# Theoritical Probability Dist

Probability Mass function

DRV

1. Binomal Dist

2. Poissen Dist

Probability density function

CRV

1. Normal Dist

## ① Binomial Dist :→

It is due to James Bernolli (1700) is a discrete probability dist. The Bernolli process has the following properties

(i) An experiment is repeated $n$ number of times called $n$ trials where $n$ is a fixed integer

$n \leq 30$

$n \geq 30$

(ii) The outcome of each trial is satisfying two mutually exclusive categories arbitrarily called success and failure

(iii) Probability of success denoted by $p$ remain constant for each trial & failure $q$

$$\boxed{p + q = 1}$$

$$\boxed{q = 1 - p}$$

(iv) The outcomes are independent. Each trial in the Bernolli process is known as Bernolli trial. The Binomial random variable $X$ is the number of success in $n$ trial.

Binomial dist is thus probability dist of this discrete random variable $x$ and is given by.

$$b(x; n, p) = {}^{n}C_{x} \, p^{x} \, q^{n-x}, \quad x = 0, 1, \ldots \ldots n$$

$$= 0 \quad , \text{ otherwise}$$

where $n$ no. of trials

$p$ probability of success in any trial

$q$ probability of Failure.

$x$ success  $p \; p \; p \ldots x$ times

$= p^{x}$

for remaining $n-x$ failure

$q \; q \; q \ldots \; n-x$ times

$= q^{n-x}$

This can be happen in  ${}^{n}C_{x} = \frac{n!}{x!\,(n-x)!}$

By multiplication rule the probability of $x$ success

$$\boxed{{}^{n}C_{x}\ p^{x}\ q^{n-x}}$$

$${}^{4}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{4-3}$$

4-Coins $\qquad 2^4 = 16$

| $X$ | $P(x)$ |
| --- | --- |
| 0 | ${}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = \frac{1}{16}$ |
| 1 | ${}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 = \frac{4}{16}$ |
| 2 | ${}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{6}{16}$ |
| 3 | ${}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = \frac{4}{16}$ |
| 4 | ${}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = \frac{1}{16}$ |

| | |
| --- | --- |
| HHHH | THHH ✓ |
| HHHT ✓ | THHT |
| HHTH ✓ | THTH |
| HHTT | THTT |
| HTHH ✓ | TTHH |
| HTHT | TTHT |
| HTTH | TTTH |
| HTTT | TTTT |

$$= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3}$$

| $x$ | $P(x)$ | $x\,P(x)$ | $x^2$ | $x^2 P(x)$ |
|---|---|---|---|---|
| 0 | $1/16$ | 0 | 0 | 0 |
| 1 | $4/16$ | $4/16$ | 1 | $4/16$ |
| 2 | $6/16$ | $12/16$ | 4 | $24/16$ |
| 3 | $4/16$ | $12/16$ | 9 | $36/16$ |
| 4 | $1/16$ | $4/16$ | 16 | $16/16$ |
| | | $\frac{32}{16}$ | | $\frac{80}{16}$ |

$$E(x) = \frac{32}{16} = 2$$

$$E(x^2) = \frac{80}{16} = 5$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= 5 - 2^2$$

$$= 5 - 4$$

$$= 1$$

**Remarks** ① The binomial dist is characterized by two parameters $n$ the number of trials and $p$ the probability of success in each trial therefore binomial dist. is biparametric dist

$${}^{n}C_{x}\ p^{x}\ q^{n-x} \qquad x = 0, 1, 2 \ldots n$$

$$q = 1 - p$$

(2) The mean of binomial dist is $np$

$$E(X) = \text{mean} = \bar{x} = np$$

$n = 4 \quad p = \frac{1}{2} \qquad \bar{x} = 4 \times \frac{1}{2} = 2$

(3) The variance of binomial dist is $npq$

$$V(x) = E(x^2) - (E(x))^2 = npq$$

$$npq = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$$

## 4) Mode of Binomial Dist

(i) When $(n+1)p$ is an integral value then binomial dist is bimodal. there are Two mode

$$M_1 = (n+1)p \qquad M_2 = (n+1)p - 1$$

(ii) When $(n+1)p$ is non-integral value then binomial dist is unimodal There is only one mode which is integral part of $(n+1)p$

E (i) $n = 16$ $p = \frac{1}{3}$

$(n+1)p = (16+1)\frac{1}{3} = \frac{17}{3} = 5.666\ldots$

$\boxed{\text{Mode} = 5}$

(2) $n = 15$ $p = \frac{1}{2}$ $(n+1)p = (15+1) \times \frac{1}{2}$

$= \frac{16}{2} = 8$

$\text{Mode}_1 = 8$ $\text{Mode}_2 = 8 - 1 = 7$

Ex ③

$n = 11 \qquad p = \frac{1}{4} \qquad q = 1 - \frac{1}{4} = \frac{3}{4}$

$$\text{Mean} = np = 11 \times \frac{1}{4} = \frac{11}{4}$$

$$\text{Variance} = npq = 11 \times \frac{1}{4} \times \frac{3}{4} = \frac{33}{16}$$

$$SD = \sqrt{V(x)} = \sqrt{npq} = \sqrt{\frac{33}{16}}$$

Mode

$$(n+1)p = (11+1)\frac{1}{4} = \frac{12}{4} = 3$$

$$\boxed{\begin{array}{l} \text{Mode}_1 = 3 \\ \text{Mode}_2 = 3-1 = 2 \end{array}}$$

(5) $f(x) = b(x; n, p) \geq 0$

$${}^{n}C_{x}\, p^{x} q^{n-x} \geq 0$$

(6) $$\sum_{x=0}^{n} b(x; n, p) = 1$$

$$\sum_{x=0}^{n} f(x) = 1 = \sum_{x=0}^{n} p(x)$$

$$\sum_{x=0}^{n} b(x; n, p) = b(0; n, p) + b(1, n, p) + b(2, n, p) + \cdots + b(n, n, p)$$

$$\left\{ b(x; n, p) = {}^{n}C_{x}\, p^{x} q^{n-x} \right\}$$

$$\sum_{x=0}^{n} b(x; n, p) = {}^{n}C_{0}\, p^{0} q^{n-0} + {}^{n}C_{1}\, p^{1} q^{n-1} + {}^{n}C_{2}\, p^{2} q^{n-2} + \cdots + {}^{n}C_{n}\, p^{n} q^{0}$$

$$\left\{ (a+b)^n = {}^nC_0\, a^0 b^n + {}^nC_1\, a^1 b^{n-1} + {}^nC_2\, a^2 b^{n-2} + \cdots\, {}^nC_n\, a^n b^0 \right\}$$

Binomial Expansion

$$\sum_{x=0}^{n} b(x; n, p) = (p+q)^n = 1^n = 1$$

$$\{\, p+q = 1$$

$$\sum_{x=0}^{n} b(x; n, p) = 1$$

$$b(0, n, p) + b(1; n, p) + \cdots + b(n, n, p) = 1$$

$$p(0) + p(1) + p(2) + \cdots + p(n) = 1$$

$$\sum_{x=0}^{n} p(x) = 1$$