20CS6037: Machine Learning
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SMO Algorithm

Assigned 10/20/2016
Due 11/03/2016 (11:59PM on BB)

You are to write in MATLAB your own implementation of SMO algorithm along the following lines:

- Given $S = \{(x_i, y_i); y_i = \pm 1; i = 1, \ldots, l\}$ linearly separable training set, $\epsilon > 0$, find the optimal separating hyperplane using the steps 1-10 shown below.
- Let K denote a kernel on $S \times S$. Here, since the data is linearly separable, you can use $K_{ij} = K(x_i, x_j) = x_i \cdot x_j$, where $\cdot$ denotes the dot product.
- Denote by E_i the following expression:

$$E_i = \left(\sum_{j=1}^{l} \alpha_j y_j K(x_i, x_j) + b \right) - y_i, i = 1, 2$$

1. Initialize $\alpha = \{\alpha_1, \ldots, \alpha_l\}$ randomly subject to constraint $\sum_{i=1}^{l} y_i \alpha_i = 0$. Set $b = 0$.
2. Calculate the weight vector $w = \sum_{i=1}^{l} \alpha_i y_i x_i$;
3. Calculate KKT conditions:
$$KKT(i) = \alpha(i)\{y_i(\langle w, x_i \rangle + b) - 1\}$$
4. Pick x_1, x_2:
   (a) Let $i_1 = argmax_{i=1,\ldots,l} KKT(i)$.
   (b) Pick $x_1 = x_{i_1}$.
   (c) Calculate $e(i) = E(1) - E(i) = \sum_{j=1}^{l} \alpha_j y_j (K_{j1} - K_{ji}) + y_i - y_1 K_{ij}$
   (d) Let $i_2 = argmax_i |e(i)|$.
   (e) Pick $x_2 = x_{i_2}$.
   (f) Calculate $k = K_{11} + K_{22} - 2 * K_{12}$
5. Update α_2:
$$\alpha_2^{new} = \alpha_2^{old} + \frac{y_2 E(2)}{k}$$
6. Update α_1:
$$\alpha_1^{new} = \alpha_1^{old} + y_1 y_2 (\alpha_2^{old} - \alpha_2^{new})$$
7. For $i = 1, \ldots, l$, if $\alpha_i < \epsilon$, $\alpha_i \leftarrow 0$;
8. Select $\alpha_i > 0$, calculate b (from KKT conditions)
9. Test for classification;
10. Repeat from Step 2. until classified.

Please document your program.
I will supply you with a data set on which to run your final result. Please upload your program along with results on Blackboard.