CS536

MORE Parsing

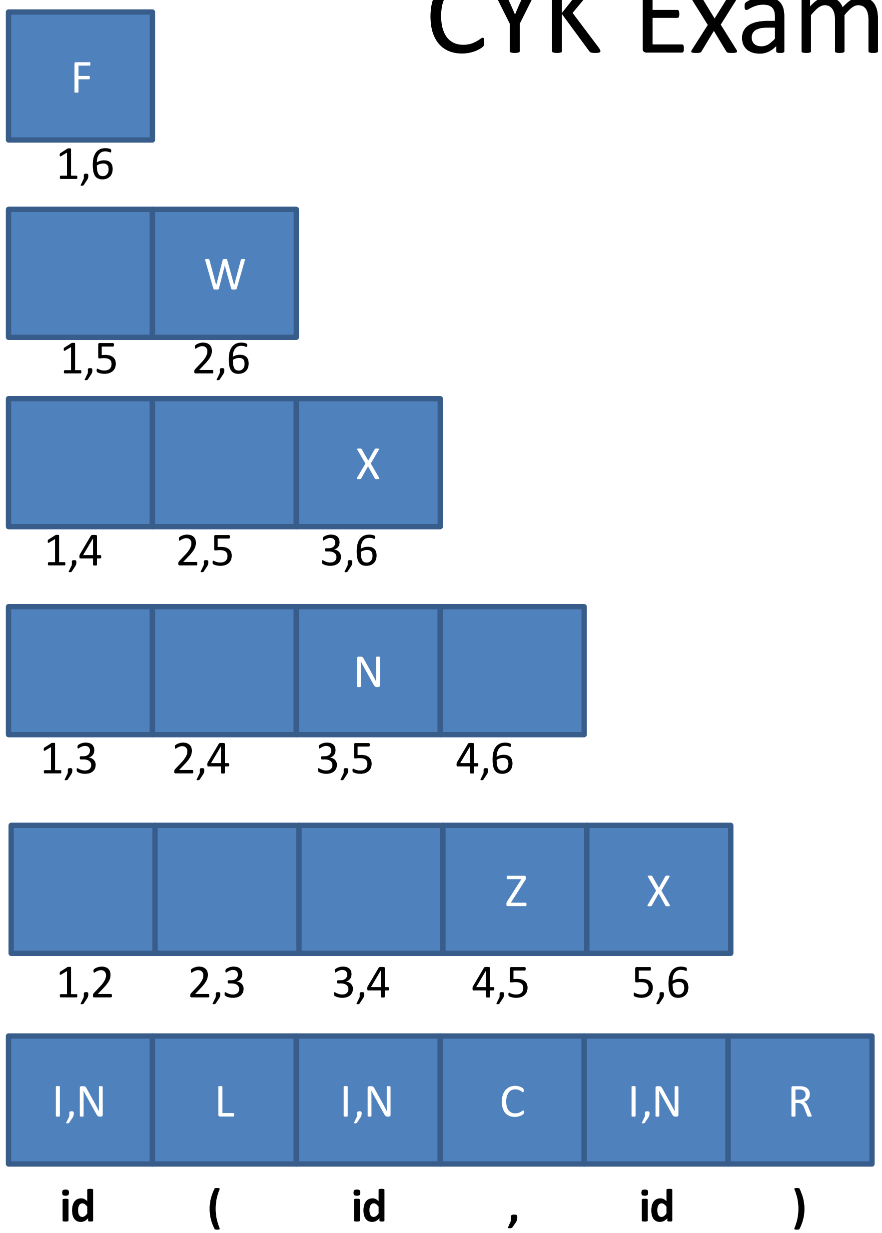
Last Time

- CYK
  - Step 1: get a grammar in Chomsky Normal Form
  - Step 2: Build all possible parse trees bottom-up
    - Start with runs of 1 terminal
    - Connect 1-terminal runs into 2-terminal runs
    - Connect 1- and 2- terminal runs into 3-terminal runs
    - Connect 1- and 3- or 2- and 2- terminal runs into 4 terminal runs
    - …
    - If we can connect the entire tree, rooted at the start symbol, we've found a valid parse

Some Interesting properties of CYK

- Very old algorithm
  - Already well known in early 70s
- No problems with ambiguous grammars:
  - Gives a solution for *all* possible parse tree simultaneously

