Chapter 12

Kinematics of Particle

Engineering Mechanics : Dynamics

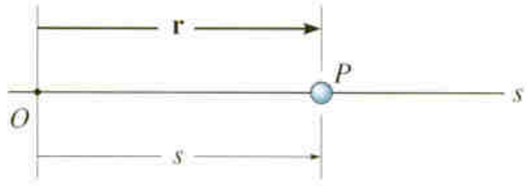
R.C. Hibbeler

Kinematics of particle that moving along a rectilinear or straight line path

Position

A particle travels along a straight-line path defined by the coordinate axis s.

The position of the particle at any instant, relative to the origin, O, is defined by the position vector ***r*** , or the scalar s. Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m) or feet (ft).



Position

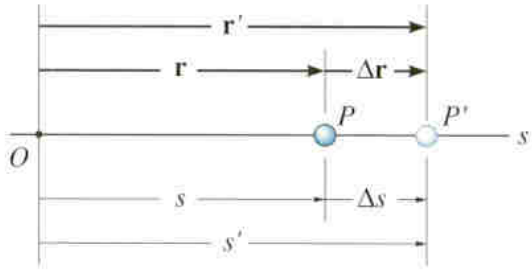
Displacement

The displacement of the particle is defined as its change in position.

Vector form: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

Scalar form: $\Delta s = s' - s$

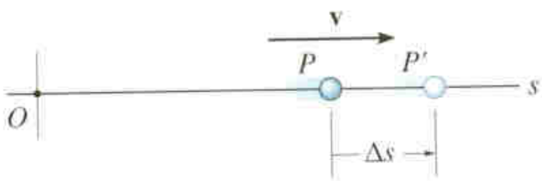
The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.



Displacement

Velocity

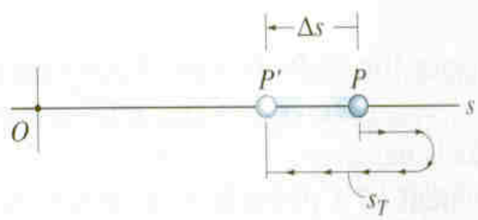
Velocity is a measure of the rate of change in the position of a particle. It is a <u>vector quantity</u> (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.



Velocity

The **average velocity** of a particle during a time interval Δt is $\mathbf{v}_{avg} = \dfrac{\Delta \mathbf{r}}{\Delta t}$

The **instantaneous velocity** is the time-derivative of position. $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$



Average velocity and Average speed

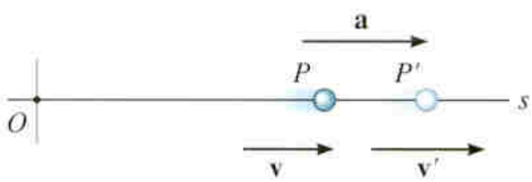
Speed is the magnitude of velocity $v = \dfrac{ds}{dt}$

Average speed is the total distance traveled divided by elapsed time: $(v_{sp})_{avg} = \dfrac{s_T}{\Delta t}$

Acceleration

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical unit is m/s2.

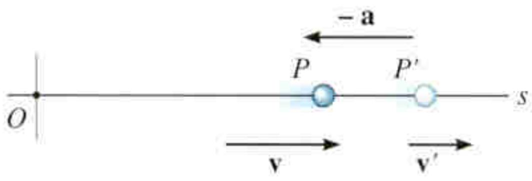
The **instantaneous acceleration** is the time derivative of velocity.



Acceleration

Vector form: $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$

Scalar form: $a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$



Deceleration

Acceleration can be positive (speed increasing) or negative (speed decreasing).

As the book indicates, the derivative equations for velocity and acceleration can be manipulated to get $ads = vdv$

SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

	Differentiate	Integrate
Position	s	$\int_{s_0}^{s} ds = \int_0^t v dt$
velocity	$v = \dfrac{ds}{dt}$	$\int_{v_0}^{v} dv = \int_0^t a dt$ or $\int_{v_0}^{v} v dv = \int_{s_0}^{s} a ds$
acceleration	$a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt} = v\dfrac{dv}{ds}$	a

Note that s_0 and v_0 represent the **initial** position and velocity of the particle at t = 0.

Constant Acceleration

The three kinematics equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2$ downward. These equations are:

$$\int_{v_0}^{v} dv = \int_{0}^{t} a_c dt \qquad\qquad v = v_0 + a_c t$$

$$\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + a_c t)\ dt \qquad\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

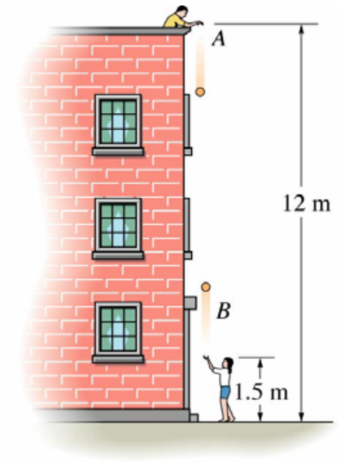
$$\int_{v_0}^{v} v dv = \int_{s_0}^{s} a_c ds \qquad\qquad v^2 = v_0^2 + 2a_c (s - s_0)$$

Example 12.1
A motorcyclist travels along a straight road at a speed of 27 m/s. When the brakes are applied, the motorcycle decelerates at a rate of -6t m/s^2.
Find: The distance the motorcycle travels before it stops.

Example 12.2

Ball A is released from rest at a height of 12 m at the same time that ball B is thrown upward, 1.5 m from the ground. The balls pass one another at a height of 6 m.

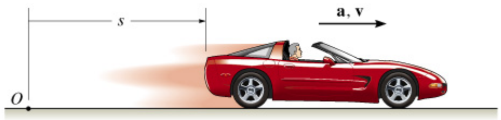
Find: The speed at which ball B was thrown upward.



Example 12.3

The car moves in a straight line such that for a shot time its velocity is defined by $v = (3t^2 + 2t)$ *ft/s*, where *t* is in seconds.

Determine its **position** and **acceleration** when *t=3* s. when *t=0,s=0*



Example 12.4

A bicycle starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is **constant**. Also, how long does it take to reach the speed of 30 km/h? **[a_c= 1.74 m/s^2, t = 4.80 s]**

Example 12.5

A baseball is thrown downward from a 12.5-m tower with an initial speed of 4.5 m/s. Determine the speed at which it hits the ground and the time of travel.

