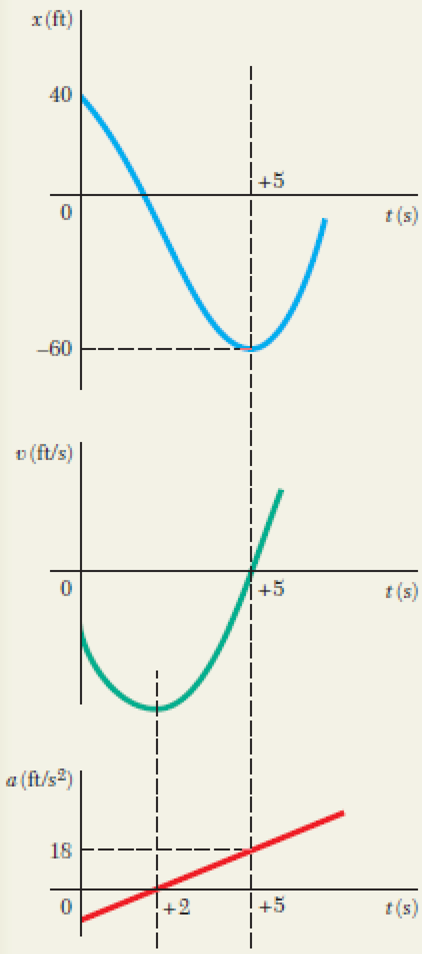
Tutorial sheet 5

1. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine (*a*) the time at which the velocity will be zero, (*b*) the position and distance traveled by the particle at that time, (*c*) the acceleration of the particle at that time, (*d*) the distance traveled by the particle from t = 4 s to t = 6 s. **[a) t=5 sec, b) x=-60 m, distance= 100 m, c) a= 18 m/sec², d) x= 18 m]**



The equations of motion are

$$x = t^3 - 6t^2 - 15t + 40 \quad (1)$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \quad (2)$$

$$a = \frac{dv}{dt} = 6t - 12 \quad (3)$$

a. Time at Which $v = 0$. We set $v = 0$ in (2):

$$3t^2 - 12t - 15 = 0 \qquad t = -1 \text{ s} \qquad \text{and} \qquad t = +5 \text{ s}$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$, the particle moves in the negative direction; for $t > 5$ s, $v > 0$, the particle moves in the positive direction.

b. Position and Distance Traveled When $v = 0$. Carrying $t = +5$ s into (1), we have

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60 \text{ ft}$$

The initial position at $t = 0$ was $x_0 = +40$ ft. Since $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, we have

$$\text{Distance traveled} = x_5 - x_0 = -60 \text{ ft} - 40 \text{ ft} = -100 \text{ ft}$$

Distance traveled = 100 ft in the negative direction

c. Acceleration When $v = 0$. We substitute $t = +5$ s into (3):

$$a_5 = 6(5) - 12 \qquad a_5 = +18 \text{ ft/s}^2$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals will be computed separately.

From $t = 4$ s to $t = 5$ s: $\quad x_5 = -60$ ft

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

$$\text{Distance traveled} = x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft}$$
$$= 8 \text{ ft in the negative direction}$$

From $t = 5$ s to $t = 6$ s: $\quad x_5 = -60$ ft

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$

$$\text{Distance traveled} = x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft}$$
$$= 10 \text{ ft in the positive direction}$$

Total distance traveled from $t = 4$ s to $t = 6$ s is 8 ft + 10 ft $= 18$ ft

2. The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when a = 0. **[t=0.667 sec, $x_{2/3}$=0.259 m and $v_{2/3}$=-8.56 m/sec]**

SOLUTION

We have $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$

Then $v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$

and $a = \frac{dv}{dt} = 72t^2 - 12t - 24$

When $a = 0$: $72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$

or $(3t - 2)(2t + 1) = 0$

or $t = \frac{2}{3}\text{ s}$ and $t = -\frac{1}{2}\text{ s}$ (Reject) $t = 0.667\text{ s}$ ◀

At $t = \frac{2}{3}$ s: $x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3$ or $x_{2/3} = 0.259\text{ m}$ ◀

$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3$ or $v_{2/3} = -8.56\text{ m/s}$ ◀

3. The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x is expressed in meters and t in seconds. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero. **[(a) t=4 sec, (b) t=2.5 sec, $x_{2.5}$= 1.5 m and distance traveled= 24.5 m]**

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

$$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(*a*) $\underline{v = 0 \text{ when}}$ $\quad 6t^2 - 30t + 24 = 0$

$6(t-1)(t-4) = 0 \qquad t = 1.000 \text{ s} \quad \text{or} \quad t = 4.00 \text{ s}$ ◀

