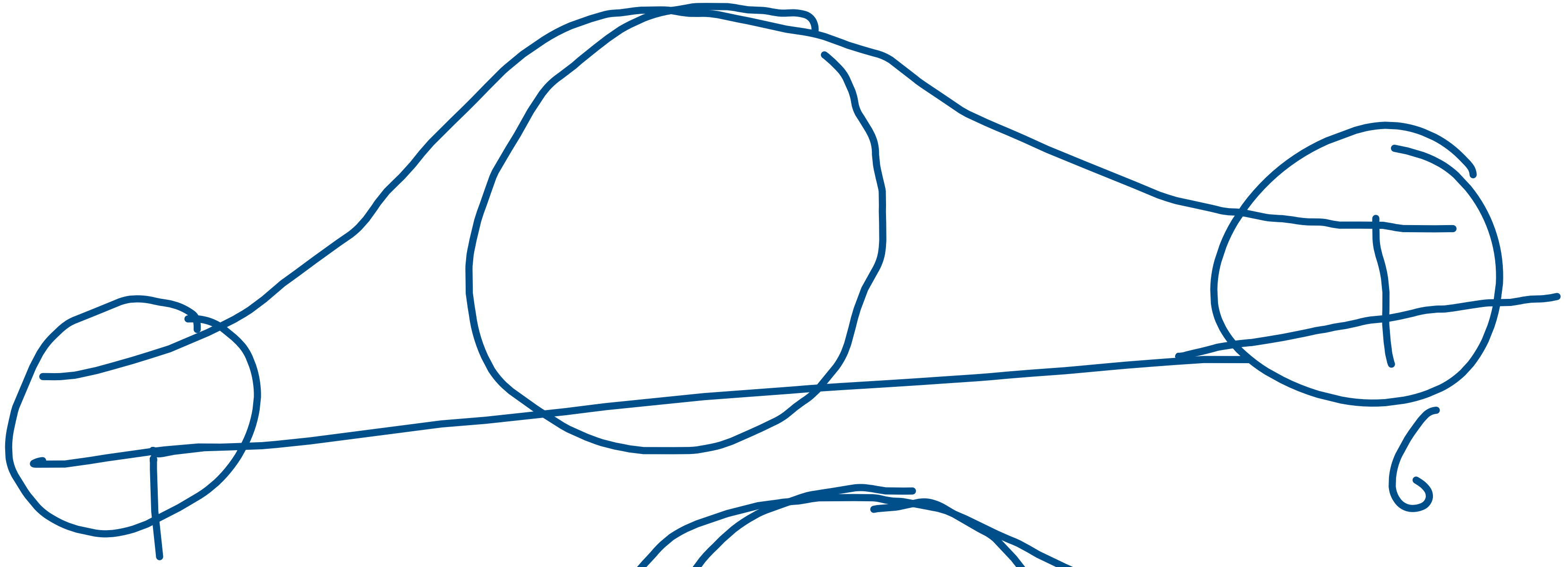
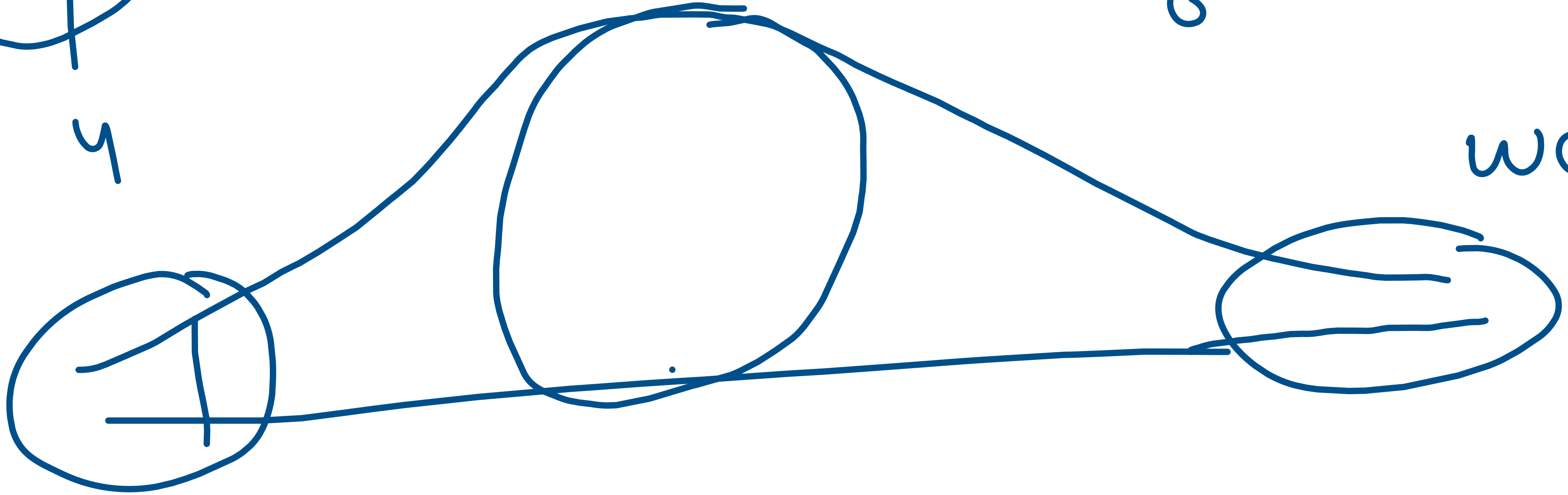
# Normal Dist



4

6

Height



wages

Normal probability dist or Normal Dist is the probability dist of continuous random variable $x$, known as normal variable or normal variate

It is given by

$$\boxed{N(\bar{x}, \sigma) = f(x) = p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}, \quad -\infty < x < \infty}$$

$\bar{x}$ = Arithmetic mean.

$\sigma$ = standard deviation

are two parameters of Normal Dist

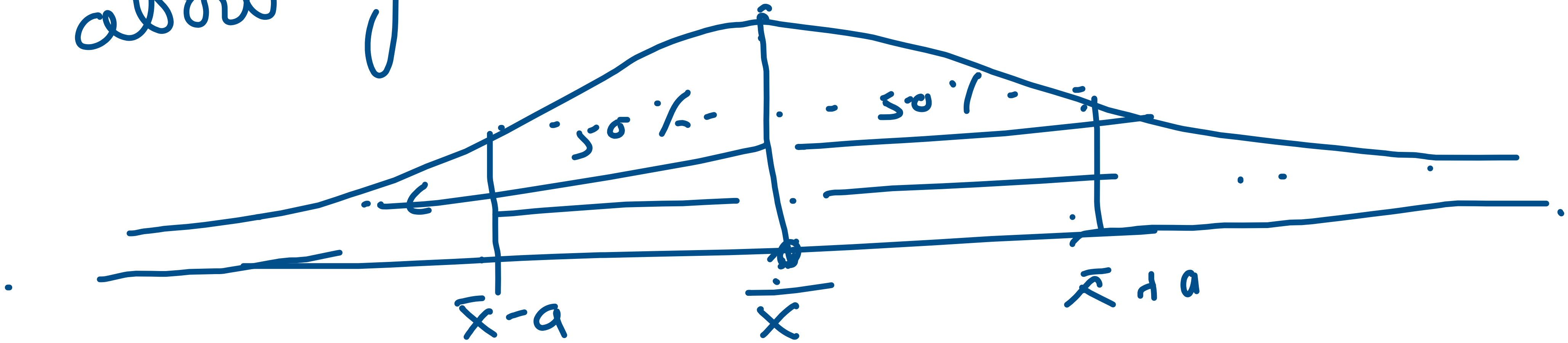
so normal dist is called

biparametric dist.