CYK Example



$F \longrightarrow I\ W$

$F \longrightarrow I\ Y$

$W \longrightarrow L\ X$

$X \longrightarrow N\ R$

$Y \longrightarrow L\ R$

$N \longrightarrow$ **id**

$N \longrightarrow I\ Z$

$Z \longrightarrow C\ N$

$I \longrightarrow$ **id**

$L \longrightarrow$ **(**

$R \longrightarrow$ **)**

$C \longrightarrow$ **,**

Thinking about Language Design

- Balanced considerations
  - Powerful enough to be useful
  - Simple enough to be parseable
- Syntax need not be complex for complex behaviors
  - Guy Steele's "Growing a Language"

  https://www.youtube.com/watch?v=_ahvzDzKdB0

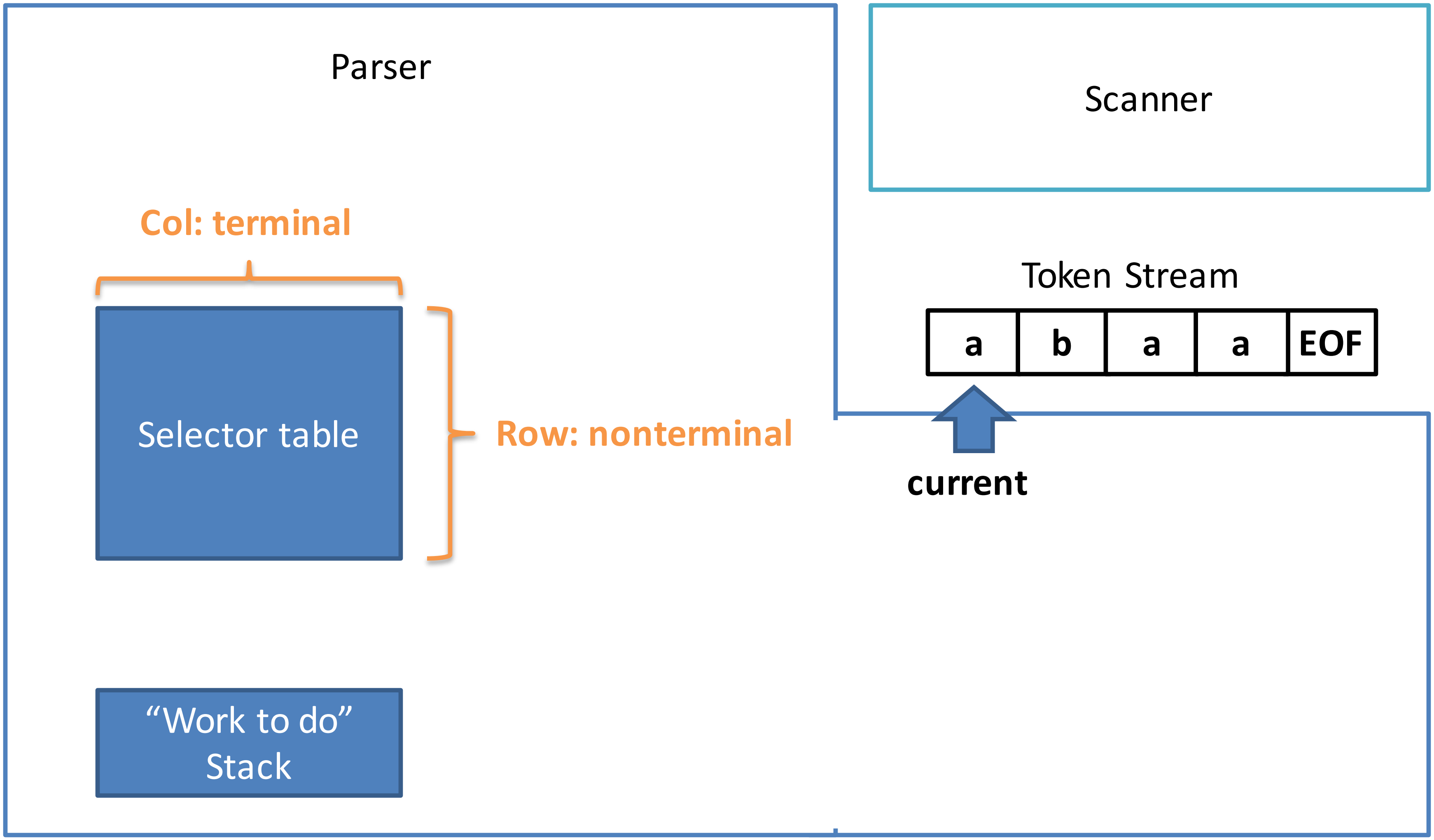
Restricting the Grammar

- By restricting our grammars we can
  - Detect ambiguity
  - Build linear-time, O(n) parsers
- LL(1) languages
  - Particularly amenable to parsing
  - Parseable by Predictive (top-down) parsers
    - Sometimes called recursive descent