(*b*) $\underline{a = 0 \text{ when}}$ $\quad 12t - 30 = 0 \qquad t = 2.5 \text{ s}$

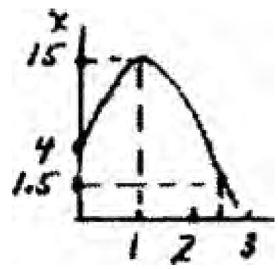
For $t = 2.5$ s: $\quad x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

$x_{2.5} = +1.500 \text{ m}$ ◀

To find total distance traveled, we note that

$\underline{v = 0 \text{ when } t = 1 \text{ s:}}$ $\quad x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4$

$x_1 = +15 \text{ m}$



For $t = 0$, $\quad x_0 = +4 \text{ m}$

Distance traveled

From $t = 0$ to $t = 1$ s: $\quad x_1 - x_0 = 15 - 4 = 11 \text{ m} \rightarrow$

From $t = 1$ s to $t = 2.5$ s: $\quad x_{2.5} - x_1 = 1.5 - 15 = 13.5 \text{ m} \leftarrow$

Total distance traveled $= 11 \text{ m} + 13.5 \text{ m} = 24.5 \text{ m}$ ◀

4. The motion of a particle is defined by the relation $x = 6t^2 - 8 + 40\cos\pi t$, where x and t are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration when $t = 6$ s **[x_6= 248 m, v=72 m/s and a=-383 m/sec^2]**

SOLUTION

We have $x = 6t^2 - 8 + 40\cos \pi t$

Then $v = \dfrac{dx}{dt} = 12t - 40\pi \sin \pi t$

and $a = \dfrac{dv}{dt} = 12 - 40\pi^2 \cos \pi t$

At $t = 6$ s: $x_6 = 6(6)^2 - 8 + 40\cos 6\pi$ or $x_6 = 248$ m ◀

$v_6 = 12(6) - 40\pi \sin 6\pi$ or $v_6 = 72.0$ m/s ◀

$a_6 = 12 - 40\pi^2 \cos 6\pi$ or $a_6 = -383$ m/s^2 ◀

5. The acceleration of a particle is defined by the relation a = kt^2. (a) Knowing that v = -32 ft/s when t = 0 and that v = 32 ft/s when t = 4 s, determine the constant k. (b) Write the equations of motion, knowing also that x = 0 when t = 4. **[k=3, v= t^3- 32, x=t^4/4 -32t+ 64]**

The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

SOLUTION

We have $a = kt^2 \quad k = \text{constant}$

Now $$\frac{dv}{dt} = a = kt^2$$

At $t = 6$ s, $v = 18$ m/s: $$\int_{18}^{v} dv = \int_{6}^{t} kt^2\, dt$$

or $$v - 18 = \frac{1}{3}k(t^3 - 216)$$

or $$v = 18 + \frac{1}{3}k(t^3 - 216)(\text{m/s})$$

Also $$\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$$

At $t = 0$, $x = 24$ m: $$\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right] dt$$

or $$x - 24 = 18t + \frac{1}{3}k\left(\frac{1}{4}t^4 - 216t\right)$$

Now

At $t = 6$ s, $x = 96$ m: $$96 - 24 = 18(6) + \frac{1}{3}k\left[\frac{1}{4}(6)^4 - 216(6)\right]$$

or $$k = \frac{1}{9}\ \text{m/s}^4$$

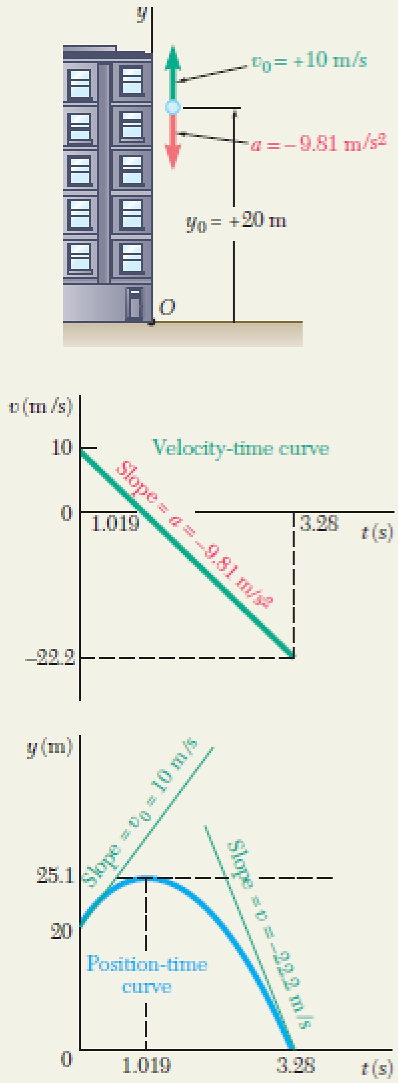
Then $$x - 24 = 18t + \frac{1}{3}\left(\frac{1}{9}\right)\left(\frac{1}{4}t^4 - 216t\right)$$

