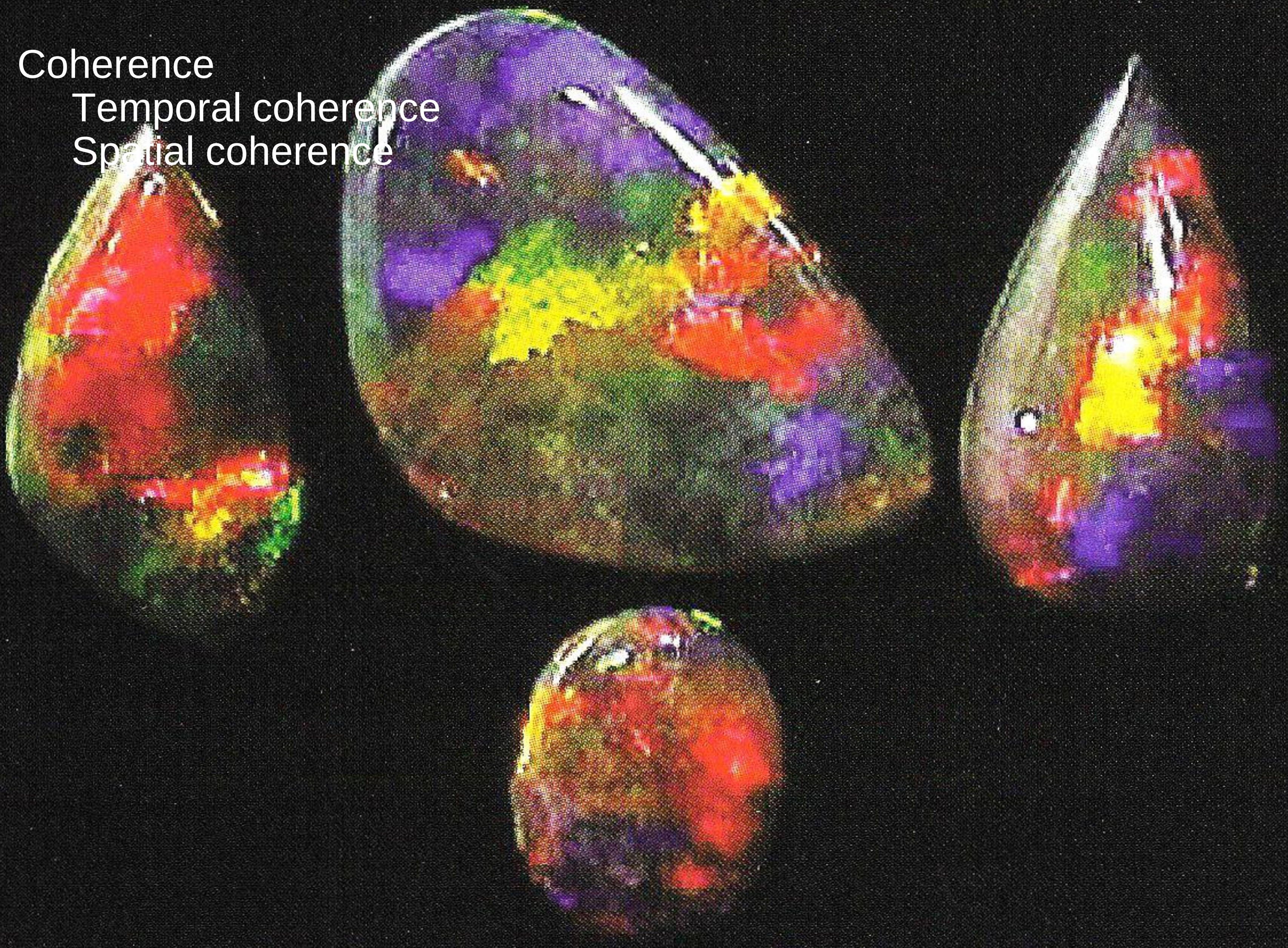
Coherence

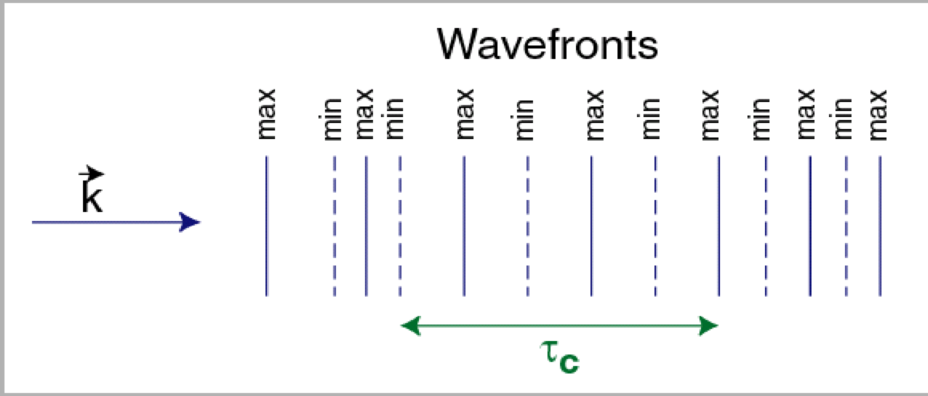
- Coherence
  - Temporal coherence
  - Spatial coherence