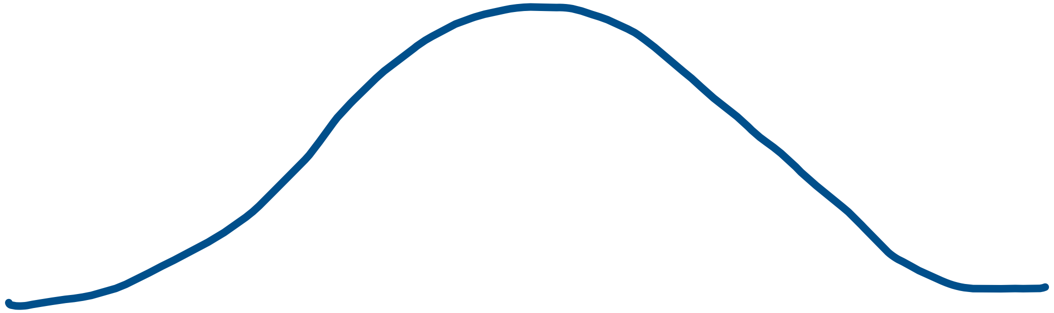
Normal dist is also called Gaussian dist.

# Properties of Normal dist

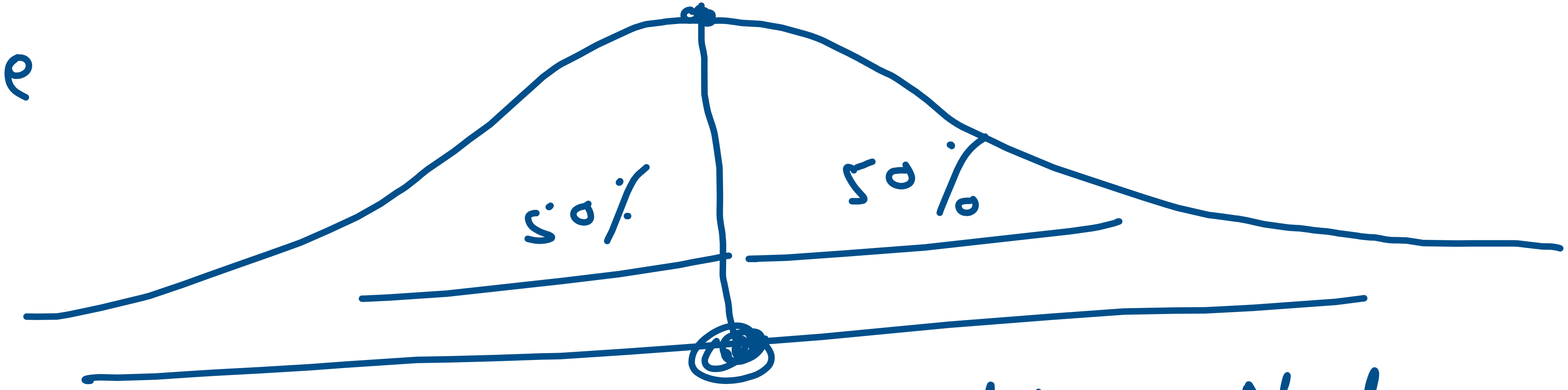
① The graph of Normal dist. $y = f(x)$ in $x$-$y$ plane is known as normal curve. is symmetric curve about $y$-axis



It is bell shaped



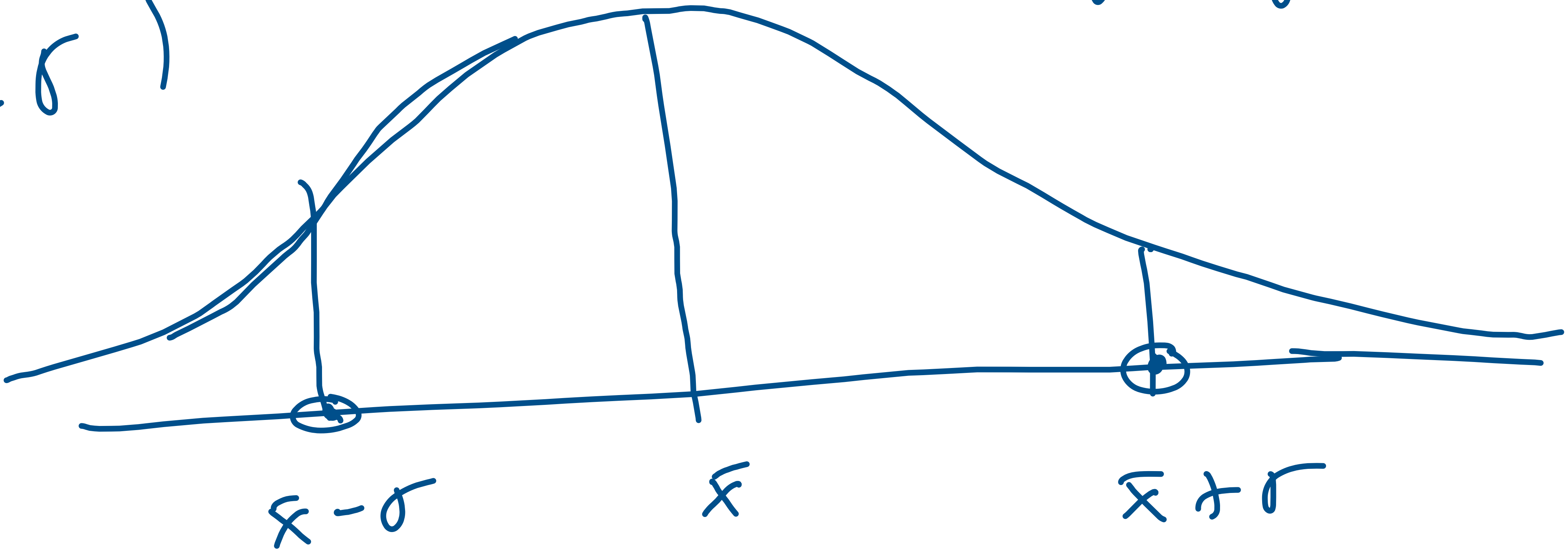
The mean, median and mode coincide



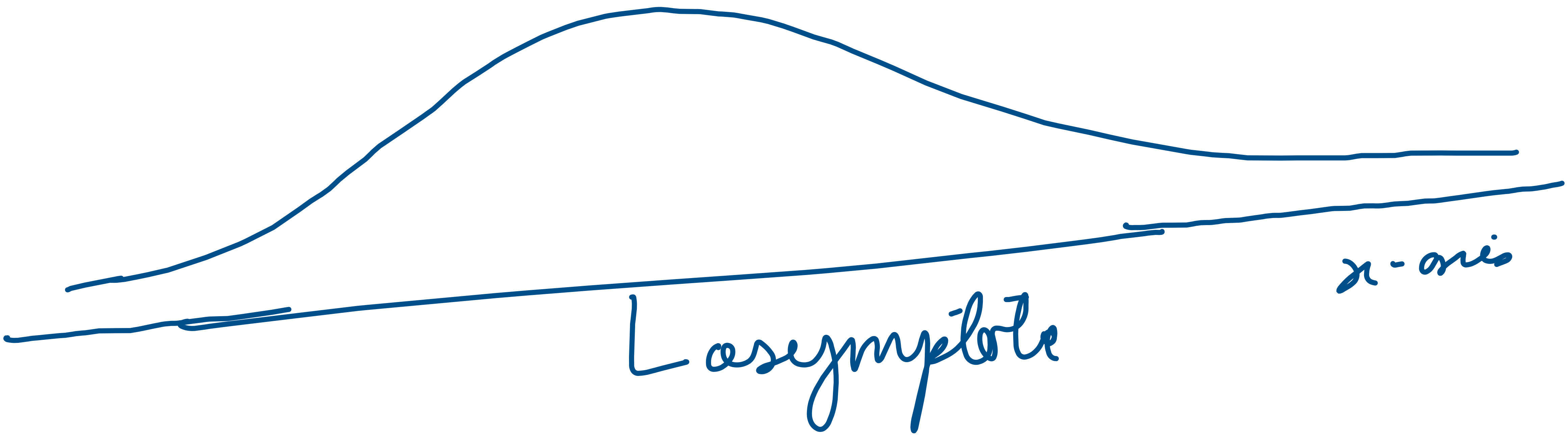

$\bar{x}$ = Median = Mode

There are two points of inflexion

$(\bar{x} \pm \sigma)$



Normal curve is asymptotic to x-axis



The area under the normal curve is unity

1 = 100%

$$\int_{-\infty}^{\infty} f(x)\,dx = 1, \quad \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}\, dx = 1$$

$$f(x) \geq 0$$

$$N(\bar{x}, \sigma) \geq 0$$

$$p(x) \geq 0$$

Probability that the continuous random variable $X$ lie between $x_1$ and $x_2$ i.e