Top-Down Parsers

- Start at the Start symbol
- “predict” what productions to use
  - Example: if the current token to be parsed is an id, no need to try productions that start with integer literal
  - This might seem simple, but keep in mind multiple levels of productions that have to be used

Predictive Parser Sketch



Algorithm

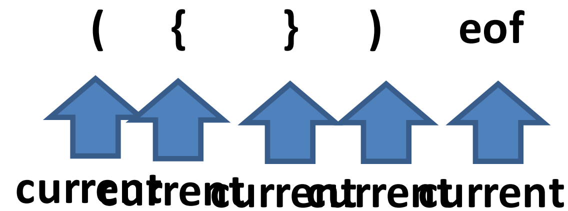
```
stack.push(eof)
stack.push(Start non-term)
t = scanner.getToken()
Repeat
  if stack.top is a terminal y
    match y with t
    pop y from the stack
    t = scanner.next_token()
  if stack.top is a nonterminal X
    get table[X,t]
    pop X from the stack
    push production's RHS (each symbol from Right to Left)
Until one of the following:
  stack is empty                                      accept
  stack.top is a terminal that doesn't match t
  stack.top is a non-term and parse table entry is empty
```

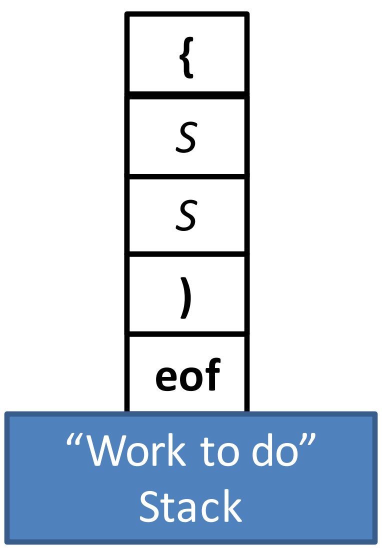
reject

Example

$S \rightarrow (\ S\ )\ |\ \{\ S\ \}\ |\ \varepsilon$

	(	)	{	}	eof
S	(S)	ε	{ S }	ε	ε





Example 2, bad input: You try

$S \rightarrow$ **(** S **)** | **{** S **}** | **ε**

	(	**)**	**{**	**}**	**eof**
S	(S)	ε	{ S }	ε	ε

INPUT

((} eof

This Parser works great!

- Given a single token we always knew exactly what production it started

	(	)	{	}	eof
S	(S)	ε	{ S }	ε	ε

Two Outstanding Issues

1. How do we know if the language is LL(1)
   – Easy to imagine a Grammar where a single token is not enough to select a rule

$$S \rightarrow \textbf{(}\ S\ \textbf{)} \mid \textbf{\{}\ S\ \textbf{\}} \mid \boldsymbol{\varepsilon} \mid \textbf{(}\ \textbf{)}$$

1. How do we build the selector table?
   – It turns out that there is one answer to both:

If our selector table has 1 production per cell, then grammar is LL(1)

LL(1) Grammar Transformations

- Necessary (but not sufficient conditions) for LL(1) Parsing:
  - Free of left recursion
    - No nonterminal loops for a production
    - Why? Need to look past list to know when to cap it
  - Left factored
    - No rules with common prefix
    - Why? We'd need to look past the prefix to pick rule

Left-Recursion

- Recall, a grammar such that $X \overset{+}{\Rightarrow} X\ \alpha$ is left recursive
- A grammar is immediately left recursive if this can happen in one step:

$$A \rightarrow A\ \alpha \mid \beta$$

Fortunately, it's always possible to change the grammar to remove left-recursion without changing the language it recognizes

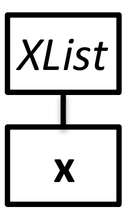
Why Left Recursion is a Problem (Blackbox View)

CFG snippet: $XList \longrightarrow XList$ **x** | **x**

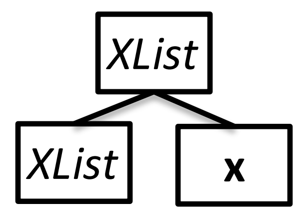
Current parse tree: $\boxed{XList}$

Current token: **x**

How should we grow the tree top-down?