The Temporal Coherence Time and the Spatial Coherence Length

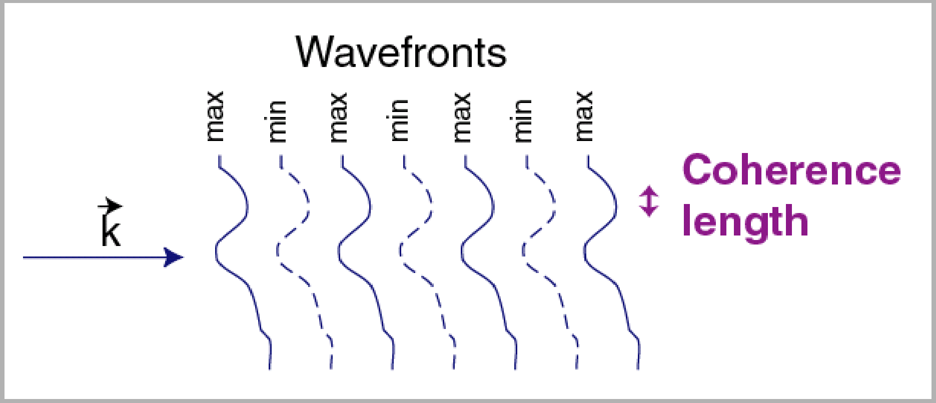
The temporal coherence time is the time the wave-fronts remain equally spaced. That is, the field remains sinusoidal with one wavelength:

Temporal Coherence Time, τ_c



The spatial coherence length is the distance over which the beam wave-fronts remain flat:

Spatial Coherence Length



Since there are two transverse dimensions, we can define a coherence area.

Spatial and Temporal Coherence

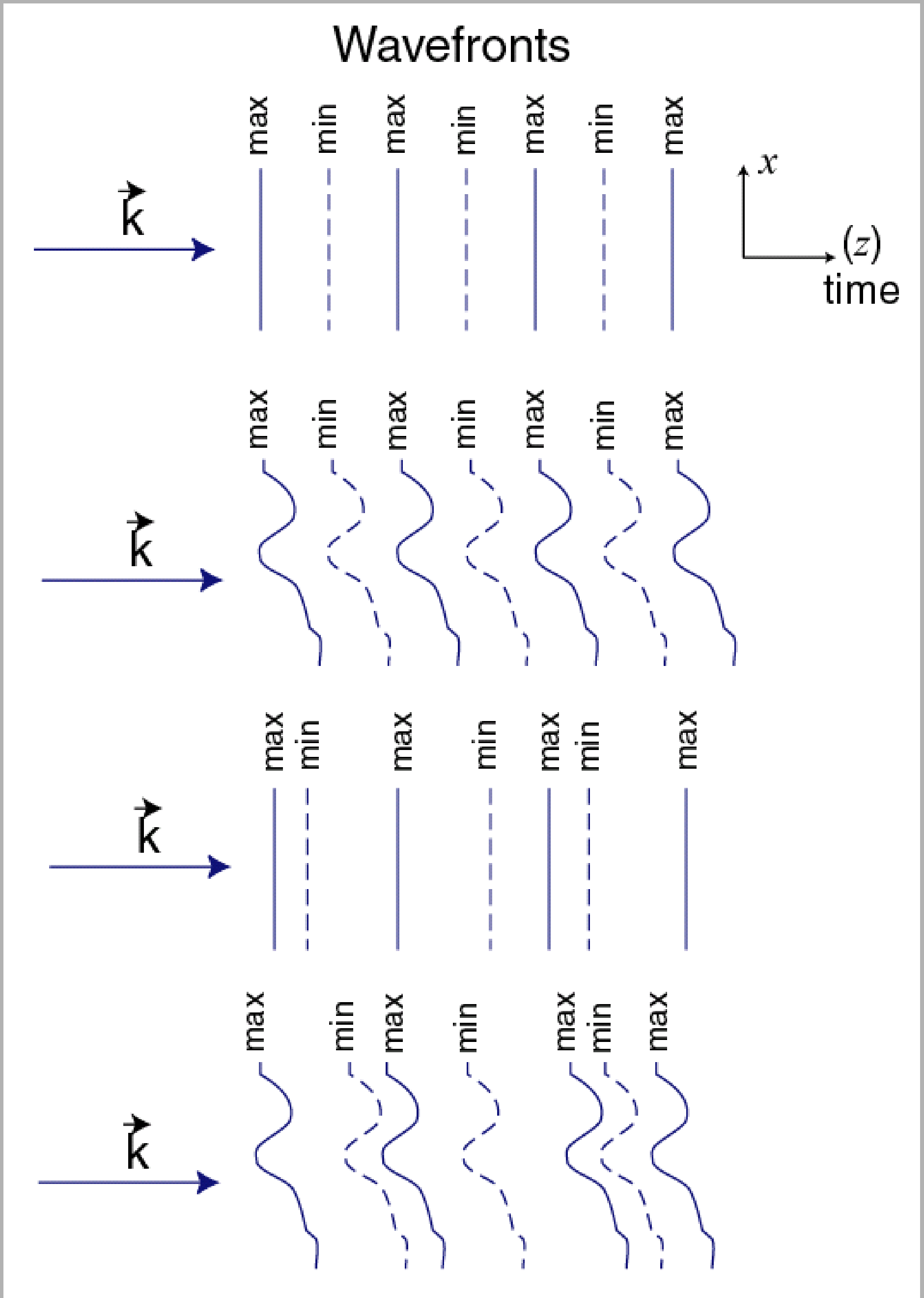
Beams can be coherent or only partially coherent (indeed, even incoherent) in both space and time.