$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\, dx$$

$$= \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}\, dx$$

# Standard Normal distribution ( Z )

$\bar{x} = 0 \qquad \sigma = 1$

when in normal distribution mean = 0 and standard deviation = 1 then normal dist transformed to standard normal dist Z given

$$N(\bar{x}, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

$\bar{x} = 0 \qquad \sigma = 0$

$$N(0,1) = \frac{1}{1(\sqrt{2\pi})} e^{-\frac{(x-0)^2}{2(1)^2}}$$

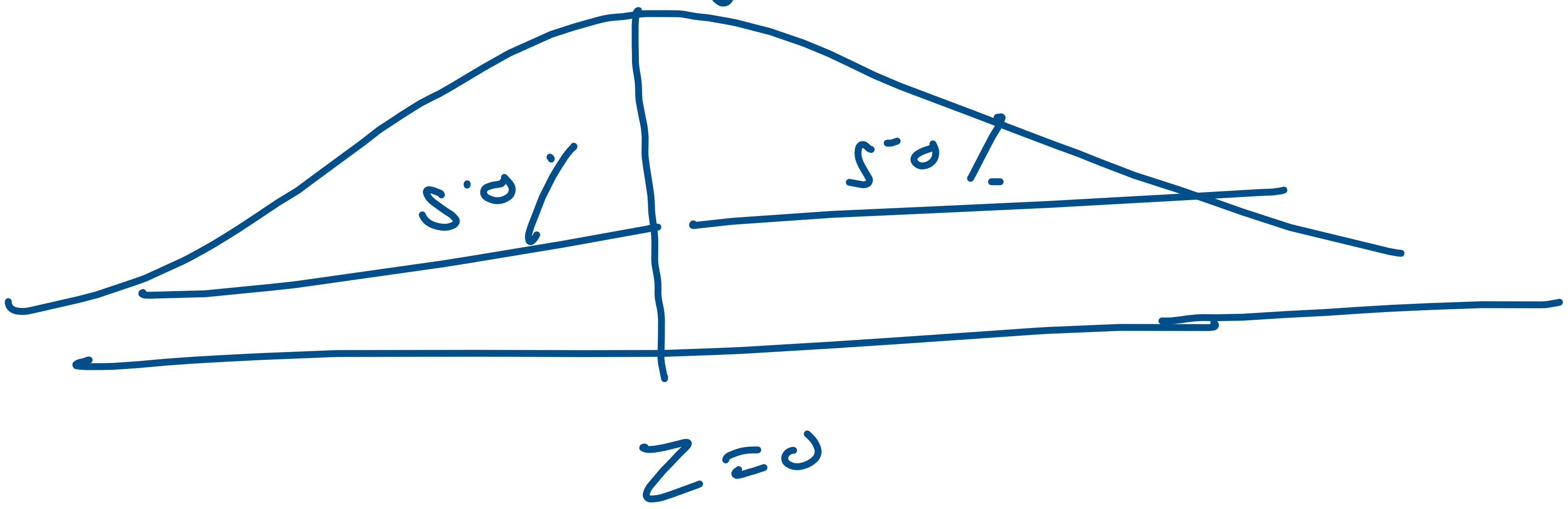
$$N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$x \rightarrow z$$

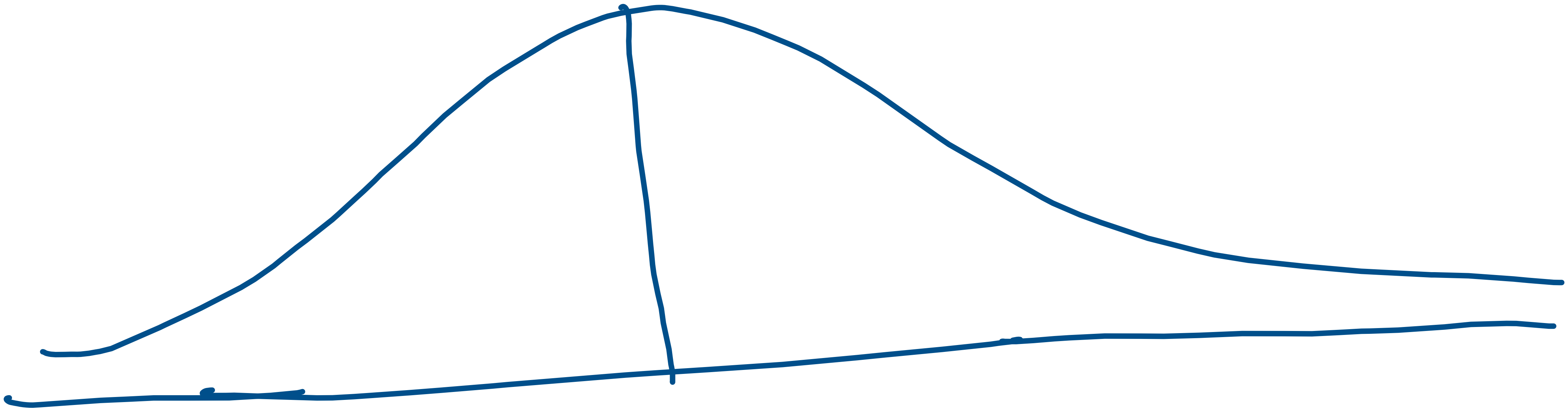
$$\boxed{N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}}$$

$$-\infty < z < \infty$$

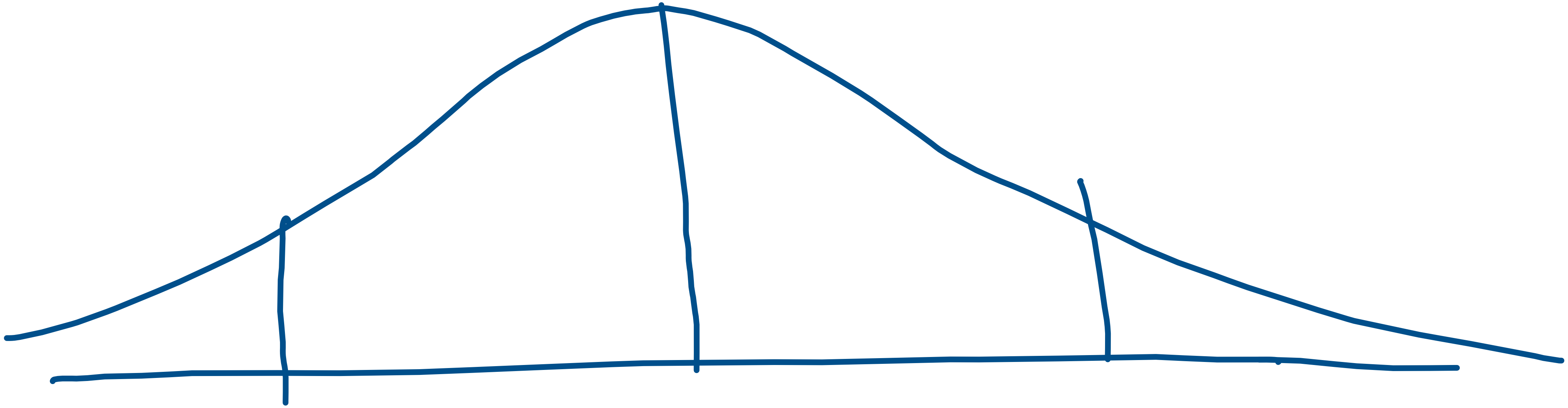
The graph of standard normal dist is called standard normal curve, which is symmetric about y-axis,



