Advanced Machine Learning

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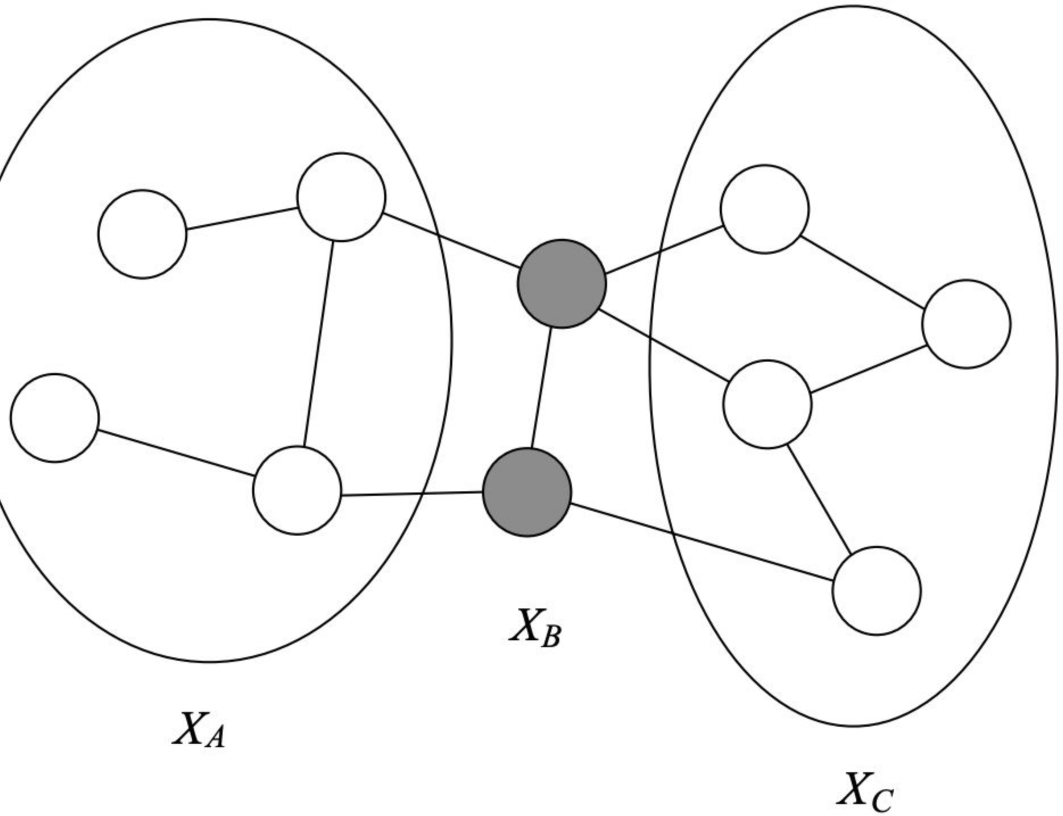
Markov Network - Global Independencies

We say that a set of nodes $\boldsymbol{Z}$ separates $\boldsymbol{X}$ *and* $\boldsymbol{Y}$ *in* $\mathcal{H}$, *denoted* $\mathrm{sep}_{\mathcal{H}}(\boldsymbol{X};\boldsymbol{Y} \mid \boldsymbol{Z})$, *if there is no* $\boxed{\textit{active path}}$ *between any node* $X \in \boldsymbol{X}$ *and* $Y \in \boldsymbol{Y}$ *given* $\boldsymbol{Z}$. *We define the* global independencies *associated with* $\mathcal{H}$ *to be:*

$$\mathcal{I}(\mathcal{H}) = \{(\boldsymbol{X} \perp \boldsymbol{Y} \mid \boldsymbol{Z}) \ : \ \mathrm{sep}_{\mathcal{H}}(\boldsymbol{X};\boldsymbol{Y} \mid \boldsymbol{Z})\}.$$

Let $\mathcal{H}$ *be a Markov network structure, and let* $X_1 — \ldots — X_k$ *be a path in* $\mathcal{H}$. *Let* $\boldsymbol{Z} \subseteq \mathcal{X}$ *be a set of* observed variables. *The path* $X_1 — \ldots — X_k$ *is* active *given* $\boldsymbol{Z}$ *if none of the* X_i*'s*, $i = 1, \ldots, k$, *is in* $\boldsymbol{Z}$.