Spatial and Temporal Coherence:

Temporal Coherence; Spatial **In**coherence

Spatial Coherence; Temporal **In**coherence

Spatial **and** Temporal **In**coherence



The coherence time is the reciprocal of the bandwidth.

The coherence time is given by:

$$\boxed{\tau_c = 1/\Delta\nu}$$

where $\Delta\nu$ is the light bandwidth (the width of the spectrum).

Sunlight is temporally very incoherent because its bandwidth is very large (the entire visible spectrum).

Lasers can have coherence times as long as about a second, which is amazing; that's $>10^{14}$ cycles!

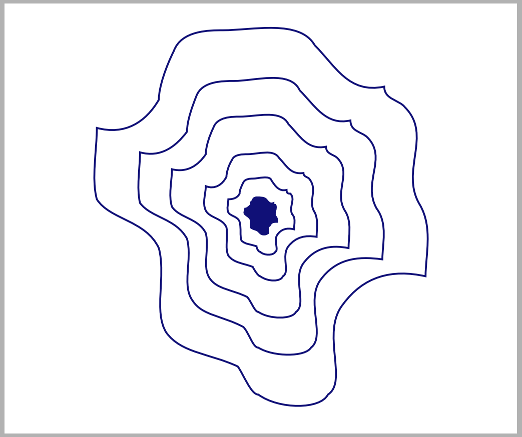
The spatial coherence depends on the emitter size and its distance away.

The van Cittert-Zernike Theorem states that the spatial coherence area A_c is given by:

$$\mathrm{A}_c = \frac{D^2 \lambda^2}{\pi d^2}$$

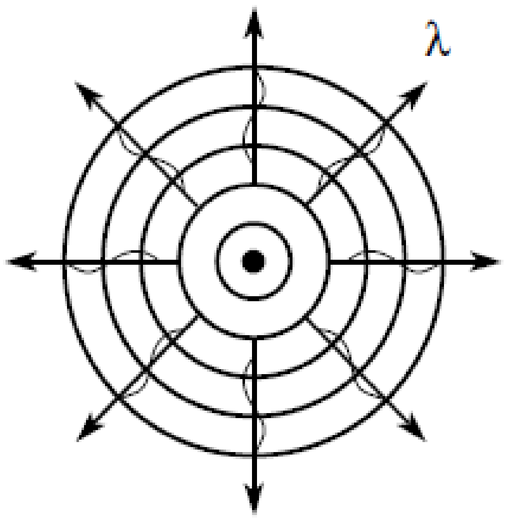
where d is the diameter of the light source and D is the distance away.

Basically, wave-fronts smooth out as they propagate away from the source.

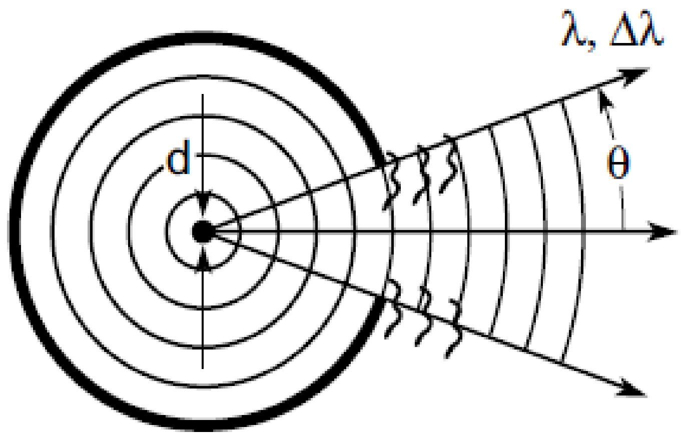


Starlight is spatially very coherent because stars are very far away.

