Università della Svizzera italiana | Faculty of Informatics | Institute of Computing CI

Numerical Computing **2022**

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Solution for Project 6 Due date: Wednesday, December 21, 2022, 11:59 PM

The purpose of this project is to implement the Simplex Method to find the solution to linear programs, involving both the minimization and the maximization of the objective function.

1. Graphical Solution of Linear Programming Problems [20 points]

This exercise contains 2 subsections. It asks explicitly to solve the system of inequalities produced by the second sub-question. Looking at the provided data for this sub-question, the following system of inequalities arises:

$$\begin{aligned} \max : 85\mathrm{x}_1 + 110x_2 - 25x_1 - 40x_2 \\ \text{subject to: } 25x_1 + 40x_2 \leq 7000 \\ \text{where: } x_1 + x_2 \leq 265 \\ \text{with } x_1,\, x_2 \geq 0 \end{aligned}$$

As can be seen from the above system of inequalities, the definition of the exercise has been taken into account and has been translated into an equivalent system of inequalities. The goal of the exercise is to maximize the net profit of the tailor, hence the **max** preceding the inequalities system. The second line of the above system denotes that the tailor can spend up to 7000 (CHF) for x_1 and x_2 combined. Moreover, in the third line, it is stated that the monthly demand can reach up to 265 pairs of trousers. Last but not least, we must make sure that the quantities are non-negative.
After solving the above inequalities system, starting from the top, the following gets yielded:

$$x_2 \leq 175 - 0.625x_1,$$
$$x_2 \leq 265 - x_1,$$
$$x_2 \leq - x_1$$

Regarding the first line of the original systems of equations, what can be commented is that:

$$60x_1 + 70x_2$$

In the following section, the feasible region identified by the aforementioned constraints is visualized:

To find the optimal solution effectively means that the distribution between the 2 types of trousers must be found, such that selling them shall result in maximum profit for the tailor. The value of the objective function that maximizes the profit is computed in a separate Julia file, called **exercise1.jl** The visualized constraints applicable to this exercise are depicted in the images below.

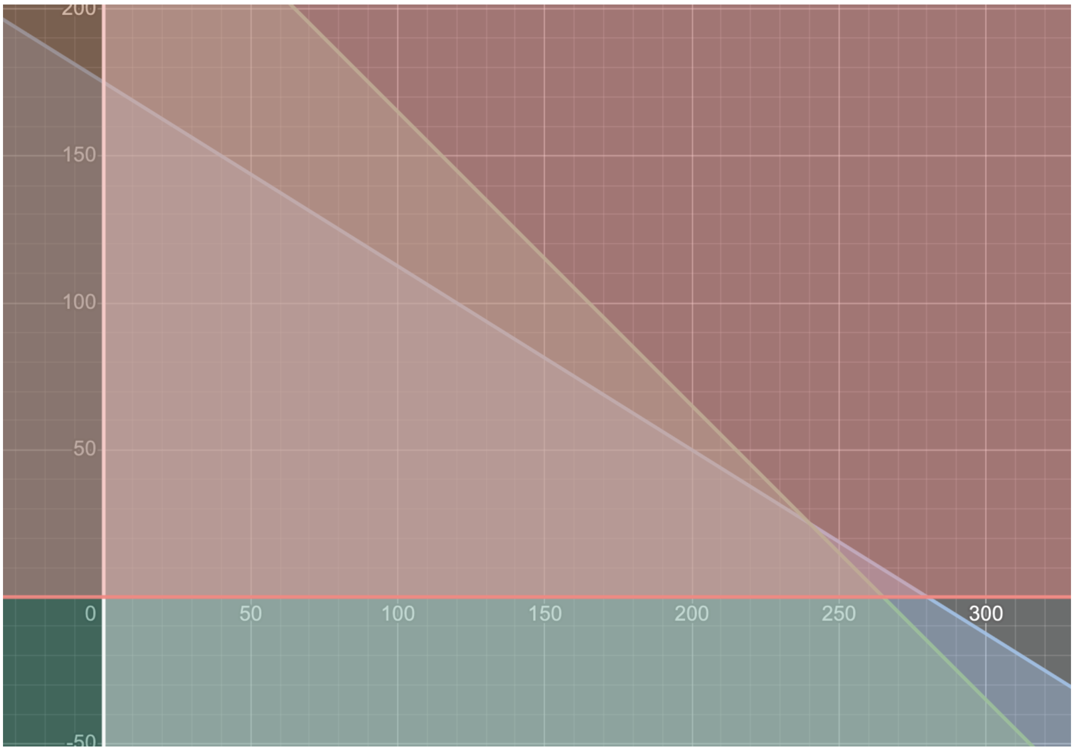

Figure 1: Graph-Represented System of Inequalities

One thing that might not be clear in this scenario is that the visualized line for x1 $\geq$ 0 overlaps the other visualized line for x2 $\geq$ 0.

