# Multiplication Law of Probability and Conditional Probability

*For two events A and B*

$$P(A \cap B) = P(A) \,.\, P(B \mid A), \; P(A) > 0$$
$$= P(B) \,.\, P(A \mid B), \; P(B) > 0$$

*where $P(B \mid A)$ represents the conditional probability of occurrence of B when the event A has already happened and $P(A \mid B)$ is the conditional probability of happening of A, given that B has already happened.*

## Dependent event

when B depends on A

If event A already happened, then the probability of event B when B depends on A is denoted by $P(B/A)$ and is called the conditional probability of B when B depends on A

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

$P(A) > 0$

If event B already happened then the probability of event A when A depends on B is denoted by P(A/B) and is called conditional probability of A when A depends on B

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0}$$

Thus the conditional probabilities $P(B|A)$ and $P(A|B)$ are defined if and only if $P(A) \neq 0$ and $P(B) \neq 0$, respectively.

(i) For $P(B) > 0$, $P(A|B) \leq P(A)$

(ii) The conditional probability $P(A|B)$ is not defined if $P(B) = 0$.

(iii) $P(B|B) = 1$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$A \rightarrow B$

$$P(B|B) = \frac{P(B \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$B \rightarrow A$

$$P(A|A) = \frac{P(A \cap A)}{P(A)}$$

$$= \frac{P(A)}{P(A)} = 1$$

Q Two cards are drawn then which is dependent event

(a) Two card K and Q drawn with replacement

52      52

K      Q

$\frac{4}{52}$      $\frac{4}{52}$

independent event

(b) Two card K and Q drawn without replacement

52      51

K      Q ✓

$\frac{4}{52}$      $\frac{4}{51}$

dependent events

when A and B are independent events

Then

$P(A/B) \longrightarrow P(A)$ $\qquad$ $P(B/A) \longrightarrow P(B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \longrightarrow P(A) = \frac{P(A \cap B)}{P(B)} \checkmark$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \longrightarrow P(B) = \frac{P(A \cap B)}{P(A)} \checkmark$$

$$\boxed{P(A \cap B) = P(A)\, P(B)}$$

(1) For independent events A and B

$$P(A \cap B) = P(A)\, P(B)$$ ✓

(2) For mutually exclusive events A and B

$$P(A \cap B) = 0$$ ✓

*If A and B are independent events then A and $\bar{B}$ are also independent events.*

(I) A and $\bar{B}$ are also independent

(II) $\bar{A}$ and B are also independent

(III) $\bar{A}$ and $\bar{B}$ are also independent

$P(A \cap B) = P(A)\,P(B)$

$P(A \cap \bar{B}) = P(A)\,P(\bar{B})$

$P(\bar{A} \cap B) = P(\bar{A})\,P(B)$

$P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

Two dice, one green and the other red, are thrown. Let A be the event that the sum of the points on the faces shown is odd, and B be the event of at least one ace (number '1').

(a) Describe the (i) complete sample space, (ii) events $A$, $B$, $\bar{B}$, $A \cap B$, $A \cup B$, and $A \cap \bar{B}$ and find their probabilities assuming that all the 36 sample points have equal probabilities.

(b) Find the probabilities of the events :

(i) $(\bar{A} \cup \bar{B})$ (ii) $(\bar{A} \cap \bar{B})$ (iii) $(A \cap \bar{B})$ (iv) $(\bar{A} \cap B)$ (v) $(\overline{A \cap B})$ (vi) $(\bar{A} \cup B)$ (vii) $(\overline{A \cup B})$ (viii) $\bar{A} \cap (A \cup B)$ (ix) $A \cup (\bar{A} \cap B)$ (x) $(A \mid B)$ and $(B \mid A)$, and (xi) $(\bar{A} \mid \bar{B})$ and $(\bar{B} \mid \bar{A})$.

(a) (i) $S = \{ (1,1), (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6), (2,1)\ (2,2),$
$(2,3)\ (2,4)\ (2,5)\ (2,6)\ (3,1)\ (3,2)\ (3,3)\ (3,4)$
$(3,5)(3,6)(4,1)\ (4,2)\ (4,3)\ (4,4)(4,5), (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)(5,5)(5,6), (6,1)\ (6,2), (6,3)\ (6,4)(6,5)\ (6,6) \}$

$6 \times 6 = 36$

A

| Sum | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| no. | 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 2 | 1 |

$n(A) = 18$

$P(A) = \frac{18}{36} = \frac{1}{2}$

$A = \{(2,1), (1,2), (2,3), (3,2), (1,4), (4,1), (6,1), (1,6), (3,4), (4,3), (5,2), (2,5), (4,5), (5,4), (6,3), (3,6), (5,6), (6,5)\}$

