4·4·1. Sets and Elements of Sets. A set is a well defined collection or aggregate of all possible objects having given properties and specified according to a well defined rule. The objects comprising a set are called elements, members or points of the set. Sets are often denoted by capital letters, *viz.*, A, B, C, etc. If x is an element of the set A, we write symbolically $x \in A$ (x belongs to A). If x is not a member of the set A, we write $x \notin A$ (x does not belong to A). Sets are often described by describing the properties possessed by their members. Thus the set A of all non-negative rational numbers with square less than 2 will be written as $A = \{x : x \text{ rational}, x \geq 0, x^2 < 2\}$.

$A = \{a, e, i, o, u\}$ $\quad$ $a \in A$, $e \in A$, $i \in A$ .....

$B = \{1, 2, 3, 4, 5, 6\}$ $\quad$ $1 \in B$, $2 \in B$, ... , $6 \in B$

Roaster form

$A = \{a, e, i, o, u\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$C = \{H, T\}$

Set builder form

$A = \{x : x$ is a vowel of english alphabet$\}$

$B = \{x : x$ is a natural number, $x \leq 6\}$

$C = \{x : x$ is an outcome of tossing a coin$\}$

Subset

If every element of the set A belongs to the set B, *i.e.*, if $x \in A \Rightarrow x \in B$, then we say that A is a subset of B and write symbolically $A \subseteq B$ (A is contained in B) or $B \supseteq A$ (B contains A). Two sets A and B are said to be *equal* or *identical* if $A \subseteq B$ and $B \subseteq A$ and we write $A = B$ or $B = A$.

A *null* or an *empty* set is one which does not contain any element at all and is denoted by ϕ. $= \{ \}$

$x \in A \Rightarrow x \in B$

$A \subseteq B$ — superset

subset

$B = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ $\quad A \subseteq B$

$A \subseteq B$ and $B \subseteq A$

$\Rightarrow \boxed{A = B}$

$A \subseteq A$

Remarks. 1. Every set is a subset of itself.

$\phi \subseteq A$

2. An empty set is subset of every set.

3. A set containing only one element is conceptually distinct from the element itself, but will be represented by the same symbol for the sake of convenience.

4. As will be the case in all our applications of set theory, especially to probability theory, we shall have a fixed set S (say) given in advance, and we shall be concerned only with subsets of this given set. The underlying set S may vary from one application to another, and it will be referred to as *universal set* of each particular discourse.

$\{1\} \longrightarrow$ singleton set

$\{H\}$

$S \rightarrow$ Universal set (Sample space)

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

$E \subseteq S$

Cardinal number of a set $\rightarrow$ No. of elements of a set A denoted by $n(A)$

