L42 23 April 2021

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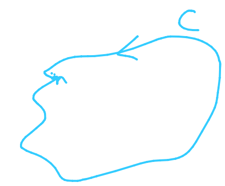
Circulation :- A line integral of a vector field $\vec{v}$ around a simple closed curve C is defined as the circulation of $\vec{v}$ around C



$$\text{Circulation} = \oint_C \vec{v}\cdot d\vec{s} = \oint_C \vec{v}\cdot \frac{d\vec{s}}{ds}\, ds$$

$$= \oint_C \vec{v}\cdot \vec{T}\, ds$$

where $\vec{T}$ is tangent vector to C

unit tangent $s(t) = x\hat{i} + y\hat{j} + z\hat{k}$

$$\frac{s'(t)}{|s'(t)|}$$

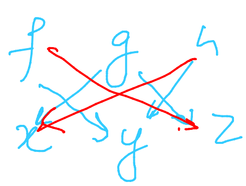
$$\frac{s'(t)}{\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}}$$

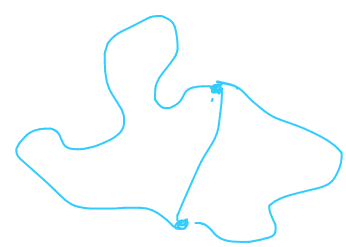
Line Integral independent of the Path.

Let C be a curve in a domain D. Let f, g and h be continuous functions having first partial derivatives in D. Then

$\int_C f\,dx + g\,dy + h\,dz$ is independent of path C if and only if

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}, \quad \frac{\partial g}{\partial z} = \frac{\partial h}{\partial y} \quad \text{and} \quad \frac{\partial f}{\partial z} = \frac{\partial h}{\partial x}$$





If $\vec{F} = f\hat{i} + g\hat{j} + h\hat{k}$ then

$$\int_C f\,dx + g\,dy + h\,dz = \int_C \vec{F}\cdot d\vec{s}$$

If the line integral is path independent, then $\vec{F} = \text{grad}(\phi)$

$$\text{Curl}\,\vec{F} = \text{curl}(\text{grad}\,\phi)$$

$$= 0$$

$\Rightarrow$ **Conservative field**

If $\vec{F}$ is a conservative field then work done along any

If $\vec{F}$ is a Conservative field then work done along any closed path is 0.

Show that the line integral is independent of path of integration. Evaluate the integral.

$$\int_P^Q \underbrace{2xy^2}_{f}\,dx + \underbrace{(2x^2y+1)}_{g}\,dy$$

$P: (-1,2)$ $Q\,(2,3)$

$\int_C \underline{f\,dx + g\,dy}$ is independent of path if and only if

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial y} = 4xy \qquad \frac{\partial g}{\partial x} = 4xy \quad \Rightarrow \text{ line integral is path independent.}$$

$$\int_C f\,dx + g\,dy = \int \vec{F}\cdot d\vec{s}$$

$$\int d(\phi) = \underline{M\,dx + N\,dy}$$

Exact

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\vec{F} = 2xy^2\hat{i} + (2x^2y+1)\hat{j}$$

$$\vec{F} = \text{grad}.\,A$$

$$2xy^2\hat{i} + (2x^2y+1)\hat{j} = \frac{\partial A}{\partial x}\hat{i} + \frac{\partial A}{\partial y}\hat{j}$$

$$\frac{\partial A}{\partial x} = 2xy^2, \qquad \frac{\partial A}{\partial y} = \underline{2x^2y+1}$$

Integrate w.r.t. x (y Const)

$$A(x,y) = \cancel{2}\frac{x^2}{\cancel{2}}y^2 + B(y)$$

$$\frac{\partial A}{\partial y} = \cancel{2x^2y} + \frac{dB}{dy} = \cancel{2x^2y} + 1 \Rightarrow \int \frac{dB}{dy} = 1$$

$$B = y + C$$

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$A(x,y) = x^2y^2 + y + C$

$\int f\,dx + g\,dy = \int \cancel{\text{grad}} \left(x^2y^2 + y + C\right)\Big|_P^Q$

$\int (x\,dy + y\,dx)$

$\int_P^Q d(xy) = ?$

$\left|x^2y^2 + y + C\right|_{(-1,2)}^{(2,3)} = (36 + 3 + C) - (4 + 2 + C)$

$= 39 - 6 = 33$

$\int_P^Q f\,dx + g\,dy + h\,dz$

$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ - - - - -

$\vec{F} \cdot d\vec{s} = |\phi|_P^Q$

$\vec{F} = \text{grad}\,\phi$

$\phi = ?$

Q25. If F is conservative vector field, F= gradf, then which of the following is correct

a. work done is independent of path of integration.
b. work done is dependent of path of integration.
c. work done is constant
d. Work done is zero

Q68. If $\int_P^Q \vec{F}.d\vec{r}$ is independent of the path of integration if

(a) Curl $\vec{F}$=0 (b) div $\vec{F}$ = 0 (c) grad $|\vec{F}|$ = 0 (d) none of these