

Assuming a multivariate normal distribution for the underlying factors, f , the likelihood function is

$$L = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(f-\mu)^T \Sigma^{-1} (f-\mu)}$$

where n : number of factors

Σ : covariance matrix of factors

μ : mean of factors

If we maximize log-likelihood, with respect to f , it is equivalent to maximizing

$$-\frac{1}{2}(f-\mu)^T \Sigma^{-1} (f-\mu)$$

as the rest does not depend on f

We also have the constraint $Af = v$, which defines the set of views specified by the user.

To solve this optimization problem, we write the Lagrangian function such as

$$\mathcal{L}(f, d) = -\frac{1}{2}(f-\mu)^T \Sigma^{-1} (f-\mu) - d^T (Af - v) \Big|_{f \text{ and } d} \Rightarrow \text{maximize wrt}$$

$$\textcircled{1} -(\Sigma^{-1} f - \Sigma^{-1} \mu) - A^T d = 0$$

$$\textcircled{2} Af = v$$

\Rightarrow Multiplying $\textcircled{1}$ by $A\Sigma$ from left and rearranging, we get

$$\textcircled{3} Af + (A\Sigma A^T) d = A\mu$$

$$\Rightarrow d = (A\Sigma A^T)^{-1} (A\mu - v) \text{ by replacing } Af \text{ with } v$$

Then replacing d in $\textcircled{1}$ with this equation we write

$$-(\Sigma^{-1} f - \Sigma^{-1} \mu) - A^T (A\Sigma A^T)^{-1} (A\mu - v) = 0$$

$$\Rightarrow f = \mu - \Sigma A^T (A\Sigma A^T)^{-1} (A\mu - v)$$