mean = median = mode = 0



# Point of inflexion ($Z = \pm 1$)



$\bar{x} - \sigma$ $\quad$ $\bar{x}$ $\quad$ $\bar{x} + \sigma$

$0 - 1$ $\quad$ $Z = 0$ $\quad$ $0 + 1$

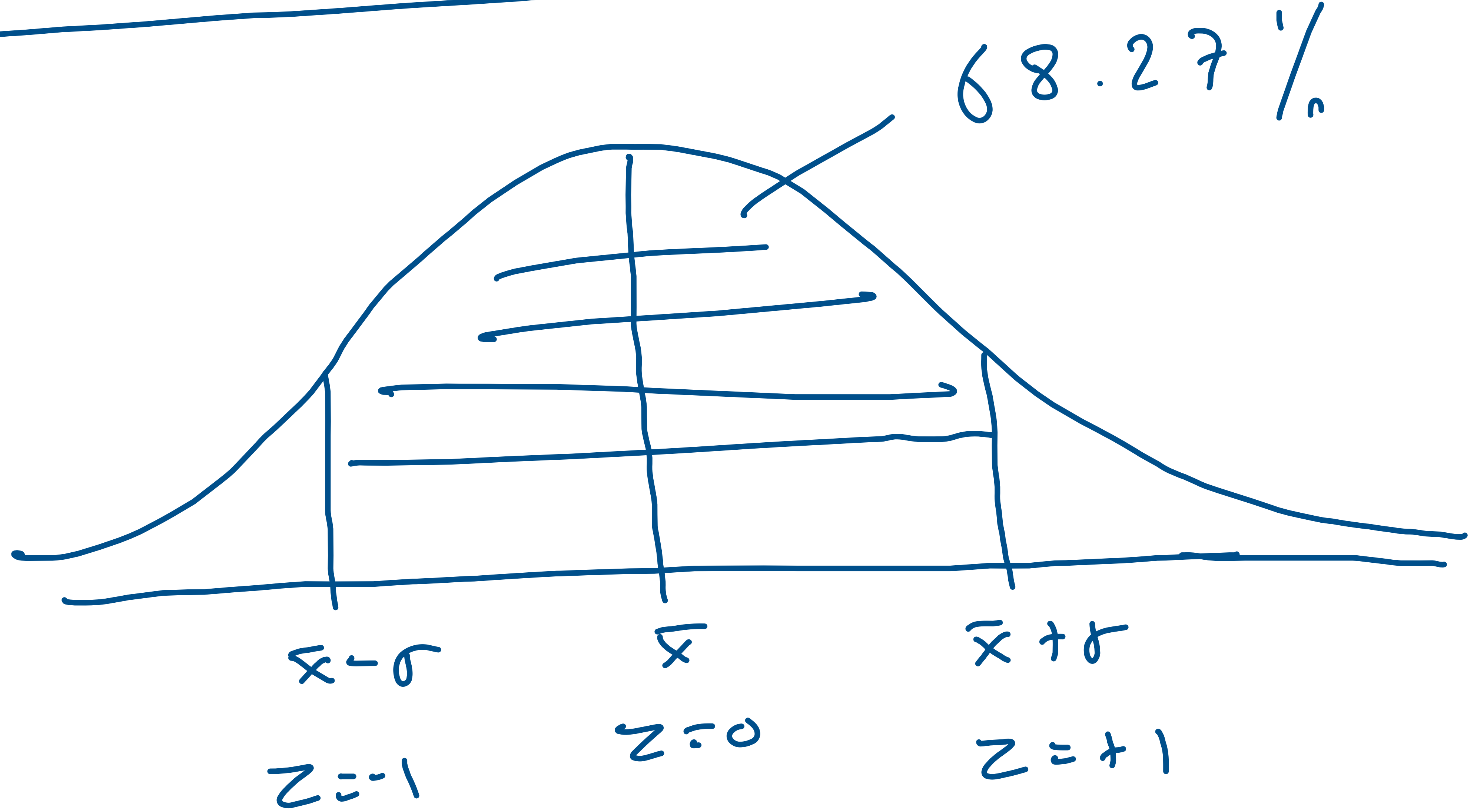
$\underline{\underline{z = -1}}$ $\quad$ $\underline{\underline{Z = 1}}$

# Z - transformation

$$\boxed{z = \frac{x - \bar{x}}{\sigma}}$$

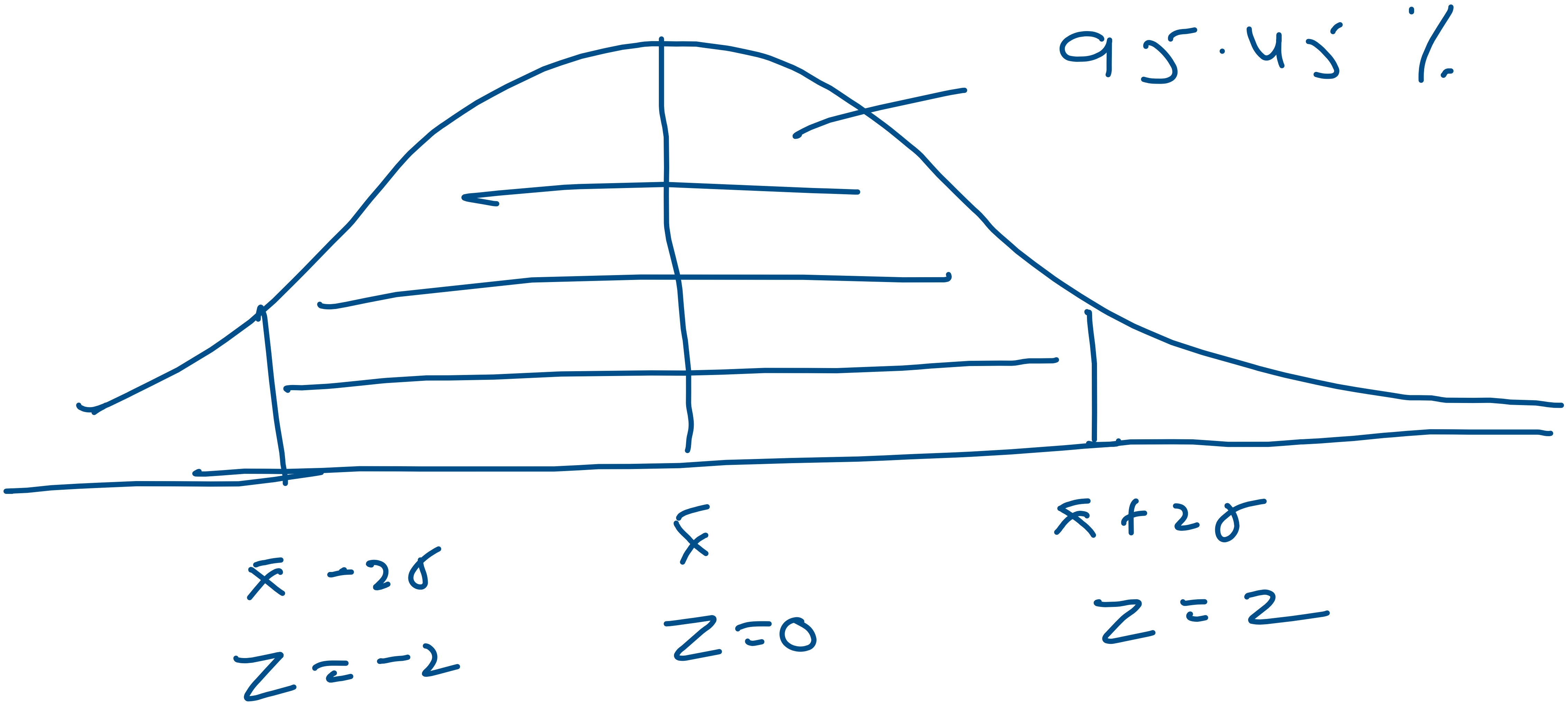
which transform normal variable x to standard normal variable Z