[v_2 = 16.3 m/s, t = 1.20 s]

Example 12.6

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time? **[t = 30 s, s = 792 m]**

Example 12.7

The position of a particle along a straight line is given by $s = (0.3t^3 - 2.7t^2 + 4.5t)$ m, where *t* is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \leq t \leq 10$ s. **[at t = 10s, a_{max} = 12.6 m/s^2, v_{max} = 40.5 m/s]**

Example 12.8

A Particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begin to decelerate at the rate of $a = (-1.5v^{1/2})$ m/s^2 ,where *v* is in m/s, determine the distance it travels before it stops.

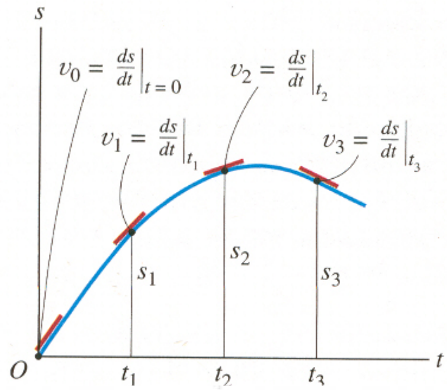
Graphical method

Graphing provides a good way to handle complex motions that would be difficult to describe with formulas. Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics.

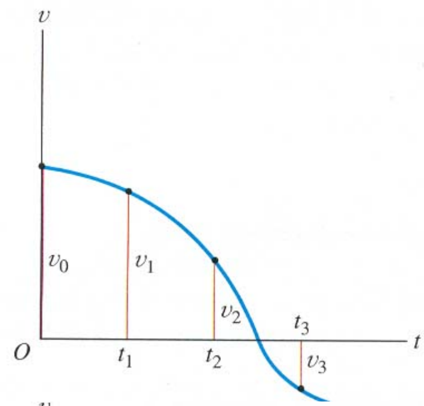
The approach builds on the facts that slope and differentiation are linked and that integration can be thought of as finding the area under a curve.

S-T GRAPH

Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or $v = \dfrac{ds}{dt}$).

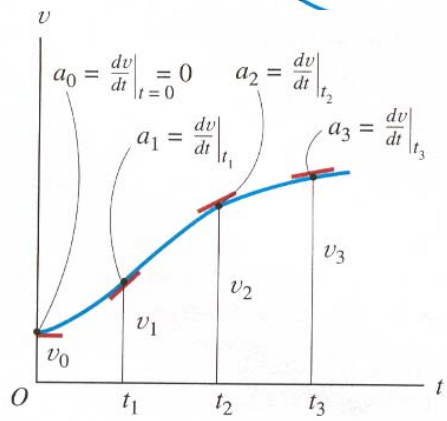


Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.



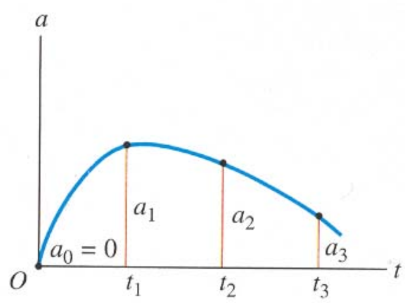
V-T GRAPH

Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point (or $a = \dfrac{dv}{dt}$).



Therefore, the a-t graph can be constructed by finding the slope at various points along the v-t graph.

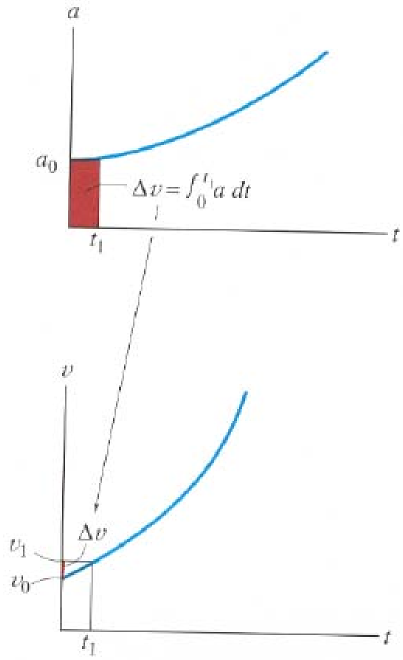
Also, the distance moved (displacement) of the particle is the area under the v-t graph during time Δt.



A-T GRAPH

Given the a-t curve, the change in velocity (Δv) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle.

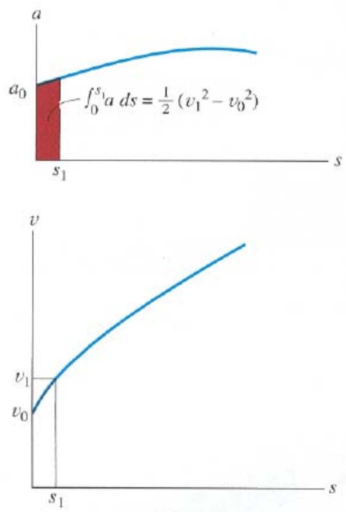


A-S GRAPH

A more complex case is presented by the a-s graph. The area under the acceleration versus position curve represents the change in velocity
(recall $\int ads = \int vdv$).

$$\frac{1}{2}(v_1^2 - v_0^2) = \int_{s_1}^{s_2} ads = \text{area under the a-s graph}$$

This equation can be solved for v1, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance.

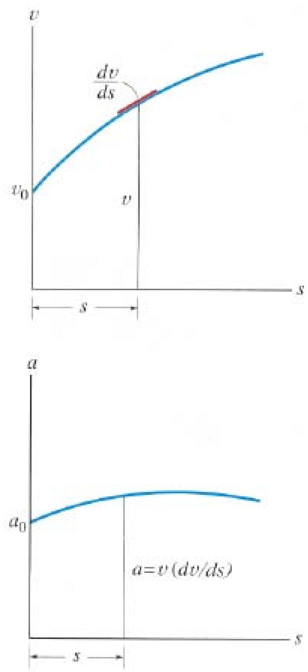


V-S GRAPH

Another complex case is presented by the v-s graph. By reading the velocity v at a point on the curve and multiplying it by the slope of the curve (dv/ds) at this same point, we can obtain the acceleration at that point.

$$a = v(\frac{dv}{ds})$$

Thus, we can obtain a plot of a vs. s from the v-s curve.