Coherence, Partial Coherence, and Incoherence



Point source oscillator
$-\infty < t < \infty$



Source of finite size,
divergence, and duration

Spatial and Temporal Coherence

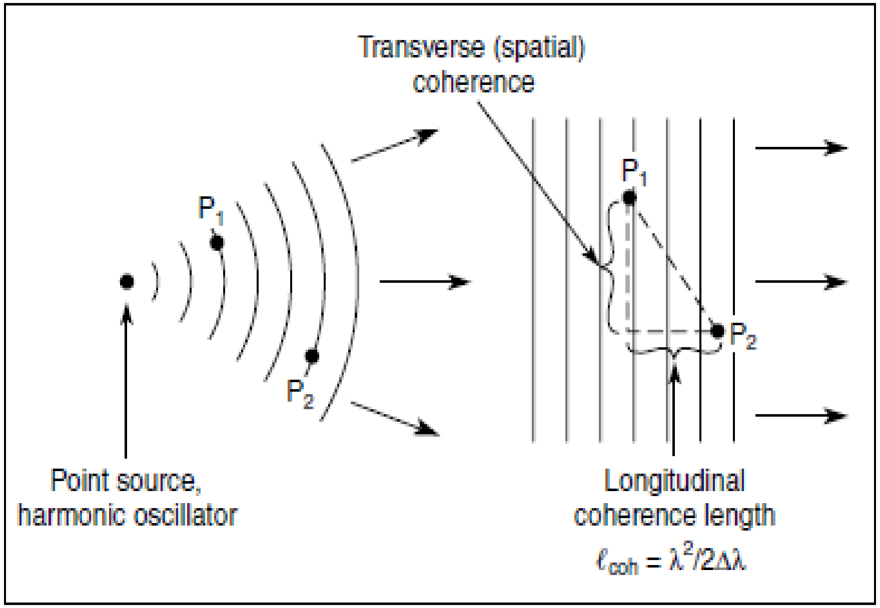
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t+\tau)E_2^*(t)\rangle \qquad (8.1)$$

Normalize degree of spatial coherence (complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}} \qquad (8.12)$$

A high degree of coherence ($\mu \to 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \to 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



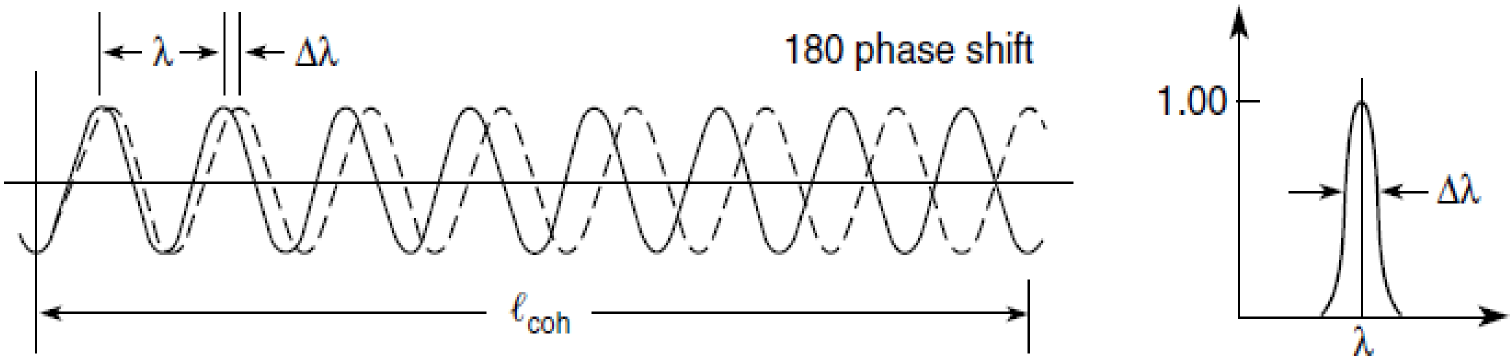
Longitudinal (temporal) coherence length

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

Full spatial (transverse) coherence

$$\boxed{d \cdot \theta = \lambda/2\pi} \qquad (8.5)$$

Spectral Bandwidth and Longitudinal Coherence Length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})(\lambda + \Delta\lambda)$$

Equating the two

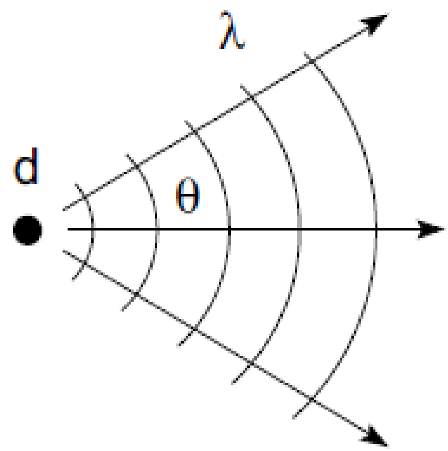
$$N = \lambda/2\Delta\lambda$$

so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

A Practical Interpretation of Spatial Coherence

- Associate spatial coherence with a spherical wavefront.
- A spherical wavefront implies a point source.
- How small is a "point source"?



