L15 20 Feb 2021

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Case No:-2

If $s(x) =$ Sin ax or Cos ax

$$\frac{1}{f(D)}(\text{Sin } ax \text{ or } \text{Cos } ax)$$

$$= \frac{1}{f(D^2=-a^2)}.(\text{Sin } ax \text{ or } \text{Cos } ax), \quad \text{provided } f(D^2=-a^2) \neq 0$$

$D^2 \to -a^2$

Case of failure

If $f(D^2=-a^2)=0$

$$y_p = x.\frac{1}{f'(D^2=-a^2)}(\text{Sin } ax \text{ or } \text{Cos } ax)$$

Find the general solution of: $y'' - 16y = \cos 2x$

C.F

A.E $D^2-16=0$

$D^2=16$

$D=\pm 4$

$y_c = c_1 e^{-4x} + c_2 e^{4x}$

P.I

$$\frac{1}{f(D)}\text{Cos } 2x = \frac{1}{D^2-16}.\text{Cos } 2x$$

$D^2 = -a^2 = -4$

$$y_p = \frac{1}{-4-16}\text{Cos } 2x$$

$$= \frac{-1}{20}\text{Cos } 2x$$

General sol

$$y = y_c + y_p = c_1 e^{-4x} + c_2 e^{4x} - \frac{1}{20}\text{Cos } 2x \quad \text{Ans}$$

Find the general solution of: $y'' + 9y = \sin 3x$

C.F

A.E $D^2+9=0$

$D^2=-9$

$D=\pm 3i$

$y_c = C_1 \text{Cos } 3x + C_2 \text{Sin } 3x$

P.I

$$\frac{1}{D^2+9}\text{Sin } 3x \quad (D^2=-9)$$

$$= \frac{1}{\cancel{-9}+\cancel{9}}\text{Sin } 3x \to (\text{Case of failure})$$

$$= x.\frac{1}{2D}\text{Sin } 3x$$

$$= \frac{x}{2}\frac{1}{D}(\text{Sin } 3x) = \frac{x}{2}\left(-\frac{\text{Cos } 3x}{3}\right)$$

General sol

$$-\frac{x}{2}\ \frac{1}{D}(\ldots) - \frac{1}{2}\left(\ldots\frac{1}{3}\right)$$

General sol

$$y = y_p + y_c = \boxed{c_1\cos 3x + c_2\sin 3x - \frac{x}{6}\cos 3x}$$

$$y_p = -\frac{x}{6}\cos 3x \qquad \frac{1}{D}$$

Find the general solution of: $\underbrace{2y'' - 5y' + 3y}$ = sin x

C.F

A.E $2D^2 - 5D + 3 = 0$

$2D^2 - 2D - 3D + 3 = 0$

$2D(D-1) - 3(D-1) = 0$

$(2D-3)(D-1) = 0$

$D = 1, \frac{3}{2}$

$$y_c = c_1 e^{x} + c_2 e^{3/2 x}$$

P.I

$$\frac{1}{2D^2 - 5D + 3}\sin x \qquad D^2 \to -1$$

$$= \frac{1}{-2 - 5D + 3}\sin x = \frac{1}{1 - 5D}\sin x$$

$$\boxed{= \frac{1}{1-5D} \times \frac{(1+5D)}{(1+5D)}\sin x} \to \text{note}$$

$$= \frac{1+5D}{1 - 25D^2}\sin x \qquad D^2 \to -1$$

$$= \frac{1+5D}{1 - 25(-1)}\sin x = \frac{1+5D}{26}\sin x$$

$$y = y_c + y_p = ?$$

$$= \frac{1}{26}\left[(1+5D)\sin x\right]$$

note this step

$$\boxed{= \frac{1}{26}\left[\sin x + 5\,D(\sin x)\right]}$$

$$y_p = \frac{1}{26}\left[\sin x + 5\cos x\right]$$

\# Find P.I $(D^3 - 3D^2 + 9D - 27)y = \cos 3x$

$$D^2 \to -9$$

$$\frac{1}{D^3 - 3D^2 + 9D - 27}\cos 3x = \frac{1}{\cancel{-9D} - 3(-9) + \cancel{9D} - \cancel{27}}\cos 3x$$

