Q1. The length of the curve traced by $r(t) = a\cos t\, \hat{i} + a\sin t\, \hat{j}$, $a > 0, 0 \le t \le \frac{\pi}{4}$ is

(a) $2\pi a^2$ (b) $\frac{\pi a}{2}$ (c) $4\pi a^2$ (d) $\frac{\pi a}{4}$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \cdot dt = \int_0^{\pi/4} \sqrt{(-a\sin t)^2 + (a\cos t)^2} \cdot dt$$

$$= \int_0^{\pi/4} a\, dt = a \cdot \left[\frac{\pi}{4}\right]$$

Q2. The parametric representation of the curve x+z=3, y-z=0 is

(a) x=3-2t , y=t, z=t (b) x=0 , y=3/2, z=3/2 (c) x=3-t, y=t, z=t (d) 3x=y=z

$x+z=3$, $y-z=0$

$y=z=t$

$x=3-z$

$x=3-t$

Q4. The position vector of a moving particle is $\vec{r}(t) = (\cos t + \sin t)\hat{i} + (\sin t - \cos t)\hat{j} + 2t\hat{k}$. Its speed is

(a) $\sqrt{6}$ (b) t (c) $\sqrt{3}$ (d) 1

$$\bar{r}(t) = (\cos t + \sin t)\hat{i} + (\sin t - \cos t)\hat{j} + 2t\hat{k}$$

$$\bar{v}(t) = \frac{d\bar{r}(t)}{dt} = (-\sin t + \cos t)\hat{i} + (\cos t + \sin t)\hat{j} + 2$$

$$\text{Speed} = |\bar{v}(t)| = \sqrt{\sin^2 t + \cos^2 t - \cancel{2\sin t\cos t} + \cos^2 t + \sin^2 t + \cancel{2\sin t\cos t} + 4}$$

$$= \sqrt{1+1+4} = \sqrt{6}$$

Q5. If the unit tangent vector of the curve $x = t, y = t^2, z = t^3$, at $t = 1$ is $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ then $\beta =$

(a) 0 (b) 2 (c) $2/\sqrt{14}$ (d) $1/\sqrt{14}$

$$\bar{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$$

$$r'(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

$$r'(t)\Big|_{t=1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\frac{r'(t)}{|r'(t)|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{1+4+9}} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}} = \quad \beta = \frac{2}{\sqrt{14}}$$

Gradient of a scalar field :-

Let $f(x,y,z)$ be a real valued function defining a scalar field.

vector operator ∇

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y}$$

and $$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

The gradient of a scalar field $f(x,y,z)$ denoted ∇f or grad(f)

$$\underbrace{\nabla f}_{\text{vector field}} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

Find the gradient of the scalar function

① $f = x^3 - 3x^2y^2 + y^3$

$$\nabla f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^3 - 3x^2y^2 + y^3\right)$$

$$= \hat{i}\left(3x^2 - 6y^2x\right) + \hat{j}\left(-6x^2y + 3y^2\right)$$

gradient of f at specified point

gradient of f at specified point

$f = x^3 + y^3 \sin 4y + z^2 \qquad (1, \pi/3, 1)$

$$\nabla f = \hat{i}\frac{\partial}{\partial x}(x^3+y^3\sin 4y+z^2) + \hat{j}\frac{\partial}{\partial y}(x^3+y^3\sin 4y+z^2) + \hat{k}\frac{\partial}{\partial z}(x^3+y^3\sin 4y+z^2)$$

$$= \hat{i}(3x^2) + \hat{j}\left[y^3(\cos 4y \cdot 4) + \sin 4y \cdot 3y^2\right] + \hat{k}\left[2z\right]$$

$$\nabla f\Big|_{(1,\pi/3,1)} = \hat{i}(3) + \hat{j}\left[\frac{4\pi^3}{27}\cos\frac{4\pi}{3} + \sin\frac{4\pi}{3}\cdot 3\cdot\frac{\pi^2}{9}\right] + \hat{k}\left[2\right]$$

If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ and $|\vec{r}| = r$ $\qquad r = \sqrt{x^2+y^2+z^2}$

$$\text{grad}\left(\frac{1}{r}\right) = \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)\left(\frac{1}{r}\right) \qquad \frac{1}{x}$$

$$= \hat{i}\left(\frac{-1}{r^2}\right)\frac{\partial r}{\partial x} + \hat{j}\left(\frac{-1}{r^2}\right)\frac{\partial r}{\partial y} + \hat{k}\left(\frac{-1}{r^2}\right)\frac{\partial r}{\partial z}$$

$r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$= \frac{-1}{r^2}\left[\hat{i}\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}\right] + (\quad) + (\quad)$$

$$= \frac{-1}{r^2}\left[\hat{i}\frac{x}{r} + \hat{j}\frac{y}{r} + \hat{k}\frac{z}{r}\right] = \frac{-(x\hat{i}+y\hat{j}+z\hat{k})}{r^3}$$

$$\boxed{\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}} = \frac{-1}{r^2}\left[\frac{\vec{r}}{r}\right] = \frac{-\hat{r}}{r^2}$$