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size "d" you can resolve, with wavelength λ and half angle θ, is

$$\mathbf{d} \cdot \theta = \frac{\lambda}{2\pi}$$

Partially Coherent Radiation Approaches Uncertainty Principle Limits

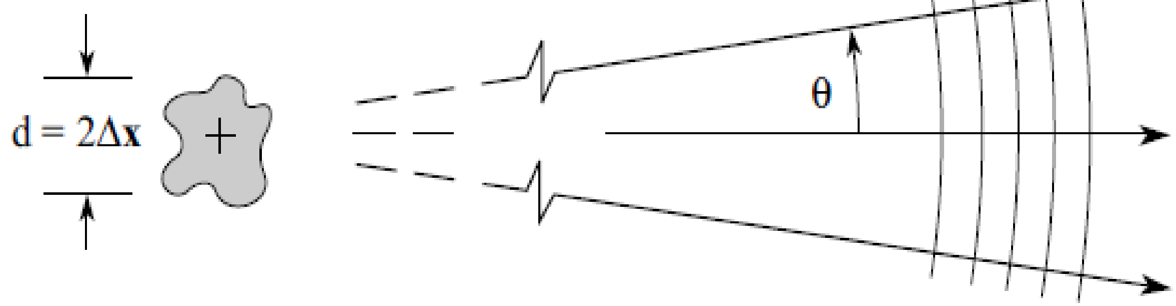
$\Delta \mathbf{x} \cdot \Delta \mathbf{p} \geq \hbar/2 \quad (8.4)$

Standard deviations of Gaussian distributed functions (Tipler, 1978, pp. 174-189)

$\Delta \mathbf{x} \cdot \hbar \Delta \mathbf{k} \geq \hbar/2$

$\Delta \mathbf{x} \cdot \mathbf{k} \Delta\theta \geq 1/2$

$2\Delta \mathbf{x} \cdot \Delta\theta \geq \lambda/2\pi$

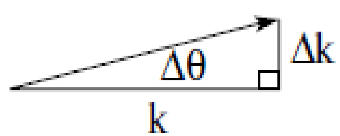


Spherical wavefronts occur in the limiting case

Note:

$\Delta \mathbf{p} = \hbar \Delta k$

$\Delta k = k \Delta\theta$

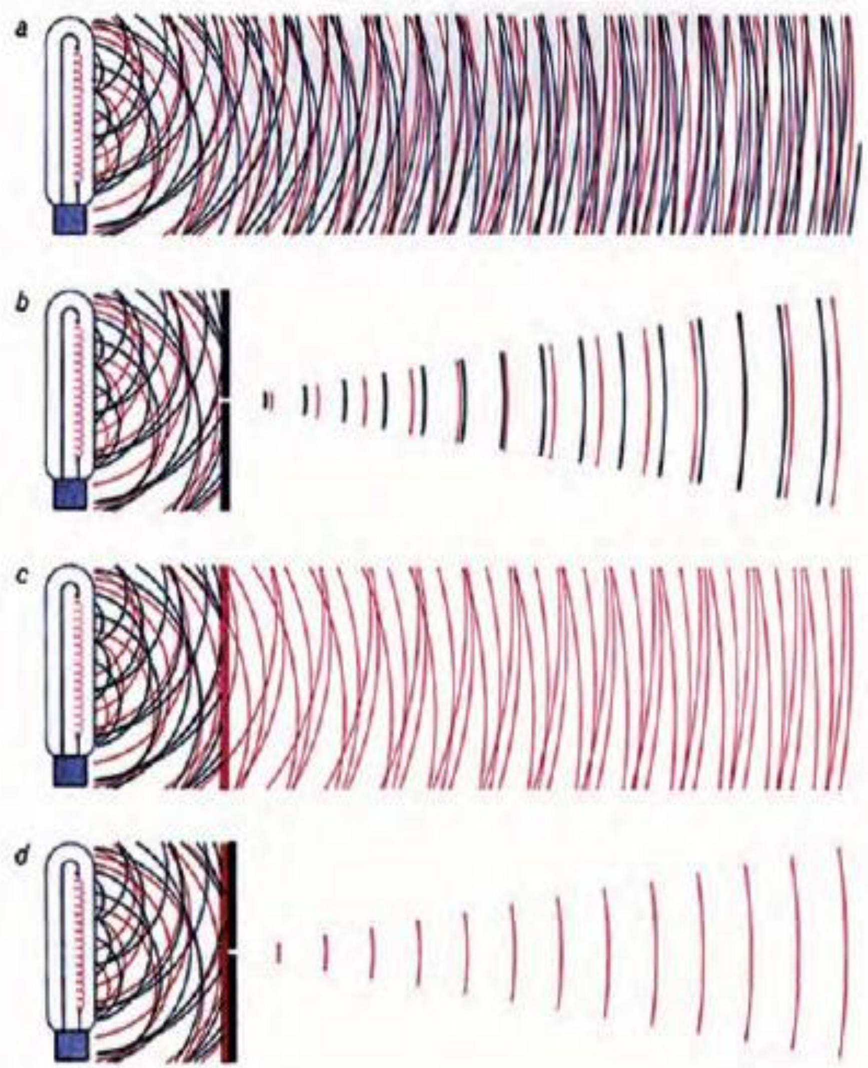


$$\left.\begin{array}{c}\boxed{d \cdot \theta = \lambda/2\pi} \\ \text{(spatially coherent)}\end{array}\right\} \frac{1}{\sqrt{e}} \text{ quantities}$$