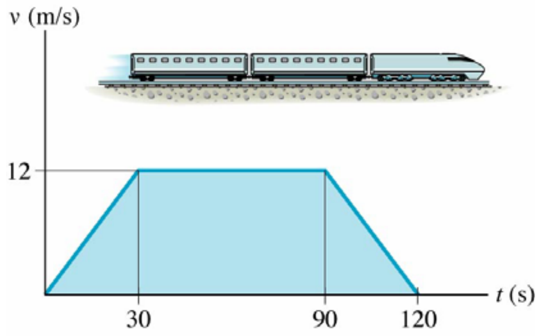


Example 12.9

Given: v-t graph for a train moving between two stations

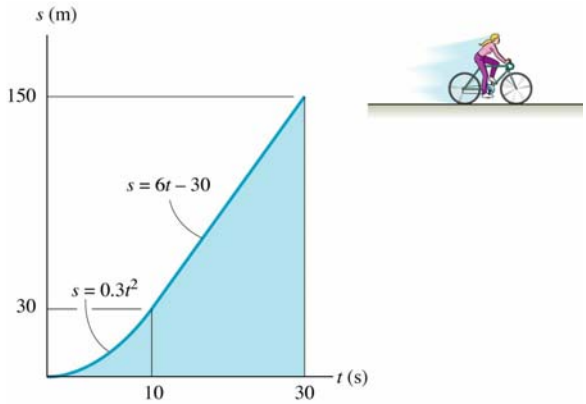
Find : a-t graph over this time interval

: distance between the stations and average speed



Example 12.10

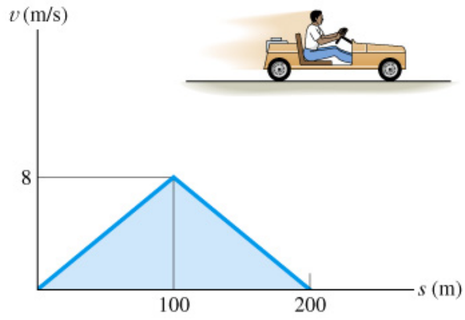
A bicycle moves along a straight road such that its position is described by the graph shown. Construct the *v-t* and *a-t* graph between t = 0 – 30 s



(a)

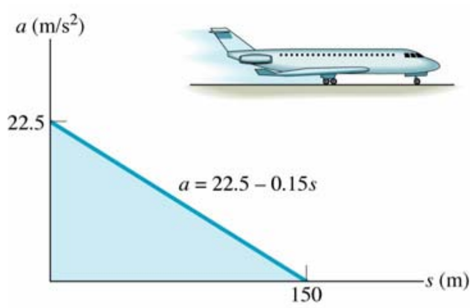
Example 12.11

The *v-s* graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at s = 50m and s= 150m. Draw the *a-s* graph.



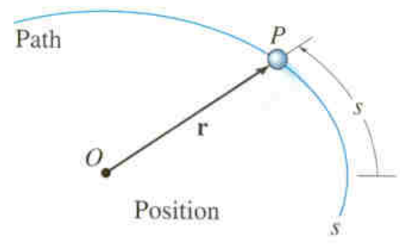
Example 12.12

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 60 m.



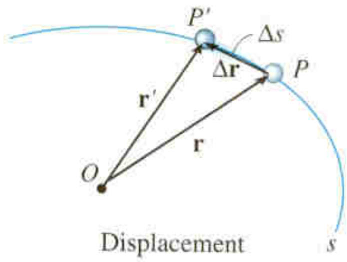
General Curvilinear Motion

Position and Displacement



A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, vectors are used to describe the motion.

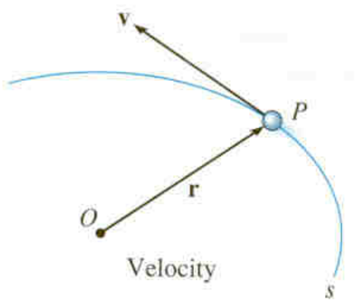
A particle moves along a curve defined by the path function, s.



The position of the particle at any instant is designated by the vector ***r*** = ***r***(t). Both the magnitude and direction of ***r*** may vary with time.

If the particle moves a distance Δs along the curve during time interval Δt, the displacement is determined by vector subtraction: Δ ***r*** = ***r'*** - ***r***

Velocity



Velocity represents the <u>rate of change in the position</u> of a particle.

The **average velocity** of the particle during the time increment Δt is

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

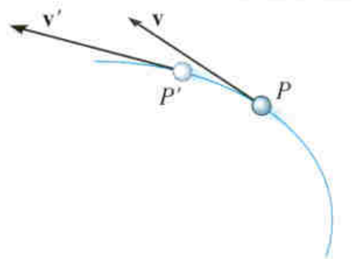
The **instantaneous velocity** is the time-derivative of position

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

The velocity vector, ***v***, is <u>always tangent</u> to the path of motion.

The magnitude of ***v*** is called the speed. Since the arc length Δs approaches the magnitude of Δ***r*** as t→0, the speed can be obtained by differentiating the path function (v = ds/dt). Note that this is not a vector!

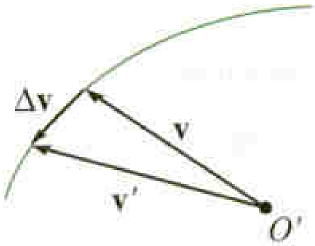
Acceleration



Acceleration represents the rate of change in the velocity of a particle.

If a particle's velocity changes from ***v*** to ***v'*** over a time increment Δt, the **average acceleration** during that increment is:

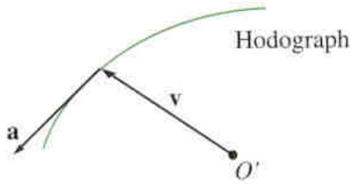
$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{(v - v')}{\Delta t}$$

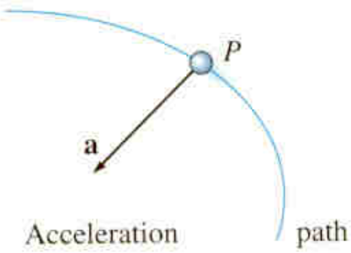


The **instantaneous** acceleration is the time-derivative of velocity:

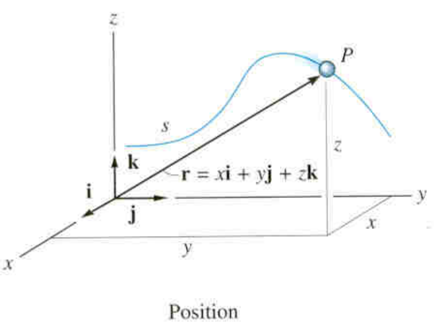
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

A plot of the locus of points defined by the arrowhead of the velocity vector is called a hodograph. The acceleration vector is tangent to the **hodograph**, but not, in general, tangent to the path function.