or $$x(t) = \frac{1}{108}t^4 + 10t + 24 \quad ◀$$

and $$v = 18 + \frac{1}{3}\left(\frac{1}{9}\right)(t^3 - 216)$$

or $$v(t) = \frac{1}{27}t^3 + 10 \quad ◀$$

6. A ball is tossed with a velocity of 10 m/s directed vertically upward from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine (*a*) the velocity *v* and elevation *y* of the ball above the ground at any time *t*, (*b*) the highest elevation reached by the ball and the corresponding value of *t*, (*c*) the time when the ball will hit the ground and the corresponding velocity. **[t= 1.012 sec, y=25.1m]**



a. Velocity and Elevation. The y axis measuring the position coordinate (or elevation) is chosen with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated. Substituting for a in $a = dv/dt$ and noting that at $t = 0$, $v_0 = +10$ m/s, we have

$$\frac{dv}{dt} = a = -9.81 \text{ m/s}^2$$

$$\int_{v_0=10}^{v} dv = -\int_0^t 9.81\, dt$$

$$[v]_{10}^{v} = -[9.81t]_0^t$$

$$v - 10 = -9.81t$$

$$v = 10 - 9.81t \quad (1)$$

Substituting for v in $v = dy/dt$ and noting that at $t = 0$, $y_0 = 20$ m, we have

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0=20}^{y} dy = \int_0^t (10 - 9.81t)\, dt$$

$$[y]_{20}^{y} = [10t - 4.905t^2]_0^t$$

$$y - 20 = 10t - 4.905t^2$$

$$y = 20 + 10t - 4.905t^2 \quad (2)$$

b. Highest Elevation. When the ball reaches its highest elevation, we have $v = 0$. Substituting into (1), we obtain

$$10 - 9.81t = 0 \qquad t = 1.019 \text{ s}$$

Carrying $t = 1.019$ s into (2), we have

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \qquad y = 25.1 \text{ m}$$

c. Ball Hits the Ground. When the ball hits the ground, we have $y = 0$. Substituting into (2), we obtain

$$20 + 10t - 4.905t^2 = 0 \qquad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s}$$

Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into (1), we have

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \qquad v = 22.2 \text{ m/s} \downarrow$$

7. A stone is thrown vertically upwards, from the ground, with a velocity 49 m/sec. after 2 seconds, another stone is thrown vertically upwards from the same place. If both the stone strike the ground at the same time, find the velocity, with which the second stone was thrown upwards **[velocity=39.2 m/sec]**

Solution. First of all, consider the upwards motion of the first stone. In this case, initial velocity $(u) = -49$ m/s (Minus sign due to upward motion) and final velocity $(v) = 0$ (because stone is at maximum height)

Let t = Time taken by the stone to reach maximum height.

We know that final velocity of the stone (v),

$$0 = u + gt = -49 + 9.8\,t \quad \text{...(Minus sign due to upwards motion)}$$

$$\therefore \quad t = \frac{49}{9.8} = 5 \text{ s}$$

It means that the stone will take 5 s to reach the maximum height and another 5 s to come back to the ground.

$\therefore$ Total time of flight $= 5 + 5 = 10$ s

Now consider the motion of second stone. We know that time taken by the second stone for going upwards and coming back to the earth

$$= 10 - 2 = 8 \text{ s}$$

and time taken by the second stone to reach maximum height

$$= \frac{8}{2} = 4 \text{ s}$$

Now consider the upward motion of the second stone. We know that final velocity of the stone (v),

$$0 = u + gt = -u + 9.8 \times 4 = -u + 39.2$$

$$\therefore \quad u = 39.2 \text{ m/s} \quad \textbf{Ans.}$$

8. A stone, dropped into well, is heard to strike the water after 4 seconds. Find the depth of well, if the velocity of sound is 350 m/sec. **[s=70.8 m]**

Solution. First of all, consider the downward motion of the stone. In this case, initial velocity (u) = 0 (because it is dropped)

Let t = Time taken by the stone to reach the bottom of the well.