As seen when running the aforementioned Julia file, the optimal solution shall be when the profit of the seller is maximized. This has been computed using a for-loop in Julia, and as a result, when running the code in Julia's REPL, it is indicated that the maximum profit of the tailor is 18540 CHF. To calculate the value of the objective function, we need to find the optimal solution. Since the optimal solution found is 18540, one can confidently state that the value of the objective function is thereby 18540.

Looking at the depicted graphs above, one can confidently state that the intersection point of those graphs, is the place where the respective intersecting graphs have the same x and y (in this scenario, x1, and x2) values (even though they are not visible on this graph but at some point they intersect). Additionally, it can also be stated that the intersection point is the optimal solution to the given linear programming problem.

2. Implementation of the Simplex Method [30 points]

Both the **standardize** and the **simplex** functions have not only been implemented but also sufficiently commented on with respect to their functionalities and characteristics. The code is located in the respective Julia files.

3. Applications to Real-Life Example: Cargo Aircraft [25 points]

(1) The first (theoretical) part of this exercise shall be answered here whereas the necessary script used to solve this exercise is written in a file called **exercise3.jl**. Specifically, the given exercise program logic shall be translated into an equivalent linear program. Effectively, the goal of these real-life applications is to maximize the cost of some performed actions.

The constraints of the specific linear program are depicted in the following system of inequalities. For the first part, the cargo weights and the compartment's capacity constraints shall be denoted:

$$\begin{aligned}16x_1 + 32x_2 + 40x_3 + 28x_4 &\leq 18,\\ 16x_1 + 32x_2 + 40x_3 + 28x_4 &\leq 32,\\ 16x_1 + 32x_2 + 40x_3 + 28x_4 &\leq 25,\\ 16x_1 + 32x_2 + 40x_3 + 28x_4 &\leq 17,\end{aligned}$$

Likewise, for the cargo volumes and the compartment's capacity, the following constraints shall be taken into account:

$$\begin{aligned}320x_1 + 510x_2 + 630x_3 + 125x_4 &\leq 11930,\\ 320x_1 + 510x_2 + 630x_3 + 125x_4 &\leq 22552,\\ 320x_1 + 510x_2 + 630x_3 + 125x_4 &\leq 11209,\\ 320x_1 + 510x_2 + 630x_3 + 125x_4 &\leq 5870,\end{aligned}$$

Moreover, the profit constraints are:

$$0.1x_2 + 0.2x_3 + 0.3x_4 \leq 1,$$

Lastly,

$$x_1 + x_2 + x_3 + x_4 \geq 0$$

By definition, in linear programming problems, the objective function refers to the real-valued function whose value has to be either maximized or minimized according to the constraints that are defined on the specified linear programming problem over a set of possible solutions. The objective function, in this scenario, is to maximize

$$\mathbf{135}x_1 + \mathbf{200}x_2 + \mathbf{410}x_3 + \mathbf{520}x_4 + \mathbf{0.1}x_2 + \mathbf{0.2}x_3 + \mathbf{0.3}x_4$$

In other words, to maximize the profit when allocating cargo to compartments.

(2) The following image visualizes the optimal allocation of cargo such that the profit is maximized.