or

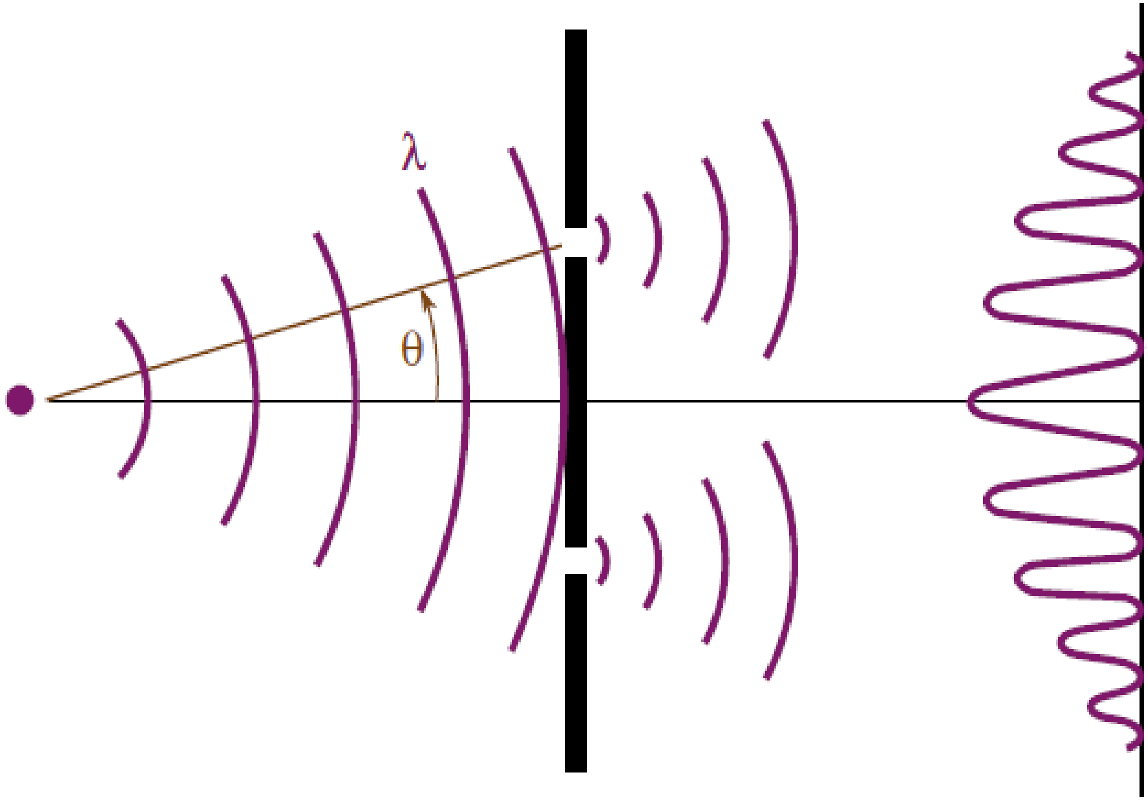
$$(d \cdot 2\theta)_{FWHM} \simeq \lambda/2 \;\Big\}\; \text{FWHM quantities}$$