We know that depth of the well,

$$s = ut + \frac{1}{2} gt^2 = 0 + \frac{1}{2} \times 9.8 \times t^2 = 4.9\, t^2 \quad ...(i)$$

and time taken by the sound to reach the top

$$= \frac{\text{Depth of the well}}{\text{Velocity of sound}} = \frac{s}{350} = \frac{4.9\, t^2}{350} \quad ...(ii)$$

Since the total time taken (*i.e.* stone to reach the bottom of the well and sound to reach the top of the well) is 4 seconds, therefore

$$t + \frac{4.9\, t^2}{350} = 4$$

$$4.9\, t^2 + 350\, t = 1400$$

or $$4.9\, t^2 + 350\, t - 1400 = 0$$

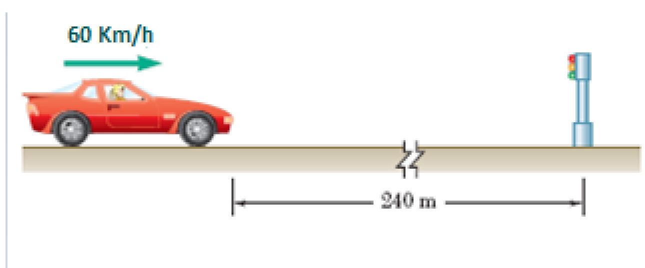
This is a quadratic equation in t,

$$\therefore \quad t = \frac{-350 \pm \sqrt{(350)^2 + 4 \times 4.9 \times 1400}}{2 \times 4.9} = 3.8 \text{ s}$$

Now substituting the value of t in equation (i),

$$s = 4.9\, t^2 = 4.9\, (3.8)^2 = 70.8 \text{ m} \qquad \textbf{Ans.}$$

9. A driver is driving his car at 60 km/hr when he observes that a traffic light 250 m ahead turns red. The traffic light is timed to remain red for 20 seconds before it turns green. The driver wishes to pass the traffic lights without stopping to wait for it turns green. Calculate (a) the required uniform acceleration of the car, (b) the speed of the car as it passes the traffic light. **[a= -0.417 m/sec2, v= 8.33 m/sec]**



(a) Acceleration of the car.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)}$$

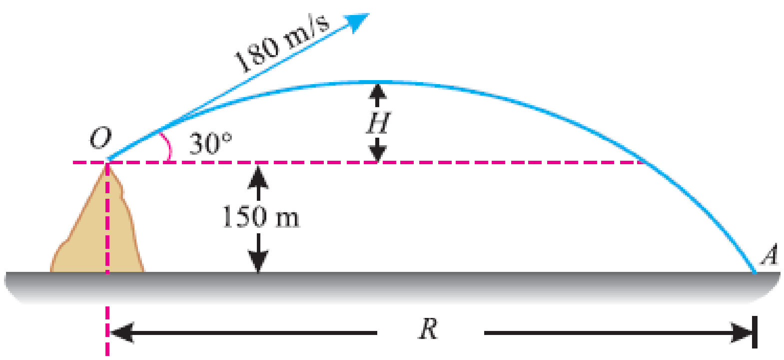
a=-0.417 m/sec^2

(*b*) <u>Speed at *B*</u>.

$$v_B = v_A + at$$

v= 8.33 m/sec

10. A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile **[(a)s=563.3 m, (b)horizontal distance=3102 m]**



1. *The greatest elevation above the ground reached by the projectile*

We know that maximum height to which the projectile will rise above the edge *O* of the cliff,

$$H = \frac{u^2 \sin^2 \alpha}{2g} = \frac{(180)^2 \sin^2 30^\circ}{2 \times 9.8} = \frac{(180)^2 \times (0.5)^2}{19.6} = 413.3 \text{ m}$$

$\therefore$ Greatest elevation above the ground reached by the projectile,

$$s = 413.3 + 150 = 563.3 \text{ m} \quad \textbf{Ans.}$$

2. *The horizontal distance from the gun to the point, where the projectile strikes the ground.*

First of all, consider motion of the projectile from the edge of the cliff to the maximum height. We know that the time taken by the projectile to reach maximum height from the edge of the cliff,

$$t_1 = \frac{u \sin \alpha}{g} = \frac{180 \sin 30^\circ}{9.8} = \frac{180 \times 0.5}{9.8} = 9.2 \text{ s}$$

Now consider vertical motion of the projectile from the maximum height to the ground due to gravitational acceleration only. In this case, $u = 0$ and $s = 563.3$ m.

Let t_2 = Time taken by the projectile to reach the ground from the maximum height.