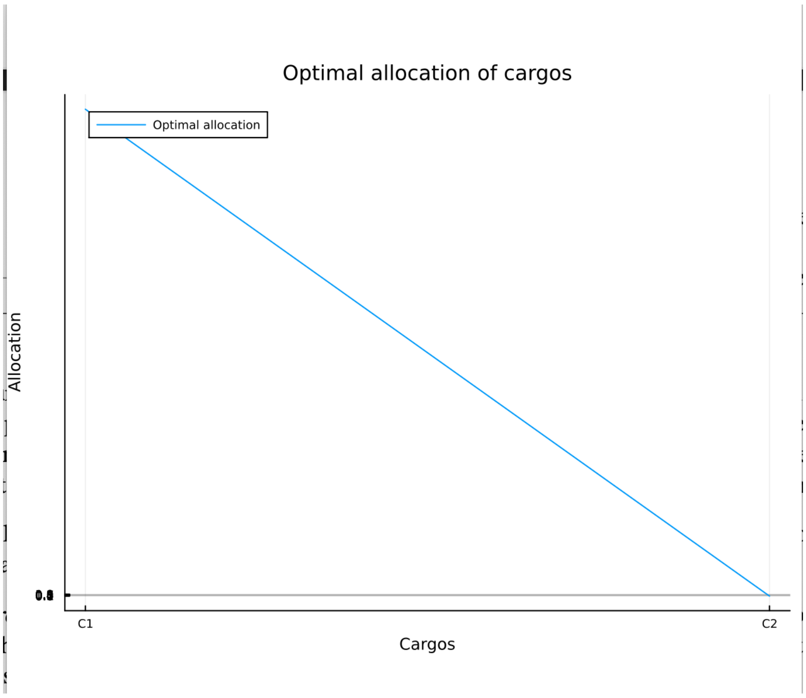

Figure 2: Optimal Cargo Solution Visualization

The objective function is given by the following by maximizing $\mathbf{135}x_1 + \mathbf{200}x_2 + \mathbf{410}x_3 + \mathbf{520}x_4$. It is clearly visible that the optimal solution tends to monotonically decrease and this is no surprise as the more the useful (i.e. profitable) containers get filled, the less profitable containers remain and thus the profits tends to increase, but in an declining manner.

4. Cycling and Degeneracy [10 points]

(1) The Julia code for this sub-question has been extensively commented on, written, and stored in the **exercise4.jl** file. Convergence is not achieved within the maximum number of iterations. Some unwanted behavior occurs, meaning the indices go out of bounds, and if we continue iterating, the program shall crash. For this reason, it is advisable to stop iterating beforehand. This happens because the algorithm is not converging, and the maximum number of iterations is not enough to find the optimal solution, therefore we should stop iterating beforehand in order to prevent such unwanted behavior.

(2) The following screenshot visualizes all constraints contained in the provided system of inequalities.

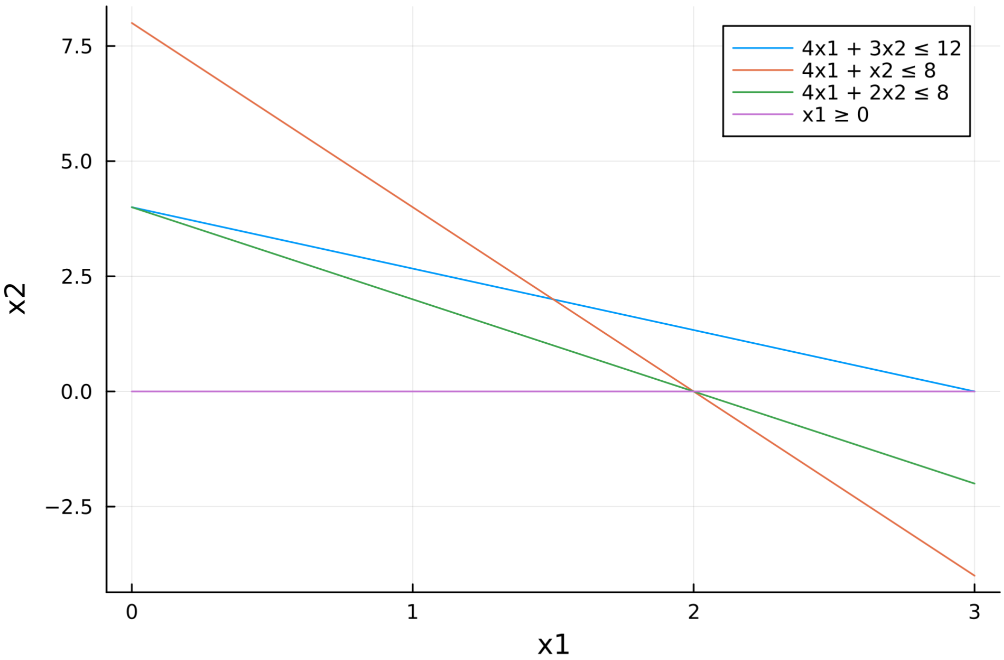

Figure 3: Graph-Represented System of Inequalities

Looking at the depicted graphs above, one can confidently state that the intersection point of those graphs, is the place where the respective intersecting graphs have the same x and y (in this scenario, x_1 and x_2) values. Additionally, it can also be stated that the intersection point is the optimal solution to the given linear programming problem.

Looking at the given system of linear equations, one can clearly observe that the first 3 inequalities are quite similar. The unique thing that changes among them is the constant with which the x and y (x_1 and x_2 in this scenario) get multiplied, and hence, their positioning in the graph.

Looking at the generated graph, some conclusions can be derived with respect to the behavior of the script-written solver. Effectively, it is noticeable that not all inequalities have a point at which **all** of them intersect. This is actually problematic, and explains the previous conclusion that convergence is not achieved within the span of the maximum iterations.