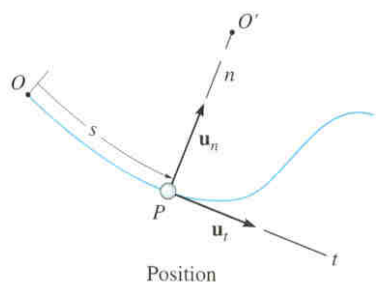




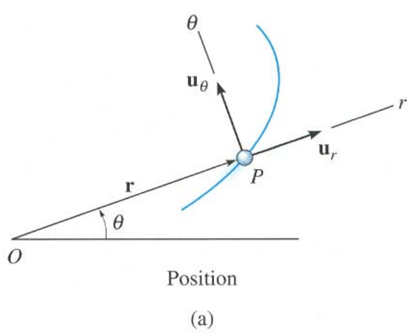
Curvilinear Motion



Rectangular Component



Normal and Tangential Component
n-t

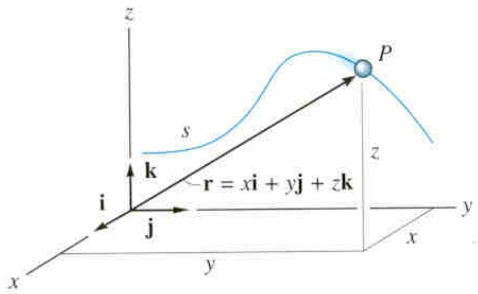


Cylindrical Component
r-θ

Curvilinear Motion: Rectangular Component

Position

It is often convenient to describe the motion of a particle in terms of its x, y, z or rectangular components, relative to a fixed frame of reference.



Position

The position of the particle can be defined at any instant by the position vector

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} .$$

The x, y, z components may all be functions of time, i.e.,

$$x = x(t),\ y = y(t),\ \text{and}\ z = z(t) .$$

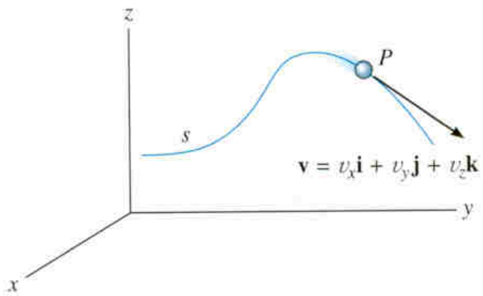
The magnitude of the position vector is:

$$r = \sqrt{x^2 + y^2 + z^2}$$

The direction of $\boldsymbol{r}$ is defined by the unit vector: $\boldsymbol{u_r} = (1/r)\boldsymbol{r}$

Velocity

The velocity vector is the time derivative of the position vector:



Velocity

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d(x\mathbf{i})}{dt} + \frac{d(y\mathbf{j})}{dt} + \frac{d(z\mathbf{k})}{dt}$$

Since the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are constant in magnitude and direction, this equation reduces to $v = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$ where

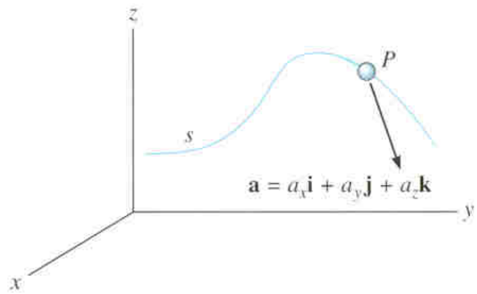
$$v_x = \dot{x} = \frac{dx}{dt}, v_y = \dot{y} = \frac{dy}{dt}, v_z = \dot{z} = \frac{dz}{dt}$$

The **magnitude** of the velocity vector is

$$v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$$

The direction of $\boldsymbol{v}$ is <u>tangent</u> to the path of motion.

Acceleration



Acceleration

The acceleration vector is the time derivative of the velocity vector (second derivative of the position vector):

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

where

$$a_x = \dot{v}_x = \ddot{x} = \frac{dv_x}{dt}, \qquad a_y = \dot{v}_y = \ddot{y} = \frac{dv_y}{dt}, \qquad a_z = \dot{v}_z = \ddot{z} = \frac{dv_z}{dt}$$

The **magnitude** of the acceleration vector is

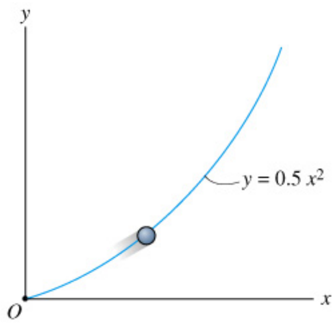
$$a = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$$

The direction of ***a*** is **usually <u>not</u> tangent** to the path of the particle.

Example 12.13
The flight path of the helicopter as it takes off from A is defined by the parametric equation $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in second. Determine the **distance** the helicopter is from point A and the **magnitudes of its velocity and acceleration** when t = 10s.

Example 12.14

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ m/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when t = 1s. When t = 0, x = 0, y = 0.

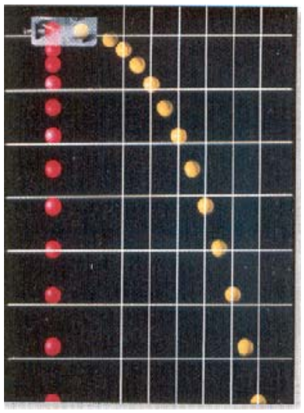


Example 12.15

A particle is moving along the curve $y = x - (x^2/400)$, where x and y are in meter. If the velocity component in the x direction is v_x = 2 m/s and remains *constant*, determine the magnitudes of the velocity and acceleration when x =20 m.