We know that the vertical distance (s),

$$563.3 = u t_2 + \frac{1}{2} g t_2^2 = 0 + \frac{1}{2} \times 9.8 t_2^2 = 4.9 t_2^2$$

or $$t^2 = \frac{563.3}{4.9} = 115 \quad \text{or} \quad t_2 = 10.7 \text{ s} \qquad ...(ii)$$

$\therefore$ Total time taken by the projectile to reach the ground from the edge of the cliff

$$= t_1 + t_2 = 9.2 + 10.7 = 19.9 \text{ s}$$

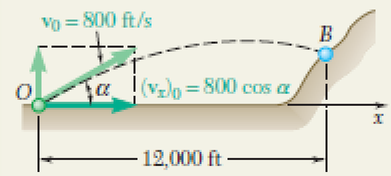
and horizontal distance from the gun to the point, where the projectile strikes the ground,

$$R = \text{Horizontal components of velocity} \times \text{Time}$$

$$= 180 \cos 30^\circ \times 19.9 = (180 \times 0.866) \times 19.9 \text{ m}$$

$$= 3102 \text{ m} = 3.102 \text{ km} \quad \textbf{Ans.}$$

11. A projectile is fired with an initial velocity of 800 ft/s at a target *B* located 2000 ft above the gun *A* and at a horizontal distance of 12,000 ft. Neglecting air resistance, calculate the value of the firing angle α **[α=29.5° and α=70°]**



The horizontal and the vertical motion will be considered separately.

Horizontal Motion. Placing the origin of the coordinate axes at the gun, we have

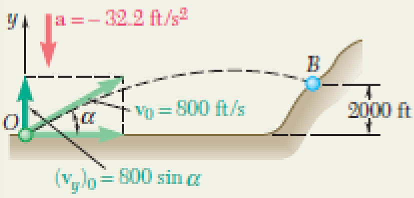
$$(v_x)_0 = 800 \cos \alpha$$

Substituting into the equation of uniform horizontal motion, we obtain

$$x = (v_x)_0 t \qquad x = (800 \cos \alpha)t$$

The time required for the projectile to move through a horizontal distance of 12,000 ft is obtained by setting x equal to 12,000 ft.

$$12{,}000 = (800 \cos \alpha)t$$
$$t = \frac{12{,}000}{800 \cos \alpha} = \frac{15}{\cos \alpha}$$



Vertical Motion

$$(v_y)_0 = 800 \sin \alpha \qquad a = -32.2 \text{ ft/s}^2$$

Substituting into the equation of uniformly accelerated vertical motion, we obtain

$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = (800 \sin \alpha)t - 16.1t^2$$

Projectile Hits Target. When $x = 12{,}000$ ft, we must have $y = 2000$ ft. Substituting for y and setting t equal to the value found above, we write

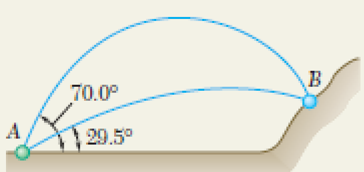
$$2000 = 800 \sin \alpha \frac{15}{\cos \alpha} - 16.1\left(\frac{15}{\cos \alpha}\right)^2$$

Since $1/\cos^2 \alpha = \sec^2 \alpha = 1 + \tan^2 \alpha$, we have

$$2000 = 800(15) \tan \alpha - 16.1(15^2)(1 + \tan^2 \alpha)$$
$$3622 \tan^2 \alpha - 12{,}000 \tan \alpha + 5622 = 0$$

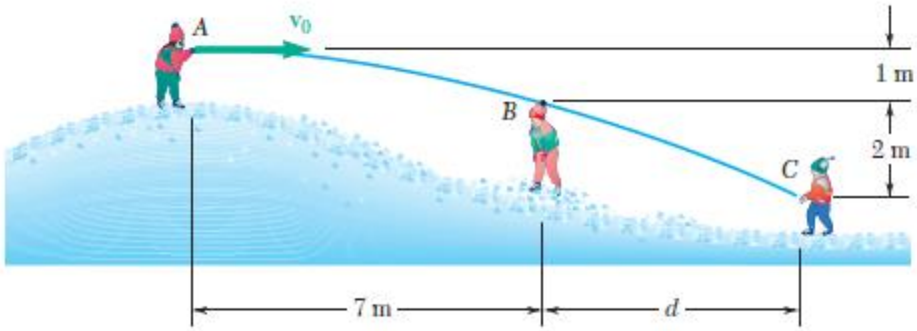
Solving this quadratic equation for $\tan \alpha$, we have

$$\tan \alpha = 0.565 \qquad \text{and} \qquad \tan \alpha = 2.75$$
$$\alpha = 29.5° \qquad \text{and} \qquad \alpha = 70.0° \quad ◀$$



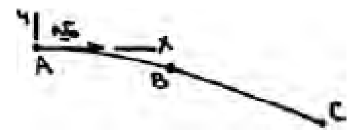
The target will be hit if either of these two firing angles is used (see figure).