Markov Network - Global Independencies



Markov Network - Global Independencies

Let $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ be three disjoint subsets of variables such that $\mathcal{X} = \boldsymbol{X} \cup \boldsymbol{Y} \cup \boldsymbol{Z}$.

$$P(\mathcal{X}) = \phi_1(\boldsymbol{X}, \boldsymbol{Z})\phi_2(\boldsymbol{Y}, \boldsymbol{Z})$$

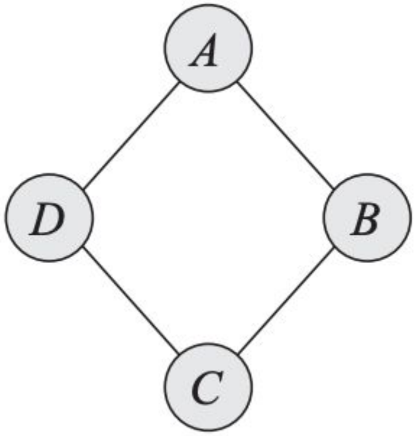
$$\boxed{\boldsymbol{X} \perp \boldsymbol{Y} \mid \boldsymbol{Z} \text{ ?}}$$

Markov Network - Local Independencies

Pairwise Independency:

Let $\mathcal{H}$ be a Markov network. We define the pairwise independencies *associated with $\mathcal{H}$ to be:*

$$\mathcal{I}_p(\mathcal{H}) = \{(X \perp Y \mid \mathcal{X} - \{X, Y\}) \ : \ X—Y \notin \mathcal{H}\}.$$

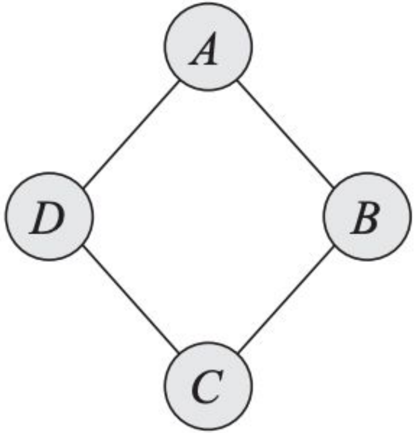


Markov Network - Local Independencies

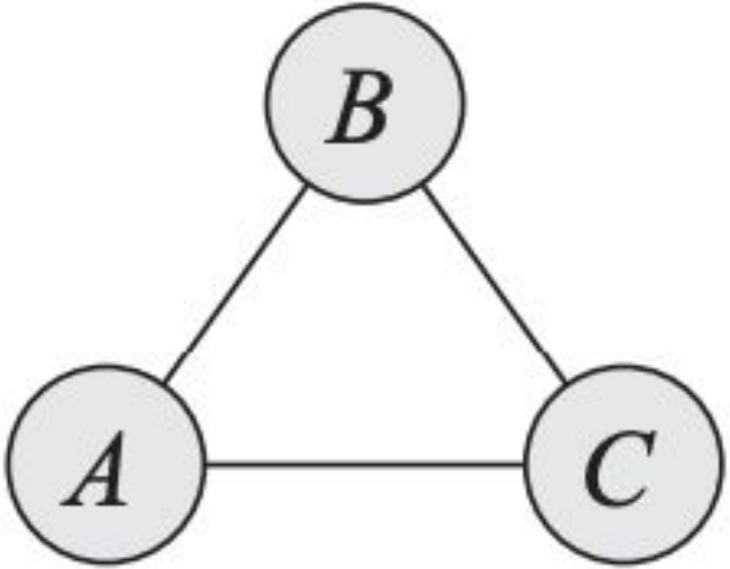
Markov Blanket:

For a given graph $\mathcal{H}$, *we define the* Markov blanket *of* X *in* $\mathcal{H}$, *denoted* $\mathrm{MB}_{\mathcal{H}}(X)$, *to be the neighbors of* X *in* $\mathcal{H}$. *We define the* local independencies *associated with* $\mathcal{H}$ *to be:*

$$\mathcal{I}_\ell(\mathcal{H}) = \{(X \perp \mathcal{X} - \{X\} - \mathrm{MB}_{\mathcal{H}}(X) \mid \mathrm{MB}_{\mathcal{H}}(X)) \ : \ X \in \mathcal{X}\}.$$



Factor Graphs



Factor Graphs