[v = 2.69 m/s, a = 0.02 m/s^2]

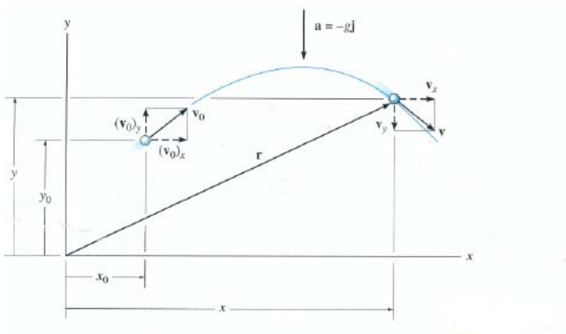
Motion of a projectile



Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity).

For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant.

Horizontal Motion



Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the x direction can be determined by:

$$x = x_0 + (v_{0x})(t)$$

Vertical Motion

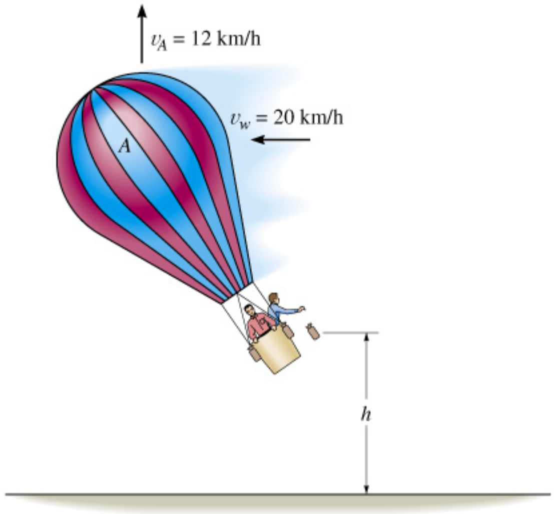
Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

$$v_y = v_{0y} - g(t)$$

$$y = y_0 + (v_{0y})(t) - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

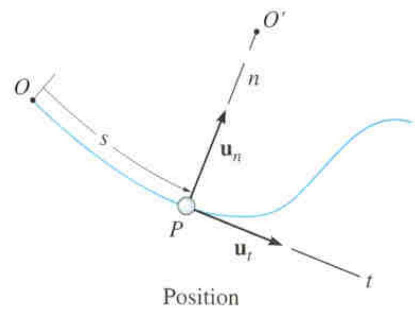
Example 12.16



The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_W = 20$ km/h. If a ballast bag is dropped from the balloon at the instance h = 50 m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?

Curvilinear Motion: Normal and Tangential Component

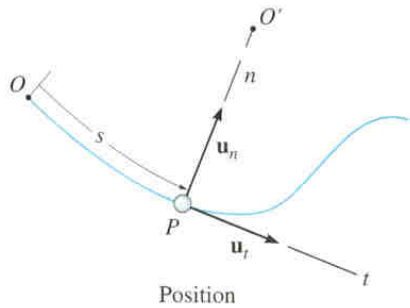
When a particle moves along a curved path, it is sometimes convenient to describe its motion using coordinates other than Cartesian. When the path of motion is known, normal (n) and tangential (t) coordinates are often used.



Position

In the n-t coordinate system, the origin is located on the particle (the origin moves with the particle).

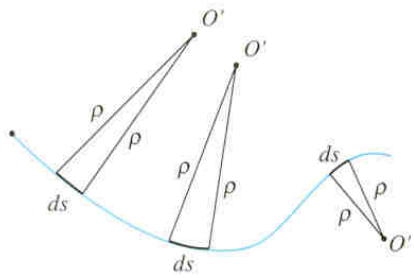
The t-axis is tangent to the path (curve) at the instant considered, positive in the direction of the particle's motion. The n-axis is perpendicular to the t-axis with the positive direction toward the center of curvature of the curve.



Position

The positive n and t directions are defined by the unit vectors $\boldsymbol{u}_n$ and $\boldsymbol{u}_t$, respectively.

The center of curvature, O', always lies on the **concave** side of the curve.

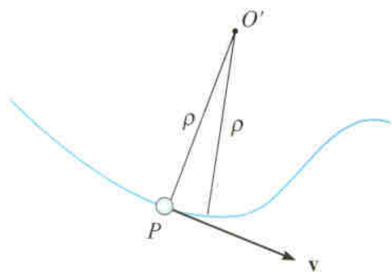


Radius of curvature

The radius of curvature, ρ, is defined as the perpendicular distance from the curve to the center of curvature at that point.

The position of the particle at any instant is defined by the distance, s, along the curve from a fixed reference point.

Velocity in the *n-t* Coordinate System



Velocity

The velocity vector is always **tangent** to the path of motion (t-direction).

The magnitude is determined by taking the time derivative of the path function, s(t).

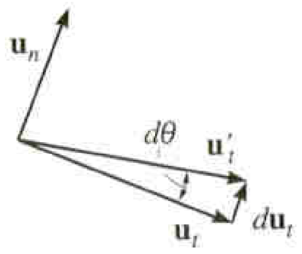
$$\mathbf{v} = v\mathbf{u}_t \quad \text{where} \quad v = \dot{s} = \frac{ds}{dt}$$

Here v defines the **magnitude** of the velocity (speed) and $\boldsymbol{u}_t$ defines the **direction** of the velocity vector.

Acceleration in the n-t Coordinate system

Acceleration is the time rate of change of velocity:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{u}_t)}{dt} = \dot{v}\mathbf{u}_t + v\dot{\mathbf{u}}_t$$



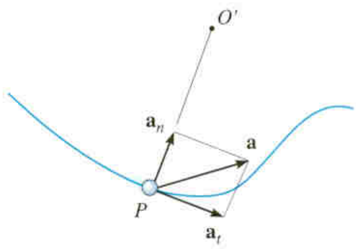
Here v represents the change in the magnitude of velocity and $\dot{\mathbf{u}}_t$ represents the rate of change in the direction of $\mathbf{u}_t$.

After mathematical manipulation, the acceleration vector can be expressed as:

$$\mathbf{a} = \dot{v}\mathbf{u}_t + \left(\frac{v^2}{\rho}\right)\mathbf{u}_n = a_t\mathbf{u}_t + a_n\mathbf{u}_n$$

There are two components to the acceleration vector:

$$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n$$



Acceleration

The tangential component is tangent to the curve and in the direction of increasing or decreasing velocity.

$$a_t = \dot{v} \text{ or } a_t ds = v dv$$

The normal or centripetal component is always directed toward the center of curvature of the curve.

$$a_n = \frac{v^2}{\rho}$$

The magnitude of the acceleration vector is

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$

SPECIAL CASES OF MOTION

1) The particle moves along a straight line.