12. Three children are throwing snow-balls at each other. Child *A* throws a snowball with a horizontal velocity $\mathbf{V}_0$. If the snowball just passes over the head of child *B* and hits child *C*, determine (*a*) the value of $\mathbf{V}_0$, (*b*) the distance *d*. **[V₀=15.5 m/s and d=5.12 m]**



SOLUTION

(*a*) Vertical motion. (Uniformly accelerated motion)



$$y = 0 + (0)t - \frac{1}{2}gt^2$$

At *B*: $$-1\text{ m} = -\frac{1}{2}(9.81\text{ m/s}^2)t^2 \quad \text{or} \quad t_B = 0.451524\text{ s}$$

Horizontal motion (Uniform)

$$x = 0 + (v_x)_0 t$$

At *B*: $$7\text{ m} = v_0(0.451524\text{ s})$$

or $$v_0 = 15.5031\text{ m/s}$$

$v_0 = 15.50$ m/s ◀

(*b*) Vertical motion: At *C*: $$-3\text{ m} = -\frac{1}{2}(9.81\text{ m/s}^2)t^2$$

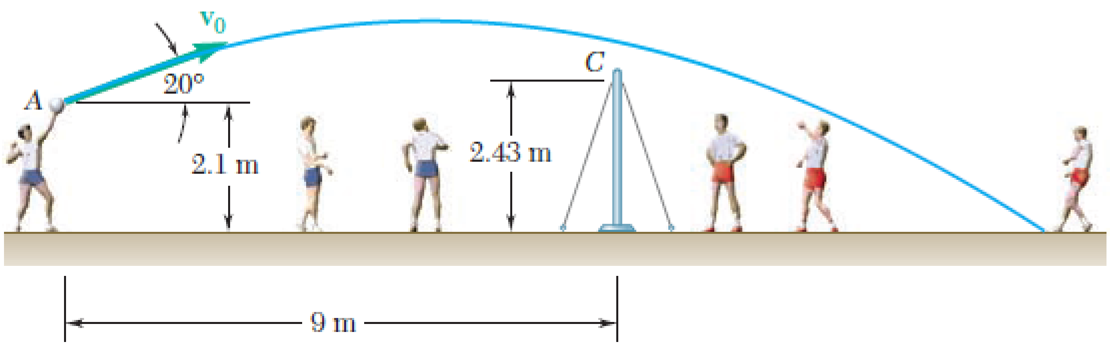
or $$t_C = 0.782062\text{ s}$$

Horizontal motion.

At *C*: $$(7 + d)\text{ m} = (15.5031\text{ m/s})(0.782062\text{ s})$$

or $d = 5.12$ m ◀

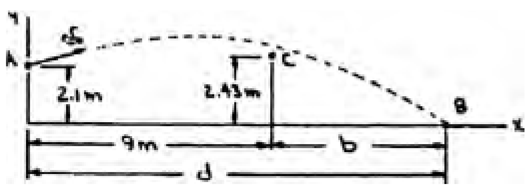
13. A volleyball player serves the ball with an initial velocity **V**$_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.**[(a) yes, the ball will clear the net; y_c>2.43 m (b) 7.01 m from the net]**



SOLUTION

First note $(v_x)_0 = (13.40 \text{ m/s}) \cos 20° = 12.5919 \text{ m/s}$

$(v_y)_0 = (13.40 \text{ m/s}) \sin 20° = 4.5831 \text{ m/s}$



(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At C $\quad 9 \text{ m} = (12.5919 \text{ m/s})t \quad \text{or} \quad t_C = 0.71475 \text{ s}$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

At C:
$$y_c = 2.1 \text{ m} + (4.5831 \text{ m/s})(0.71475 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.71475 \text{ s})^2$$
$$= 2.87 \text{ m}$$

$y_C > 2.43 \text{ m}$ (height of net) $\Rightarrow$ ball clears net ◀

(*b*) At B, $y = 0$: $\quad 0 = 2.1 \text{ m} + (4.5831 \text{ m/s})t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$

Solving $\quad t_B = 1.271175 \text{ s}$ (the other root is negative)

Then $\quad d = (v_x)_0 t_B = (12.5919 \text{ m/s})(1.271175 \text{ s})$
$= 16.01 \text{ m}$

The ball lands $\quad b = (16.01 - 9.00) \text{ m} = 7.01 \text{ m}$ from the net ◀

14. The motion of a cam is defined by the relation θ = $2t^3+0.5$, where θ is expressed in radians and *t* in seconds. Determine the displacement, the angular velocity, and the angular acceleration of the cam when *t* = 2 s. **[θ=16.5 rad, ώ=24 rad/sec and α=24 rad/s^2]**

Solution. Given : Equation for angular displacement $\theta = 2t^3 + 0.5$ ...(*i*)

