as shown in Fig. 5.9(*b*). Consider the equilibrium of 750 N block.

Σ Forces normal to the plane = 0 $\rightarrow$

$$N_1 - 750 \cos 60 = 0 \quad \therefore \; N_1 = 375 \text{ newton} \qquad ...(i)$$

Since the motion is impending, from law of friction,

$$F_1 = \mu N_1 = 0.2 \times 375 = 75 \text{ newton} \qquad ...(ii)$$

Σ Forces parallel to the plane = 0 $\rightarrow$

$$T - F_1 - 750 \sin 60 = 0$$

$$\therefore \quad T = 75 + 750 \sin 60 = 724.5 \text{ newton.} \qquad ...(iii)$$

Consider the equilibrium of 500 N block.

$$\sum F_V = 0 \rightarrow$$

$$N_2 - 500 + P \sin 30 = 0$$

i.e., $$N_2 + 0.5P = 500 \qquad ...(iv)$$

From law of friction,

$$F_2 = \mu N_2 = 0.2\,(500 - 0.5P) = 100 - 0.1P \qquad ...(v)$$

$$\sum F_H = 0 \rightarrow$$

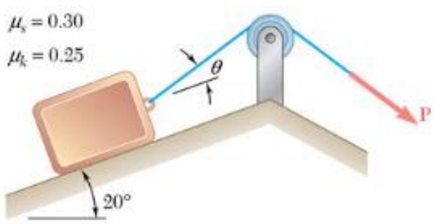
$$P \cos 30 - T - F_2 = 0$$

i.e., $$P \cos 30 - 724.5 - 100 + 0.1P = 0$$

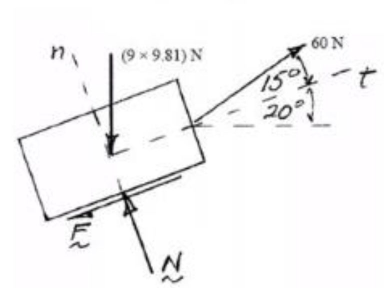
$$\therefore \quad P = 853.5 \text{ N}$$

12. Determine whether the 9 Kg block shown is in equilibrium, find the magnitude and direction of the friction force when P=60 N and θ = 15°? **(Not in equilibrium and F=16.86 N)**

Solution



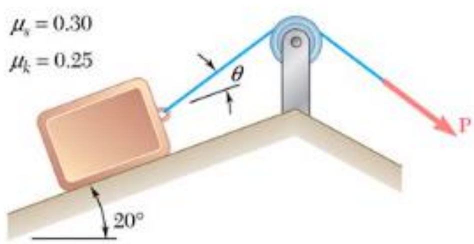
Free Body Diagram



$\nwarrow \Sigma F_n = 0: \quad N - (9 \times 9.81\text{ N})\cos 20° + (60\text{ N})\sin 15° = 0$

$N = 67.436\text{ N}$

$F_{max} = \mu_s N = (0.3)(67.436\text{ N}) = 20.231\text{ N}$



$\nwarrow \Sigma F_n = 0: \quad N - (9 \times 9.81\text{ N})\cos 20° + (60\text{ N})\sin 15° = 0$

$N = 67.436\text{ N}$

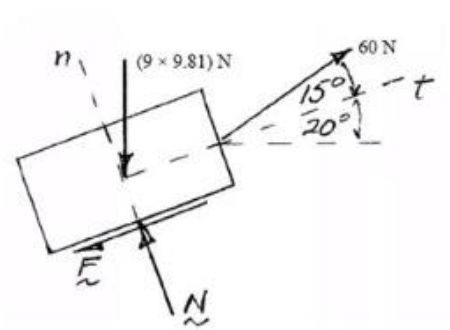
$F_{max} = \mu_s N = (0.3)(67.436\text{ N}) = 20.231\text{ N}$

Assume equilibrium:

$\nearrow \Sigma F_t = 0: \quad (60\text{ N})\cos 15° - (9 \times 9.81\text{ N})\sin 20° - F = 0$

$F = 27.759\text{ N} = F_{eq.}$

but $F_{eq.} > F_{max}$ impossible, so block slides up ◀



and

$F = \mu_k N = (0.25)(67.436\text{ N})$

$\text{F} = 16.86\text{ N} \swarrow$ ◀