$r \rightarrow \infty$ => $a_n = v^2/r = 0$ => $a = a_t = \dot{v}$

The tangential component represents the time rate of change in the magnitude of the velocity.

2) The particle moves along a curve at **constant** speed.

$a_t = \dot{v} = 0$ => $a = a_n = v^2/r$

The normal component represents the time rate of change in the direction of the velocity.

3) The tangential component of acceleration is constant, at = (at)c.
In this case,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$
$$v = v_0 + a_c t$$
$$v^2 = v_0^2 + 2a_c(s - s_0)$$

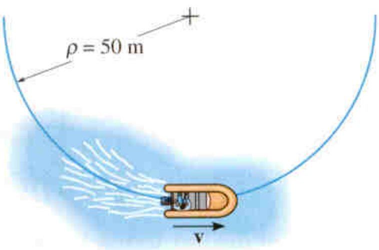
As before, s_o and v_o are the initial position and velocity of the particle at t = 0.
How are these equations related to projectile motion equations?

4) The particle moves along a path expressed as y = f(x).
The radius of curvature, r, at any point on the path can be calculated from

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{\left| d^2 y / dx^2 \right|}$$

Example 12.17

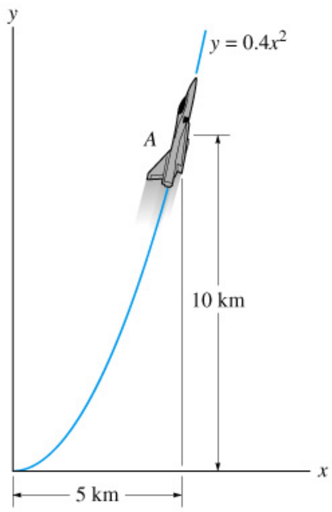
Starting from rest, a motorboat travels around a circular path of r = 50 m at a speed that increases with time, v = (0.2 t^2) m/s.



Find: The magnitudes of the boat’s velocity and acceleration at the instant t = 3 s.

Example 12.18

The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s^2. Determine the magnitude of acceleration of the plane when it is at point A.



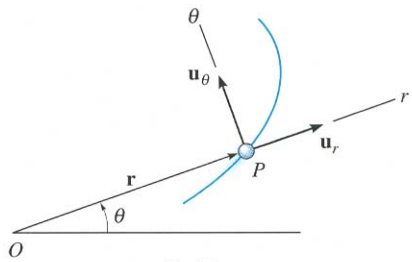
Curvilinear Motion: Cylindrical Component

Position

We can express the location of P in polar coordinates as

$$\mathbf{r} = r\mathbf{u}_r$$

Note that the radial direction, r, extends outward from the fixed origin, O, and the transverse coordinate, θ, is measured counter-clockwise (CCW) from the horizontal.



Position

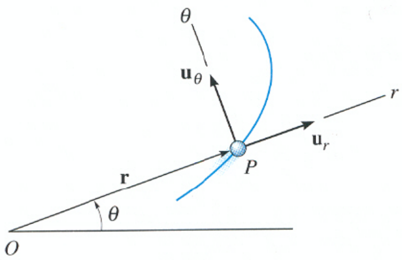
(a)

Velocity

The instantaneous velocity is defined as:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d(r\mathbf{u}_r)}{dt}$$

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\frac{d\mathbf{u}_r}{dt}$$



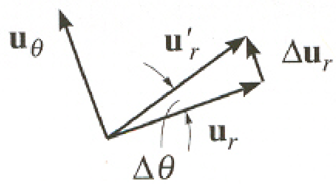
Position

Using the chain rule:

$$\frac{d\mathbf{u}_r}{dt} = \left(\frac{d\mathbf{u}_r}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

We can prove that

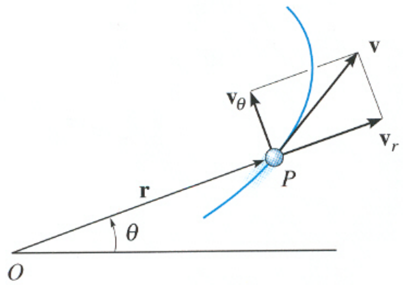
$$\frac{d\mathbf{u}_r}{d\theta} = \mathbf{u}_\theta \qquad \text{so} \qquad \frac{d\mathbf{u}_r}{dt} = \dot{\theta}\,\mathbf{u}_\theta$$



Therefore:

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta$$

Thus, the velocity vector has two components: $\dot{r}$, called the radial component, and $r\dot{\theta}$, called the transverse component. The speed of the particle at any given instant is the sum of the squares of both components or



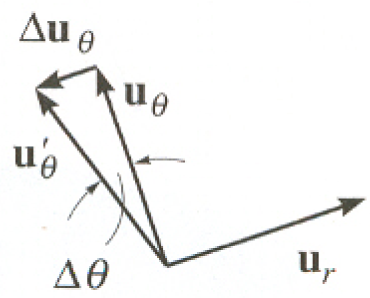
Velocity

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$

Acceleration

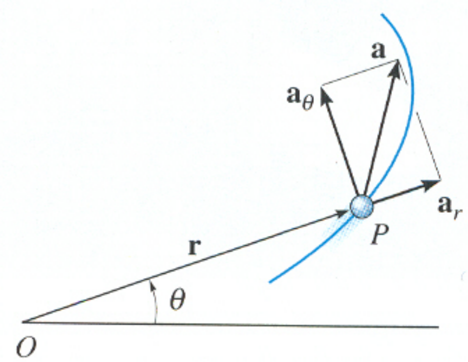
The instantaneous acceleration is defined as:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt}(\dot{r}\,\mathbf{u_r} + r\dot{\theta}\,\mathbf{u_\theta})$$



After manipulation, the acceleration can be expressed as

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u_\theta}$$



Acceleration

The term $(\ddot{r} - r\dot{\theta}^2)$ is the radial acceleration or a_r.