$A = \{2, 4, 6, 8, 10\}$ $\qquad n(A) = 5$

$B = \{H, T\}$ $\qquad n(B) = 2$

$C = \{\ \} = \phi$ $\qquad n(C) = 0$

For a set A with n elements the total number of subsets are 2^n

$A = \{a, b\}$, $n(A) = 2$, $2^2 = 4$

$\phi = \{\ \}$

$\{a\}$

$\{b\}$

$\{a, b\}$

Q1 $S = \{H, T\}$ outcome on tossing a coin

then what is the possible no. of event

$n(S) = 2$ $\qquad 2^2 = 4$

(a) 1

(b) 2

(c) 3

✓ (d) 4

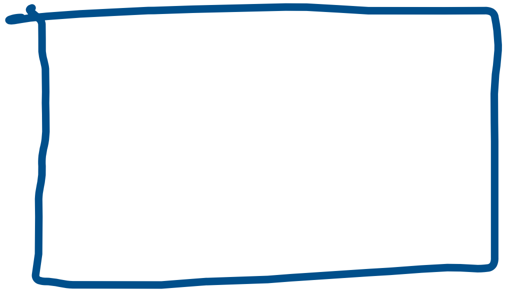
$\phi = \{\ \}$ ⟶ impossible event

$\{H\}$

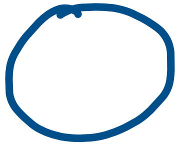
$\{T\}$

$\{H, T\}$ ⟶ sure / certain event

Venn diagram

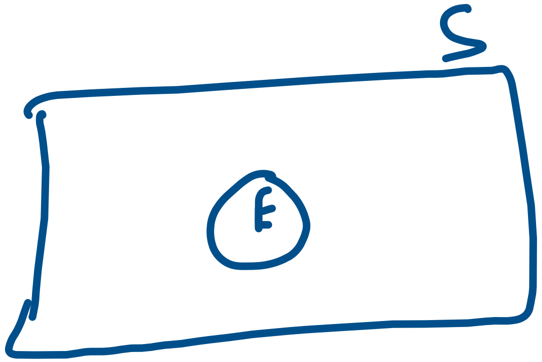


— S universal set



— Event (E)

$E \subseteq S$



Operation on Sets

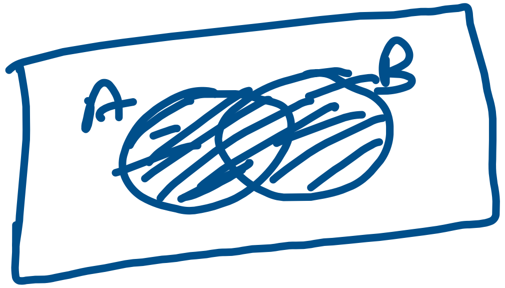
The union of two given sets A and B, denoted by $A \cup B$, is defined as a set consisting of all those points which belong to either A or B or both. Thus symbolically,

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}.$$

Similarly

$$\bigcup_{i=1}^{n} A_i = \{ x : x \in A_i \text{ for at least one } i = 1, 2, \ldots, n \}$$

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \ldots\ldots \cup A_n$$



$A = \{1, 2, 4, 6, 8\}$

$B = \{2, 3, 4, 5, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

The *intersection* of two sets A and B, denoted by $A \cap B$, is defined as a set consisting of all those elements which belong to both A and B. Thus

$$A \cap B = \{ x : x \in A \text{ and } x \in B \}.$$

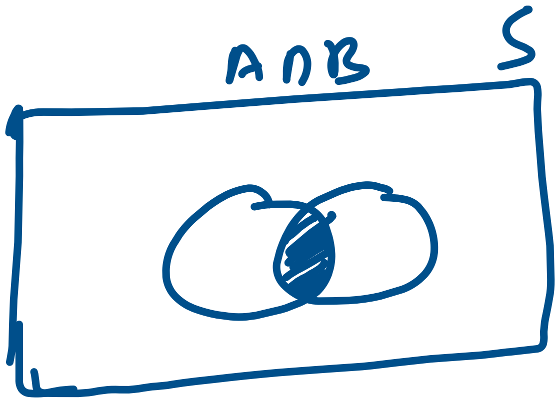
Similarly

$$\bigcap_{i=1}^{n} A_i = \{ x : x \in A_i \text{ for all } i = 1, 2, \ldots, n \}$$

For example, if $A = \{1, 2, 5, 8, 10\}$ and $B = \{2, 4, 8, 12\}$, then

$$A \cup B = \{1, 2, 4, 5, 8, 10, 12\} \text{ and } A \cap B = \{2, 8\}.$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$



If A and B have no common point, *i.e.*, $A \cap B = \phi$, then the sets A and B are said to be *disjoint*, *mutually exclusive* or *non-overlapping*.

The *relative difference* of a set A from another set B, denoted by $A-B$ is defined as a set consisting of those elements of A which do not belong to B. Symbolically,

$$A - B = \{ x : x \in A \text{ and } x \notin B \}.$$

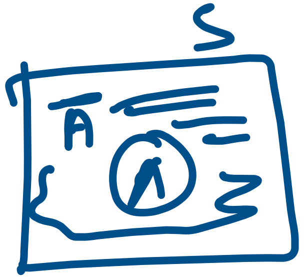
The *complement* or *negative* of any set A, denoted by $\bar{A}$ is a set containing all elements of the universal set S, (say), that are not elements of A, *i.e.*, $\bar{A} = S - A$.