$$D^3 = D^2 \cdot D$$

$$\frac{1}{D^3-3D^2+9D-27}\cdot \cos 3x = \frac{1}{-\cancel{9D}-3(-9)+\cancel{9D}-\cancel{27}} \cos 3x \qquad D^3 = D^2\cdot D \to -9D$$

$$= \frac{1}{0}\cos 3x \rightarrow (\text{Case of failure})$$

$$x\cdot\frac{1}{3D^2-6D+9}\cos 3x = x\cdot\frac{1}{3(-9)-6D+9}\cos 3x \qquad D^2 \to -9$$

$$-27+9$$

$$= x\cdot\frac{1}{-18-6D}\cos 3x$$

$$= -\frac{x}{6}\cdot\frac{1}{D+3}\cos 3x$$

$$= -\frac{x}{6}\cdot\left[\frac{D-3}{D^2-9}\cos 3x\right]$$

$$= -\frac{x}{6}\left[\frac{D-3}{-9-9}\cos 3x\right]$$

$$= +\frac{x}{6\times 18}\left[D(\cos 3x) - 3\cos 3x\right]$$

$$= \frac{x}{108}\left[-3\sin 3x - 3\cos 3x\right]$$

$$= -\frac{x}{36}\left[\sin 3x + \cos 3x\right] \quad \text{Ans}$$

#

$$\frac{1}{D^3+1}\cos^2 \frac{x}{2} \qquad \cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$= \frac{1}{D^3+1}\left(\frac{1+\cos x}{2}\right)$$

$$= \frac{1}{\;}\cdot\frac{1}{2} + \frac{1}{\;}\frac{1}{\;}\cdot\cos x$$

$$= \underbrace{\frac{1}{D^3+1}\cdot\frac{1}{2}} + \frac{1}{2}\frac{1}{D^3+1}\cdot \cos x$$

$$= \frac{1}{D^3+1}\cdot\frac{1}{2}e^{0x} + \frac{1}{2}\cdot\frac{1}{D^3+1}\cdot\cos x$$

$$= \frac{1}{2}\cdot\frac{1}{1}\cdot e^{0x} + \frac{1}{2}\cdot\frac{1}{-D+1}\cdot\cos x$$

$$= \frac{1}{2} + \frac{1}{2}\cdot\frac{1+D}{1-D^2}\cdot\cos x = \frac{1}{2}+\frac{1}{2}\cdot\frac{1+D}{2}\cos x$$

$$= \frac{1}{2}+\frac{1}{4}(\cos x - \sin x)$$

Q27. The Particular Integral (P.I.) for the differential equation $y'' + y' - 6y = 5e^{-3x}$ is:

(a) $-e^{-3x}$ (b) $-xe^{-3x}$ (c) xe^{-3x} (d) e^{-3x}

$D \to -3$

$$5\frac{1}{D^2+D-6}e^{-3x} = 5\cancel{x}\cdot\frac{1}{2D+1}e^{-3x}$$

Case of failure

$$5x\cdot\frac{1}{-6+1}\cdot e^{-3x} = -xe^{-3x}$$

Q38. The Particular Integral of $\frac{d^2y}{dx^2} + y = \sin x$, is

(a) $-\frac{x}{2}\sin x$ (b) $\frac{x}{2}\sin x$ (c) $\frac{x}{2}\cos x$ (d) $-\frac{x}{2}\cos x$

$D^2 \to -1$

$$\frac{1}{D^2+1}\cdot\sin x = \frac{1}{\cancel{-1}+1}\sin x = \frac{1}{0} \quad \text{Case of fail.}$$

$$x\cdot\frac{1}{2D}\sin x = \frac{x}{2}(-\cos x)$$

The lowest possible order of homogeneous linear differential equation whose pa

$1+x+e^{x}-3e^{3x}$ is

a) 4 b)3 c) 2 d) 5

- A
- B
- C
- D

Clear Response

$(1+x)\cdot e^{0x}+e^{x}-3e^{3x}$

A.E 3, 1, 0, 0