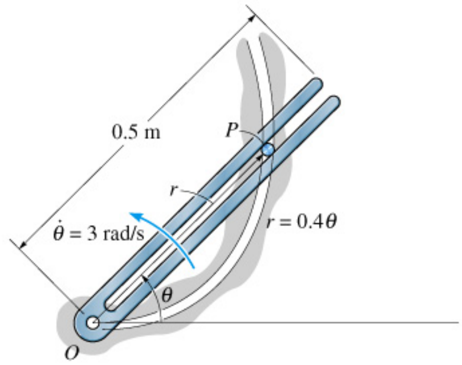
The term $(r\ddot{\theta} + 2\dot{r}\dot{\theta})$ is the transverse acceleration or a_θ

The magnitude of acceleration is

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$

Example 12.19

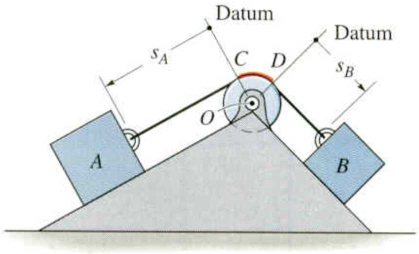
The slotted link is pined at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a shot distance along the spiral guide $r = (0.4)\theta$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instance $\theta = \pi/3$ rad.



Absolute Dependent Motion Analysis of Two Particles

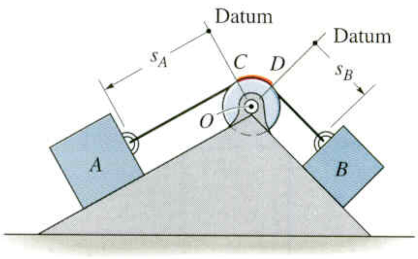
Dependent Motion

In many kinematics problems, the motion of one object will depend on the motion of another object.



The blocks in this figure are connected by an inextensible cord wrapped around a pulley. If block *A* moves downward along the inclined plane, block *B* will move up the other incline.

The motion of each block can be related mathematically by defining position coordinates, s_A and s_B. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.



In this example, position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks *A* and *B*.

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_r is the total cord length and l_{CD} is the length of cord passing over arc CD on the pulley.

The velocities of blocks A and B can be related by differentiating the position equation. Note that l_{CD} and l_T remain constant, so

$$\frac{dl_{CD}}{dt} = \frac{dl_T}{dt} = 0$$

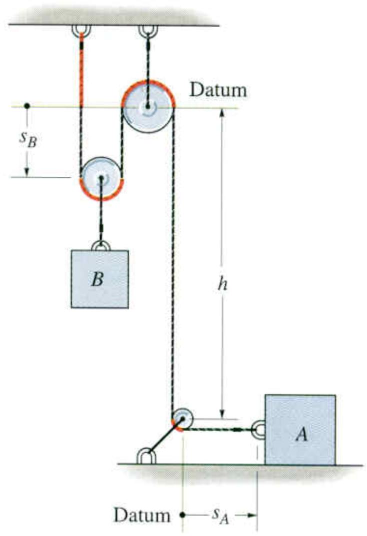
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \Rightarrow \qquad v_B = -v_A$$

The negative sign indicates that as A moves down the incline (positive s_A direction), B moves up the incline (negative s_B direction).

Accelerations can be found by differentiating the velocity expression.

$$a_B = -a_A$$

Consider a more complicated example. Position coordinates (s_A and s_B) are defined from fixed datum lines, measured along the direction of motion of each block.



Note that s_B is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, h is a constant.

The red colored segments of the cord remain constant in length during motion of the blocks.

The position coordinates are related by the equation

$$2s_B + h + s_A = l$$

Where l is the total cord length minus the lengths of the red segments.

Since l and h remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

$$2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A$$

When block B moves downward ($+s_B$), block A moves to the left ($-s_A$). Remember to be consistent with the sign convention!

This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley.

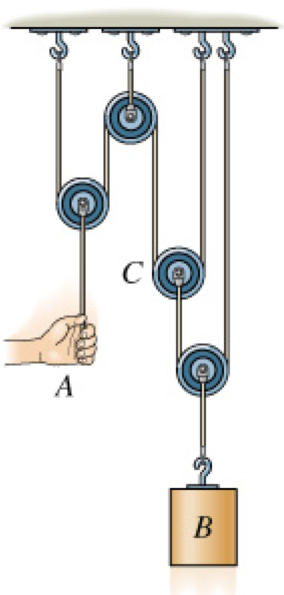
The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l$$

$$2v_B = v_A$$

$$2a_B = a_A$$

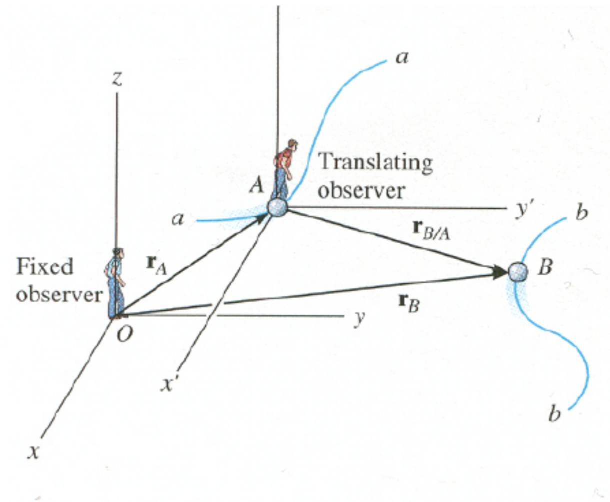
Example12.20
Determine the displacement of the block at B if A is pulled down 1 m.



Relative Motion Analysis

Position

The absolute position of two particles A and B with respect to the fixed x, y, z reference frame are given by $\mathbf{r_A}$ and $\mathbf{r_B}$. The position of B relative to A is represented by



$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$$

Therefore, if $\quad \mathbf{r}_B = (10\mathbf{i} + 2\mathbf{j})$ m

And $\quad \mathbf{r}_A = (4\mathbf{i} + 5\mathbf{j})$ m

Then $\quad \mathbf{r}_{B/A} = (6\mathbf{i} - 3\mathbf{j})$ m

Velocity

To determine the relative velocity of B with respect to A, the time derivative of the relative position equation is taken.

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$

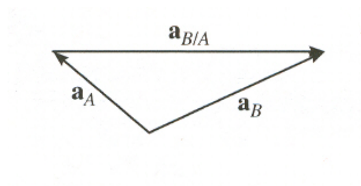
Or
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

In these equations, v_B and v_A are called absolute velocities and $v_{B/A}$ is the relative velocity of B with respect to A.

Note that $\quad \mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$

Acceleration

The time derivative of the relative velocity equation yields a similar vector relationship between the absolute and relative accelerations of particles A and B.



