L19 01 Mar 2021

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Method of variation of parameters.

to solve non Homogeneous LDE with Constant Coefficient.

Let us Consider 2nd order Non Homogeneous LDE with Constant Coefficient.

$$ay'' + by' + cy = s(x)$$

AE $(aD^2 + bD + c) = 0$

$D = ?$

Complimentary function

$$\boxed{y_c = c_1 y_1 + c_2 y_2}$$

Let us suppose the particular Integral

$$\boxed{y_p = A(x) y_1 + B(x) y_2}$$

Wronskian $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$\boxed{y_p = y_1 \cdot \underbrace{-\int \frac{y_2\, s(x)}{W}\, dx}_{A(x)} + y_2 \underbrace{\int \frac{y_1\, s(x)}{W}\, dx}_{B(x)}}$$

Where $A(x) = -\int \frac{y_2\, s(x)}{W} dx$ and $B(x) = \int \frac{y_1\, s(x)}{W} dx$

\# $\frac{d^2y}{dx^2} + y = \tan x$

A.E $(D^2+1) y = \tan x$

$D^2 + 1 = 0 \Rightarrow D^2 = -1 \Rightarrow D = \pm i \Rightarrow \boxed{y_c = c_1 \cos x + c_2 \sin x}$

$y_1 = \cos x \quad y_2 = \sin x \quad s(x) = \tan x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$W = 1$$

$$y_p = -y_1 \int \frac{y_2\, g(x)}{W}\, dx + y_2 \int \frac{y_1\, g(x)}{W}\, dx$$

$$y_p = -\cos x \int \sin x \tan x \, dx + \sin x \int \cos x \cdot \tan x \, dx$$

$$= -\cos x \int \frac{\sin^2 x}{\cos x}\, dx + \sin x \int \sin x \, dx$$

$$= -\cos x \int \frac{1-\cos^2 x}{\cos x}\, dx + \sin x(-\cos x)$$

$$= -\cos x \int (\sec x - \cos x)\, dx - \sin x \cos x$$

$$= -\cos x \left[\log(\sec x + \tan x)\right] + \cancel{\cos x(\sin x)} - \cancel{\sin x \cos x}$$

$$y_p = -\cos x \log(\sec x + \tan x)$$

$(D^2+4)y = 4 \cdot \sec^2 2x$

A.E $\quad D^2+4=0 \Rightarrow D^2=-4 \Rightarrow D=\pm 2i \Rightarrow y_c = c_1 \underbrace{\cos 2x}_{y_1} + c_2 \underbrace{\sin 2x}_{y_2}$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2(\cos^2 2x + \sin^2 2x) = 2.$$

$y_1 = \cos 2x, \quad y_2 = \sin 2x \quad g(x) = 4 \cdot \sec^2 2x \qquad W = 2$

$$y_p = -y_1 \int \frac{y_2\, g(x)}{W}\, dx + y_2 \int \frac{y_1\, g(x)}{W}\, dx$$

$$= -\cos 2x \int \frac{\sin 2x \cdot 4\sec^2 2x}{2}\, dx + \sin 2x \int \frac{\cos 2x \cdot 4 \cdot \sec^2 2x}{2}\, dx$$

$$= \text{[illegible]} + 2\sin 2x \int \sec 2x \, dx$$

$$= -\cos 2x \int \frac{2\sin 2x}{\cos^2 2x}\,dx + 2\sin 2x \int \sec 2x\,dx$$

$$= -\cos 2x \int -\frac{dt}{t^2} + \cancel{2}\sin 2x \left[\frac{\log(\sec 2x + \tan 2x)}{\cancel{2}}\right]$$

$\cos 2x = t$

$-\sin 2x \cdot 2\,dx = dt$

$$= \cancel{-\cos 2x}\left(\frac{1}{\cancel{\cos 2x}}\right) + \sin 2x \log(\sec 2x + \tan 2x)$$

$\frac{1}{t^2} = t^{-2}$

$\frac{t^{-2+1}}{-2+1}$

$-\frac{t^{-1}}{-1}$

$$y_p = -1 + \sin 2x \log(\sec 2x + \tan 2x)$$

~~$y_p = B(x)y_1 + A(x)y_2$~~

$y_p = A(x)y_1 + B(x)y_2$

1 The value of parameter A(x) for LDE $y''-2y'-3y = e^x$ using method of variation of parameters when $y_1 = e^{-x}$ and $y_2 = e^{3x}$ is

(a) $-\frac{e^{2x}}{8} + c$ (b) $\frac{3}{2}e^x + c$ (c) $\frac{2}{3}e^{3x} + c$ (d) $-\frac{e^{5x}}{18} + c$

$y = A(x)y_1 + B(x)y_2$

$$y'' - 2y' - 3y = e^x$$

$y_1 = e^{-x}$ $y_2 = e^{3x}$

$$A(x) = -\int \frac{y_2 S(x)}{W}\,dx$$

$$W = \begin{vmatrix} e^{-x} & e^{3x} \\ -e^{-x} & 3e^{3x} \end{vmatrix}$$

$$= -\int \frac{e^{3x} \cdot e^x}{4e^{2x}}\,dx = -\frac{1}{4}\int e^{2x}\,dx$$

$$= -\frac{1}{4}\frac{e^{2x}}{2} = \frac{-1}{8}e^{2x}$$

$W = 3e^{2x} + e^{2x}$

$W = 4e^{2x}$

Q19. By the method of variation of parameter if $A(x)\cos x + B(x)\sin x$ be the particular integral of the differential equation

$y'' + y = \sec x$ then $A(x)$ is

(a) - log|cos x| (b) log|cos x| (c) log|sin x| (d) - log|sin x|

$$y_p = -y_1 \int \frac{y_2 S(x)}{W}\,dx + y_2 \int \frac{y_1 S(x)}{W}\,dx$$

$$\int \frac{\sin x}{\cos x}\,dx$$

$$A(x) = -\int \frac{\sin x \cdot \sec x}{1}\, dx = -\int \tan x \, dx$$

$$= +\left[\log(\cos x)\right]$$