Spatial and Spectral Filtering to Produce Coherent Radiation



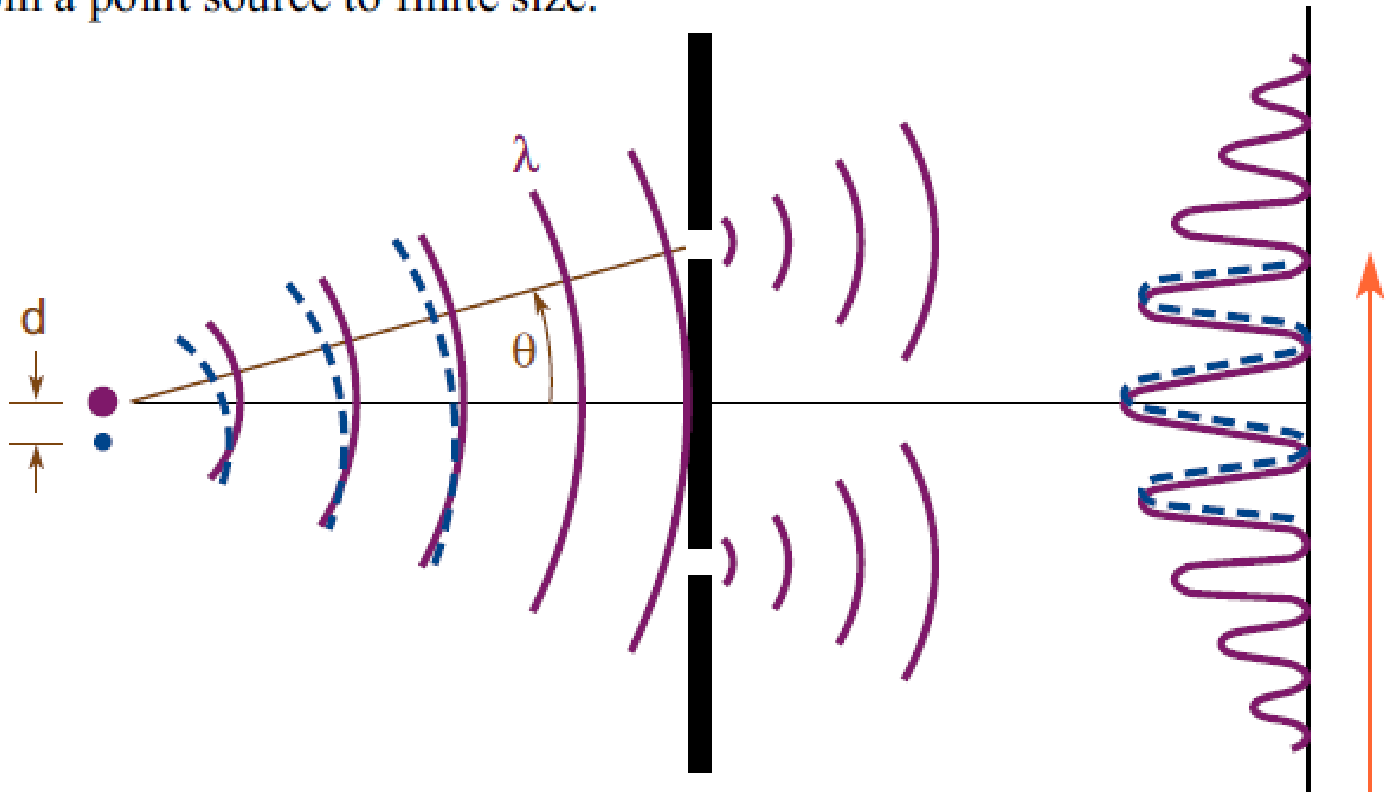
Courtesy of A. Schawlow, Stanford.

Young's Double Slit Experiment: Spatial Coherence and the Persistence of Fringes



Young's Double Slit Experiment: Spatial Coherence and the Persistence of Fringes

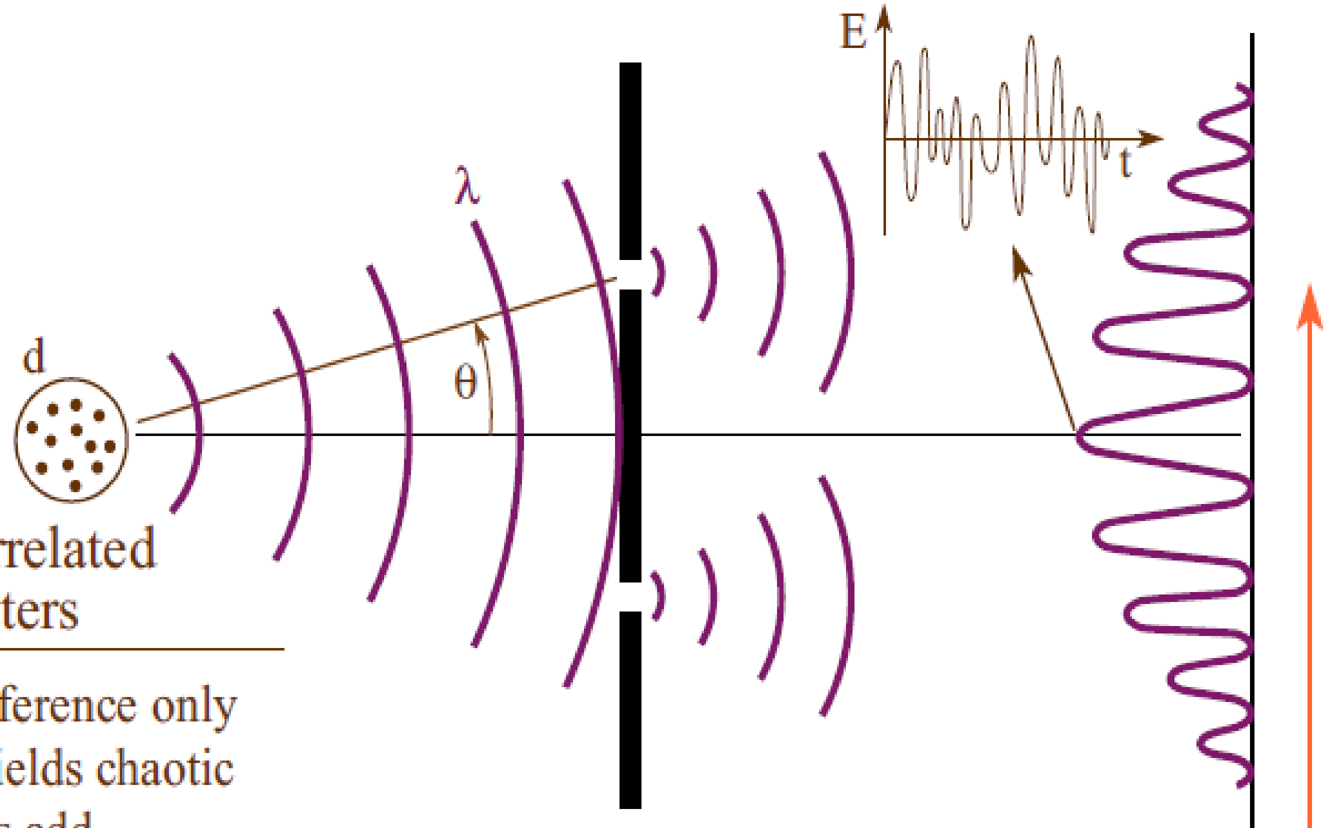
Persistence of fringes as the source grows from a point source to finite size.



