Some theorems on Random Variables. Here we shall state (without proof) some of the fundamental results and theorems on random variables.

1. A function $X(\omega)$ from S to R $(-\infty, \infty)$ is a random variable if and only if

$$\{\omega : X(\omega) < a\} \in \mathbf{B}$$

(S, B, P)

CX

$X_1 + X_2$

$X_1 X_2$

$C_1 X_1 + C_2 X_2$

$X_1 - X_2$

2. If X_1 and X_2 are random variables and C is a constant then CX_1, $X_1 + X_2$, X_1X_2 are also random variables.

Remark. It will follow that $C_1X_1 + C_2X_2$ is a random variable for constants C_1 and C_2. In particular $X_1 - X_2$ is a r.v.

$\frac{X_1}{X_2}$ not always r.v

If X is a r.v. and $f(\cdot)$ is a continuous function, then $f(X)$ is a r.v.

If X is a r.v. and $f(\cdot)$ is an increasing function, then $f(X)$ is a r.v.

$X_1 < X_2$ $f(X_1) < f(X_2)$

Distribution Function. Let X be a r.v. on (S,B,P). Then the function :

$$F_X(x) = P(X \le x) = P\{\omega : X(\omega) \le x\}, \quad -\infty < x < \infty$$

is called the distribution function (d.f.) of X.

If clarity permits, we may write $F(x)$ instead of $F_X(x)$.

$$F(x) = P(X \le x) = P(-\infty < X \le x)$$

Q1

x	$p(x)$	$F(x)$
0	$1/8$	$F(0) = P(x \le 0) = P(0) = \frac{1}{8}$
1	$3/8$	$F(1) = P(x \le 1) = p(0) + p(1) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$
2	$3/8$	$F(2) = p(x \le 2) = p(0) + p(1) + p(2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$
3	$1/8$	$F(3) = p(x \le 3) = p(0) + p(1) + p(2) + p(3) = \frac{7}{8} + \frac{1}{8} = 1$

Q2 A random variable X has the following probability distribution :

x :	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6)$, $P(X \geq 6)$, and $P(0<X<5)$, (iii) If $P(X \leq c) > \frac{1}{2}$, find the minimum value of c, and (iv) Determine the distribution function of X.

Sol

$$\sum p(x) = 1 = 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k$$

$$9k + 10k^2 = 1$$

$$10k^2 + 9k - 1 = 0$$

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k+1) - 1(k+1) = 0$$

$$(k+1)(10k-1) = 0$$

$$\cancel{k = -1} \qquad \boxed{k = \frac{1}{10}}$$

$$p(x<6) = p(0)+p(1)+p(2)+p(3)+p(4)+p(5)$$

$$= 0 + k + 2k + 2k + 3k + k^2$$

$$= 8k + k^2 = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$$

$$p(x \geq 6) = 1 - p(x<6) = 1 - \frac{81}{100} = \frac{19}{100}$$

$$p(0<x<5) = p(1)+p(2)+p(3)+p(4)$$

$$= k + 2k + 2k + 3k = 8k = \frac{8}{10} = \frac{4}{5}$$

X	$p(x)$
0	$0 = 0$
1	$K = 1/10$
2	$2K = 2/10$
3	$2K = 2/10$
4	$3K = 3/10$
5	$K^2 = 1/100$
6	$2K^2 = 2/100$
7	$7K^2 + K = \frac{7}{100} + \frac{1}{10} = \frac{17}{100}$

$F(x)$

$F(0) = P(x \leq 0) = 0$

$F(1) = P(x \leq 1) = \frac{1}{10}$

$F(2) = P(x \leq 2) = 3/10$

$F(3) = P(x \leq 3) = 5/10 = \frac{1}{2}$

$F(4) = P(x \leq 4) = 8/10 > \frac{1}{2}$

$F(5) = P(x \leq 5) = \frac{81}{100} > \frac{1}{2}$

$F(6) = P(x \leq 6) = 83/100 > \frac{1}{2}$

$F(7) = P(x \leq 7) = \frac{100}{100} = 1 > \frac{1}{2}$

(iii) $P(X \le c) > \frac{1}{2}$.

$$p(x \le 4) = \frac{8}{10} > 1/2$$

$$p(x \le 5) = \frac{81}{100} > 1/2$$

$$p(x \le 6) = \frac{83}{100} > 1/2$$

$$p(x \le 7) = 1 > 1/2$$

$c = 4, 5, 6, 7$

$\min c = 4$

4

Q 3

From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Answer the following when the sample is drawn without replacement.

