L4 29 Jan 2021

(3) If $Mdx + Ndy = 0$ can be written in the form $f(xy)\,y\,dx + g(xy)\,x\,dy = 0$

then. $I.F = \dfrac{1}{Mx - Ny}$, $Mx - Ny \neq 0$

$f(xy)$

$\boxed{f(x,y) = x^2 + y^2}$

$$(x^2y^3 - y)dx + (x^3y^2 + x)dy = 0$$

$M = x^2y^3 - y$ $\quad \dfrac{\partial M}{\partial y} = 3x^2y^2 - 1$

$N = x^3y^2 + x$ $\quad \dfrac{\partial N}{\partial x} = 3x^2y^2 + 1$

$\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

$\Rightarrow$ is not exact.

$f(xy) = x^2y^2 - 2$

$f(xy) \cancel{= x^2y^2} - 4$

$xy = t$

$\to (x^2y^2 - 1)ydx + (x^2y^2 + 1)xdy = 0$

$x^2y^2 - 1$

$t^2 - 1$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(x^2y^3 - y)x - (x^3y^2 + x)y} = \frac{1}{\cancel{x^3y^3} - xy - \cancel{x^3y^3} - xy}$$

$$I.F = \frac{1}{-2xy}$$

$$\left[(x^2y^3 - y)dx + (x^3y^2 + x)dy = 0\right] \times \left(\frac{-1}{2xy}\right)$$

$$\underbrace{\left(\frac{-xy^2}{2} + \frac{1}{2x}\right)}_{M} dx + \underbrace{\left(\frac{x^2y}{-2} - \frac{1}{2y}\right)}_{N} dy = 0$$

$\dfrac{\partial M}{\partial y} = \dfrac{-\cancel{2}xy}{\cancel{2}} = -xy$

$\dfrac{\partial N}{\partial x} = \dfrac{\cancel{2}xy}{\cancel{-2}} = -xy$

Solution

$$\int_{y\,\text{const.}} \left(-\frac{xy^2}{2} + \frac{1}{2x}\right)dx + \int \left(\frac{-1}{2y}\right)dy = c$$

$$-\frac{y^2x^2}{2 \times 2} + \frac{1}{2}\log x - \frac{1}{2}\log y = C \quad \text{Ans}$$

Q13. The integrating factor to reduce (1+xy)ydx+(1-xy)xdy=0 as an exact equation is:

(a) $\frac{1}{x^2y^2}$ **(b)** $\frac{1}{2x^2y^2}$ **(c)** $\frac{1}{2x^3y^2}$ **(d)** $\frac{1}{3x^3y^3}$

$f(xy)\,y\,dx + g(xy)\,x\,dy = 0$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(1+xy)yx - (1-xy)xy} = \frac{1}{\cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2}$$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(1+xy)yx - (1-xy)xy} = \frac{1}{\cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2}$$

$$\boxed{I.F = \frac{1}{2x^2y^2}}$$

(4) For $M\,dx + N\,dy = 0$

i) If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then $I.F = e^{\int f(x)\,dx}$

(ii) If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = -f(y)$ then $I.F = e^{\int f(y)\,dy}$

$$(4xy + 3y^2 - x)dx + (x^2 + 2xy)dy = 0$$

$M = 4xy + 3y^2 - x$ $\quad \frac{\partial M}{\partial y} = 4x + 6y$

$N = x^2 + 2xy$ $\quad \frac{\partial N}{\partial x} = 2x + 2y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\Rightarrow$ is not exact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{(4x+6y) - (2x+2y)}{N} = \frac{2x+4y}{N} = \frac{2\cancel{(x+2y)}}{x\cancel{(x+2y)}} = \frac{2}{x}\ (f(x))$$

$$I.F = e^{\int f(x)\,dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = \cancel{e}^{\cancel{\log}\, x^2} = x^2.$$

$$(4xy + 3y^2 - x)x^2\,dx + (x^2 + 2xy)x^2\,dy = 0$$

$$\int_{y\,\text{can}} M\,dx + \int (\text{Terms of } N \text{ free from } x)\,dy = c$$

Sol

$$\int_{y\,\text{const.}} \underbrace{(4x^3y + 3x^2y^2 - x^3)}_{M}\,dx + \underline{0} = c$$

$$= \frac{4x^4}{4}y + \frac{3x^3}{3}y^2 - \frac{x^4}{4} = C$$

For what values of a, given differential equ.: $(x^2 - ay)dx + (y^2 - ax)dy = 0$ is exact:

$\frac{\partial M}{\partial y} = -a$ $\frac{\partial N}{\partial x} = -a$

$a = 1$

$a = \frac{1}{\sqrt{10}}$

(A) ~~For all values of a~~

(B) There does not exist any value of a for which given diff. equ. is exact.

(C) Only for $a = 1$

(D) None of these.

$$(xy^3 + y)dx + (2x^2y^2 + 2x + 2y^4)dy = 0$$

$M = xy^3 + y$ $\frac{\partial M}{\partial y} = 3xy^2 + 1$

$N = 2x^2y^2 + 2x + 2y^4$ $\frac{\partial N}{\partial x} = 4xy^2 + 2$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = 3xy^2 + 1 - 4xy^2 - 2$$
$$= -xy^2 - 1$$
$$= \frac{-(xy^2+1)}{y(xy^2+1)}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{-1}{y}$$

$$I.F = e^{\int f(y)dy}$$

$$e^{\int \frac{1}{y} dy} = e^{\log y} = y.$$

$\Rightarrow$ given $\Rightarrow$ $(xy^4 + y^2)dx + (2x^2y^3 + 2xy + 2y^5)dy = 0$

Sol

$$\int_{y\,\text{const}} (xy^4 + y^2)dx + \int 2y^5 dy = c$$

$$\frac{x^2}{2}y^4 + y^2x + \frac{\cancel{2}y^6}{\cancel{6}\,3} = c$$

Home work

① Under what conditions, the differential eqns are exact

i) $xy^3dx + ax^2y^2dy = 0$

ii) $(ax + y)dx + (kx + by)dy = 0$

② Find I.F and solve

i) $(x^3 + y^3 + 1)dx + xy^2dy = 0$

Q2 Find I.F and solve

i) $(x^3+y^3+1)\,dx + xy^2\,dy = 0$

ii) $(2y^3xe^y + y^2 + y)\,dx + (y^3x^2e^y - xy - 2x)\,dy = 0$

iii) $y(1+xy^2)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$

iv) $(5x^3+12x^2+6y^2)\,dx + 6xy\,dy = 0$