A and B are Mutually Exclusive

$A \cap B = \phi$

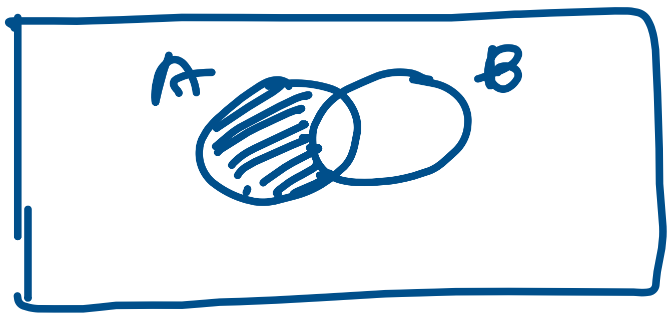
$\bar{A} = S - A$

$= \{x : x \in S$ and $x \notin A\}$



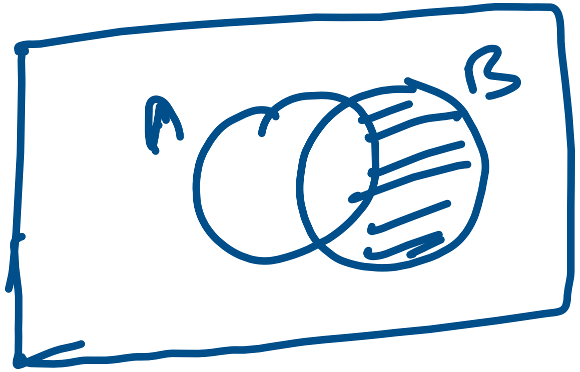
$A - B = A - (A \cap B) = \text{only } A = A \cap \overline{B}$

$A - B$



$B - A = B - (A \cap B) = \text{only } B = \overline{A} \cap B$

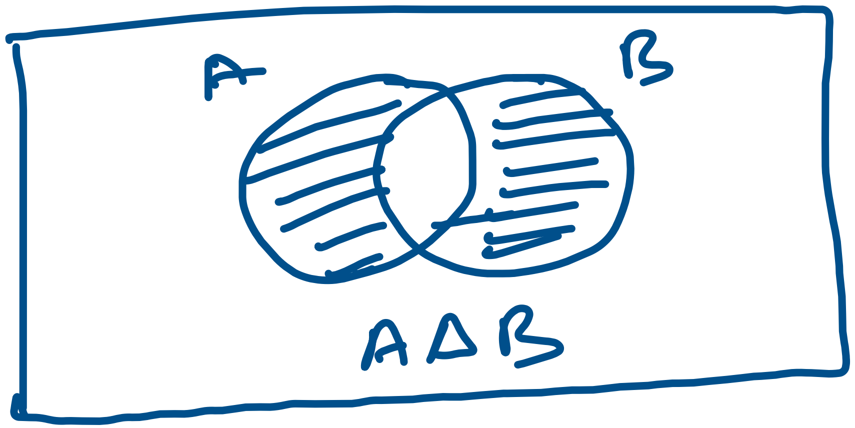
$B - A$



Q2 Symmetric difference (Δ) of A and B

$$A \Delta B = (A - B) \cup (B - A)$$

$$= (A \cup B) - (A \cap B)$$



Algebra of Sets

Now we state certain important properties concerning operations on sets. If A, B and C are the subsets of a universal set S, then the following laws hold:

Commutative Law : $A \cup B = B \cup A$, $A \cap B = B \cap A$

Associative Law : $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Law : $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Complementary Law : $A \cup \overline{A} = S$, $A \cap \overline{A} = \phi$