Angular displacement after 2 seconds

Substituting $t = 2$ in equation (*i*),

$$\theta = 2\,(2)^3 + 0.5 = 16.5 \text{ rad} \quad \textbf{Ans.}$$

Angular velocity after 2 seconds

Differentiating both sides equation (*i*) with respect to *t*,

$$\frac{d\theta}{dt} = 6\,t^2 \qquad \text{...}(ii)$$

or velocity, $\omega = 6\,t^2$ ...(*iii*)

Substituting $t = 2$ in equation (*iii*),

$$\omega = 6\,(2)^2 = 24 \text{ rad/sec} \quad \textbf{Ans.}$$

Angular acceleration after 2 seconds

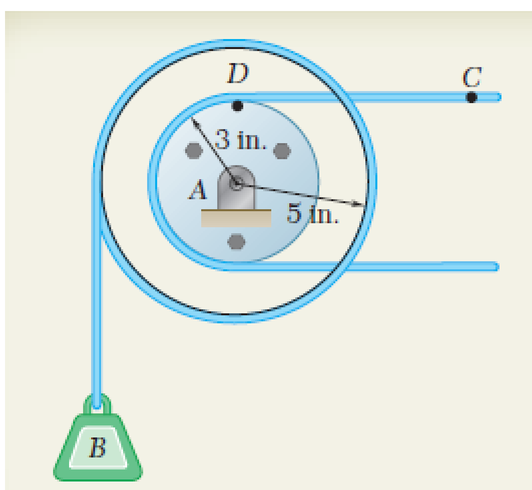
Differentiating both sides of equation (*iii*) with respect to *t*,

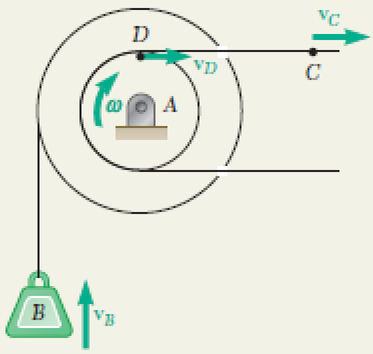
$$\frac{d\omega}{dt} = 12t \text{ or Acceleration } \alpha = 12t$$

Now substituting $t = 2$ in above equation,

$$\alpha = 12 \times 2 = 24 \text{ rad/sec}^2 \quad \textbf{Ans.}$$

15. Load B is connected to a double pulley by one of the two inextensible cables shown in fig 1. The motion of the pulley is controlled by cable C, which has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right. Determine (a) the number of revolutions executed by the pulley in 2s, (b) the velocity and change in position of the load B after 2 s, and (c) the acceleration of point D on the rim of the inner pulley at t = 0. **[(a) n=2.23 rev. (b) v=50 in/s and Δy= 70 in (c) a=48.8 in/sec^2]**





a. Motion of Pulley. Since the cable is inextensible, the velocity of point D is equal to the velocity of point C and the tangential component of the acceleration of D is equal to the acceleration of C.

$$(\mathbf{v}_D)_0 = (\mathbf{v}_C)_0 = 12 \text{ in./s} \rightarrow \qquad (\mathbf{a}_D)_t = \mathbf{a}_C = 9 \text{ in./s}^2 \rightarrow$$

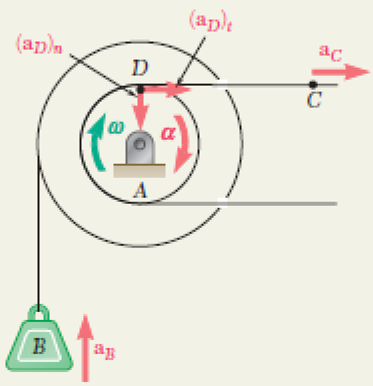
Noting that the distance from D to the center of the pulley is 3 in., we write

$$(v_D)_0 = r\omega_0 \qquad 12 \text{ in./s} = (3 \text{ in.})\omega_0 \qquad \boldsymbol{\omega}_0 = 4 \text{ rad/s} \downarrow$$
$$(a_D)_t = r\alpha \qquad 9 \text{ in./s}^2 = (3 \text{ in.})\alpha \qquad \boldsymbol{\alpha} = 3 \text{ rad/s}^2 \downarrow$$

Using the equations of uniformly accelerated motion, we obtain, for $t = 2$ s,

$$\omega = \omega_0 + \alpha t = 4 \text{ rad/s} + (3 \text{ rad/s}^2)(2 \text{ s}) = 10 \text{ rad/s}$$
$$\boldsymbol{\omega} = 10 \text{ rad/s} \downarrow$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4 \text{ rad/s})(2 \text{ s}) + \tfrac{1}{2}(3 \text{ rad/s}^2)(2 \text{ s})^2 = 14 \text{ rad}$$
$$\theta = 14 \text{ rad} \downarrow$$

$$\text{Number of revolutions} = (14 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 2.23 \text{ rev}$$



b. Motion of Load *B*. Using the following relations between linear and angular motion, with $r = 5$ in., we write