$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

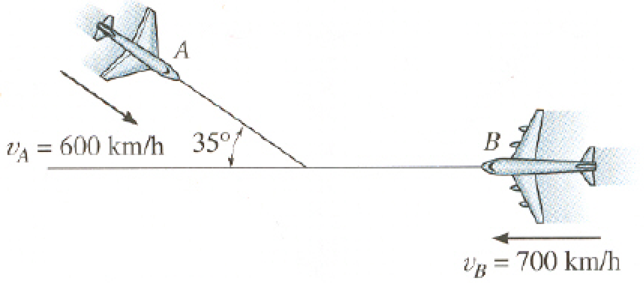
Or
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Example 12.21

Given: $v_A = 600$ km/hr

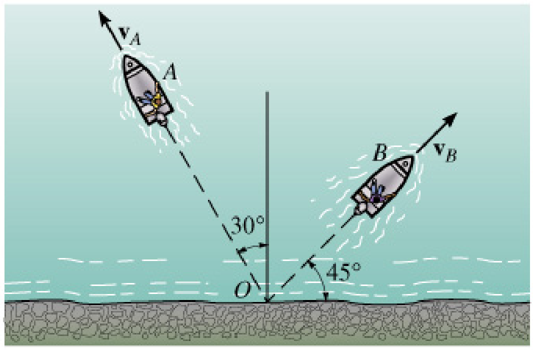
$v_B = 700$ km/hr

Fine: $v_{A/B}$



Example 12.22

Two boats leave the shore at the same time and travel in the direction shown.
If $v_A = 6$ m/s and $v_B = 4.5$ m/s, determine the speed of boat A with respect to boat B. How long after leaving the shore will the boats be 240 apart?



Example 12.23

Given: $v_A = 10$ m/s

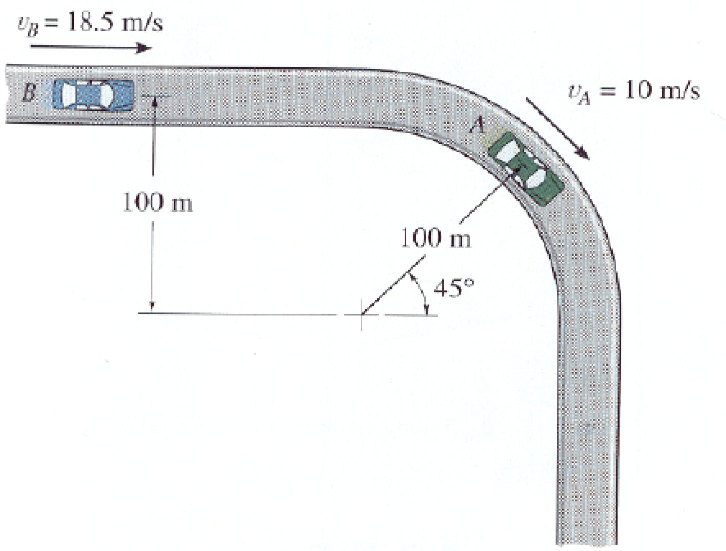
$v_B = 18.5$ m/s

$a_{t)A} = 5$ m/s2

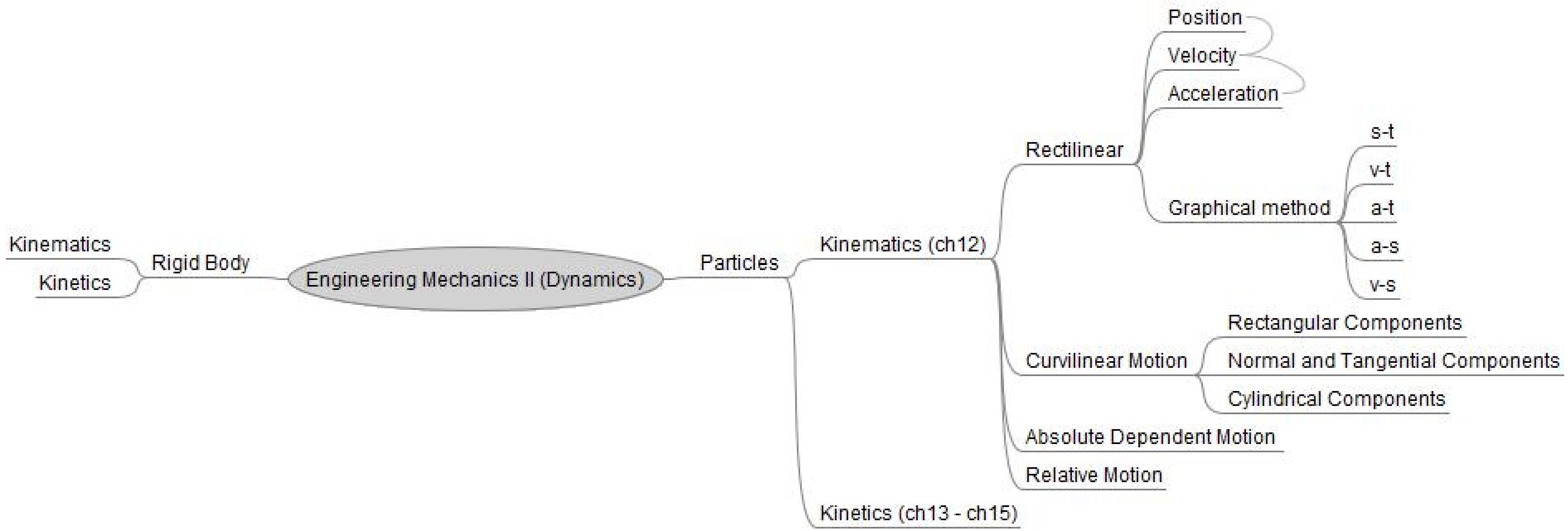
$a_B = 2$ m/s2

Find: $\boldsymbol{v_{A/B}}$

$\boldsymbol{a_{A/B}}$



Chapter 12



Engineering Mechanics II (Dynamics)
Rigid Body
Kinematics
Kinetics
Particles
Kinematics (ch12)
Rectilinear
Position
Velocity
Acceleration
Graphical method
s-t
v-t
a-t
a-s
v-s
Curvilinear Motion
Rectangular Components
Normal and Tangential Components
Cylindrical Components
Absolute Dependent Motion
Relative Motion
Kinetics (ch13 - ch15)