(OR)



Correct if there are no more **x**s

Correct if there are more **x**s

We don't know which without more lookahead

Why Left Recursion is a Problem (Whitebox View)

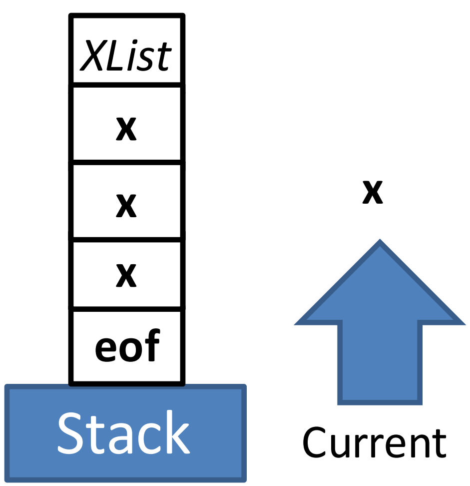
CFG snippet: *XList* $\longrightarrow$ *XList* **x** | **x**

Current parse tree: $\boxed{XList}$

Current token: **x**

Parse table:

	x	**eof**
XList	*XList* **x**	**ε**



(Stack overflow)

Removing Left-Recursion

(for a single immediately left-recursive rule)

$A \rightarrow A\ \alpha \mid \beta$

$$\begin{aligned} A &\rightarrow \beta\ A' \\ A' &\rightarrow \alpha\ A' \\ &\quad \mid\ \varepsilon \end{aligned}$$

Where β does not begin with A

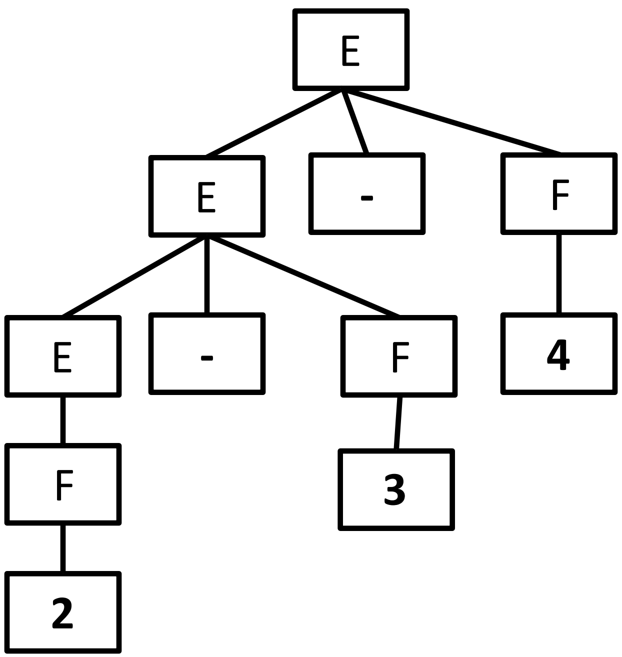
Example

$$A \rightarrow A\ \alpha \mid \beta \quad \Longrightarrow \quad \begin{aligned} A &\rightarrow \beta\ A' \\ A' &\rightarrow \alpha\ A' \\ &\mid\ \varepsilon \end{aligned}$$