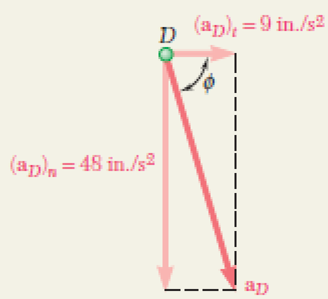
$$v_B = r\omega = (5 \text{ in.})(10 \text{ rad/s}) = 50 \text{ in./s} \qquad \mathbf{v}_B = 50 \text{ in./s} \uparrow$$
$$\Delta y_B = r\theta = (5 \text{ in.})(14 \text{ rad}) = 70 \text{ in.} \qquad \Delta y_B = 70 \text{ in. upward}$$

c. Acceleration of Point *D* at *t* = 0. The tangential component of the acceleration is

$$(\mathbf{a}_D)_t = \mathbf{a}_C = 9 \text{ in./s}^2 \rightarrow$$

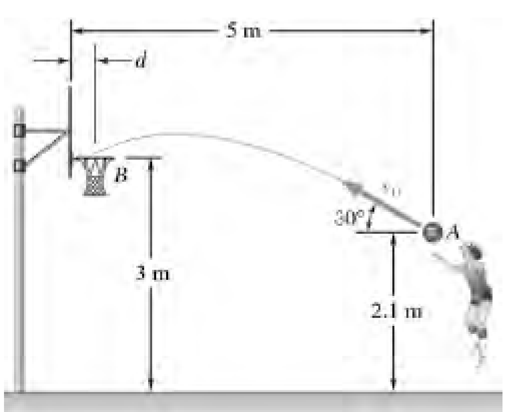
Since, at $t = 0$, $\omega_0 = 4$ rad/s, the normal component of the acceleration is

$$(a_D)_n = r_D\omega_0^2 = (3 \text{ in.})(4 \text{ rad/s})^2 = 48 \text{ in./s}^2 \qquad (\mathbf{a}_D)_n = 48 \text{ in./s}^2 \downarrow$$



The magnitude and direction of the total acceleration can be obtained by writing

$$\tan\phi = (48 \text{ in./s}^2)/(9 \text{ in./s}^2) \qquad \phi = 79.4°$$
$$a_D \sin 79.4° = 48 \text{ in./s}^2 \qquad a_D = 48.8 \text{ in./s}^2$$
$$\mathbf{a}_D = 48.8 \text{ in./s}^2 \measuredangle 79.4°$$

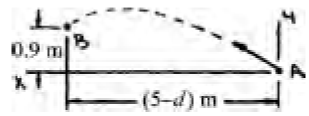


PROBLEM 11.106

A basketball player shoots when she is 5 m from the backboard. Knowing that the ball has an initial velocity $\mathbf{v}_0$ at an angle of 30° with the horizontal, determine the value of v_0 when d is equal to (*a*) 225 mm, (*b*) 425 mm.

SOLUTION

First note



$$(v_x)_0 = v_0 \cos 30^\circ$$
$$(v_y)_0 = v_0 \sin 30^\circ$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$(5-d) = (v_0 \cos 30^\circ)\, t \quad \text{or} \quad t_B = \frac{5-d}{v_0 \cos 30^\circ}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At B: $$0.9 = (v_0 \sin 30^\circ)\, t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B $$0.9 = (v_0 \sin 30^\circ)\left(\frac{5-d}{v_0 \cos 30^\circ}\right) - \frac{1}{2} g \left(\frac{5-d}{v_0 \cos 30^\circ}\right)^2$$

or $$v_0^2 = \frac{2g(5-d)^2}{3\left[\frac{1}{\sqrt{3}}(5-d) - 0.9\right]} \quad d \sim \text{m}$$

(*a*) $d = 225$ mm $= 0.225$ m $$v_0^2 = \frac{2(9.81)(5-0.225)^2}{3\left[\frac{1}{\sqrt{3}}(5-0.225) - 0.9\right]}$$

or $$v_0 = 8.96 \text{ m/s} \blacktriangleleft$$

(*b*) $d = 425$ mm $= 0.425$ m $$v_0^2 = \frac{2(9.81)(5-0.425)^2}{3\left[\frac{1}{\sqrt{3}}(5-0.425) - 0.9\right]}$$

or $$v_0 = 8.87 \text{ m/s} \blacktriangleleft$$