# Area under Normal Curve

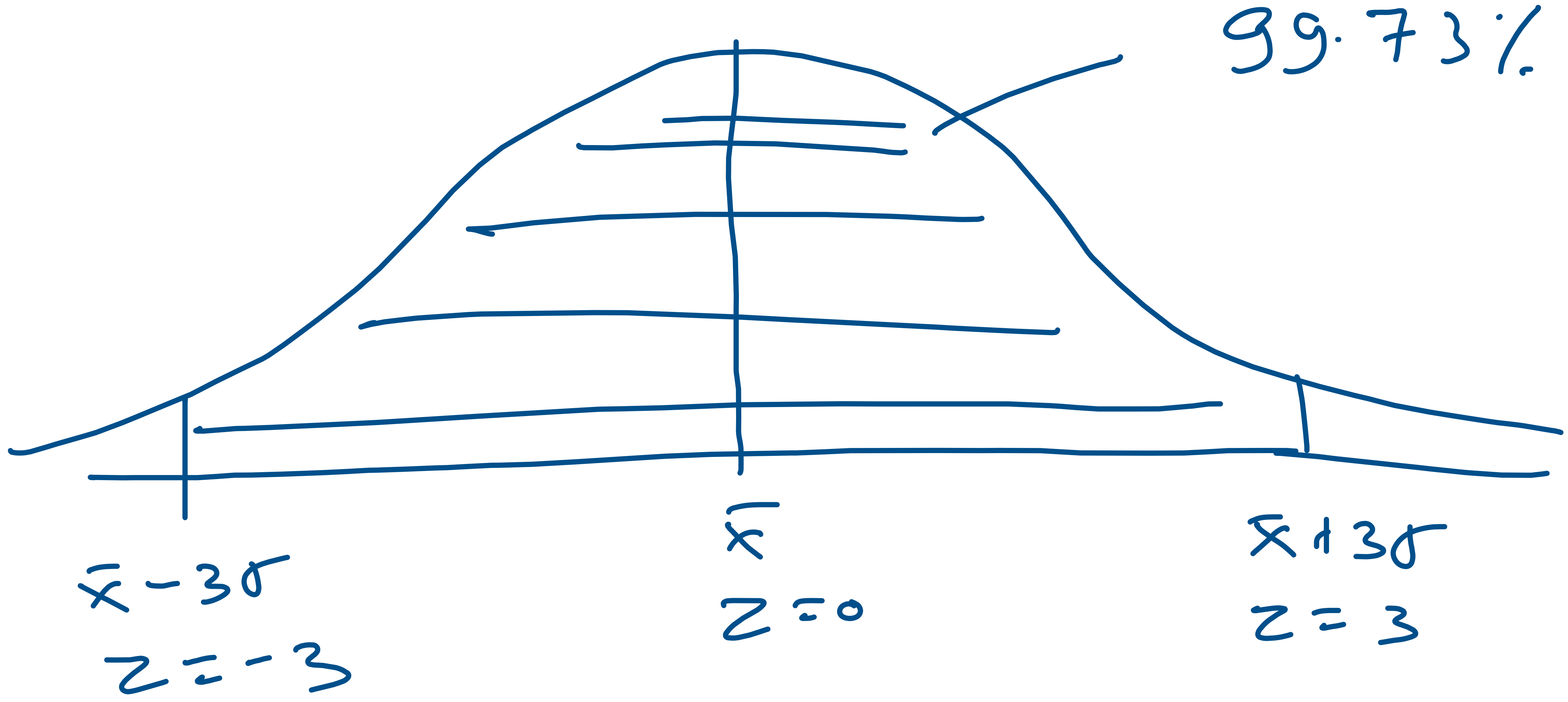


$$P(\bar{x} - \sigma \leq x \leq \bar{x} + \sigma) = 0.6827$$

$$P(-1 \leq z \leq 1) = 0.6827$$




$$p(x-2\sigma \le x \le \bar{x}+2\sigma) = p(-2 \le Z \le 2) = 0.9545$$



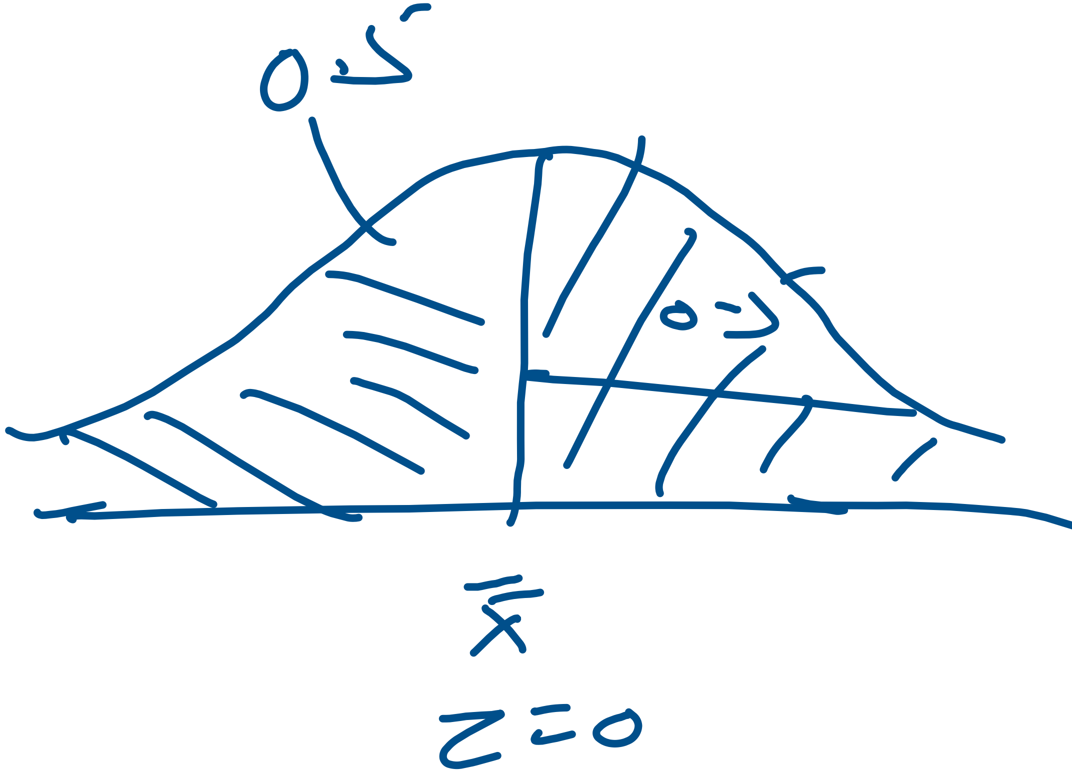

$$P(\bar{x}-3\sigma \le x \le \bar{x}+3\sigma) = P(-3 \le z \le 3) = 0.9973$$

$$P(x \geq \bar{x}) = 0.5$$

$$P(x \leq \bar{x}) = 0.5$$

$$P(Z \leq 0) = 0.5$$

$$P(Z \geq 0) = 0.5$$



Ex ①  If $Z$ is normally distributed with mean $= 0$  variance $= 1$

find

(a) $P(Z \geq -1.64)$

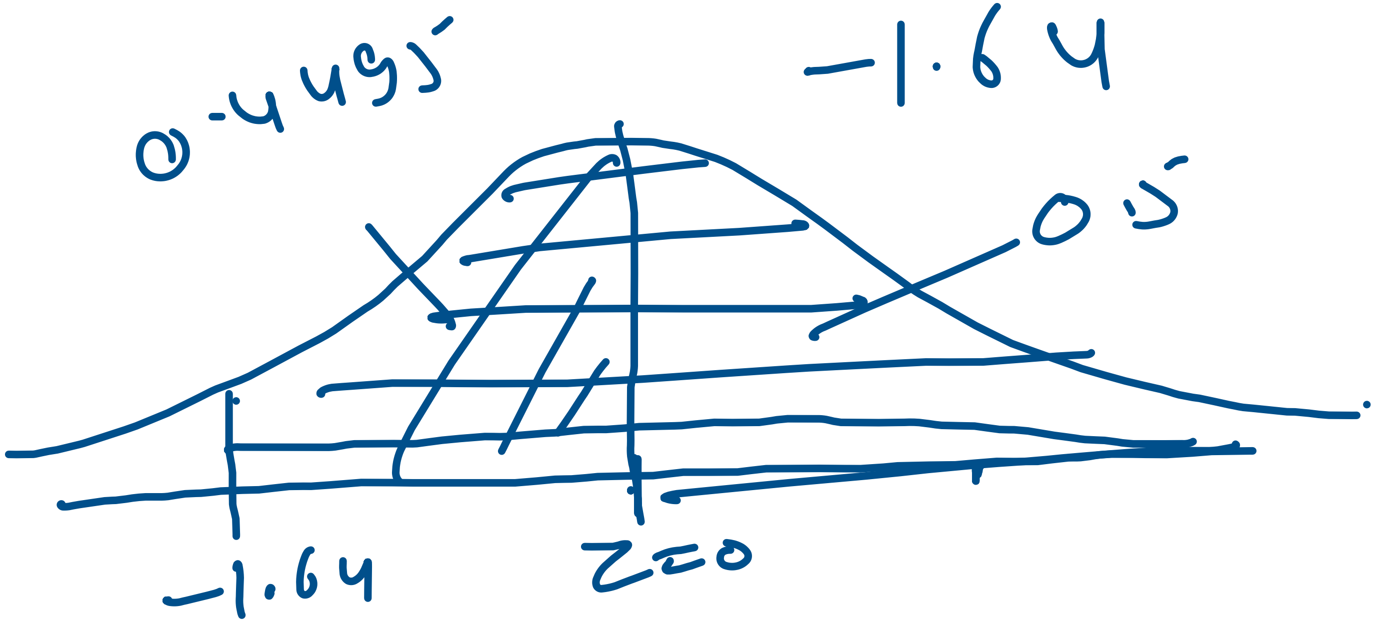
(b) $P(-1.96 \leq Z \leq 1.96)$

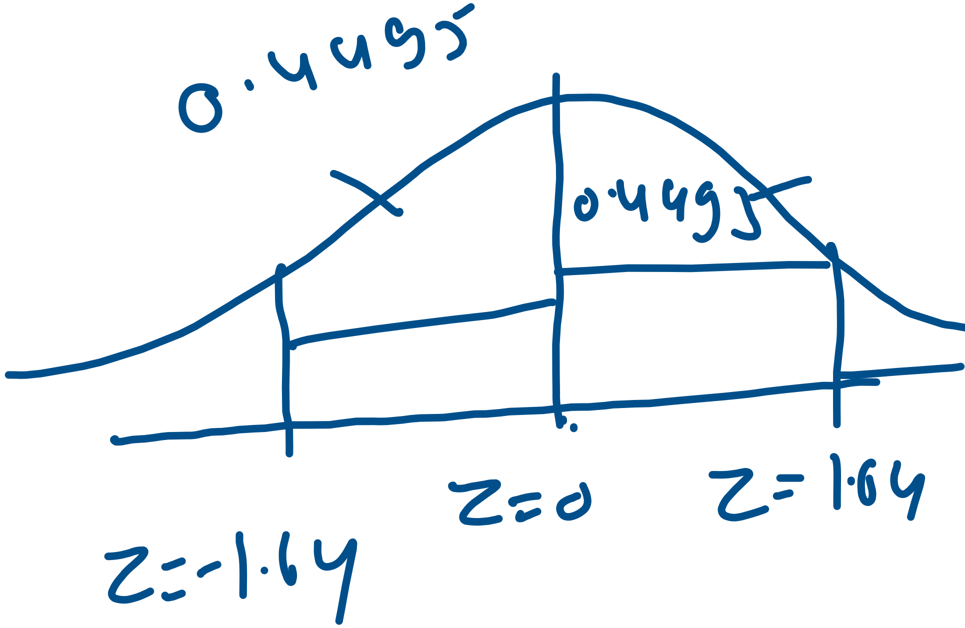
(c) $P(-0.8 \leq Z \leq 1.53)$

(d) $P(Z \leq 1.83)$

(a) $P(Z \geq -1.64) = P(-1.64 \leq Z < \infty)$

$$= \int_{-1.64}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}} \, dz$$



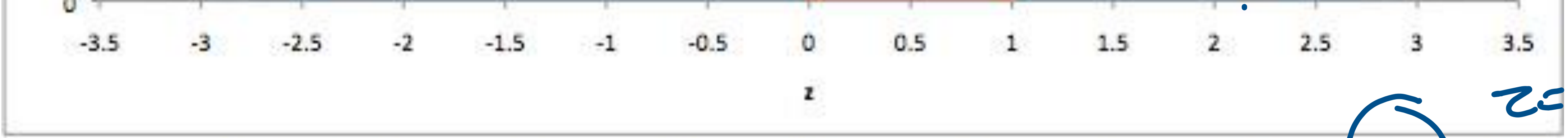



$$P(Z \geq -1.64) = P(-1.64 \leq Z < \infty)$$

$$= P(-1.64 \leq Z \leq 0) + P(Z \geq 0)$$

$$= 0.4495 + 0.5$$

$$= 0.9495$$




z=1.28

| | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| **0.1** | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| **0.2** | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| **0.3** | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| **0.4** | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| **0.5** | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| **0.6** | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| **0.7** | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| **0.8** | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| **0.9** | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| **1.0** | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| **1.1** | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| **1.2** | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| **1.3** | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| **1.4** | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| **1.5** | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| **1.6** | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| **1.7** | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| **1.8** | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| **1.9** | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| **2.0** | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| **2.1** | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |

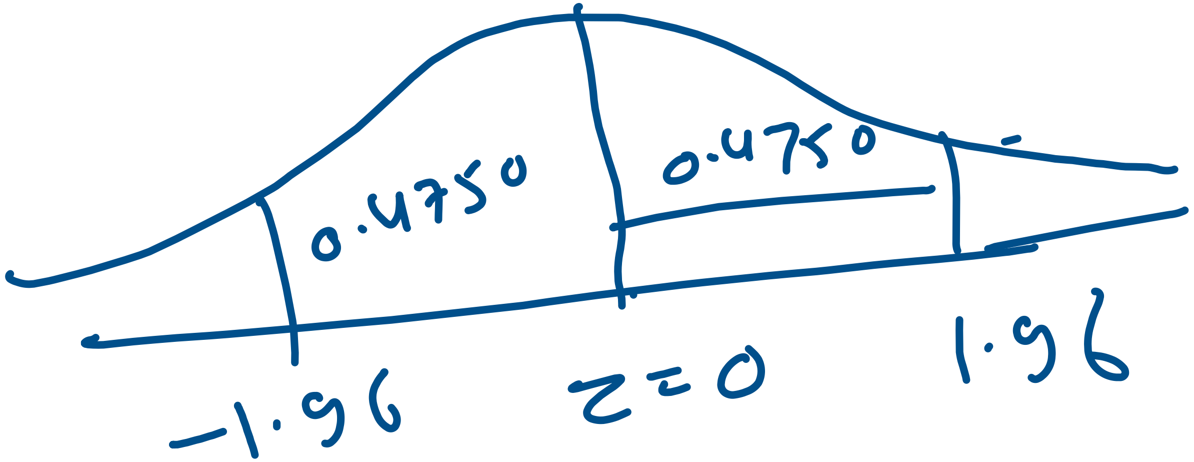
0.4

$$P(-1.96 \le Z \le 1.96)$$

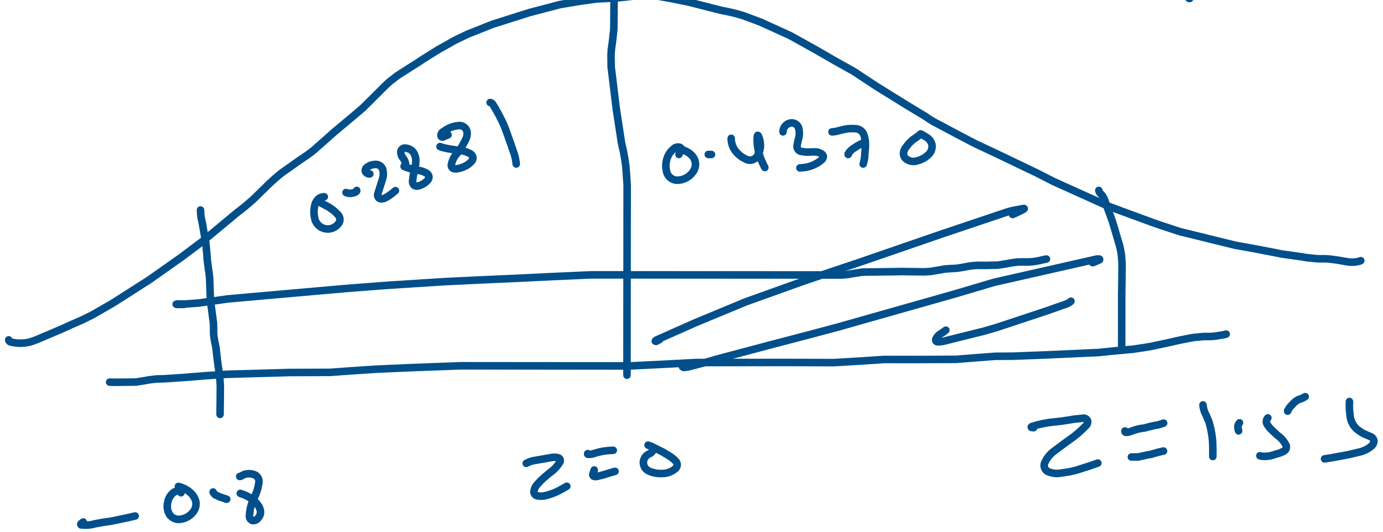
$$= \int_{-1.96}^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz$$