$B = \{(1,1), (1,2), (2,1), (1,3), (3,1), (1,4), (4,1), (1,5), (5,1), (1,6), (6,1)\}$

$n(B) = 11$

$P(B) = \frac{11}{36}$

$\bar{B} = S - B$

$$n(\bar{B}) = n(S) - n(B)$$
$$= 36 - 11$$
$$= 25$$

$$P(\bar{B}) = \frac{25}{36}.$$

$A \cap B = \{(1,2), (2,1), (4,1), (1,4), (1,6), (6,1)\}$

$$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

$\underline{A \cup B}$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 18 + 11 - 6$$

$$= 23$$

$$P(A \cup B) = \frac{23}{36}$$

$A \cap \bar{B}$ = only $A = A - A \cap B$

$P(A \cap \bar{B})$

$$= P(A) - P(A \cap B)$$

$$= \frac{18}{36} - \frac{6}{36}$$

$$= \frac{12}{36} = \frac{1}{3}$$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$$

By De-Morgans Law

(a) $1/2$

(b) $1/4$

(c) $1/6$

✓(d) $5/6$

$$= 1 - P(A \cap B)$$

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

$$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$$

$$= 1 - \frac{23}{36} = \frac{13}{36}$$

$$P(\bar{A}\cap B) = P(\text{only } B) = P(B) - P(A\cap B)$$

$$= \frac{11}{36} - \frac{6}{36} = \frac{5}{36}$$

$$P(\overline{A\cap B}) = 1 - P(A\cap B) = 1 - \frac{1}{6} = \frac{5}{6}$$

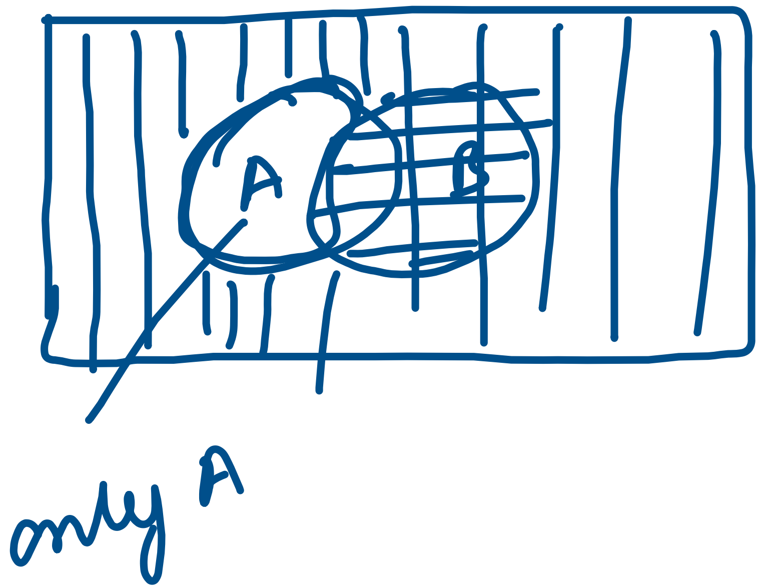
$$P(\bar{A} \cup B) = 1 - P(\text{only } A) = 1 - [P(A \cap \bar{B})]$$

$$= 1 - [P(A) - P(A \cap B)]$$

$$= 1 - \left[\frac{18}{36} - \frac{6}{36}\right]$$

$$= 1 - \left[\frac{12}{36}\right]$$

$$= 1 - \left[\frac{1}{3}\right] = \frac{2}{3}$$



$$P(\bar{A} \cup B) = P(\bar{A} \cup \bar{\bar{B}})$$

$$= P(\overline{A \cap \bar{B}})$$

$$= 1 - P(A \cap \bar{B})$$

$$= 1 - [P(A) - P(A \cap B)]$$

$$= 1 - \left[\frac{18}{30} - \frac{6}{30}\right] = \frac{2}{3}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{6/36}{11/36} = \frac{6}{11}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{6/36}{18/36} = \frac{6}{18} = \frac{1}{3}$$

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 23/36}{1 - 11/36} = \frac{13/36}{25/36} = \frac{13}{25}$$

$$P(\bar{B}/\bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$$ By De-Morgan Law

$$= \frac{1 - P(A \cup B)}{1 - P(A)} = \frac{1 - 23/36}{1 - 18/36} = \frac{13/36}{18/36}$$

$$= \frac{13}{18}$$