L24 12 Mar 2021

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$a_0 y'' + a_1 y' + a_2 y = 0$

$(1-x^2) y'' - 2xy' + n(n+1) y = 0$, n is an integer.

$\boxed{a_0 = 1-x^2}$ $\quad a_1 = -2x \quad a_2 = n(n+1)$

Continuous on $\quad (-\infty, \infty) \quad (-\infty, \infty) \quad (-\infty, \infty)$ $\quad (-\infty, \infty)$

$\boxed{a_0 \neq 0}$ $\quad 1 - x^2 \neq 0$

$\downarrow \{-1, 1\}$

$1 - x^2 = 0$

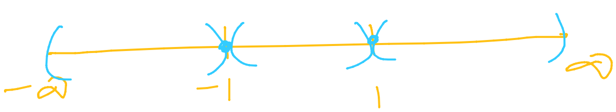
$x^2 = 1$

$x = \pm 1$

$\{-1, 1\}$

$(-\infty, -1) \quad (-1, 1) \quad (1, \infty)$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$



~~$(-\infty, -1)$~~ ~~$(-1, \infty)$~~

$\boxed{x \in \mathbb{R} - \{-1, 1\}}$

$\sqrt{x}\, y'' + 6x y' + 15 y = 0$

$a_0 = \sqrt{x} \quad a_1 = 6x \quad a_2 = 15$

$[0, \infty) \quad (-\infty, \infty) \quad (-\infty, \infty) \longrightarrow [0, \infty) - \{0\}$

$(0, \infty)$

$a_0 \neq 0$

$\sqrt{x} = 0$

$x = 0$

$2y'' - 3y' - y = \log x$

$a_0 = 2 \quad a_1 = -3 \quad a_2 = -1 \qquad f(x) = \log x$

$$(-\infty,\infty) \quad (-\infty,\infty) \quad (-\infty,\infty) \quad (0,\infty)$$

$$\downarrow$$

$$(0,\infty)$$

$$(x^2-1)y'' + 2xy' + y = x\log x$$

$a_0 = x^2-1$ $\quad a_1 = 2x$ $\quad a_2 = 1$ $\quad s(x) = x\log x$

$$(-\infty,\infty) \quad (-\infty,\infty) \quad (-\infty,\infty) \quad (0,\infty) \rightarrow (0,\infty)$$

$$(0,1) \quad (1,\infty)$$

$a_0 \neq 0$ $\quad a_0 = 0$, $x^2 = 1$, $x = \pm 1$

Lowest possible order

10.) The lowest possible order of homogeneous linear differential equation whose particular solution is $\boxed{1 + x + e^{x} - 3e^{3x}}$ is

a) 4 $\qquad$ b)3 $\qquad$ c) 2 $\qquad$ d) 5

$$(1+x)e^{0x} + e^{1x} - 3e^{3x}$$

$$D = 0, 0, 1, 3$$

$1+x \quad D = 2, 2$

$y_c = (c_1 + c_2 x)e^{2x}$

$D = 0, 0$

$y_c = (c_1 + c_2 x)e^{0x}$

$= c_1 + c_2 x$

$1, x, x^2, 1+x$

$$W = \begin{vmatrix} 1 & x & x^2 & 1+x \\ 0 & 1 & 2x & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0 \qquad \text{L.D}$$

$$W = \begin{vmatrix} 1 & x & x^2 & 1+x \\ 0 & 1 & 2x & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0 \qquad \underline{\underline{L.D}}$$

$$[D^2+D]y = x^2+2x+4$$

$$\frac{1}{D^2+D}(x^2+2x+4) = \frac{1}{D(1+D)} \cdot x^2+2x+4$$

$$= \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$$

D^3

$$= \frac{1}{D}(1-D+D^2)(x^2+2x+4)$$

$$= \frac{1}{D}\left[(x^2+\cancel{2x}+4)-(\cancel{2x}+\cancel{2})+\cancel{2}\right]$$

$$= \frac{1}{D}(x^2+4) = \frac{x^3}{3}+4x$$

The lowest possible order of homogeneous linear differential equation whose particular solution is $3\cos 2x + 5\sinh 3x$ is $+5\left(\frac{e^{x}-e^{-x}}{2}\right)$

a) 2 b) 3 c) 5 d) 4

$D = \alpha \pm i\beta$
$= 0 \pm i2$

$D = 1, -1$

$$\boxed{\begin{aligned} \cosh x &= \frac{e^{x}+e^{-x}}{2} \\ \sinh x &= \frac{e^{x}-e^{-x}}{2} \end{aligned}}$$