(i) Find the probability distribution of X,

(ii) Find $P(X \le 1)$, $P(X < 1)$ and $P(0 < X < 2)$

$$TC = {}^{10}C_4$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 10 \times 3 \times 7$$

10

D 3, ND 7

4

	D	ND
	0	4
	1	3
	2	2
	3	1

X	$p(x)$	$F(x)$
0	$\dfrac{{}^3C_0 \times {}^7C_4}{{}^{10}C_4} = p(0) = \dfrac{1}{6}$	$F(0) = \dfrac{1}{6}$
1	$\dfrac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4} = p(1) = \dfrac{1}{2}$	$F(1) = \dfrac{1}{6} + \dfrac{1}{2} = \dfrac{2}{3}$
2	$\dfrac{{}^3C_2 \times {}^7C_2}{{}^{10}C_4} = p(2) = \dfrac{3}{10}$	$F(2) = \dfrac{2}{3} + \dfrac{3}{10} = \dfrac{29}{30}$
3	$\dfrac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4} = p(3) = \dfrac{1}{30}$	$F(3) = \dfrac{1}{30} + \dfrac{29}{30} = \dfrac{30}{30} = 1$

$$P(X \leq 1) = P(0) + P(1) = \frac{1}{6} + \frac{1}{2} = \frac{1+3}{6}$$

$$= \frac{4}{6} = \frac{2}{3}$$

$$P(X < 1) = P(0) = \frac{1}{6}$$

$$P(0 < X < 2) = P(1) = \frac{1}{2}$$

Qu A random variable X has the following probability distribution :

Values of X, x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value of a.

(ii) Find $P(X < 3)$, $P(X \geq 3)$, $P(0 < X < 5)$.

(iii) What is the smallest value of x for which $P(X \leq x) > 0.5$? and

(iv) Find out the distribution function of X?

Sol

$$\sum p(x) = 1 = a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a$$

$$81a = 1$$

$$\boxed{a = \frac{1}{81}}$$

$$P(X<3) = P(0) + P(1) + P(2)$$
$$= a + 3a + 5a = 9a = \frac{9}{81} = \frac{1}{9}$$

$$P(X \geq 3) = 1 - P(X<3) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$P(0<X<5) = P(1) + P(2) + P(3) + P(4)$$
$$= 3a + 5a + 7a + 9a$$
$$= 24a = \cancel{24}^{8} \times \frac{1}{\cancel{81}_{27}} = \frac{8}{27}$$

X	$P(x)$
0	$a = 1/81$
1	$3a = 3/81$
2	$5a = 5/81$
3	$7a = 7/81$
4	$9a = 9/81$
5	$11a = 11/81$
6	$13a = 13/81$
7	$15a = 15/81$
8	$17a = 17/81$

$F(x)$

$F(0) = 1/81 = P(X \le 0)$

$F(1) = 4/81 = P(X \le 1)$

$F(2) = 9/81 = P(X \le 2)$

$F(3) = 16/81 = P(X \le 3)$

$F(4) = 25/81 = P(X \le 4)$

$F(5) = 36/81 = P(X \le 5)$

$F(6) = 49/81 = P(X \le 6) > \frac{1}{2}$

$F(7) = 64/81 = P(X \le 7) > \frac{1}{2}$

$F(8) = 81/81 = 1 = P(X \le 8) >$

iii

$p(X \le x) > 0.5$

$x = 6, 7, 8$

$p(X \le 6) = \frac{49}{81} > \frac{1}{2}$

Smallest 6

$p(X \le 7) = \frac{64}{81} > 1/2$

$p(X \le 8) = \frac{81}{81} = 1 > \frac{1}{2}$