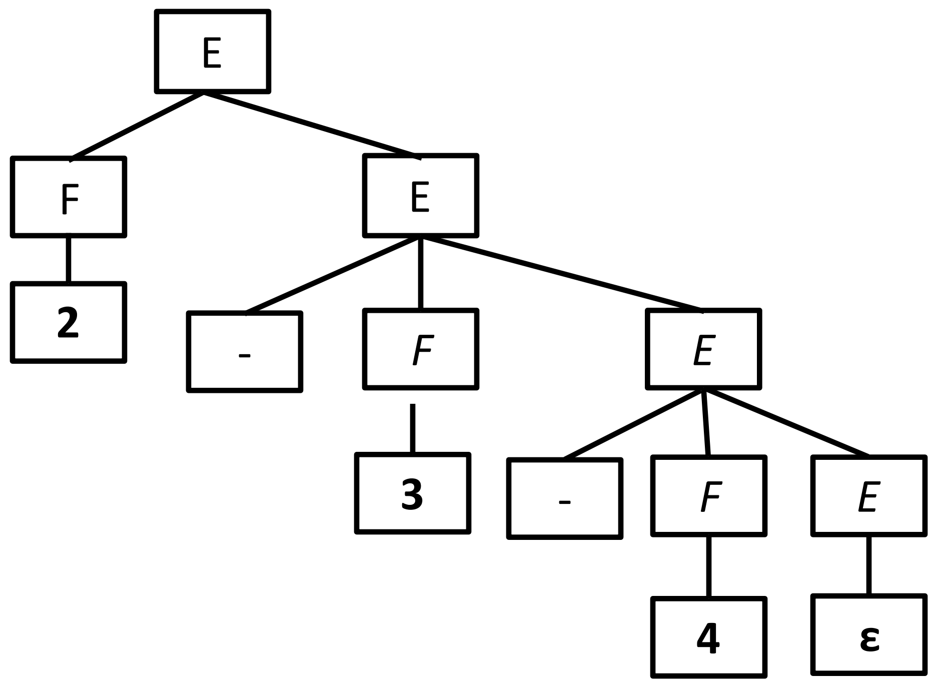
$$\begin{aligned} &\mathit{Exp} &&\rightarrow \mathit{Exp} - \mathit{Factor} \\ & &&\mid\ \mathit{Factor} \\ &\text{Factor} &&\rightarrow \textbf{intlit} \mid (\ \text{Exp}\ ) \end{aligned} \quad \Longrightarrow \quad \begin{aligned} &\mathit{Exp} &&\rightarrow \mathit{Factor}\ \mathit{Exp'} \\ &\mathit{Exp'} &&\rightarrow -\ \mathit{Factor}\ \mathit{Exp'} \\ & &&\mid\ \varepsilon \\ &\text{Factor} &&\rightarrow \textbf{intlit} \mid (\ \text{Exp}\ ) \end{aligned}$$

Let's check in on the Parse Tree…

$$\begin{aligned} \textit{Exp} &\rightarrow \textit{Exp} - \textit{Factor} \\ &\;\;|\;\; \textit{Factor} \\ \text{Factor} &\rightarrow \textbf{intlit} \mid (\text{ Exp })\end{aligned}$$



$$\begin{aligned} \textit{Exp} &\rightarrow \textit{Factor Exp'} \\ \textit{Exp'} &\rightarrow -\ \textit{Factor}\ \textit{Exp'} \\ &\;\;|\;\; \varepsilon \\ \text{Factor} &\rightarrow \textbf{intlit} \mid (\text{ Exp })\end{aligned}$$



… We'll fix that later

General Rule for Removing Immediate Left-Recursion

$A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n \mid A\ \beta_1 \mid A\ \beta_2 \mid \ldots\ A\ \beta_m$

$A \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \ldots \mid \alpha_n A'$
$A' \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots \mid \beta_m A' \mid \varepsilon$

Left Factored Grammars

- If a nonterminal has two productions whose RHS has a common prefix it is not left factored and not LL(1)

$$\mathit{Exp} \rightarrow \textbf{(}\ \mathit{Exp}\ \textbf{)} \mid \textbf{(}\ \textbf{)}$$

Not left factored

Left Factoring

- Given productions of the form

$$A \rightarrow \alpha\, \beta_1 \mid \alpha\, \beta_2$$

$$A \rightarrow \alpha\, A'$$
$$A' \rightarrow \beta_1 \mid \beta_2$$

Combined Example

Exp → **(** *Exp* **)** | *Exp Exp* | **( )**

Remove
Immediate left-recursion

Exp → **(** *Exp* **)** *Exp'* | **( )** *Exp'*
Exp' → *Exp Exp'* | ε

Left-factoring

Exp -> **(** *Exp''*
Exp'' -> *Exp*) *Exp'* |) *Exp'*
Exp' -> exp exp' | ε

Where are we at?

- We've set ourselves up for success in building the selection table
  - Two things that prevent a grammar from being LL(1) were identified and avoided
    - Not Left-Factored grammars
    - Left-recursive grammars
  - Next time
    - Build two data structures that combine to yield a selector table:
      - FIRST set
      - FOLLOW set