Hyperbolic functions

$\cosh x = \frac{e^{ix}+e^{-ix}}{}$

sinhx
coshx
tanhx
Sechx

sinx
cosx
tanx
secx

→ Trigonometric ratios

$\cos x = \frac{e^{ix}+e^{-ix}}{2}$

$\sin x = \frac{e^{ix}-e^{-ix}}{2i}$

$\tanh x$
$\operatorname{sech} x$
$\operatorname{cosech} x$
$\coth x$

$\tan x$
$\sec x$
$\operatorname{cosec} x$
$\cot x$

Session

$y = c_1e^{2x} + c_2e^{-3x} + c_4\sin 3x + c_3\cos 3x$

c_1e^{mx}

$D = 2, -3$ $\qquad D = -3$ $\qquad D = \pm 3i$

$\boxed{(D-2)(D+3)(D^2+9) = 0}$

$D^2 = -9$

(D^2+9)

Lecture 17

Ch. 1

$D^{''''}x = m^4x$

$D^{IV}x - m^4x = 0$

$(D^4 - m^4) = 0 \qquad (D^2-m^2)(D^2+m^2) = 0$

$\Rightarrow (D-m)(D+m)\,(D^2+m^2) = 0$

$\Rightarrow D = m, -m, \qquad D = \pm mi$

$x = c_1\cos mt + c_2\sin mt + c_4\cosh mt$

$y = \frac{c_1e^{mx}+c_2e^{-mx}}{2} + c_3\cos mx + c_4\sin mx$

$\cosh mt \qquad \frac{e^x+e^{-x}}{2}$

$(2D-1)y = e^{2x}$ Then the value of $(D-2)(2D-1)y$ is

A.E

$2D-1=0$

$D=\frac{1}{2}$

P.I

$\frac{1}{2D-1}\cdot e^{2x}$

$(D-2)(2D-1)y$

(e^{2x})

$2D-1=0$

$D=\frac{1}{2}$

$y_c = c_1 e^{\frac{1}{2}x}$

$\frac{1}{2D-1} \cdot e^{2x}$

$\frac{1}{2(2)-1} e^{2x}$

$y_p = \frac{1}{3} e^{2x}$

$$\boxed{y = c_1 e^{\frac{1}{2}x} + \frac{1}{3} e^{2x}}$$

$(D-2)(2D-1)y$

$(D-2)(e^{2x})$

$= 2e^{2x} - 2e^{2x} = 0$

$$(D^2+3D+2)y = e^{2x}\sin x$$

$$\frac{1}{f(D)} \cdot e^{ax} V = e^{ax} \frac{1}{f(D+a)} \cdot V$$

$(1+x,\ 1-x,\ 1,\ x^2,\ (1+x)^2)$

(a) 2, (b) 3 (c) 4 (d) 5

$$\begin{vmatrix} 1+x & 1-x & 1 & x^2 \\ 1 & -1 & 0 & 2x \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-x & 1 & x^2 \\ -1 & 0 & 2x \\ 0 & 0 & 2 \end{vmatrix}$$

$= 2$

$$(D^2-2D-3)y = 2x^2+6x$$

$$\frac{1}{D^2-2D-3} \cdot 2x^2+6x = \frac{1}{-3\left(1-\left(\frac{D^2-2D}{3}\right)\right)} \cdot 2x^2+6x$$

$$= \frac{-1}{3}\left[1-\frac{D^2-2D}{3}\right]^{-1} 2x^2+6x$$

$$= \frac{-1}{3}\left[1+\left(\frac{D^2-2D}{3}\right)+\left(\frac{D^2-2D}{3}\right)^2\right](2x^2+6x$$

$D(2x^2+6x)$

$= 4x+6$

$D^2(2x^2+6x)$

$= \frac{-1}{3}\left\{(2x^2+6x) + \frac{1}{3}(\right.$ $\longrightarrow$?

$D^2(2x^2+6x) = 4$

$y_1' = 2y_1 + y_2$ $\quad$ $y_2' = y_1 + 2y_2$

$y_2 = y_1' - 2y_1$
$y_2' = y_1'' - 2y_1'$

Second order ~~diff~~ C.D.E. satisfied by y_1

$y_1'' - 2y_1' = y_1 + 2(y_1' - 2y_1)$

$y_1'' - 2y_1' = y_1 + 2y_1' - 4y_1$

$$\boxed{y_1'' - 4y_1' + 3y_1 = 0}$$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ ——

$x = e^t$ $\quad$ $t = \log x$

$xD = \theta$

$x^2D^2 = \theta(\theta - 1)$

P.I $\quad$ $\underline{g(x) = e^{ax} \quad \sin \quad x^m \quad e^{ax}\sin bx}$

Limit 72 $\quad$ $y_c =$ $\quad$ A.E $\quad$ D^3 $\quad$ $D = ?$

$y =$

Mandatory $\quad$ D^4 ——

$I.F = \frac{1}{Mx + Ny}$

② $f(xy)\, y\,dx + g(x,y)\, x\,dy \Rightarrow$ $I.F = \frac{1}{Mx - Ny}$