$A \cup S = S$, $(\because A \subseteq S)$, $A \cap S = A$

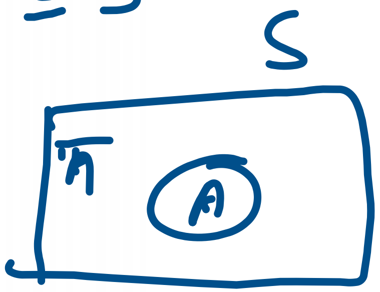
$A \cup \phi = A$, $A \cap \phi = \phi$

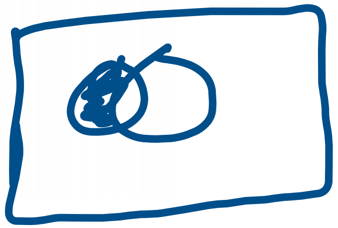
Difference Law : $A - B = A \cap \overline{B}$

$A - B = A - (A \cap B) = (A \cup B) - B$

$A - (B - C) = (A - B) \cup (A - C)$.

$A \subseteq S$





$$(A \cup B) - C = (A - C) \cup (B - C)$$

$$A - (B \cup C) = (A - B) \cap (A - C)$$

$$(A \cap B) \cup (A - B) = A \,,\; (A \cap B) \cap (A - B) = \phi$$

De-Morgan's Law — Dualization Law.

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B} \text{ and } \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

More generally

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i} \text{ and } \overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \bigcup_{i=1}^{n} \overline{A_i}$$

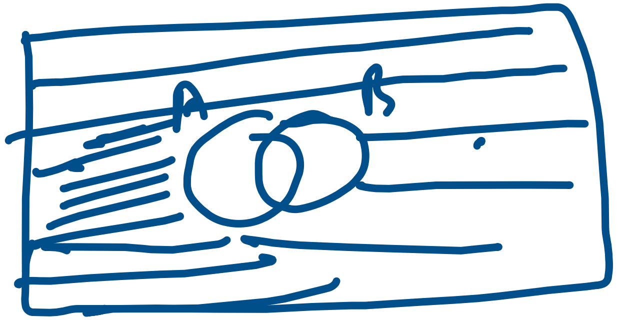
Involution Law : $\overline{(\overline{A})} = A$

Idempotency Law : $A \cup A = A \,,\; A \cap A = A$

De-Morgan's Law

A and B

(1) $\overline{A \cup B} = \overline{A} \cap \overline{B}$



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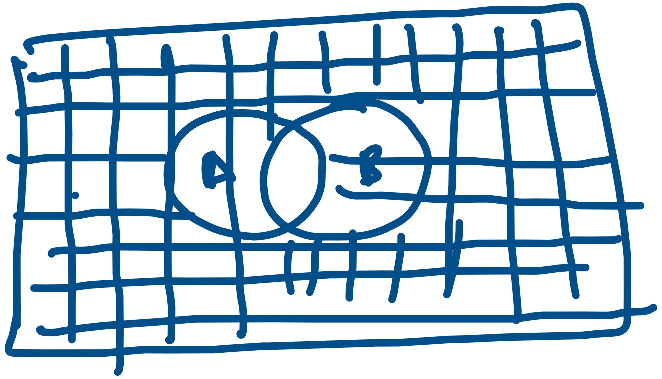

(2) $$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$$

$$\overline{(A_1 \cup A_2 \cup A_3)} = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3}$$

③ $$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

④ $$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$

$$\overline{A_1 \cap A_2 \cap A_3} = \overline{A_1} \cup \overline{A_2} \cup \overline{A_3}$$

A, B and C are three orbitrary events. Find expressions for the events noted below, in the context of A, B and C.

(i) only A occurs, $A \cap \bar{B} \cap \bar{C}$

(ii) Both A and B, but not C, occur, $A \cap B \cap \bar{C}$

(iii) All three events occur, $A \cap B \cap C$

(iv) At least one occurs, $A \cup B \cup C$

(v) At least two occur,

(vi) One and no more occurs,

(vii) Two and no more occur,

(viii) None occurs.

(v) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (A \cap B \cap C)$

(vi) $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

(vii) $(A \cap B \cap C) \cup (A \cap \overline{B} \cap C) \cup (A \cap B \cap \overline{C})$

(viii) $\overline{A} \cap \overline{B} \cap \overline{C} = \overline{A \cup B \cup C}$