$$d \cdot 2\theta|_{\mathrm{FWHM}} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2}N_{coh}\lambda$$

Young's Double Slit Experiment with Random Emitters: Young did not have a laser



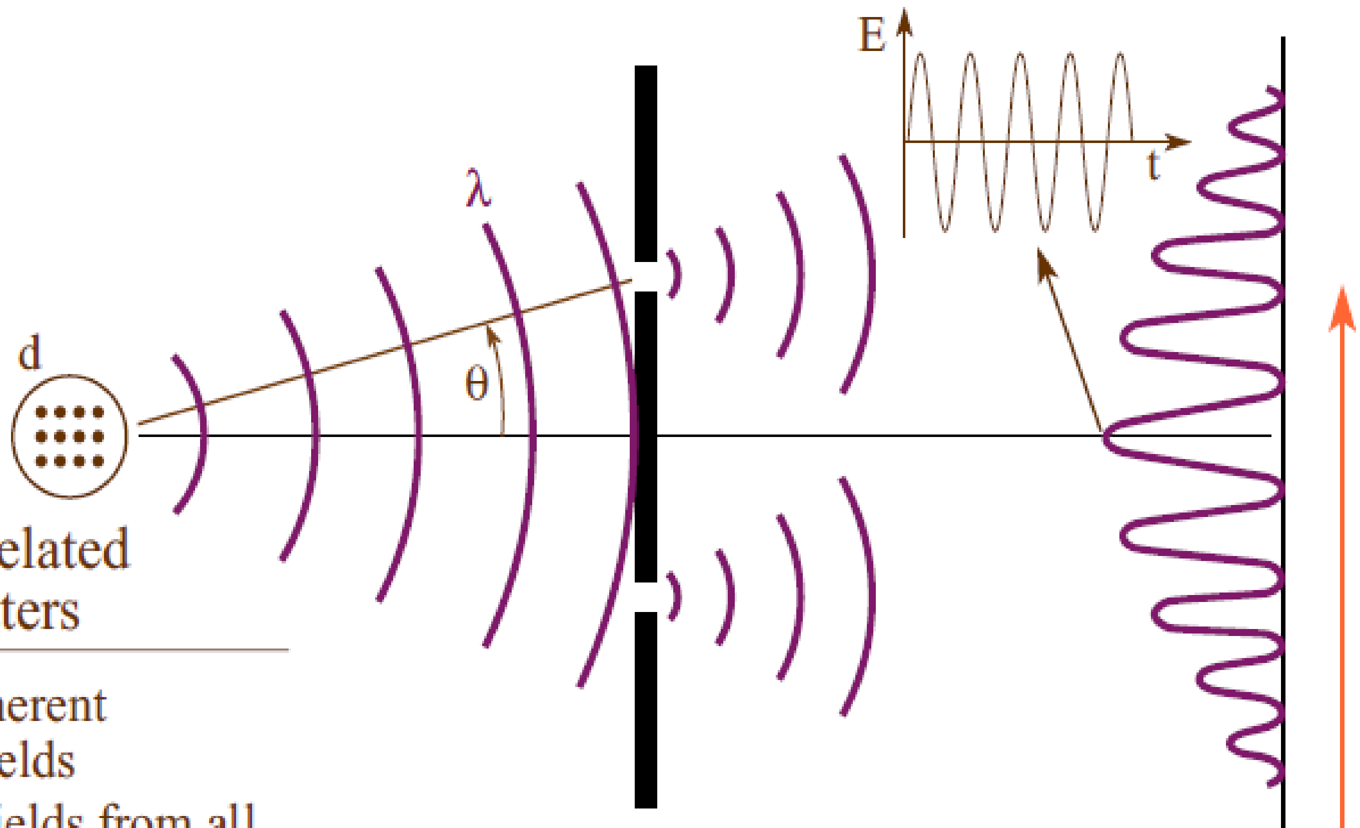
N uncorrelated emitters

- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power ~ N

$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

Young's Double Slit Experiment with Phase Coherent Emitters (some lasers, or properly seeded FELs)



N correlated emitters

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power ~ N^2
- New phase sensitive probing of matter possible

$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$