$$P(-1.96 \le Z \le 1.96) = 0.4750 + 0.4750$$

$$= 0.95$$



$$P(-0.8 \le Z \le 1.53) = \int_{-0.8}^{1.53} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz$$

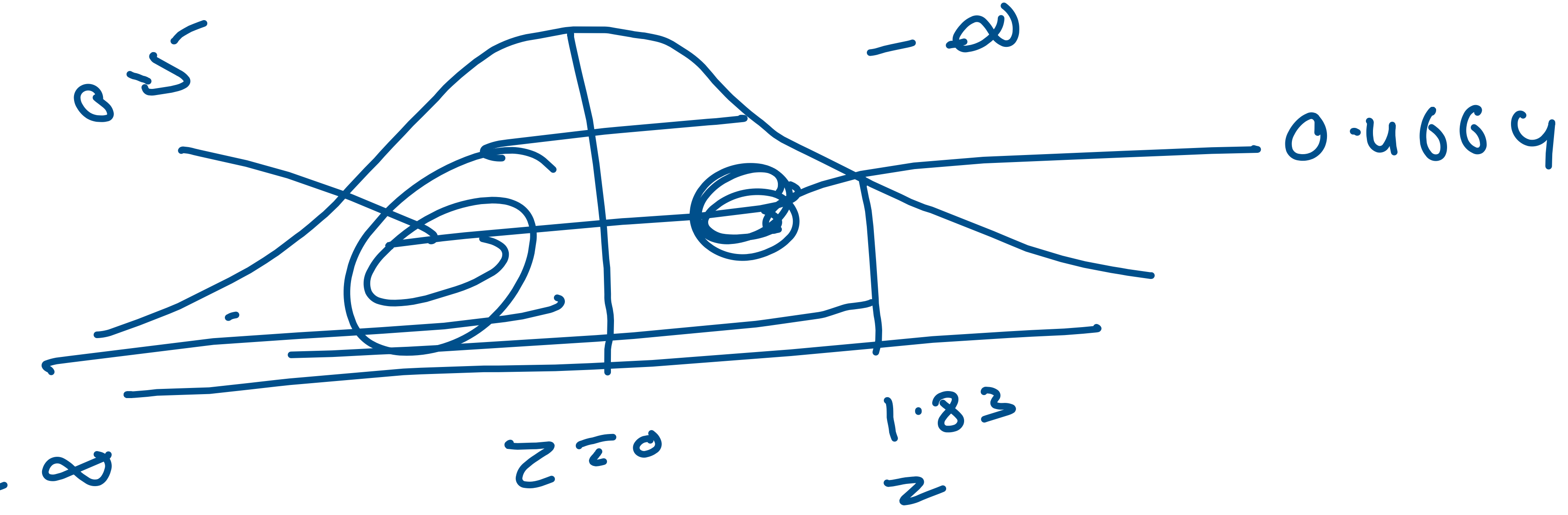



$$P(-0.8 \le Z \le 1.53) = 0.2881 + 0.4370$$

$$= 0.7251$$

$$P(Z \leq 1.83) = P(-\infty < Z \leq 1.83)$$

$$= \int_{-\infty}^{1.83} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz$$




$$P(-\infty < Z \leq 1.83) = 0.5 + 0.4664$$

$$= 0.9664$$

E 2 Determine the minimum marks a student must get in order to receive an A grade if the top 10% of the student are awarded A grade in an examination where mean marks is 72 and standard deviation is 9.

