L14 17 Feb 2021

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① $(\sin y - \sin xy)\,dx + (x\cos y - x\sin xy)\,dy = 0$

② $(f(x,y), y') = 0$ $\qquad y' = f(x,y)$

ⓐ $\cancel{g(x,y)=0}$ ⓑ $g(x,y,1)=0$ ⓒ $g(x,y,c)=0$

ⓓ $y = mx + C$

Sol $\qquad M\,dx + N\,dy = 0$

$M = \sin y - \sin xy$ $\qquad \frac{\partial M}{\partial y} = \cos y - \cos xy\,[x] = \cos y - x\cos xy$

$N = x\cos y - x\sin xy$ $\qquad \frac{\partial N}{\partial x} = \cos y - x(\cos xy \cdot y) - \sin xy.$

$= \cos y - xy\cos xy - \sin xy$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \cancel{\cos y} - x\cos xy - \cancel{\cos y} + xy\cos xy + \sin xy$

$= -x\cos y + xy\cos xy + \sin xy$

③ $D^3 - 6D + 13 = 0$ $\qquad y''' - 6y' + 13 = 0$

unit 3

Solution of Non-homogeneous LDE with constant coefficients Using Operator Method:

$$a.\frac{d^2y}{dx^2} + b.\frac{dy}{dx} + cy = R(x) \quad —①$$

S.F $\qquad a.D^2y + bDy + cy = R(x)$

$\frac{d}{dx} \to D$ $\qquad (aD^2 + bD + c)y = R(x)$

The General solution of ① Consists of two parts (i) Complementary function (C.F)

(ii) Particular Integral.

$$\boxed{y = y_c + y_p}$$

$y_c \to C.F$

$y_p \to P.I$

$$\boxed{y = y_c + y_p}$$

$y_c \to C.F$

$y_p \to P.I$

Complementary function (C.F) y_c

C.F is nothing but the general sol. of ① when $s(x)=0$

i.e $(aD^2+bD+c)y=0$

$$f(D).y=0$$

Particular Integral

$$f(D)y = s(x)$$

$$\cancel{(f(D))^{-1}}\cancel{(f(D))}\,y = (f(D))^{-1} s(x)$$

$$\boxed{y_p = \frac{1}{f(D)}\cdot s(x)}$$

$\frac{d}{dx} \to D$

$Dx = 1$

$\frac{1}{D}x = \frac{x^2}{2}$

$\frac{1}{D^2}x = \frac{1}{2}\cdot\frac{x^3}{3} = \frac{x^3}{6}$

Case No.-1

If $s(x) = e^{ax}$ then

P.I $\quad \frac{1}{f(D)}s(x) = \frac{1}{f(D)}e^{ax} \qquad (D \to a)$

$$= \frac{1}{f(a)}\cdot e^{ax}, \quad f(a) \neq 0$$

Case of failure

If $f(a)=0$ then

P.I $\quad \frac{1}{f(D)}e^{ax} = x\cdot\frac{1}{(f(D))'}\cdot e^{ax}$

symbolic form

$\int 2x\,dx$

P.I $\frac{1}{f(D)} e = x \cdot \frac{1}{(f(D))'} \cdot$

Symbolic form

$\int 2x\,dx$
$2 \cdot \int x\,dx$

Find the general solution of: $y'' + 5y' + 4y = 18e^{2x}$

$y = y_c + y_p$

S.F

C.F

$(D^2+5D+4)y = 18e^{2x}$

A.E $D^2+5D+4=0$

$D^2+4D+D+4=0$

$D(D+4)+1(D+4)=0$

$(D+1)(D+4)=0$

$D=-1,-4$

$y_c = c_1e^{-x}+c_2e^{-4x}$

P.I $y_p = \frac{1}{f(D)} g(x) = \frac{1}{D^2+5D+4} \cdot 18e^{2x}$

$= 18 \cdot \frac{1}{D^2+5D+4} \cdot e^{2x}$ $(D \to 2)$

$= 18 \cdot \frac{1}{4+5(2)+4} \cdot e^{2x}$

$y_p = \frac{\cancel{18}}{\cancel{18}} \cdot e^{2x} = e^{2x}$

General sol $y = y_c + y_p$

$y = (c_1e^{-x}+c_2e^{-4x}) + e^{2x}$ Ans

Find the general solution of: $y'' + y' - 6y = e^{2x}$

C.F

A.E $D^2+D-6=0$

$D^2+3D-2D-6=0$

$D(D+3)-2(D+3)=0$

$(D+3)(D-2)=0$

$D=2,-3$

$y_c = c_1e^{-3x}+c_2e^{2x}$

P.I $\frac{1}{f(D)}e^{2x} = \frac{1}{D^2+D-6} \cdot e^{2x}$ $(D \to 2)$

$= \frac{1}{\cancel{4+2-6}} \cdot e^{2x} = \frac{1}{0} \cdot e^{2x}$ (case of failure)

$\frac{1}{f(D)}e^{2x} = x \cdot \frac{1}{2D+1} \cdot e^{2x}$

$y_p = x \cdot \frac{1}{5} \cdot e^{2x}$

$y = y_c + y_p = c_1e^{-3x}+c_2e^{2x} + \frac{x}{5}e^{2x}$ Ans

Find the general solution of: $y'' - 6y' + 9y = 14e^{3x}$

$D^2 - 6D + 9 = 0$ $\frac{1}{\;} \cdot 14 \cdot e^{3x} = \frac{1}{\;}$

Find the general solution of: $y'' - 6y' + 9y = 14e^{3x}$

$D^2 - 6D + 9 = 0$

$D = 3, 3$

$$\frac{1}{D^2 - 6D + 9} \cdot 14 \cdot e^{3x} = \frac{1}{0}$$

$\cancel{9} - \cancel{18} + \cancel{9}$

$$x \cdot \frac{1}{2D - 6} \cdot 14 \cdot e^{3x} = x \cdot \frac{1}{\cancel{6} - \cancel{6}} 14 \cdot e^{3x}$$

$$x^2 \cdot \frac{1}{2} \cdot 14 \cdot e^{3x}$$

$$y_p = \frac{\overset{7}{\cancel{14}}}{\cancel{2}} \cdot x^2 e^{3x}$$