If any student is selected at random what is the probability of

(a) $p(x \le 60)$

(b) $p(60 \le x \le 90)$

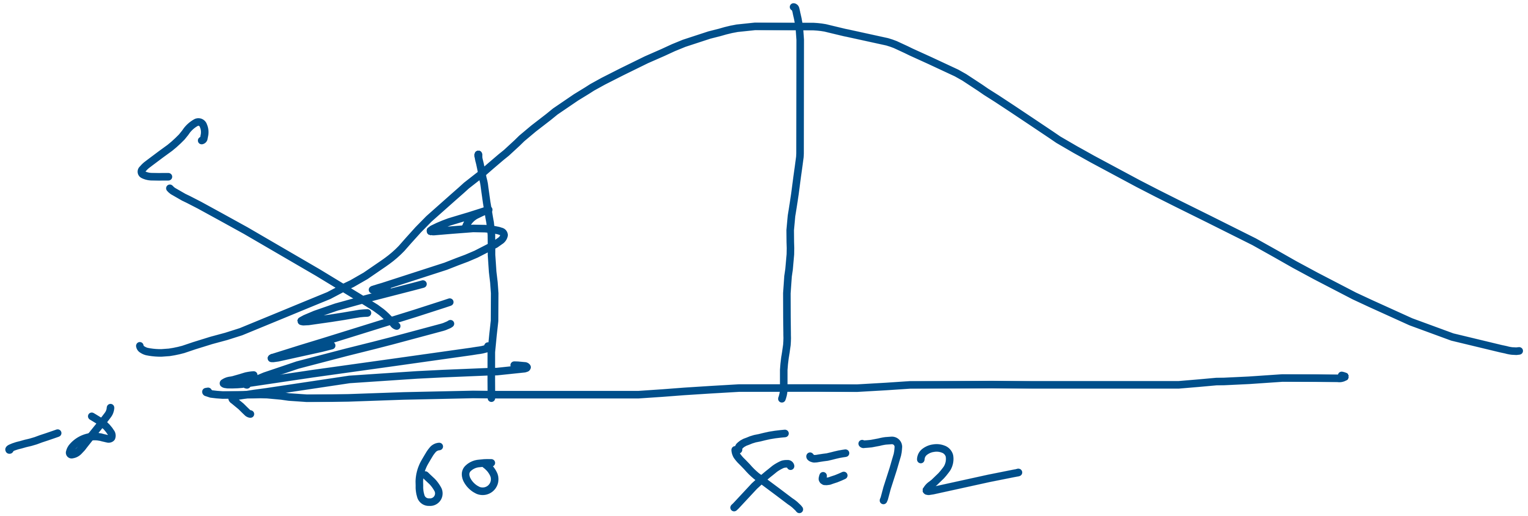
(c) $p(\geq 80)$

(a) $p(x \le 60) = p(-\infty < x \le 60)$

$\bar{x} = 72$

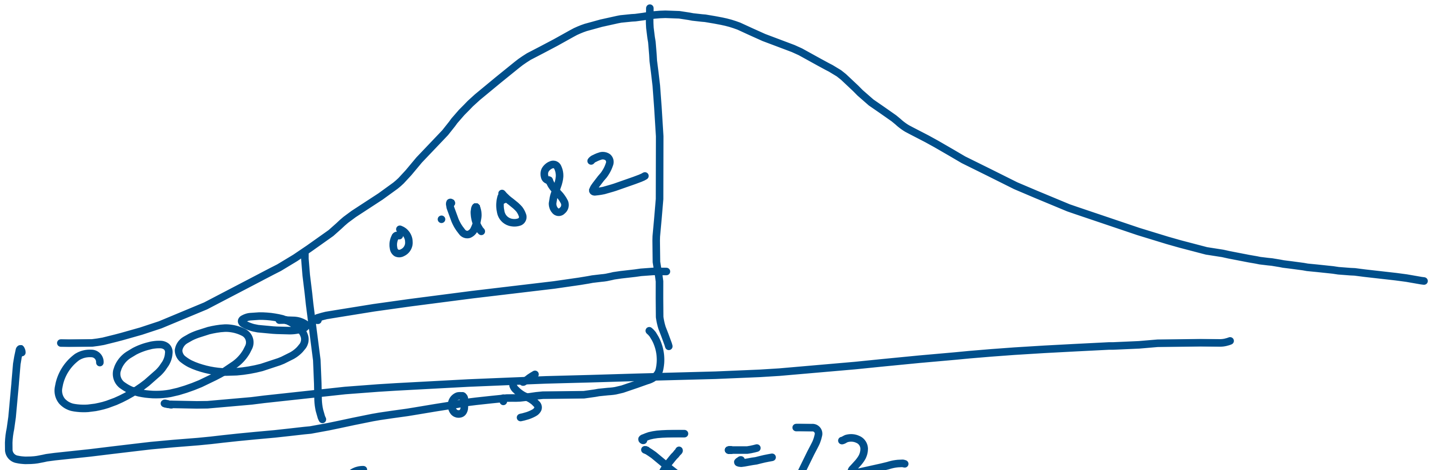
$\sigma = 9$

$$= \int_{-\infty}^{60} \frac{1}{9\sqrt{2\pi}} e^{-\frac{(x-72)^2}{2(9)^2}} \, dx$$




$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

By Z - transformation



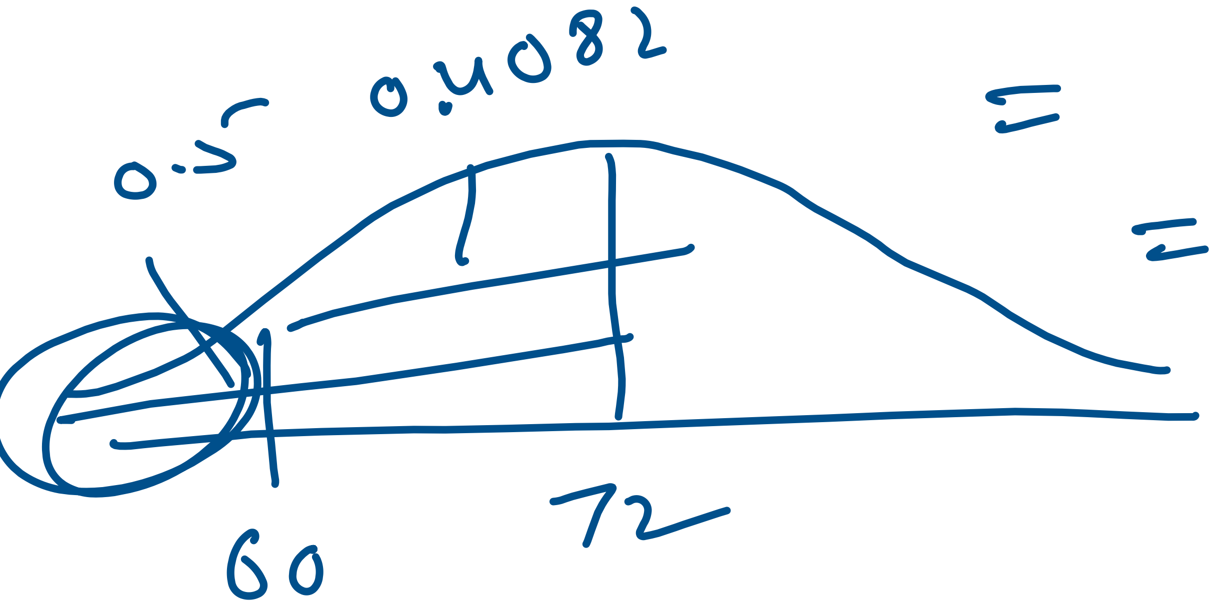

$x = 60$ $\quad\quad$ $\bar{x} = 72$

$z = -1.33$ $\quad\quad$ $z = 0$

$$Z = \frac{x - \bar{x}}{\sigma} \Rightarrow Z = \frac{60 - 72}{9} = \frac{-12}{9}$$

$$= -1.333$$

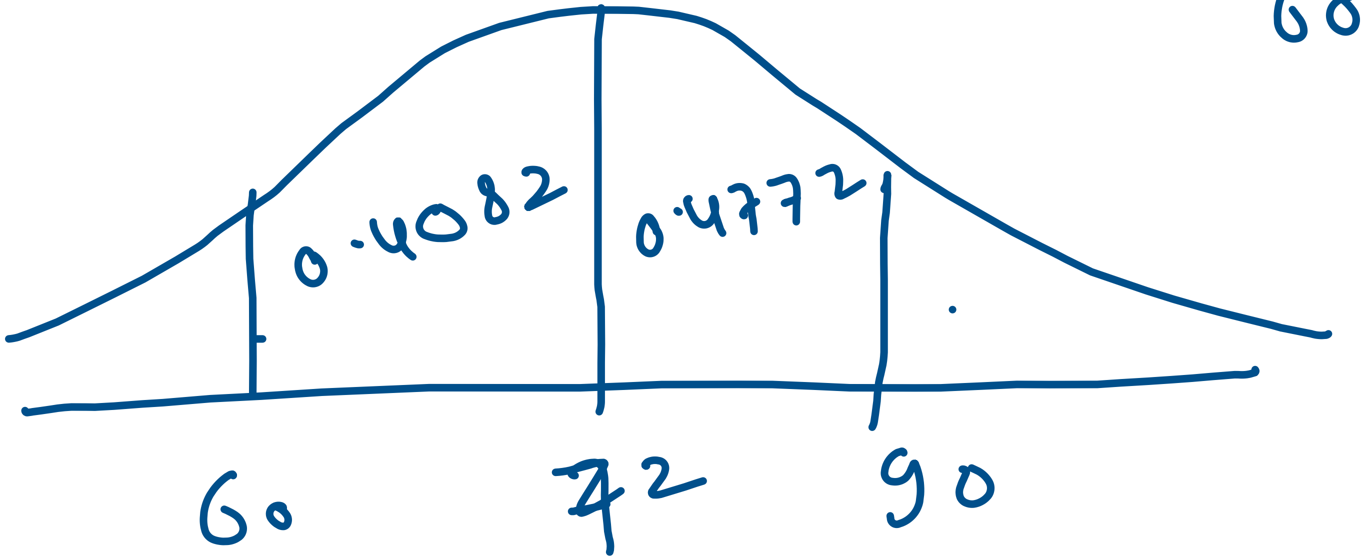
$$P(X \leq 60) = P(X \leq 72) - P(60 \leq X \leq 72)$$




$$= 0.5 - 0.4082$$

$$= 0.0918$$

$$P(60 \le x \le 90) = \int_{60}^{90} \frac{1}{9\sqrt{2\pi}} e^{-\frac{(x-72)^2}{2(9^2)}}\, dx$$




$Z$

$Z = 1.33$

$$Z = \frac{90-72}{9}$$

$$= \frac{18}{9}$$

$$= 2.00$$

$$P(60 \le x \le 90) = 0.4082 + 0.4772$$

$$=$$

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$$P(x \geq 80) = \int_{80}^{\infty} \frac{1}{9\sqrt{2\pi}} e^{\frac{-(x-72)^2}{2(9)^2}} \, dx$$


$\bar{x} = 72$ 80

$z = 0$ $z = 0.89$

$$z = \frac{80 - 72}{9} = \frac{8}{9} = 0.89$$

$$p(x \geq 80) = 0.5 - 0.3133$$


$\bar{x}$ 72

$z = 0$

$\bar{x} = ? = 83.5$

A

$Z = 1.28$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$1.28 = \frac{X - 72}{9}$$

$$x = 1.28 \times 9 + 72$$

$$\bar{x} = 83.52$$